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Pion dressing of nucleons and nuclear forces: A nonperturbative approach

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We develop a nonperturbative method for obtaining pion dressing for strong interaction through minimisation of total energy. As an illustration, using coherent states we consider the pion-nucleon system with the physical nucleon represented by the bare nucleon and a calculable meson cloud. We next extend the method to obtain pion dressing for the two-nucleon system with deuteron as an example. The analysis de facto replaces the scalar-isoscalar potential in nuclear physics by multipion condensates and yields approximately correct results.

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I. INTRODUCTION

Obtaining nuclear forces at a microscopic level is an important problem, since it shall be the basis for nucleons in finite nuclei as well as in nuclear matter. Such a problem is not yet solvable if we consider a quark-gluon description with nuclear forces emerging as an effective residual interaction. The alternative approach is to tackle the problem through meson interaction. This also entails a major technical problem with $G^2_{NN\pi}/4\pi = 14.6$, which makes perturbative methods unreliable. A nonperturbative method through the mean-field approximation has also been developed [1], which is theoretically attractive and also gives reasonable results. This however suffers from two defects. Firstly, the meson fields are classical. Secondly, it has to bring in the hypothetical σ meson, which could be an effect of multipion exchange. Thus on aesthetic as well as phenomenological grounds, alternative methods shall add to our understanding.

It was recently seen that for the description of nuclear matter pion scalar isosinglet pairs [2, 3] which are off shell can simulate the effect of σ mesons. This is not surprising since the coherent state [4] type of construction is a very natural quantum mechanical formalism for classical fields. Such a construct also includes higher order effects [2–5]. The method has also been applied to a few other problems with interesting results indicative of its versatility for different type of problems [6]. It would be desirable to develop the same for finite nuclei.

When the interaction is strong, it is clear that the physical particle will always be dressed. We shall here consider a state for a physical nucleon with an off mass shell coherent state of scalar isoscalar pion pairs in a manner similar to Ref. [2] and obtain the "dressing" by minimising energy. We then construct the bound state of two nucleons which naturally changes the pion dressing and again minimise energy to obtain the structure of the two body bound state. In this paper we develop the algebra for this dressing. We consider here one pion exchange and ω repulsion in addition to dressing through pion pairs.

The paper is organised as follows. In Sec. II we develop a general approach for the construction of the physical or "dressed" nucleon along with the algebra for pion dressing. We demonstrate here that the dressing of the nucleon with pion pairs becomes equivalent to a space-dependent Bogoliubov transformation. In Sec. III we obtain the energy expectation value of a single nucleon as a functional of pion dressing. In Section IV we consider the single nucleon with pion dressing as determined through energy minimisation. Section V is devoted to the consideration of two-nucleon bound system. In Section VI we calculate the energy expectation values for deuteron. In Section VII we minimise the energy and calculate some physical properties of deuteron.

The present analysis is a theoretical examination of a new nonperturbative approximation scheme in quantum field theory as applied to nuclear physics, as well as the consideration of possible signatures of the same in a realistic environment. It is an extension of mean field approach of Walecka where classical fields are replaced by quantum coherent states. It is also different from the meson exchange or similar attempts [7] in the sense that σ-meson exchange is replaced by off shell pion quanta.

II. GENERAL THEORY

The Lagrangian for the pion nucleon system is taken as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - M + G\gamma_5\phi)\psi + \frac{1}{2}[(\partial_\mu\phi_i)(\partial^\mu\phi_i) - m^2\phi_i\phi_i]. \tag{2.1}$$

We shall consider the same in the nonrelativistic limit. Then, the Hamiltonian density for the nucleons with the interaction is given as

$$\mathcal{H}(\mathbf{x}) = \mathcal{H}^0_N(\mathbf{x}) + \mathcal{H}_{\rm int}(\mathbf{x}) + \mathcal{H}_M(\mathbf{x}) \tag{2.2}$$

with

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$$\mathcal{H}_N^0(\mathbf{x}) = \psi_I(\mathbf{x})^\dagger \Big(M - \frac{\nabla^2}{2M}\Big)\psi_I(\mathbf{x}), \tag{2.3a}$$

and, including the term with the contact interaction [2],

$$\mathcal{H}_{\rm int}(\mathbf{x}) = i\frac{G}{2M}\psi_I(\mathbf{x})^\dagger(\boldsymbol{\sigma}\nabla\phi(\mathbf{x}))\psi_I(\mathbf{x}) + \frac{G^2}{2M}\psi_I(\mathbf{x})^\dagger\phi_i(\mathbf{x})\phi_i(\mathbf{x})\psi_I(\mathbf{x}), \tag{2.3b}$$

along with the meson term as

$$\mathcal{H}_M(\mathbf{x}) = \frac{1}{2}\big[(\partial_0\phi_i(\mathbf{x}))^2 + (\nabla\phi_i(\mathbf{x}))^2 + m^2(\phi_i(\mathbf{x}))^2\big]. \tag{2.3c}$$

We note that here we are to take the ordered products for the above expressions, so that the vacuum energy density is zero. In the above, $\psi_I(\mathbf{x})$ is the nonrelativistic two component spin and isospin quartet nucleon field operator and the matrix ϕ is given as $\phi = \tau_i\phi_i(\mathbf{x})$. In the present analysis, we shall consider only the second term on the right hand side of equation (2.3b) in a nonperturbative manner, and shall retain as usual only the perturbative contribution for the first term. We shall first construct the theory with the above, and later include the effect of other mesons.

We expand the field operator $\phi_i(\mathbf{z})$ in terms of the creation and annihilation operators of off-mass-shell mesons satisfying equal time algebra [5] as

$$\phi_i(\mathbf{z}) = \frac{1}{\sqrt{2\omega_{\mathbf{z}}}}\big(a_i(\mathbf{z})^\dagger + a_i(\mathbf{z})\big) \tag{2.4a}$$

and

$$\dot{\phi}_i(\mathbf{z}) = i\sqrt{\frac{\omega_{\mathbf{z}}}{2}}\big(a_i(\mathbf{z})^\dagger - a_i(\mathbf{z})\big). \tag{2.4b}$$

In the perturbative basis we have $\omega_{\mathbf{z}} = \big(m^2 - \nabla^2\big)^{\frac{1}{2}}$. We shall still use this, but we note that since we shall take an arbitrary number of pions in $U(\mathbf{x})$ in equation (2.7) as given later, the results shall be nonperturbative. We then substitute [2]

$$B(\mathbf{x})^\dagger = \frac{1}{2}\int f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2)a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2)^\dagger d\mathbf{z}_1 d\mathbf{z}_2. \tag{2.5}$$

