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Coriolis coupling in two-quasiparticle rotational bands of deformed even-even nuclei

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The experimental data on two-quasiparticle rotational bands of doubly-even nuclei in the rare-earth region are analyzed; both the $K_-=|\Omega_1-\Omega_2|$ and many of the $K_+=(\Omega_1+\Omega_2)$ bands exhibit an odd-even staggering in energy. A formalism for Coriolis coupling within the framework of the two-quasiparticle plus axially symmetric rotor model is presented. A detailed application of the model is made to the 27 known two-quasiparticle bands in ^{168}Er. The odd-even staggering in all the bands is reproduced very well. A discussion of the mixing effects in some of the bands is also presented. The Coriolis mixing is able to resolve the apparent violation of the Gallagher rule in the configuration $\{\frac{5}{2}[642]_n\otimes\frac{1}{2}[521]_n\}$.

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I. INTRODUCTION

Rotational bands based on two-quasiparticle (2qp) intrinsic excitations in the deformed odd-odd nuclei have recently been studied in great detail [1] and exhibit a number of interesting features like odd-even staggering in the $K_-=|\Omega_p-\Omega_n|$ bands and almost no staggering in the $K_+=(\Omega_p+\Omega_n)$ bands. In contrast the $K_+=(\Omega_p+\Omega_n)$ bands, which originate from the high-j neutron-proton orbitals, are observed to exhibit a large staggering and sometimes a signature inversion. Most of these features can be understood within the framework of a two-quasiparticle plus rotor model calculations [2,3]. Two-quasiparticle intrinsic excitations are also possible in the even-even nuclei. Although the experimental data for these states and the rotational bands based on them are small in number as compared to the odd-odd nuclei, a recent compilation [4] lists at least 50 Gallagher doublets and in several cases the associated rotational bands.

The 2qp intrinsic states in the even-even nuclei differ from those of the odd-odd nuclei in many ways: Firstly, the residual n-p interaction splits the $K_\pm=(\Omega_1\pm\Omega_2)$ states and puts the singlet member lower than the triplet member which is just opposite to that in the odd-odd nuclei. Secondly the odd-even splitting of the $K=0$ bands (the Newby shift) in the even-even nuclei is also expected to be opposite in sign to that observed in the odd-odd nuclei. Moreover, the splitting energies are expected to be quite large in magnitude in the even-even nuclei. The residual n-p interaction parameters are also different from the parameters used in the odd-odd nuclei. These features radically change the 2qp band structure of the even-even nuclei as compared to the odd-odd nuclei. An examination of the experimental data on 2qp rotational bands of the even-even nuclei in the rare-earth region reveals a significant odd-even staggering in the $K_-=|\Omega_1-\Omega_2|$ bands; however, some odd-even staggering is also seen in many $K_+=(\Omega_1+\Omega_2)$ bands in contrast to the $K_+=(\Omega_p+\Omega_n)$ bands in the odd-odd nuclei which mostly exhibit a smooth behavior. Besides, the $K_+=(\Omega_1+\Omega_2)$ and the $K_-=|\Omega_1-\Omega_2|$ bands originating from the high-j orbitals are observed to exhibit many additional features. In this paper, we present a model for detailed Coriolis coupling of the 2qp rotational bands in the even-even nuclei. A detailed application of this model has been made to ^{168}Er where a large number of 2qp rotational bands have been experimentally observed. The model is able to explain the odd-even staggering in the $K_+=(\Omega_1+\Omega_2)$ and $K_-=|\Omega_1-\Omega_2|$ bands very well. In Sec. II we present an analysis of the empirical data on 2qp rotational bands in the even-even rare-earth nuclei where we highlight some of the important and unusual features observed by us. In Sec. III we give a brief description of the model. In Sec. IV we present a detailed application of the model to the 27 2qp rotational bands seen in ^{168}Er and also point out the mechanisms responsible for the odd-even staggering. We summarize the results in Sec. V.

II. ODD-EVEN STAGGERING IN THE 2qp ROTATIONAL BANDS

Figures 1 and 2 present some of the experimental data for odd-even staggering in the $K_-=|\Omega_1-\Omega_2|$ and the $K_+=(\Omega_1+\Omega_2)$ rotational bands of the even-even rare-earth nuclei, respectively. Varying degree of odd-even staggering is observed in both the K_- and K_+ bands. It is rather interesting to note that the K_+ bands, which originate from the high-j configuration, exhibit a greater odd-even staggering although K is very large; for example, the $K_+^\pi=4^-,\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ band in ^{170}Yb shows a greater odd-even effect than the $K_-^\pi=3^-,\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ band. Moreover, the signature dependence in the $K_+^\pi=4$ band is quite irregular. Even more interesting is the observation that the $K_-^\pi=3$ and $K_+^\pi=4$ bands have an opposite phase of staggering, while normally one would expect the same phase. Large irregularities are also seen in the $K_+^\pi=7^-,\{\frac{7}{2}[633]_n\otimes\frac{7}{2}[514]_n\}$ band of ^{180}Os.

III. THE MODEL AND THE METHODOLOGY

A. The model

We have used a two-quasiparticle plus rotor model (TQPRM) where a nearly complete Coriolis mixing of all the known and unknown bands may be carried out. The model is developed on a line similar to the one recently developed for the odd-odd nuclei [1]; some differences arise because of the antisymmetric nature of the intrinsic wave function in an even-even nucleus. In brief, the TQPRM Hamiltonian is

$$H = H_{\text{intr}} + H_{\text{rot}} , \tag{1}$$

where the first term is the intrinsic part and the second term is the rotational part of the Hamiltonian. The intrinsic part consists of a deformed axially symmetric average field H_{av}, a short-range residual interaction H_{pair}, and a short-range neutron-neutron/proton-proton residual interaction V_{12} so that

$$H_{\text{intr}} = H_{\text{av}} + H_{\text{pair}} + V_{12} . \tag{2}$$

Here, we have assumed that the core is always in its vibrational ground state and therefore the long-range vibrational interaction has been neglected. Since the 2qp states in the even-even nuclei occur above the pairing gap, a coupling with vibrational phonons is more probable than in the odd-odd nuclei. This assumption therefore requires that we must confine our calculations to only those 2qp bands where vibrational admixture is known to be very small. For an axially symmetric rotor,