We also take

$$\beta(\mathbf{x}) = B(\mathbf{x})^\dagger - B(\mathbf{x}). \tag{2.6}$$

Clearly $\beta(\mathbf{x})$ is an anti-Hermitian field operator. We now consider the matrix

$$U(\mathbf{x}) = \exp(\beta(\mathbf{x})). \tag{2.7}$$

We note that $U(\mathbf{x})$ is a unitary operator which, operating on vacuum, creates an arbitrarily large number of scalar isospin singlet pairs of pions corresponding to squeezed coherent states. Also, for the sake of simplicity, we shall here take $f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2)$, which is clearly symmetric in the arguments $\mathbf{z}_1$ and $\mathbf{z}_2$, to be a real function. From translational invariance, $f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2)$ shall be a function of $\mathbf{x}-\mathbf{z}_1$ and $\mathbf{x}-\mathbf{z}_2$. We now construct

$$\psi_{I\alpha}^{(\rm phys)}(\mathbf{x})^\dagger \mid {\rm vac}> = \psi_{I\alpha}(\mathbf{x})^\dagger U(\mathbf{x}) \mid {\rm vac}> \tag{2.8}$$

as an ansatz for the state of the physical nucleon. In the above, e.g., α stand for both the spin and isospin indices. Summation over repeated indices as usual is understood. Clearly the above state contains one nucleon along with a coherent cloud of mesons. In contrast to Refs. [2] and [4], we have defined here the space of the meson cloud through unitary operators, which, as we shall see, considerably simplifies the calculations. We also further note that from equation (2.8)

$$< {\rm vac} \mid \psi_{I\beta}^{(\rm phys)}(\mathbf{x}')\psi_{I\alpha}^{(\rm phys)}(\mathbf{x})^\dagger \mid {\rm vac}> = \delta(\mathbf{x}-\mathbf{x}')\delta_{\alpha\beta} \tag{2.9}$$

so that the physical nucleon states have the usual orthogonality relation. Now we have to use the above ansatz for the states to calculate the matrix elements, which, as we shall see, will involve some complicated but straightforward algebra. From equation (2.8), we define a "dressed" single nucleon state of momentum $\mathbf{p}$ as [8]

$$\mid \mathbf{p},\alpha> = \frac{1}{(2\pi)^{3/2}}\int e^{i\mathbf{p}\cdot x}\psi_\alpha(\mathbf{x})^\dagger U(\mathbf{x})d\mathbf{x} \mid {\rm vac}> . \tag{2.10}$$

This has a convenitional normalisation of

$$< \mathbf{p},\alpha \mid \mathbf{p}',\alpha'> = \delta_{\alpha\alpha'}\delta(\mathbf{p}-\mathbf{p}'). \tag{2.10$'$}$$

We note here a basic feature of the expression for the physical nucleon state in equation (2.8) above in contrast to the conventional picture. The single dressed nucleon does not have all its momentum carried by the bare nucleon, and the meson cloud does not enter as a wave function renormalisation. These carry some momentum through the meson creation operators in $U(\mathbf{x})$, and thus in the context of momentum carried, or through other interactions, the mesons may make their presence felt in specific reactions. The distinction thus is not only conceptual, but shall have experimental consequences as will be seen later. The energy expectation value of the nucleon at zero momentum for the interacting nucleon is given as [9]

$$h(f)\delta_{\alpha',\alpha} = (2\pi)^3 < \mathbf{0},\alpha' \mid \mathcal{H}(\mathbf{z}) \mid \mathbf{0},\alpha> . \tag{2.11}$$

As indicated, the above energy expectation value is a functional of the meson dressing f, and, due to translational invariance, $h(f)$ above shall be independent of $\mathbf{z}$. Evaluation of the quantities will necessitate obtaining some expressions involving the operators $U(\mathbf{x})^\dagger$ and $U(\mathbf{x})$ along with the basic operators of mesons in equations (2.4). We now proceed to discuss such expressions as above which will be needed to obtain the matrix elements for the dressed nucleon as in Eqs. (2.8) or (2.11), and discuss the corresponding algebra as will be used later.

A. The algebra of pion dressing

We shall now develop the formalism for the evaluation of expressions like those needed in equation (2.11). For notational simplicity, let us now suppress $\mathbf{x}$ in $f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2)$ and thus write

$$f(\mathbf{z}_1,\mathbf{z}_2) \equiv f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2). \tag{2.12}$$

We wish to evaluate first the expression

$U(\mathbf{x})^\dagger\, a_i(\mathbf{z})\, U(\mathbf{x})$, for finding the expectation values occurring on the right-hand side of equation (2.11). Again suppressing $\mathbf{x}$ we define, for any real λ, $U(\lambda) = e^{\lambda\beta(\mathbf{x})}$, and substitute

$$F(\lambda) = U(\lambda)^\dagger a_i(\mathbf{z}) U(\lambda). \tag{2.13}$$

We then obtain that

$$\frac{dF(\lambda)}{d\lambda} = U(\lambda)^\dagger \left[a_i(\mathbf{z}), B^\dagger\right] U(\lambda), \tag{2.14}$$

and that

$$\frac{d^2F(\lambda)}{d\lambda^2} = U(\lambda)^\dagger \left[B, \left[a_i(\mathbf{z}), B^\dagger\right]\right] U(\lambda). \tag{2.15}$$

We note that the substitution $F(\lambda)$ is made in a generic way for evaluating some specific expressions, and the definition for the same *will* change depending on what we wish to derive. We use that $f(\mathbf{z}_1,\mathbf{z}_2)$ is symmetric and real. We then obtain from equation (2.5) that

$$\left[a_i(\mathbf{z}_1), B(\mathbf{x})^\dagger\right] = \int f(\mathbf{z}_1,\mathbf{z}_2) a_i(\mathbf{z}_2)^\dagger\, d\mathbf{z}_2. \tag{2.16}$$

Proceeding in the same manner as above, we next obtain

$$\left[B, \left[a_i(\mathbf{z}_1), B^\dagger\right]\right] = \int f(\mathbf{z}_1,\mathbf{z}_2) d\mathbf{z}_2 f(\mathbf{z}_2,\mathbf{z}_3) a_i(\mathbf{z}_3) d\mathbf{z}_3. \tag{2.17}$$

We now define the generalised matrix $\hat{f}$ *continuous components* $\mathbf{z}_1, \mathbf{z}_2$ given as

$$\hat{f}(\mathbf{z}_1,\mathbf{z}_2) = f(\mathbf{z}_1,\mathbf{z}_2). \tag{2.18}$$

We thus have, e.g.,

$$\hat{f}^2(\mathbf{z}_1,\mathbf{z}_2) = \int \hat{f}(\mathbf{z}_1,\mathbf{z}_3)\, d\mathbf{z}_3\, \hat{f}(\mathbf{z}_3,\mathbf{z}_2).$$

Clearly the matrix $\hat{f}$ is symmetric and real. We also regard $\hat{a}_i$ as a generalised vector with continuum components $\mathbf{z}$, such that we substitute

$$\hat{a}_i(\mathbf{z}) = a_i(\mathbf{z}). \tag{2.19}$$

We shall also have a parallel equation for $a_i(\mathbf{z})^\dagger$. We can then write equation (2.16) as

$$\left[\hat{a}_i(\mathbf{z}), B^\dagger\right] = (\hat{f}\, \hat{a}_i^\dagger)(\mathbf{z}). \tag{2.20}$$

Similarly equation (2.17) becomes

$$\left[B, \left[\hat{a}_i(\mathbf{z}), B^\dagger\right]\right] = (\hat{f}^2\, \hat{a}_i)(\mathbf{z}). \tag{2.21}$$