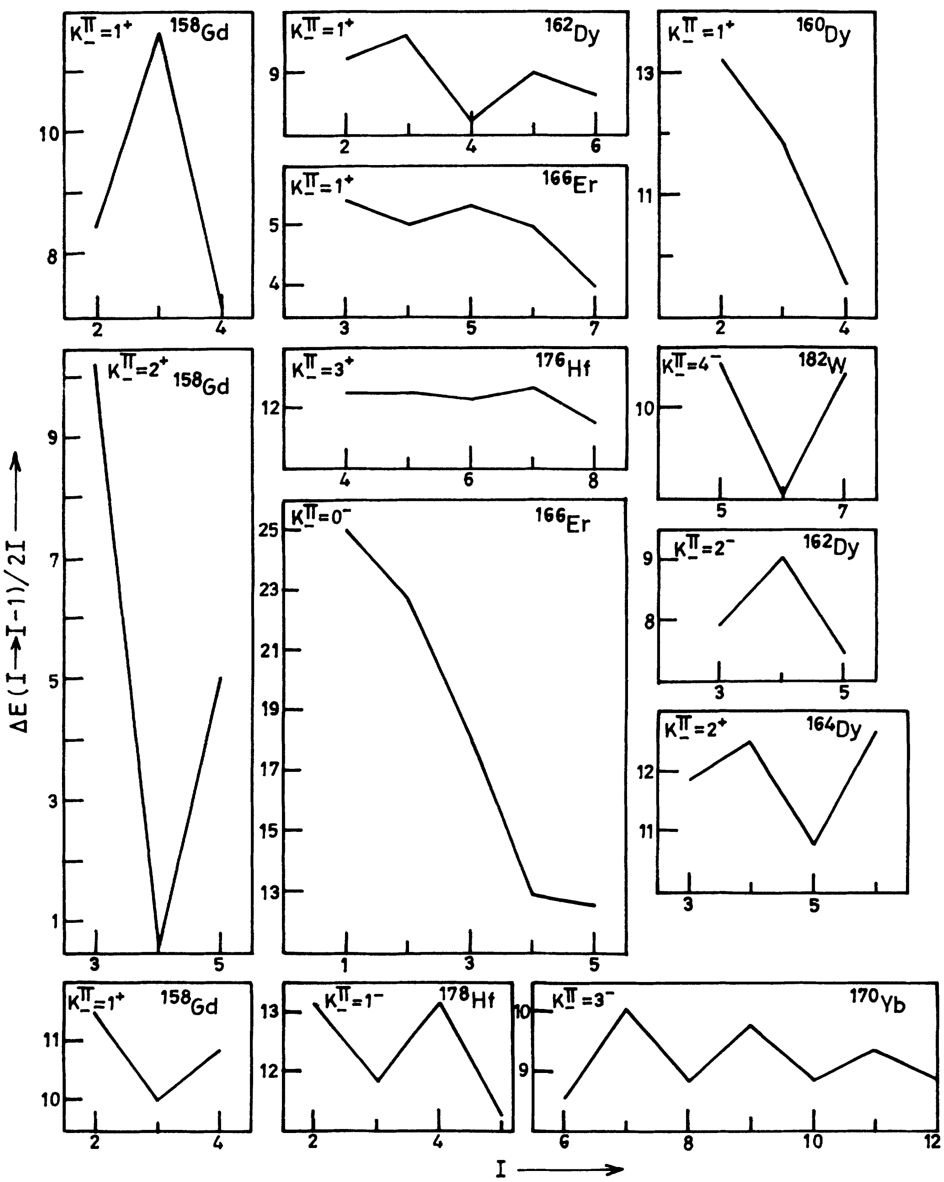

FIG. 1. The experimental plots of $\Delta E(I \to I-1)/2I$ vs I for some of the $K_- = |\Omega_1 - \Omega_2|$ bands where sufficient data are available. The data are taken from Sood *et al.* [4].

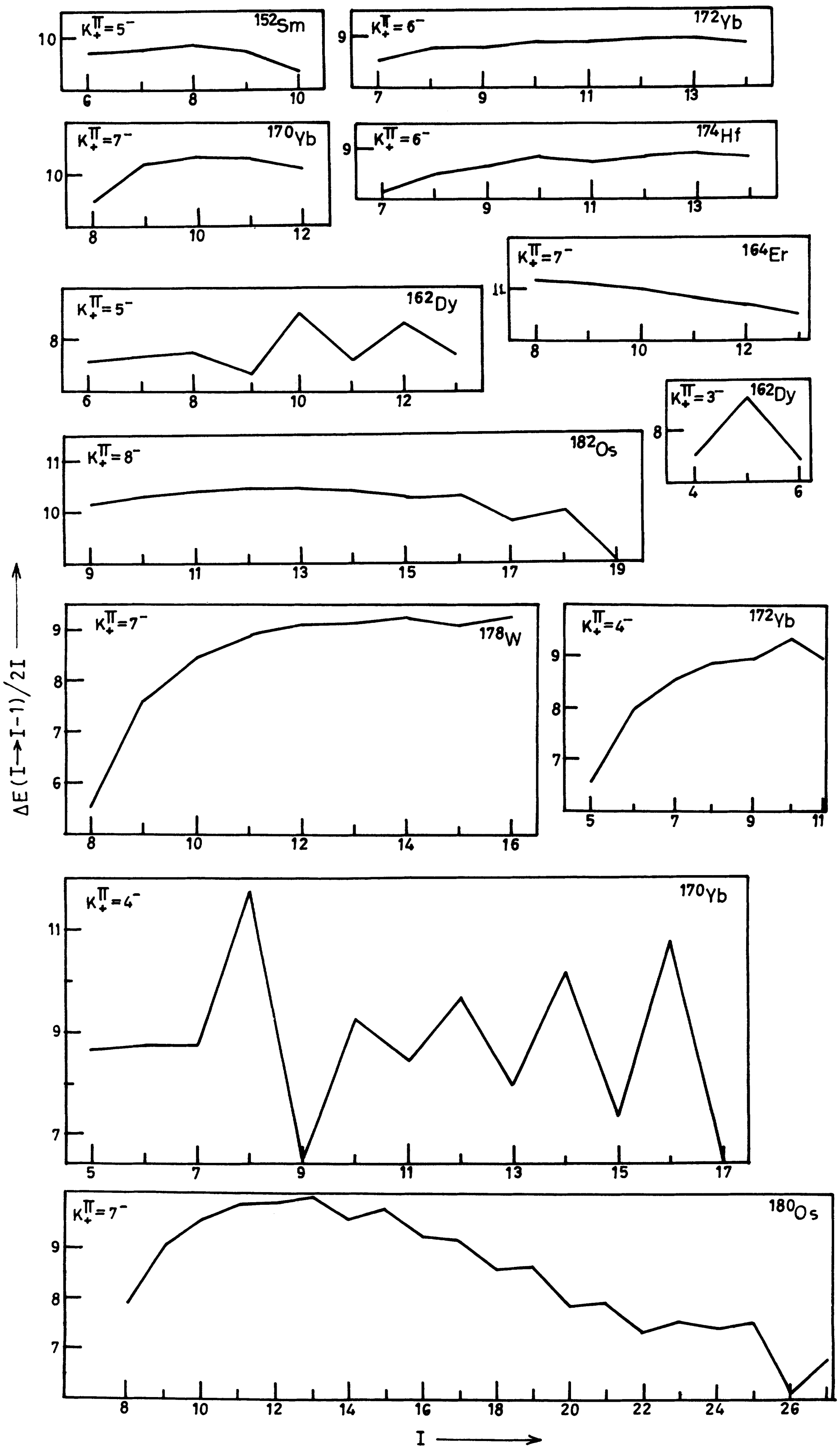

FIG. 2. Same as Fig. 1 but for some of the $K_+=(\Omega_1+\Omega_2)$ bands.

we can write

$$H_{\rm rot}=[\hbar^2/2\mathcal{J}](I^2-I_z^2)+H_{\rm cor}+H_{\rm ppc}+H_{\rm irrot}\ , \tag{3}$$

where

$$H_{\rm cor}=(-\hbar^2/2\mathcal{J})[I_+j_-+I_-j_+]\ , \tag{4a}$$

$$H_{\rm ppc}=(\hbar^2/2\mathcal{J})[j_{1+}j_{2-}+j_{1-}j_{2+}]\ , \tag{4b}$$

$$H_{\rm irrot}=(\hbar^2/2\mathcal{J})[(j_1^2-j_{1z}^2)+(j_2^2-j_{2z}^2)]\ . \tag{4c}$$

Here, $\mathcal{J}$ is the moment of inertia with respect to the rotation axis. In Eqs. (4) the subscripts 1 and 2 represent the particle 1 and the particle 2, respectively, both of which may be either neutron or proton. Other terms have their usual meaning.

The basis eigenfunctions may be written as

$$|IMK\alpha\rangle=\left[\frac{(2I+1)}{32\pi^2(1+\delta_{K0})}\right]^{1/2}\{D^I_{MK}(|K\alpha\rangle-|K^A\alpha^A\rangle)+(-1)^{I+K}D^I_{M-K}R_i(|K\alpha\rangle-|K^A\alpha^A\rangle)\}\ , \tag{5}$$

$$|K\alpha\rangle=|\rho_1(1)k_1(1)\rangle|\rho_2(2)k_2(2)\rangle\ , \tag{6a}$$

$$|K^A\alpha^A\rangle=|\rho_1(2)k_1(2)\rangle|\rho_2(1)k_2(1)\rangle\ , \tag{6b}$$

where the index α $(=\rho_1\rho_2)$ characterizes the configuration of the two neutrons or the two protons. K is the projection of the intrinsic angular momentum on the symmetry axis.

For the $K=0$ band, the intrinsic wave function for the even-even nucleus may be written as

$$\{|K=0\alpha\rangle-|K^A=0\alpha^A\rangle\}=(1/2)^{1/2}[\{|\rho_1(1)k(1)\rangle|\rho_2(2)-k(2)\rangle-j_\alpha|\rho_1(1)-k(1)\rangle|\rho_2(2)k(2)\rangle\}$$
$$-\{|\rho_1(2)k(2)\rangle|\rho_2(1)-k(1)\rangle-j_\alpha|\rho_1(2)-k(2)\rangle|\rho_2(1)k(1)\rangle\}]\ . \tag{7}$$

This may be compared with the $K=0$ band wave function in the odd-odd nucleus which has the following form:

$$|K=0\alpha\rangle=(1/2)^{1/2}[|\rho_1(1)k(1)\rangle|\rho_2(2)-k(2)\rangle-j_\alpha|\rho_1(1)-k(1)\rangle|\rho_2(2)k(2)\rangle]\ . \tag{8}$$

A rotational band can be developed on each intrinsic state $\{|K\alpha\rangle-|K^A\alpha^A\rangle\}$. Either a parallel or an antiparallel coupling is possible giving rise to Gallagher doublets [5,6]

$$K_+=(k_1+k_2)\ \text{and}\ K_-=|k_1-k_2|\ . \tag{9}$$

The residual interaction V_{12} splits the two members of the doublet and a rotational band can be built on each of the members. We use the symbol $\sigma=\pm$ to denote the two types of bands $K_\pm$.

We further define the following quantities:

$$E_{\alpha+}=\varepsilon_{\rho_1}+\varepsilon_{\rho_2}+\{\langle\rho_1(1)k(1)|\langle\rho_2(2)k(2)|V_{12}|\rho_1(1)k(1)\rangle|\rho_2(2)k(2)\rangle\}$$
$$-\langle\rho_1(2)k(2)|\langle\rho_2(1)k(1)|V_{12}|\rho_1(1)k(1)\rangle|\rho_2(2)k(2)\rangle+\langle K\alpha^+|H_{\rm irrot}|K\alpha^+\rangle\ , \tag{10a}$$

$$E_{\alpha-}=\varepsilon_{\rho_1}+\varepsilon_{\rho_2}+\{\langle\rho_1(1)k(1)|\langle\rho_2(2)-k(2)|V_{12}|\rho_1(1)k(1)\rangle|\rho_2(2)-k(2)\rangle\}$$
$$-\{\langle\rho_1(1)k(1)|\langle\rho_2(2)-k(2)|V_{12}|\rho_1(2)k(2)\rangle|\rho_2(1)-k(1)\rangle\}+\langle K\alpha^-|H_{\rm irrot}|K\alpha^-\rangle\ , \tag{10b}$$

$$C_\alpha=\{\langle\rho_1(1)k|\langle\rho_2(2)-k|V_{12}|\rho_1(1)-k\rangle|\rho_2(2)k\rangle\}-\{\langle\rho_1(1)k|\langle\rho_2(2)-k|V_{12}|\rho_1(2)-k\rangle|\rho_2(1)k\rangle\}\ . \tag{11}$$