We now omit these components and write equations (2.13), (2.14), and (2.15) as equations for generalised vectors as

$$F(\lambda) = U(\lambda)^\dagger\, \hat{a}_i\, U(\lambda), \tag{2.22}$$

$$\frac{dF(\lambda)}{d\lambda} = \hat{f}\, U(\lambda)^\dagger\, \hat{a}_i^\dagger\, U(\lambda), \tag{2.23}$$

and

$$\frac{d^2F(\lambda)}{d\lambda^2} = \hat{f}^2 U(\lambda)^\dagger \hat{a}_i U(\lambda) \tag{2.24}$$

$$\equiv \hat{f}^2\, F(\lambda). \tag{2.25}$$

We have used here that the matrix $\hat{f}$ *commutes* with $U(\lambda)$. The nice feature about the above equation is that the operator parts and the integrations over the function $f(\mathbf{z}_1,\mathbf{z}_2)$ with appropriate modifications corresponding to the commutator algebra have become separated. Equation (2.25) can be easily solved and yields that

$$F(\lambda) = \cosh(\lambda\hat{f})\, \hat{a}_i + \sinh(\lambda\hat{f})\, \hat{a}_i^\dagger. \tag{2.26}$$

Hence substituting $\lambda = 1$ we obtain that

$$U(\mathbf{x})^\dagger\, a_i(\mathbf{z})\, U(\mathbf{x}) = ((\cosh\hat{f})\hat{a}_i)(\mathbf{z}) + ((\sinh\hat{f})\hat{a}_i^\dagger)(\mathbf{z}), \tag{2.27}$$

which we shall call a generalisation of the Bogoliubov transformation. The dependence of the right-hand side on $\mathbf{x}$ is hidden in $\hat{f}$ through equations (2.12). The matrix structure of $\hat{f}$ is given by equation (2.18). Equation (2.27) will be repeatedly utilised to obtain the expectation values in equation (2.11) for given $\mathcal{H}(\mathbf{x})$ which will be known polynomials of the meson fields and their derivatives. We also have the parallel equation for the meson creation operator given as

$$U(\mathbf{x})^\dagger\, a_i(\mathbf{z})^\dagger\, U(\mathbf{x}) = ((\cosh\hat{f})\hat{a}_i^\dagger)(\mathbf{z}) + ((\sinh\hat{f})\hat{a}_i)(\mathbf{z}), \tag{2.28}$$

so that naming this as a generalised Bogoliubov transformation becomes transparent. In the above equations we may note that, e.g.,

$$((\cosh\hat{f})\hat{a}_i^\dagger)(\mathbf{z}_1) \equiv \int (\cosh\hat{f})(\mathbf{z}_1,\mathbf{z}_1') a_i^\dagger(\mathbf{z}_1') d\mathbf{z}_1',$$

and the $\mathbf{x}$ dependence is hidden in $\hat{f}$ through equation (2.12).

With $U(\lambda)$ as earlier, let us now define

$$F(\lambda) = (\partial_i U(\lambda)^\dagger)(\partial_i U(\lambda)). \tag{2.29}$$

We then easily see that

$$\frac{dF}{d\lambda} = (\partial_i\beta(\mathbf{x}))\left[(\partial_i U(\lambda)^\dagger)U(\lambda) - U(\lambda)^\dagger(\partial_i U(\lambda))\right]. \tag{2.30}$$

We now put

$$F_i(\lambda) = \left[(\partial_i U(\lambda)^\dagger)U(\lambda) - U(\lambda)^\dagger(\partial_i U(\lambda))\right]. \tag{2.31}$$

We then have

$$\frac{dF_i}{d\lambda} = -2(\partial_i\beta(\mathbf{x})). \tag{2.32}$$

With appropriate boundary conditions at $\lambda = 0$ we then obtain that

$$F_i(\lambda) = -2\lambda(\partial_i\beta(\mathbf{x})). \tag{2.33a}$$

and hence from equation (2.30)

$$F(\lambda) = -\lambda^2(\partial_i\beta(\mathbf{x}))(\partial_i\beta(\mathbf{x})). \tag{2.33b}$$

This yields that

$$(\partial_i U(\mathbf{x})^\dagger)(\partial_i U(\mathbf{x})) = -(\partial_i\beta(\mathbf{x}))(\partial_i\beta(\mathbf{x})). \tag{2.34}$$

and that

$$(\partial_i U(\mathbf{x})^\dagger)U(\mathbf{x}) = -U(\mathbf{x})^\dagger(\partial_i U(\mathbf{x})) = -(\partial_i\beta(\mathbf{x})), \tag{2.35}$$

where we have used that $\partial_i(U(\mathbf{x})^\dagger U(\mathbf{x})) = 0$.

III. EXPRESSION FOR THE ENERGY FUNCTIONAL

With $\mathcal{H}(\mathbf{z})$ as in equations (2.2) and (2.3), the energy operator is

$$H = \int \mathcal{H}(\mathbf{z})d\mathbf{z}. \tag{3.1}$$

As stated earlier, the energy functional is given by $h(f)$ as in equation (2.11). We are thus to find the expressions corresponding to equations (2.3) to obtain the energy functional $h(f)$. The advantage of the present approach is that with a single ansatz function for the pions, we include an arbitrary number of pion pairs in a coherent manner with equal time algebra, which makes the contributions calculable while retaining basically nonperturbative "higher order" effects without any truncation. Clearly at present we are concentrating on the relevant attractive channel corresponding to σ meson earlier [1], without using the above unphysical "particle." The picture is also completely quantum mechanical instead of classical, and this will have additional effects which we shall discuss later. We shall here first discuss the meson dressing of a single nucleon with a relatively simple ansatz for meson dressing for the function $f(\mathbf{x},\mathbf{z}_1,\mathbf{z}_2)$ in equation (2.5). This gives rise to a position-dependent generalised Bogoliubov transformation explicitly given in equations (2.27) and (2.28) which greatly simplifies the evaluation of the matrix elements.

A. Nucleon kinetic term

Let us first consider the contribution from the nucleon kinetic term of equation (2.3a). We ignore the mass term and then choose an equivalent symmetric form so that we have

$$H_N = \frac{1}{2M}\int(\partial_i\psi_I(\mathbf{z})^\dagger)(\partial_i\psi_I(\mathbf{z}))d\mathbf{z}. \tag{3.2}$$

We then need to evaluate the matrix element from equations (2.10) and (2.11) as [9]

$$\int < \text{vac}|\psi_{I\alpha'}^{(\text{phys})}(\mathbf{x}')\mathcal{H}_N(\mathbf{z})\psi_{I\alpha}^{(\text{phys})}(\mathbf{x})^\dagger|\text{vac} > d\mathbf{x}d\mathbf{x}'$$

$$= \frac{1}{2M}\delta_{\alpha'\alpha}\int < \text{vac} \mid U(\mathbf{x}')^\dagger(\partial_i^{\mathbf{z}}\delta(\mathbf{x}'-\mathbf{z}))(\partial_i^{\mathbf{z}}\delta(\mathbf{x}-\mathbf{z}))U(\mathbf{x}) \mid \text{vac} > d\mathbf{x}\,d\mathbf{x}' \tag{3.3}$$

$$= \frac{1}{2M}\delta_{\alpha'\alpha} < \text{vac}|(\partial_i U(\mathbf{z})^\dagger)(\partial_i U(\mathbf{z}))|\text{vac} > . \tag{3.4}$$