Here $E_{\alpha\sigma}$ are the band energy parameters and C_α is the odd-even shift parameter (the Newby term). In the calculations presented here, we neglect the effect of nondiagonal matrix elements of V_{12}, although in principle they can be included. With these definitions, the matrix elements of the total Hamiltonian $\langle IMK\alpha\sigma|H|IMK'\alpha'\sigma'\rangle$ become nearly identical to those of the odd-odd nuclei with appropriate change of notations in Eq. (10) of Jain *et al.* [1]; the terms which differ due to the difference in the wave function are given by

$$=\delta_{KK'}\delta_{\alpha\alpha'}\delta_{\sigma-\sigma'-}[\tfrac{1}{2}\delta_{K0}\delta_{\sigma-}(\hbar^2/2\mathcal{J})\{(-1)^{I+1}\langle\rho_1(1)k(1)|j_{1+}|\rho_1(1)-k(1)\rangle\langle\rho_2(2)k(2)|j_{2+}|\rho_2(2)-k(2)\rangle\delta_{k'1/2}$$
$$-\langle\rho_1(1)k(1)|j_{1+}|\rho_2(1)k(1)\rangle\langle\rho_1(2)k(2)|j_{2+}|\rho_2(2)-k(2)\rangle\delta_{k'1/2}\}]$$
$$+\delta_{KK'}[\delta_{K_0}\delta_{\sigma-}\delta_{\sigma'-}\tfrac{1}{2}(\hbar^2/2\mathcal{J})\{-\langle\rho_1(1)k|j_{1+}|\rho_1'(1)k'\rangle\langle\rho_2(2)k|j_{2+}|\rho_2'(2)k'\rangle\delta_{kk'+1}$$
$$+(-1)^{I+1}\langle\rho_1(1)k|j_{1+}|\rho_2'(1)k'\rangle\langle\rho_2(2)k|j_{2+}|\rho_1'(2)k'\rangle\delta_{kk'+1}$$
$$-\langle\rho_1'(1)k'|j_{1+}|\rho_1(1)k\rangle\langle\rho_2'(2)k'|j_{2+}|\rho_2(2)k\rangle\delta_{k'k+1}$$
$$+(-1)^{I+1}\langle\rho_2'(1)k'|j_{1+}|\rho_1(1)k\rangle\langle\rho_1'(2)k'|j_{2+}|\rho_2(2)k\rangle\delta_{k'k+1}$$
$$+(-1)^{I+1}\langle\rho_1(1)k|j_{1+}|\rho_1'(1)-k'\rangle\langle\rho_2'(2)k'|j_{2+}|\rho_2(2)-k\rangle\delta_{k1/2}\delta_{k'1/2}$$
$$-\langle\rho_1(1)k|j_{1+}|\rho_2'(1)-k'\rangle\langle\rho_1'(2)k'|j_{2+}|\rho_2(2)-k\rangle\delta_{k1/2}\delta_{k'1/2}\}]\ . \tag{12}$$

Some of these terms are seen to connect the $K=0$ bands having configurations $[k,k]$ and $[k+1,k+1]$.

Diagonalization of the total Hamiltonian matrix for each value of the angular momentum I gives us the energies $E_{\rm th}(I,\alpha\sigma)$ for all the bands built on the two-quasiparticle configuration $|K\alpha\sigma\rangle$ present in the basis set of eigenfunctions. The Nilsson model single-particle Hamiltonian [7,8] has been used for the average field $H_{\rm av}$. The Nilsson model parameters used in the calculations are $\kappa_p=0.0637$, $\mu_p=0.60$ and $\kappa_n=0.0637$, $\mu_n=0.42$. The pairing interaction appears only through the modification of the single-particle energies into the quasiparticle energies ε_{ρ_1} and ε_{ρ_2}. It is known that the pairing correlations do not contribute to either the Gallagher-Moszkowski (GM) splittings or the Newby shift of odd-odd nuclei [9]; likewise we have assumed their contribution to be absent in the even-even nuclei.

B. The choice of parameters in the calculations

A correct choice of the set of the basis functions is very important as all the states which may couple together, and influence each other's behavior, should be included in the calculations. A nearly complete Coriolis coupling calculation thus requires a knowledge of a large number of 2qp states which are often unknown. We have therefore estimated the excitation energies of the important unidentified bands by using a simple semiempirical formulation [10]. In this formulation, the known properties of the quasiparticle configurations involved are taken from neighboring odd-A nuclei. The band energies, rotational parameters, and sometimes the $\langle j_+\rangle$ matrix elements as calculated from the Nilsson model were treated as free parameters and were adjusted within physically meaningful limits to fit the experimental data. A small admixture of vibrational components may also be absorbed by a minor renormalization of the intrinsic matrix elements of $j_{1\pm}$ and $j_{2\pm}$. The Newby shifts for the $K=0$ bands were also treated as free parameters as almost no data exist on the Newby shift in even-even nuclei.

IV. RESULTS AND DISCUSSION

A. The application of the model to ^{168}Er

We have applied this model to ^{168}Er which is one of the best studied well-deformed nucleus with almost 41 assigned rotational bands [4]. Almost 27 of these 41 bands have been assigned a 2qp configuration. This nucleus therefore becomes a natural testing ground for our model calculations. We have succeeded in reproducing all the 27 rotational bands very well. To facilitate the fitting of the data, the calculations were done in three separate parts: two-quasineutron bands having negative parity, two-quasineutron bands having positive parity, and two-quasiproton bands with both the parities. However, care was taken so that the $\langle j_+\rangle$ coupling matrix elements had the same value in all the three subcalculations. In total 74 2qp rotational bands were fitted in the calculations of which 27 rotational bands comprising 88 energy levels were experimentally observed. A total of 23 $\langle j_+\rangle$ single-particle matrix elements coupled these states and were taken from the Nilsson model. Of these only nine were changed during the calculations to fit the experimental data. All the unidentified bands were initially given a rotational parameter of $\hbar^2/2\mathcal{J}=12.0$ keV which was later adjusted if necessary. In Table I we list the calculated parameters obtained from the fitting of the energy levels for all the known and some unknown bands which are seen to interact rather strongly. Parameter values for certain $K=0$ bands are also given. It is highly interesting to note the very large magnitude of the Newby shifts for most of the $K=0$ bands.

In Fig. 3 we show the quality of fit to those experimental bands which exhibit an odd-even staggering. A correct reproduction of this odd-even shift, which varies wildly in magnitude from very large in the $K=1$ bands to very small in the $K=2,3$ bands, is a significant outcome of our calculations and also gives us some confidence in the model. Although a large number of parameters are involved in fitting the data, their choice is severely constrained by the physical considerations and the empirical data. Only a small number of these parameters are actually adjusted during the fitting procedure. In the following we discuss some of the bands individually.

B. The negative parity bands

The $\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ *and* $\{\frac{7}{2}[523]_p\otimes\frac{1}{2}[411]_p\}$ *bands.* Both these configurations give rise to $K_+^\pi=4^-$ and $K_-^\pi=3^-$ Gallagher doublets. Burke *et al.* [11,12] have suggested a significant mixing between the two $K_+^\pi=4^-$ bands. Experimental evidence of a mixing of two-quasiproton and two-quasineutron components in states with large K in doubly even deformed nuclei have been analyzed by Sood and Sheline [13]. Recently Soloviev and Sushkov [14] have explored the role of high-multipolarity ($\lambda\geq4$) interactions in producing these mixing effects; they calculate the 1094 keV, $K_+^\pi=4^-,\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ band to lie at 1.0 MeV with an 18% admixture of $\{\frac{7}{2}[523]_p\otimes\frac{1}{2}[411]_p\}$ band. For the 1905 keV, $K_+^\pi=4^-,\{\frac{7}{2}[523]_p\otimes\frac{1}{2}[411]_p\}$ band, Soloviev *et al.* calculate an energy of 1.6 MeV and again an 18% admixture of the $\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ band. Our model does not include any mixing between the two-quasiproton and two-quasineutron states. However, we do find a 10 to 15% Coriolis mixing of $K_+^\pi=5^-,\{\frac{7}{2}[523]_p\otimes\frac{3}{2}[411]_p\}$ in the two-quasiproton $K^\pi=4^-$ band and a 5–10% admixture of the $K_-^\pi=3^-,\{\frac{5}{2}[642]_n\otimes\frac{1}{2}[521]_n\}$ band in the two-quasineutron $K^\pi=4^-$ band. Only five members of the two-quasineutron $K^\pi=4^-$ band and three members of the two-quasiproton $K^\pi=4^-$ band are known and no odd-even shift can be definitely established from the data. No proton-neutron mixing is suggested in the two-quasineutron and two-quasiproton $K^\pi=3^-$ bands lying at 1541 and 1999 keV, respectively. However, a significant Coriolis mixing is observed in the two-quasineutron $K^\pi=3^-$ band; the configurations which mix are $K_-^\pi=2^-,\{\frac{5}{2}[642]_n\otimes\frac{1}{2}[521]_n\}$ and $K_+^\pi=4^-,\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[510]_n\}$. It may be pointed out that the $K^{\pi}=4^-$ and $K^{\pi}=3^-$ bands having the configuration $\{\frac{7}{2}[633]_n\otimes\frac{1}{2}[521]_n\}$ are also observed in ^{170}Yb and exhibit a large odd-even effect at higher spins. It is therefore reasonable to expect a similar behavior also in ^{168}Er if these bands are followed to higher spins.

The $\{\frac{5}{2}[642]_n\otimes\frac{1}{2}[521]_n\}$ *bands.* The $K_+^{\pi}=3^-$ and $K_-^{\pi}=2^-$ bands belonging to this configuration are observed to lie at 2262 and 2230 keV, respectively. Since the $K_+^{\pi}=3^-$ band is based on the singlet configuration, it represents a violation of the Gallagher rule. However,

TABLE I. Theoretically calculated bandhead energies for all the known bands and all the interacting $K=0$ bands in ^{168}Er are compared with the experimental data. Also given are the parameter values $E_\alpha, \hbar^2/2\mathcal{J}, E_N$, and those values of $\langle j_+\rangle$ which were adjusted. The values of $\langle j_+\rangle$ in the parentheses are from the Nilsson model.