Since we are considering the nucleon to be at rest, the conventional kinetic energy as above vanishes. However, an extra contribution from the meson dressing arises as written in equation (3.4). Thus from equation (2.34) we have that the additional contribution $h_N^{(1)}(f)$ to kinetic energy is given as

$$h_N^{(1)}(f) = -\frac{1}{2M} < \text{vac}|(\partial_i\beta(\mathbf{z}))(\partial_i\beta(\mathbf{z}))|\text{vac} > . \tag{3.5}$$

From equations (2.5) and (2.6) we then easily have

$$h_N^{(1)}(f) = \frac{1}{2M} < \text{vac}|(\partial_i B(\mathbf{x}))(\partial_i B(\mathbf{x})^\dagger)|\text{vac} >$$
$$= \frac{1}{2M}\int(\partial_i f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2))(\partial_i f(\mathbf{x};\mathbf{z}_1,\mathbf{z}_2))d\mathbf{z}_1 d\mathbf{z}_2. \tag{3.6}$$

Through translational invariance as stated earlier, the above function is independent of $\mathbf{x}$. Thus the nucleon kinetic term of equation (3.2) generates a contribution through the differentiation of the functions describing meson dressing of the nucleon, in additon to the conventional contribution to kinetic energy.

Let us approximate $f(\mathbf{x},\mathbf{z}_1,\mathbf{z}_2)$ as a factorisable expression with translational invariance given as

$$f(\mathbf{x},\mathbf{z}_1,\mathbf{z}_2) = af_1(\mathbf{x}-\mathbf{z}_1)f_1(\mathbf{x}-\mathbf{z}_2), \tag{3.7}$$

where a is an arbitrary dimensionless constant and the function f_1 is spherically symmetric. We normalise the function f_1 such that

$$\int(f_1(\mathbf{x}))^2\, d\mathbf{x} = 1. \tag{3.8}$$

From equation (3.6) we then obtain that, using rotational symmetry for $f_1(\mathbf{z})$,

$$h_N^{(1)}(f) = \Big(\frac{1}{2M}a^2\Big)2\int(\partial_i f_1(\mathbf{z}))^2(f_1(\mathbf{z}'))^2\, d\mathbf{z}\, d\mathbf{z}'$$
$$= \frac{1}{M}a^2\, c_2, \tag{3.9}$$

where

$$c_2 = \int (\partial_i f_1(\mathbf{x}))^2 \, d\mathbf{x}. \tag{3.10}$$

Thus in a sense the extra contribution from fermion kinetic term is proportional to the inverse size of meson dressing. With

$$f_1(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int \tilde{f}_1(\mathbf{k}) e^{i\mathbf{k}\cdot x} d\mathbf{k}, \tag{3.11}$$

we have from equation (3.10) that

$$c_2 = \int \mathbf{k}^2 \mid \tilde{f}_1(\mathbf{k}) \mid^2 d\mathbf{k}. \tag{3.12}$$

B. Interaction term

We now calculate the matrix element corresponding to $\mathcal{H}_{\text{int}}$ of equation (2.3b). From equations (2.10) and (2.11) we then obtain in the same manner as earlier,

$$\delta_{\alpha'\alpha} h_{\text{int}}(f) = \int < \text{vac} \mid U(\mathbf{x}')^\dagger \psi_{I\alpha'}(\mathbf{x}') \mathcal{H}_{\text{int}}(\mathbf{z}) \psi_{I\alpha}(\mathbf{x}) U(\mathbf{x}) \mid \text{vac} > d\mathbf{x}\, d\mathbf{x}'. \tag{3.13}$$

Since $U(\mathbf{x})$ only contains pion operators, from the above we easily obtain that

$$h_{\text{int}}(f) = \frac{G^2}{2M} < \text{vac} \mid U(\mathbf{x})^\dagger : \phi_i(\mathbf{x})\phi_i(\mathbf{x}) : U(\mathbf{x}) \mid \text{vac} > . \tag{3.14}$$

We then use that with equation (2.4a)

$$: \phi_i(\mathbf{x})\phi_i(\mathbf{x}) := \frac{1}{\sqrt{2\omega_{\mathbf{z}_1} 2\omega_{\mathbf{z}_2}}} \left[a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2)^\dagger + a_i(\mathbf{z}_1) a_i(\mathbf{z}_2) + 2 a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2) \right] \mid_{\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{x}} . \tag{3.15}$$

Hence from equations (2.27) and (2.28) and the definition of $\hat{f}$ we obtain that

$$\begin{aligned}
&< \text{vac} \mid U(\mathbf{x})^\dagger : \phi_i(\mathbf{z}_1)\phi_i(\mathbf{z}_2) : U(\mathbf{x}) \mid \text{vac} > \\
&= \frac{1}{\sqrt{2\omega_{\mathbf{z}_1} 2\omega_{\mathbf{z}_2}}} < \text{vac} \mid \left\{ \left[(\sinh \hat{f}) \hat{a}_i \right](\mathbf{z}_1) \left[(\cosh \hat{f}) \hat{a}_i^\dagger \right](\mathbf{z}_2) \right. \\
&\quad \left. + \left[(\cosh \hat{f}) \hat{a}_i \right](\mathbf{z}_1) \left[(\sinh \hat{f}) \hat{a}_i^\dagger \right](\mathbf{z}_2) + 2 \left[(\sinh \hat{f}) \hat{a}_i \right](\mathbf{z}_1) \left[(\sinh \hat{f}) \hat{a}_i^\dagger \right](\mathbf{z}_2) \right\} \mid \text{vac} > \\
&= \left(\frac{1}{\sqrt{2\omega_{\mathbf{z}_1} 2\omega_{\mathbf{z}_2}}} \right) (3)(2) \left[(\cosh \hat{f} \, \sinh \hat{f})(\mathbf{z}_1, \mathbf{z}_2) + (\sinh^2 \hat{f})(\mathbf{z}_1, \mathbf{z}_2) \right] \\
&= \left(\frac{1}{\sqrt{2\omega_{\mathbf{z}_1} 2\omega_{\mathbf{z}_2}}} \right) 3 \left[e^{2\hat{f}} - \hat{I} \right](\mathbf{z}_1, \mathbf{z}_2).
\end{aligned} \tag{3.16}$$

In the above we have substituted

$$\hat{I}(\mathbf{z}_1, \mathbf{z}_2) = \delta(\mathbf{z}_1 - \mathbf{z}_2). \tag{3.17}$$

From equations (3.7) and (3.8) for the specific form of $f(\mathbf{x}; \mathbf{z}_1, \mathbf{z}_2)$ we clearly have

$$(\hat{f}^n)(\mathbf{z}_1, \mathbf{z}_2) = \hat{f}(\mathbf{z}_1, \mathbf{z}_2) a^{n-1}. \tag{3.18}$$

With equation (3.7) this clearly yields that

$$(e^{2\hat{f}} - \hat{I})(\mathbf{z}_1, \mathbf{z}_2) = (e^{2a} - 1) f_1(\mathbf{x} - \mathbf{z}_1) f_1(\mathbf{x} - \mathbf{z}_2) \tag{3.19}$$

where the $\mathbf{x}$ dependence of $\hat{f}$ has explicitly appeared on the right hand side above. Hence with the present approximation for meson dressing equation (3.16) simplifies to

$$h_{\text{int}}(f) = \frac{G^2}{2M} 3(e^{2a} - 1) \times \left[\frac{1}{\sqrt{2\omega_{\mathbf{z}_1} 2\omega_{\mathbf{z}_2}}} (f_1(\mathbf{x} - \mathbf{z}_1) f_1(\mathbf{x} - \mathbf{z}_2)) \right]_{\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{x}} \tag{3.20}$$