Configuration	K^π	I	E_{expt} (keV)	E_{calc} (keV)	E_α (keV)	$\hbar^2/2\mathcal{J}$ (keV)	E_N (keV)
Two-quasiproton							
$\{\frac{7}{2}[523]\otimes\frac{1}{2}[411]\}$	4^-	4	1905.1	1905.1	1865.4	9.5	
$\{\frac{7}{2}[523]\otimes\frac{1}{2}[411]\}$	3^-	3	1999.2	1998.0	1964.4	11.0	
$\{\frac{7}{2}[523]\otimes\frac{1}{2}[541]\}$	4^+	4	2055.9	2054.7	2015.0	13.5	
$\{\frac{7}{2}[523]\otimes\frac{7}{2}[404]\}$	7^-	7	2122.4	2121.5	2040.0	12.0	
$\{\frac{7}{2}[523]\otimes\frac{7}{2}[404]\}$	0^-	0		2472.5	2470.0	12.0	0.0
$\{\frac{3}{2}[411]\otimes\frac{1}{2}[411]\}$	2^+	2	2193.0	2191.4	2173.6	10.4	
$\{\frac{3}{2}[411]\otimes\frac{1}{2}[411]\}$	1^+	1	2365.3	2365.2	2351.7	11.1	
$\{\frac{3}{2}[411]\otimes\frac{7}{2}[404]\}$	5^+	5	2298.0	2298.8	2237.0	12.0	
$\{\frac{1}{2}[420]\otimes\frac{1}{2}[411]\}$	0^+	0		2330.0	3188.7	12.6	+861.0
$\{\frac{3}{2}[541]\otimes\frac{3}{2}[411]\}$	0^-	0		2560.0	2480.0	14.0	−80.0
Two-quasineutron							
$\{\frac{1}{2}[510]\otimes\frac{5}{2}[512]\}$	2^+	2	1930.3	1929.6	1909.0	10.3	
$\{\frac{1}{2}[510]\otimes\frac{1}{2}[521]\}$	0^+	0		1867.7	1987.7	12.5	120.0
$\{\frac{3}{2}[521]\otimes\frac{5}{2}[512]\}$	1^+	1	2133.7	2133.8	2122.2	11.6	
$\{\frac{3}{2}[521]\otimes\frac{5}{2}[512]\}$	4^+	4	2238.2	2238.4	2195.9	13.0	
$\{\frac{3}{2}[521]\otimes\frac{1}{2}[521]\}$	2^+	2	1848.4	1847.4	1866.4	8.5	
$\{\frac{1}{2}[521]\otimes\frac{7}{2}[514]\}$	3^+	3	2186.7	2188.2	2143.3	13.0	
$\{\frac{1}{2}[521]\otimes\frac{7}{2}[514]\}$	4^+	4	2663.0	2656.2	2631.5	12.0	
$\{\frac{5}{2}[523]\otimes\frac{5}{2}[512]\}$	5^+	5	2267.0	2267.0	2242.8	8.5	
$\{\frac{5}{2}[523]\otimes\frac{5}{2}[512]\}$	0^+	0		2299.9	2400.0	12.0	0.0
$\{\frac{5}{2}[523]\otimes\frac{1}{2}[521]\}$	2^+	2	2424.9	2425.7	2404.3	9.1	
$\{\frac{5}{2}[512]\otimes\frac{1}{2}[521]\}$	3^+	3	1653.5	1653.9	1690.0	12.3	
$\{\frac{1}{2}[521]\otimes\frac{7}{2}[633]\}$	4^-	4	1094.0	1093.5	1160.4	12.7	
$\{\frac{1}{2}[521]\otimes\frac{7}{2}[633]\}$	3^-	3	1541.6	1545.1	1619.6	14.0	
$\{\frac{3}{2}[521]\otimes\frac{7}{2}[633]\}$	5^-	5	2365.0	2352.3	2490.0	8.0	
$\{\frac{5}{2}[512]\otimes\frac{5}{2}[642]\}$	0^-	0		2247.1	1990.1	10.8	−364.9
$\{\frac{5}{2}[512]\otimes\frac{5}{2}[642]\}$	5^-	5	2477.0	2478.4	2706.0	7.0	
$\{\frac{5}{2}[512]\otimes\frac{7}{2}[633]\}$	1^-	1	1358.9	1358.8	1609.0	12.9	
$\{\frac{5}{2}[512]\otimes\frac{7}{2}[633]\}$	6^-	6	1773.2	1773.1	2023.2	14.0	
$\{\frac{1}{2}[510]\otimes\frac{7}{2}[633]\}$	3^-	3	1828.1	1826.1	2080.8	8.5	
$\{\frac{1}{2}[510]\otimes\frac{7}{2}[633]\}$	4^-	4	2060.0	2059.0	2277.5	8.9	
$\{\frac{5}{2}[523]\otimes\frac{7}{2}[633]\}$	1^-	1	1936.6	1935.3	2182.5	9.9	
$\{\frac{7}{2}[514]\otimes\frac{7}{2}[633]\}$	0^-	0		1772.4	2040.0	9.3	−30.0
$\{\frac{5}{2}[523]\otimes\frac{5}{2}[642]\}$	0^-	0		2568.7	2700.0	13.0	−50.0
$\{\frac{3}{2}[521]\otimes\frac{3}{2}[651]\}$	0^-	0		3617.8	3200.0	11.0	−500.0
$\{\frac{1}{2}[521]\otimes\frac{5}{2}[642]\}$	2^-	2	2230.4	2237.5	2319.1	7.4	
$\{\frac{1}{2}[521]\otimes\frac{5}{2}[642]\}$	3^-	3	2262.0	2261.3	2307.0	8.4	

$\langle\frac{1}{2}[411]|\frac{1}{2}[411]\rangle_p=0.308(0.941)$ $\langle\frac{3}{2}[411]|\frac{1}{2}[411]\rangle_p=0.523(0.500)$
$\langle\frac{3}{2}[411]|\frac{1}{2}[420]\rangle_p=0.829(2.826)$ $\langle\frac{7}{2}[523]|\frac{5}{2}[532]\rangle_p=1.073(5.073)$
$\langle\frac{1}{2}[541]|\frac{1}{2}[541]\rangle_p=-3.21(-3.73)$ $\langle\frac{1}{2}[510]|\frac{1}{2}[510]\rangle_n=-0.010(-0.10)$
$\langle\frac{1}{2}[521]|\frac{1}{2}[521]\rangle_n=-0.143(-0.90)$ $\langle\frac{7}{2}[633]|\frac{5}{2}[642]\rangle_n=4.65(6.108)$
$\langle\frac{9}{2}[624]|\frac{7}{2}[633]\rangle_n=4.404(5.612)$

our calculations predict the unperturbed positions of the $K^{\pi}=3^-$ and 2^- bands at 2332 and 2334 keV, respectively. A Coriolis mixing with the $K^{\pi}_{-}=3^-, \{\frac{7}{2}[633]_n \otimes \frac{1}{2}[510]_n\}$ and the $K^{\pi}_{+}=4^-, \{\frac{7}{2}[633]_n \otimes \frac{1}{2}[521]_n\}$ bands pushes the $K^{\pi}_{+}=3^-$ band above the $K^{\pi}_{-}=2^-$ band in question. The violation of the Gallagher rule is therefore only an apparent one and is resolved by our calculations.

The $\{\frac{7}{2}[633]_n \otimes \frac{5}{2}[512]_n\}$ *bands.* The $K^{\pi}_{+}=6^-$ and $K^{\pi}_{-}=1^-$ bands are observed at 1773 and 1359 keV, respectively. The $K^{\pi}_{-}=1^-$ band exhibits a very large odd-even staggering, which is reproduced very well by our calulations. This odd-even shift is due to a large admixture of $K^{\pi}_{-}=0^-, \{\frac{5}{2}[642]_n \otimes \frac{5}{2}[512]_n\}$ band which we place at 2247 keV with a large Newby shift of -364 keV. Due to the large Newby shift, the odd-spin members have a $K=0$ admixture of 16 to 25% whereas the even-spin members have an admixture of 1 to 2% only. The calculations of Sood *et al.* [15] using a zero-range delta force place the $K^{\pi}_{+}=6^-$ and $K^{\pi}_{-}=1^-$ bandheads at 1782 and 1533 keV, respectively. The unperturbed positions of these bandheads from our calculations come out to be 2116 and 1622 keV, respectively.

The $\{\frac{7}{2}[633]_n \otimes \frac{5}{2}[523]_n\}$ *bands.* Only the $K^{\pi}_{-}=1$ band of this configuration is observed at 1936 keV and exhibits a large odd-even staggering. We reproduce this band very well with an admixture of $K^{\pi}_{-}=0, \{\frac{7}{2}[633]_n \otimes \frac{7}{2}[514]_n\}$ and $K^{\pi}_{-}=0^-, \{\frac{5}{2}[642]_n \otimes \frac{5}{2}[523]_n\}$ bands.

The $\{\frac{7}{2}[633]_n \otimes \frac{1}{2}[510]_n\}$ *bands.* The $K^{\pi}_{+}=4^-$ and $K^{\pi}_{-}=3^-$ bands are seen at 2060 and 1828 keV, respectively. The $K^{\pi}_{-}=3^-$ band known only up to $I=7$ shows an irregular behavior in moment of inertia which is reproduced very well by our calculations; it has a 5 to 10% admixture of $K^{\pi}_{+}=3^-, \{\frac{5}{2}[642]_n \otimes \frac{1}{2}[521]_n\}$ configuration. However, the irregular behavior is mainly due to a successive mixing of the $K^{\pi}_{+}=3^-$, $\{\frac{5}{2}[642]_n \otimes \frac{1}{2}[521]_n\}$ and the $K^{\pi}_{+}=4^-, \{\frac{7}{2}[633]_n \otimes \frac{1}{2}[521]_n\}$ bands. The $K=4$ band, on the other hand, also has a mixing with many other configurations.

The unperturbed positions of these $K^{\pi}_{+}=4^-$ and $K^{\pi}_{-}=3^-$ band heads come out to be 2311 and 2101 keV, respectively, which may be compared with the predictions of Ray [10] at 2242 and 2006 keV, respectively.

The $\{\frac{7}{2}[633]_n \otimes \frac{7}{2}[514]_n\}$ *and* $\{\frac{7}{2}[523]_p \otimes \frac{7}{2}[404]_p\}$ *bands.* Both these configurations lead to $K^{\pi}_{+}=7^-$ and $K^{\pi}_{-}=0^-$ bands. A mixing between the two $K^{\pi}=7^-$ bands has been suggested by Burke *et al.* [11,12]. Only the two-quasiproton $K=7$ band is seen experimentally at 2122 keV. We do not observe any Coriolis mixing in the two-quasiproton $K^{\pi}=7^-$ band. The unperturbed posi-

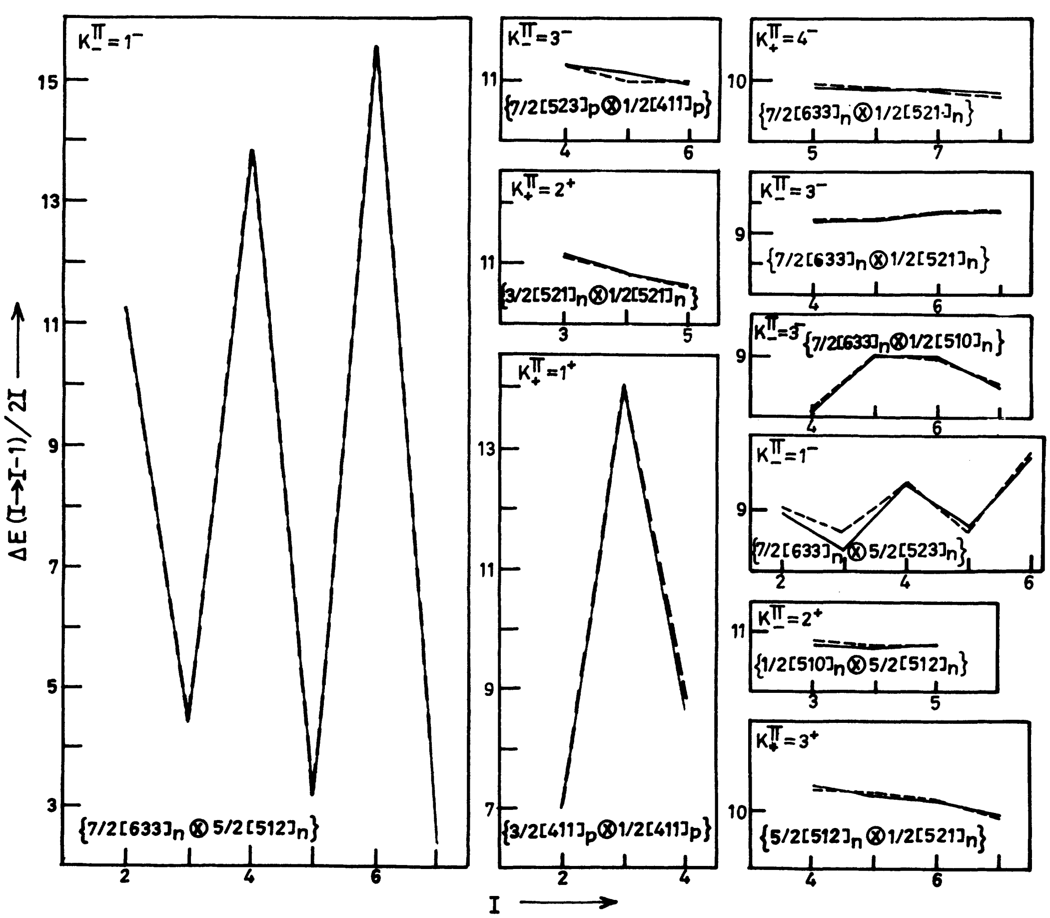

FIG. 3. Results of our calculations (dashed line) in comparison with the experimental data (solid line) for ^{168}Er.

tions of these bands in our calculations are 1898 keV for two-quasineutron and 2125 keV for two-quasiproton bands which may be compared with the predictions of Ray [10] at 1889 and 2023 keV, respectively.