Let us substitute

$$c_1 = \frac{1}{\sqrt{2\omega_{\mathbf{z}}}} f_1(\mathbf{z}) \mid_{\mathbf{z}=0} = \frac{1}{(2\pi)^{3/2}} \int \frac{1}{\sqrt{2\omega(\mathbf{k})}} \tilde{f}_1(\mathbf{k}) d\mathbf{k}. \tag{3.21}$$

Equation (3.20) then yields that

$$h_{\text{int}}(f) = \frac{G^2}{2M} 3(e^{2a} - 1) c_1^2. \tag{3.22}$$

C. Meson kinetic term

With the meson field operator expansions as in equations (2.4), we may write equation (2.3c) as

$$\mathcal{H}_M(\mathbf{z}) = a_i(\mathbf{z})^\dagger \omega_{\mathbf{z}} \, a_i(\mathbf{z}). \tag{3.23}$$

We may note that through a Bogoliubov transformation, in fact, the function $\omega(\mathbf{k})$ of equations (2.4) in field expansion might change [10], and thus we may heuristically assume that taking the meson field Hamiltonian as in equation (3.23) may not involve any real loss of generality. In any case, it corresponds here to taking the perturbative basis for the meson quanta.

In order to apply as before equation (2.11) for the energy expectation value from $\mathcal{H}_M(\mathbf{x})$, we now note that

$$< \text{vac} \mid U(\mathbf{x})^\dagger a_i(\mathbf{z})^\dagger \omega_\mathbf{z}\, a_i(\mathbf{z}) U(\mathbf{x}) \mid \text{vac} >$$

$$= 3\omega_\mathbf{z}\big(\sinh \hat{f}\, \sinh \hat{f}\big)(\mathbf{z}_1, \mathbf{z})\Big|_{\mathbf{z}_1=\mathbf{z}}$$

$$= \frac{3}{4}\frac{e^{2a}+e^{-2a}-2}{a}\omega_\mathbf{z}\tilde{f}(\mathbf{z}_1,\mathbf{z})\Big|_{\mathbf{z}_1=\mathbf{z}} \tag{3.24}$$

$$= \frac{3}{4}(e^{2a}+e^{-2a}-2) f_1(\mathbf{x}-\mathbf{z})\omega_\mathbf{z} f_1(\mathbf{x}-\mathbf{z}). \tag{3.25}$$

In writing down equations (3.24) and (3.25) we have used the specific ansatz as in equation (3.7). Equation (2.11) with equation (3.23) now yield that

$$h_M(f) = \frac{3}{4}(e^{2a}+e^{-2a}-2)\int f_1(\mathbf{x}-\mathbf{z})\omega_\mathbf{z} f_1(\mathbf{x}-\mathbf{z})\, d\mathbf{x}$$

$$= \frac{3}{4}(e^{2a}+e^{-2a}-2)c_M, \tag{3.26}$$

where

$$c_M = \int \omega(\mathbf{k}) \mid \tilde{f}_1(\mathbf{k}) \mid^2 d\mathbf{k}. \tag{3.26$'$}$$

We note that $h_M(f)$ is positive and increases very fast with an increase in the magnitude of the constant a.

D. Number density for the meson

We would like to have an intuitive picture for the meson dressing with pion pairs for the nucleon. For this purpose we note that the number density at $\mathbf{z}$ for off-shell mesons with the nucleon at $\mathbf{x}$ is given as

$$< \text{vac} \mid U(\mathbf{x})^\dagger a_i(\mathbf{z})^\dagger a_i(\mathbf{z}) U(\mathbf{x}) \mid \text{vac} >$$

$$= \frac{3}{4}(e^{2a}+e^{-2a}-2)\left[f_1(\mathbf{x}-\mathbf{z})\right]^2, \tag{3.27}$$

the derivation of the above being similar to that of equation (3.25). We also get the average number N_M of the off-mass shell pions in nucleon to be given as

$$N_M = \frac{3}{4}(e^{2a}+e^{-2a}-2). \tag{3.28}$$

The above expression gives an intuitive quantitative picture regarding the constant a as describing the pion number in the nucleon. Further, from equation (3.27) the average distance of mesons from the centre of the nucleon is given as R_{av}, where

$$R_{\text{av}}^2 = \int \left[f_1(\mathbf{z})\right]^2 \mathbf{z}^2\, d\mathbf{z}. \tag{3.29}$$

IV. DRESSED NUCLEON

We shall now discuss the dressing of a single nucleon by minimising the energy for the physical nucleon as derived in equations (3.9), (3.22), and (3.26). Here a is a free parameter and we do extremisation over the function f_1 subject to the condition of unit normalisation as in equation (3.8). For this purpose we shall parametrise $f_1(\mathbf{x})$ and extremise over the parameter space. As from equation (3.27), since $\left[f_1(\mathbf{x}-\mathbf{z})\right]^2$ is proportional to the density of pions, it should vanish when $|\mathbf{x}-\mathbf{z}|$ approaches zero or infinity. With this in mind and ensuring normalisation as in equation (3.8), we take the ansatz

$$f_1(\mathbf{x}) = \left(\frac{4}{15\pi\sqrt{\pi}R_M^7}\right)^{\frac{1}{2}} \mathbf{x}^2 \exp\left(-\frac{\mathbf{x}^2}{2R_M^2}\right). \tag{4.1}$$

The Fourier transform of the above becomes

$$\tilde{f}_1(\mathbf{k}) = \left(\frac{12R_M^3}{5\pi\sqrt{\pi}}\right)^{1/2}\left(1-\frac{R_M^2\mathbf{k}^2}{3}\right)\exp\left(-\frac{R_M^2\mathbf{k}^2}{2}\right). \tag{4.2}$$

The minimisation of energy now is to be taken for the two parameters a and R_M. We now proceed to evaluate the corresponding expressions. From equation (3.11) we obtain

$$c_2(R_M) = \frac{11}{10}\frac{1}{R_M^2}. \tag{4.3}$$

Hence from equation (3.9) the contribution from the nucleon kinetic term becomes

$$h_N(a, R_M) = \frac{11a^2}{10M}\frac{1}{R_M^2}. \tag{4.4}$$

We further note from equation (4.4), (3.22), and (3.26) that $h_N(a,R_M)$ and $h_M(a,R_M)$ are positive and even in a, where as the function $h_{\text{int}}(a,R_M)$ is negative when a is negative. Clearly

$$h(a,R_M) = h_N(a,R_M) + h_{\text{int}}(a,R_M) + h_M(a,R_M). \tag{4.5}$$

We take $m = 140$ MeV, $M = 940$ MeV and then try to find a and R_M by minimising the expression (4.5) numerically.

For a fixed negative value of a of sufficiently small magnitude, we note that $h(a,R_M)$ *decreases* indefinitely to minus infinity as $R_M \to 0$. This is effectively the reflection of the fact that self energy of the nucleon is negative infinity, and arises because of the pointlike structure of the pion and the nucleon in the effective Lagrangian. As shown earlier [4], we have to retain in a phenomenological way the fact that the hadrons are extended and composite objects, and hence when they tend to come close to each other an effective repulsion between them will be operative, and this is to be included.