The $\{\frac{7}{2}[523]_p \otimes \frac{3}{2}[411]_p\}$ *and* $\{\frac{7}{2}[633]_n \otimes \frac{3}{2}[521]_n\}$ *bands.* Both these configurations give rise to $K_-^\pi = 2^-$ and $K_+^\pi = 5^-$ bands and a strong mixing between the $K^\pi = 5^-$ triplet coupled states is suggested. Davidson *et al.* [16] observe a $K^\pi = 5^-$ band at 2365 keV, which is assigned the two-quasineutron configuration with some admixture of two-quasiproton configuration. In our calculations we have estimated unperturbed positions of the $K = 5$ two-quasineutron and two-quasiproton bandheads to be 2500 and 1905 keV. The two-quasineutron $K^\pi = 5^-$ band is found to be highly mixed with the $K_+^\pi = 5^-, \{\frac{5}{2}[523]_n \otimes \frac{5}{2}[642]_n\}$ and $K_+^\pi = 4^-, \{\frac{7}{2}[633]_n \otimes \frac{1}{2}[510]_n\}$ bands due to the particle-particle and Coriolis terms. The two-quasineutron $K^\pi = 2^-$ band also mixes with $K_-^\pi = 1^-, \{\frac{5}{2}[642]_n \otimes \frac{3}{2}[521]_n\}$ and $K_-^\pi = 0^-$, $\{\frac{3}{2}[651]_n \otimes \frac{3}{2}[521]_n\}$ configurations and exhibits an odd-even shift. After Coriolis mixing, the two-quasineutron $K = 5$ and 2 bands come out at 2367 and 2109 keV, respectively. These may be compared with the predictions of Ray [10] at 2368 and 1799 keV, respectively. The two-quasiproton $K^\pi = 5^-$ band also exhibits some Coriolis mixing with $K_+^\pi = 4^-, \{\frac{7}{2}[523]_p \otimes \frac{1}{2}[411]_p\}$ configuration. After Coriolis mixing the two-quasiproton $K = 5$ and 2 bands are predicted at 1903 and 1567 keV, respectively.

C. The positive parity bands

The $\{\frac{3}{2}[411]_p \otimes \frac{1}{2}[411]_p\}$ *and* $\{\frac{3}{2}[521]_n \otimes \frac{1}{2}[521]_n\}$ *bands.* Both these configurations lead to $K_+^\pi = 2^+$ and $K_-^\pi = 1^+$ bands and a strong mixing between the two-quasiproton and the two-quasineutron $K^\pi = 2^+$ bands is suggested. The $K_+^\pi = 2^+, \{\frac{3}{2}[521]_n \otimes \frac{1}{2}[521]_n\}$ band observed at 1848 keV exhibits a large Coriolis mixing with $K^\pi = 0^+, 1^+, \{\frac{1}{2}[510]_n \otimes \frac{1}{2}[521]_n\}$ and $K_+^\pi = 3^+$, $\{\frac{5}{2}[512]_n \otimes \frac{1}{2}[521]_n\}$ bands. The $K_-^\pi = 1^+$, $\{\frac{3}{2}[521]_n \otimes \frac{1}{2}[521]_n\}$ band also exhibits a large Coriolis mixing although it is not yet experimentally observed; we place this band at 2052 keV. The two-quasiproton $K^\pi = 2^+$ and $K^\pi = 1^+$ bands are experimentally observed at 2193 and 2365 keV, respectively. The $K^\pi = 1^+$ band exhibits a large odd-even shift due to a mixing with $K_-^\pi = 0^+, \{\frac{1}{2}[420]_p \otimes \frac{1}{2}[411]_p\}$ band, which has a large Newby shift of +861 keV. The large positive Newby shift leads to almost 50% admixture in the even-spin members whereas the odd-spin members remain nearly pure. Due to a small admixture of the $K = 0$ band, the $K^\pi = 2^+$ two-quasiproton band is also expected to show an odd-even shift.

The $\{\frac{5}{2}[512]_n \otimes \frac{1}{2}[521]_n\}$ *band.* This configuration gives rise to $K_+^\pi = 3^+$ and $K_-^\pi = 2^+$ bands of which $K^\pi = 3^+$ is experimentally observed at 1653 keV. The zero-range delta force calculations of Ray [10] predict this bandhead at 2320 keV. Both the $K^\pi = 3^+$ and 2^+ bands exhibit a significant Coriolis mixing. We do observe a mixing of $K_-^\pi = 2^+, \{\frac{5}{2}[523]_n \otimes \frac{1}{2}[521]_n\}$ band into the unknown $K = 2$ band; this mixing is greatest for $I^\pi = 3^+$ and 4^+ levels (~20%) while smaller for other spins (~5%). This gives rise to a large irregularity in the moment of inertia parameter at $I = 3$ and 4.

V. CONCLUSIONS

To conclude, we have analyzed the experimental data on two-quasiparticle states in the even-even nuclei which are found to exhibit an odd-even staggering in both the K_+ and K_- bands. Coriolis coupling seems to be responsible for the observed odd-even effect. Results of the Coriolis coupling calculations carried out for ^{168}Er, where a large number of two-quasiparticle rotational bands are experimentally known, bear out this contention by correctly reproducing the varying degree of odd-even effect observed in the various bands. Our calculations also resolve the apparent violation of the Gallagher rule in the configuration $\{\frac{5}{2}[642]_n \otimes \frac{1}{2}[521]_n\}$ of ^{168}Er. Two mechanisms are seen to be responsible for the odd-even staggering. Mainly a Coriolis coupling of first to second order with one or more $K = 0$ bands leads to the odd-even staggering. An irregular behavior seen, for example, in the $K_-^\pi = 3^-, \{\frac{7}{2}[633] \otimes \frac{1}{2}[510]\}$ band is, on the other hand, due to a mixing with another $K = 3$ and a $K = 4$ band.

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