The extra energy involved when a pion comes indefinitely near the nucleon due to their composite structure is at present not calculable. It is however clear that this structure will not permit R_M to go to zero. We shall include this effect by adding to the Hamiltonian density an effective contribution as due to pion nucleon repulsion arising from the composite nature when they come together. This contribution is taken as

$$\mathcal{H}_{nmr}(\mathbf{x}) = \int \psi_I(\mathbf{x})^\dagger \psi_I(\mathbf{x}) g(\mathbf{x}-\mathbf{z}) a_i(\mathbf{z})^\dagger a_i(\mathbf{z})\, d\mathbf{z}. \tag{4.6}$$

Parallel to the derivation of equation (3.14), we then obtain the extra contribution to energy for the dressed nucleon to be given as

$$h_{nmr}(f) = \int g(\mathbf{x}-\mathbf{z}) < \text{vac} \mid U(\mathbf{x})^\dagger a_i(\mathbf{z})^\dagger a_i(\mathbf{z}) U(\mathbf{x}) \mid \text{vac} > d\mathbf{z} = 3 \int g(\mathbf{x}-\mathbf{z}) (\sinh^2 \hat{f})(\mathbf{z},\mathbf{z})\, d\mathbf{z}. \tag{4.7}$$

Using equations (3.7) and (3.18) we then obtain that

$$h_{nmr}(f) = \frac{3}{4}(e^{2a} + e^{-2a} - 2) \int g(\mathbf{x}-\mathbf{z})(f_1(\mathbf{x}-\mathbf{z}))^2\, d\mathbf{z}. \tag{4.8}$$

Since the integrals earlier are defined in the Fourier transform space, we now take

$$g(\mathbf{x}-\mathbf{z}) = \frac{1}{(2\pi)^3} \int \tilde{g}(\mathbf{k}) e^{i\mathbf{k}(\mathbf{x}-\mathbf{z})}\, d\mathbf{k}. \tag{4.9}$$

We then obtain that in equation (4.8)

$$\int g(\mathbf{z})(f_1(\mathbf{z}))^2\, d\mathbf{z} = \frac{1}{(2\pi)^3} \int \tilde{g}(\mathbf{k}) [1 - \tfrac{1}{3} R_M^2 \mathbf{k}^2 + \tfrac{1}{60} R_M^4 (\mathbf{k}^2)^2] \times \exp(-\tfrac{1}{4} R_M^2 \mathbf{k}^2)\, d\mathbf{k}. \tag{4.10}$$

We now take a simple expression for $\tilde{g}(\mathbf{k})$ as

$$\tilde{g}(\mathbf{k}) = A_\pi (\mathbf{k}^2)^2. \tag{4.11}$$

In the above our objective is to construct phenomenologically a sufficiently repulsive term at small distances to neutralise the stated instability for small R_M. In fact, the integral of equation (4.10) simplifies to

$$c_{nmr} \equiv \int g(\mathbf{z})(f_1(\mathbf{z}))^2 d\mathbf{z} = \frac{32}{\pi\sqrt{\pi}} \frac{A_\pi}{R_M^7}, \tag{4.12}$$

which we shall see corresponds to an adequately strong repulsion for small R_M. From the above and equation (4.8) we then obtain that

$$h_{nmr}(a, R_M) = \frac{3}{4}(e^{2a} + e^{-2a} - 2) c_{nmr}. \tag{4.13}$$

The above repulsion in h_{nmr} has been our objective for the formal choice of the expression for $\tilde{g}(\mathbf{k})$ in equation (4.11). We may note that since R_M is a measure of the distance between the nucleon and the pion, the phenomenological repulsive term is such that the energy due to repulsion varies inversely as the seventh power of the distance, *which in a way is similar to the repulsion of atoms at short distances* and was a part motivation for our choice.

We now replace equation (4.4) by

$$h(a, R_M) = h_N(a, R_M) + h_{\text{int}}(a, R_M) + h_M(a, R_M) + h_{nmr}(a, R_M), \tag{4.14}$$

and then minimise $h(a, R_M)$. The relevance for the repulsion term as given by equations (4.13) with the ansatz (4.11) is that this term corresponds to an increase in energy for small distances, which prohibits the instability for R_M going to zero. This choice is mainly governed by heuristic considerations, and the function $\tilde{g}(\mathbf{k})$ is not calculable. We now take the parameter A_π of equation (4.11) or (4.13) as input and then minimise the energy expectation value to determine the variational parameters R_M and a. We note that by equations (3.29) and (4.1), the average pion distance from the nucleon is given by $R_{\text{av}} = \sqrt{3.5} R_M$.

As an example we may note that for $G^2/4\pi = 14.6$, and $A_\pi = 0.1$ fm^{-6}, through a minimisation we obtain that $R_M = 0.926$ fm with $R_{\text{av}} = 1.73$ fm, and, $a = -4.23 \times 10^{-3}$ along with the "interesting" signal that the average meson number for the dressed nucleon with squeezed coherent state as in equation (3.28) becomes about 5.4×10^{-5}. Also, the contributions to energy from h_N, h_{int}, h_M, and h_{nmr} become 0.96×10^{-4}, -0.252, 0.013, and 0.105 MeV, respectively, with the contribution to self-energy of the single nucleon being about -0.133 MeV. With this in mind we now proceed to calculate the two nucleon potential. We shall see later that even though the approximation is crude and heuristic, it shall have testable characteristic conclusions.

V. TWO-NUCLEON STATES AND PION DRESSING

We shall now find the two nucleon bound states with a generalisation of the above type of nucleon dressing, parallel to what has been considered earlier [5]. For this purpose we shall first define the two nucleon states in a manner which shall be more convenient for field theoretic calculations and then consider the energy expectation values as in the last section to obtain the nucleon wave function and the meson dressing. Clearly the two pion dressing as earlier will correspond only to the scalar isoscalar part of the two nucleon potential usually taken phenomenologically or generated through the hypothetical σ-meson exchange. We shall add to this the one pion exchange contribution and the repulsive part of the potential to make it realistic. One objective here is to show that such a picture of nuclear structure with Walecka's model generalised to include quantum two pion condensate can be a viable alternative, and we need not take separately a σ meson.

A. Two nucleon states

We shall first reorient the description for two nucleons with field operators so that we can conveniently take energy expectation values parallel to the last section. We shall now note that the nucleon field operator is $\psi_{I\alpha}(\mathbf{x})$ for both proton and neutron with $\alpha = (\pm\frac{1}{2}, \pm\frac{1}{2})$ standing for both isospin and spin indices. We then define the charge conjugate operator $\tilde{\psi}_{I\alpha}(\mathbf{x})$ as

$$\tilde{\psi}_{I\alpha}(\mathbf{x}) = (\sigma_2 \tau_2)_{\alpha\beta} \psi_{I\beta}(\mathbf{x})^\dagger. \tag{5.1}$$

From the nucleon field operator anticommutators we then have

$$\left[\psi_{I\alpha}(\mathbf{x}), \tilde{\psi}_{I\beta}(\mathbf{y})\right]_+ = (\sigma_2\tau_2)_{\beta\alpha}\delta(\mathbf{x}-\mathbf{y}). \tag{5.2}$$

It is our purpose here to illustrate the present methodology with the application to a simple physical problem. With this in mind, we shall consider as an example the state corresponding to the deuteron. With the above notation we then write the deuteron state of zero momentum as

$$|D^m(0)>=\frac{1}{2\sqrt{2}}\frac{1}{(2\pi)^{\frac{3}{2}}}\int\psi_I(\mathbf{x}_1)^\dagger U^{mn}(\mathbf{x}_1-\mathbf{x}_2)\sigma_n\tilde{\psi}_I(\mathbf{x}_2)U(\mathbf{x}_1,\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2\mid\mathrm{vac}>. \tag{5.3}$$

In the above with $\mathbf{r}=\mathbf{x}_1-\mathbf{x}_2$ we have substituted

$$U^{mn}(\mathbf{r}) = \left[\delta^{mn}u_s(r)\cos\beta + P^{mn}(\hat{r})u_d(r)\sin\beta\right], \tag{5.4}$$

with

$$P^{mn}(\hat{r}) = \frac{1}{\sqrt{2}}(3\hat{r}^m\hat{r}^n - \delta^{mn}). \tag{5.5}$$

In the above, $u_{s,d}(|\mathbf{x}_1-\mathbf{x}_2|)$ are the conventional two-nucleon s and d wave functions, β is the mixing angle, and $U(\mathbf{x}_1,\mathbf{x}_2)$ describes the meson dressing. In fact parallel to equations (2.5)–(2.7) we take $U(\mathbf{x}_1,\mathbf{x}_2)$ as

$$U(\mathbf{x}_1,\mathbf{x}_2) = \exp(\beta(\mathbf{x}_1)+\beta(\mathbf{x}_2)), \tag{5.6}$$

where $\beta(\mathbf{x})$ is given by the same equations (2.5) and (2.6). In the above the s and d wave functions have the usual normalisation given as

$$\int|u_s(r)|^2d\mathbf{r} = \int|u_d(r)|^2d\mathbf{r} = 1. \tag{5.7}$$

We again apply [9] equation (2.11) to obtain the energy expectation value of this two nucleon system. Minimisation of the same will simultaneously give both the meson dressing as well as the two nucleon wave function. In fact we have here

$$h(f)\delta^{mn} = (2\pi)^3 < D^m(0)|\mathcal{H}(\mathbf{z})|D^n(0)>. \tag{5.8}$$

As earlier, the evaluation of the above will be our objective. This will include both the effect of the nucleon wave functions and the effect of meson dressing. The space coordinate dependence of the energy expection value enables us to identify a potential [5]. Because of the choice of the interaction the potential here can only correspond to the scalar isoscalar part of the two nucleon potential. However, as we shall see later, the concept of potential will have limited validity, since nucleon wave function and meson dressing get determined simultaneously.

B. The algebra of pion dressing

Let us now recapitulate the algebra for meson dressing as in Section II. A. For this purpose we now substitute as in equation (2.12),

$$f(\mathbf{z}_1,\mathbf{z}_2) \equiv f(\mathbf{x}_1;\mathbf{z}_1,\mathbf{z}_2)+f(\mathbf{x}_2;\mathbf{z}_1,\mathbf{z}_2). \tag{5.9}$$

We then define the operator $\hat{f}$ with its matrix elements as $f(\mathbf{z}_1,\mathbf{z}_2)$ of equation (5.9). Some identities established in Section II. A then remain unaltered with $U(\mathbf{x})$ of equation (2.7) now replaced by $U(\mathbf{x}_1,\mathbf{x}_2)$ of equation (5.6). In fact we may rewrite equation (5.6) as

$$U(\mathbf{x}_1,\mathbf{x}_2) = \exp(B(\mathbf{x}_1,\mathbf{x}_2)^\dagger - B(\mathbf{x}_1,\mathbf{x}_2)), \tag{5.10}$$

where parallel to equation (2.5) we have

$$B(\mathbf{x}_1,\mathbf{x}_2)^\dagger = \int f(\mathbf{z}_1,\mathbf{z}_2)a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2)^\dagger d\mathbf{z}_1d\mathbf{z}_2 \tag{5.11}$$

with the function $f(\mathbf{z}_1,\mathbf{z}_2)$ now given by equation (5.9). It is easily seen that equation (2.28) for the Bogoliubov transformation of $a_i(\mathbf{z})^\dagger$ remains unaltered, with the only change that the matrix elements of $\hat{f}$ are now given by equation (5.9). With the same change in $\hat{f}$, also equation (3.28) remains unaltered when we replace $U(\mathbf{x})$ by $U(\mathbf{x}_1,\mathbf{x}_2)$.

We now consider the parallel of equation (3.18) for the simplification of $\hat{f}^n$, which undergoes a drastic change. With the factorisable ansatz of equation (3.7), we may rewrite equation (5.9) as

$$\begin{aligned}f(\mathbf{z}_1,\mathbf{z}_2) = a\big[\ & f_1(\mathbf{x}_1-\mathbf{z}_1)f_1(\mathbf{x}_1-\mathbf{z}_2)\\ &+f_1(\mathbf{x}_2-\mathbf{z}_1)f_1(\mathbf{x}_2-\mathbf{z}_2)\big].\end{aligned} \tag{5.12}$$

We further define the function

$$\begin{aligned}f^{(e)}(\mathbf{z}_1,\mathbf{z}_2) = a\big[\ & f_1(\mathbf{x}_1-\mathbf{z}_1)f_1(\mathbf{x}_2-\mathbf{z}_2)\\ &+f_1(\mathbf{x}_1-\mathbf{z}_2)f_1(\mathbf{x}_2-\mathbf{z}_1)\big].\end{aligned} \tag{5.13}$$

We then note that

$$\begin{aligned}\hat{f}^2(\mathbf{z}_1,\mathbf{z}_2) &= \int\hat{f}(\mathbf{z}_1,\mathbf{z}_3)\ dz_3\ \hat{f}(\mathbf{z}_3,\mathbf{z}_2)\\ &= af(\mathbf{z}_1,\mathbf{z}_2)+abf^{(e)}(\mathbf{z}_1,\mathbf{z}_2),\end{aligned} \tag{5.14}$$

where we have taken

$$b = b(\mathbf{x}_1-\mathbf{x}_2) = \int f_1(\mathbf{x}_1-\mathbf{z})f_1(\mathbf{x}_2-\mathbf{z})\ d\mathbf{z}. \tag{5.15}$$

We have also utilised the normalisation condition of equation (3.8) such that $b(\mathbf{0}) = 1$. In fact with an obvious notation we may symbolically write equation (5.14) as

$$\hat{f}^2 = a\ \hat{f}\ +\ ab\ \hat{f}^{(e)}. \tag{5.16}$$

In a parallel manner a straightforward integration will yield that

$$\hat{f}\hat{f}^{(e)} = a\ \hat{f}^{(e)}\ +\ ab\ \hat{f}. \tag{5.17}$$

Let us substitute

$$\hat{f}^n = g_n\hat{f}+g_n^{(e)}\hat{f}^{(e)}. \tag{5.18}$$

We then easily have

$$\hat{f}^{n+1} = \hat{f}(\hat{f}^n) = \hat{f}(g_n \hat{f} + g_n^{(e)} \hat{f}^{(e)})$$
$$= a\left[(g_n + b g_n^{(e)})\hat{f} + (b g_n + g_n^{(e)} \hat{f}^{(e)})\right]. \quad (5.19)$$

Hence in equation (5.14) we obtain that

$$g_{n+1} = a(g_n + b g_n^{(e)}), \quad (5.20a)$$
$$g_{n+1}^{(e)} = a(g_n^{(e)} + b g_n). \quad (5.20b)$$

Also, $g_1 = 1$ and $g_1^{(e)} = 0$, and, as a particular case of the above equation, $g_2 = a$ and $g_2^{(e)} = ab$. In fact the general solution may be easily seen to be given as

$$g_n = a^{n-1}\left((1 + b\tau_1)^{n-1}\right)_{11}, \quad (5.21a)$$

and

$$g_n^{(e)} = a^{n-1}\left((1 + b\tau_1)^{n-1}\right)_{21}, \quad (5.21b)$$

In the above, τ_1 is the usual isospin matrix. We note that the above coefficients are functions of $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ through b as in equation (5.15). From equations (5.18) and (5.21) we thus obtain that

$$\hat{f}^n = a^{n-1}\left[\left((1 + b\tau_1)^{n-1}\right)_{11} f(\mathbf{z}_1, \mathbf{z}_2) + \left((1 + b\tau_1)^{n-1}\right)_{21} f^{(e)}(\mathbf{z}_1, \mathbf{z}_2)\right]. \quad (5.22)$$

The above equation, which is the parallel of equation (3.18), is not as simple as the earlier equation, but is reasonably simple to enable us to obtain corrections corresponding to arbtitrary powers of $b(\mathbf{x}_1 - \mathbf{x}_2)$. We shall now illustrate this with the simplification of the expression $(e^{2\hat{f}} - I)(\mathbf{z}_1, \mathbf{z}_2)$ to obtain the parallel of equation (3.19). Corresponding to equation (5.22), let us define the 2×2 matrix

$$A = a + ab\tau_1. \quad (5.23)$$

We then have

$$A^{-1} = \frac{(1 - b\tau_1)}{a(1 - b^2)}. \quad (5.24)$$

From equation (5.22) we obtain

$$(e^{2\hat{f}} - I)(\mathbf{z}_1, \mathbf{z}_2) = \left[\left(A^{-1}(e^{2A} - I)\right)_{11} f(\mathbf{z}_1, \mathbf{z}_2) + \left(A^{-1}(e^{2A} - I)\right)_{21} f^{(e)}(\mathbf{z}_1, \mathbf{z}_2)\right]. \quad (5.25)$$

A straightforward matrix simplification now yields that

$$A^{-1}(e^{2A} - I) = \frac{1}{a(1 - b^2)}\left[\left((e^{2a}\cosh(2ab) - 1) - b e^{2a}\sinh(2ab)\right) + \tau_1\left(e^{2a}\sinh(2ab) - b(e^{2a}\cosh(2ab) - 1)\right)\right]. \quad (5.26)$$

Hence we obtain that

$$(e^{2\hat{f}} - I)(\mathbf{z}_1, \mathbf{z}_2) = \frac{1}{a(1 - b^2)}\left[\left((e^{2a}\cosh(2ab) - 1) - b e^{2a}\sinh(2ab)\right) f(\mathbf{z}_1, \mathbf{z}_2) + \left(e^{2a}\sinh(2ab) - b(e^{2a}\cosh(2ab) - 1)\right) f^{(e)}(\mathbf{z}_1, \mathbf{z}_2)\right]. \quad (5.27)$$

For the above matrix element, the $\mathbf{z}_1, \mathbf{z}_2$ dependence is explicitly noted, and the dependence on $\mathbf{x}_1$ and $\mathbf{x}_2$ occurs implicitly in the functions f and $f^{(e)}$ and in $b(\mathbf{x}_1 - \mathbf{x}_2)$ as in equations (5.12), (5.13), and (5.15), respectively. We shall now make use of the above identities to evaluate the matrix elements.

VI. CALCULATION OF ENERGY EXPECTATION VALUES

A. Nucleon kinetic energy

We now proceed to calculate the energy expectation values. Let us consider the contribution from the nucleon kinetic term from equation (3.2). We then need to evaluate the matrix element as from equation (5.8). Using rotational invariance we get

$$h_N(u, f) \equiv \frac{1}{3}(2\pi)^3 < D^m(0)|\mathcal{H}_N(\mathbf{z})|D^m(0) > . \quad (6.1)$$

In the above we ignore the derivative terms of $U(\mathbf{x}_1, \mathbf{x}_2)$ for meson dressing since the calculations of the last section seem to indicate that these are likely to be negligible. Hence in equation (6.1) the dependance on f disappears. Hence the energy expectation values are functionals of only $U^{mn}(\mathbf{r})$, and that the s- and d-wave contributions occur additively. On evaluation of equation (6.1) with equation (5.3) we obtain that

$$h_N(u) = \frac{1}{3M}\int d\mathbf{x}_1 (\partial_i^{(z)} U^{mn}(\mathbf{x}_1 - \mathbf{z})^*)(\partial_i^{(z)} U^{mn}(\mathbf{x}_1 - \mathbf{z})). \quad (6.2)$$

We then have

$$h_N(u) = h_N^s(u) + h_N^d(u) \quad (6.3)$$

with

$$h_N^s(u) = \frac{1}{M}\cos^2\beta \int d\mathbf{r} |\nabla u_s(r)|^2 \quad (6.4)$$

and

$$h_N^d(u) = \frac{1}{M}\sin^2\beta \int d\mathbf{r}\Big(|\nabla u_d(r)|^2 + \frac{6}{r^2}|u_d(r)|^2\Big) \tag{6.5}$$

B. Interaction energy

We next proceed to calculate the matrix element corresponding to $\mathcal{H}_{\rm int}(\mathbf{z})$ of equation (2.3b). Since $U(\mathbf{x}_1,\mathbf{x}_2)$ contains only pion operators, we then obtain that

$$\begin{aligned} h_{\rm int}(u,f) &\equiv \frac{1}{3}(2\pi)^3 < D^m(\mathbf{0})|\mathcal{H}_{\rm int}(\mathbf{z})|D^m(\mathbf{0}) > \\ &= \frac{G^2}{3M}\int d\mathbf{x}_1 \mid U^{mn}(\mathbf{x}_1-\mathbf{z}) \mid^2 < \text{vac} \mid U(\mathbf{x}_1,\mathbf{z})^\dagger : \phi_i(\mathbf{z})\phi_i(\mathbf{z}) : U(\mathbf{x}_1,\mathbf{z}) \mid \text{vac} > . \end{aligned} \tag{6.6}$$

A factor of 2 has been included in the above since an interchange of $\mathbf{x}_1$ and $\mathbf{x}_2$ gives an identical contribution. Hence using equation (3.15) with the changed definitions of U as well as of $\hat{f}$ as in equation (5.5), we obtain the parallel of equation (3.16) as

$$\begin{aligned} &< \text{vac} \mid U(\mathbf{x}_1,\mathbf{z})^\dagger : \phi_i(\mathbf{z})\phi_i(\mathbf{z}) : U(\mathbf{x}_1,\mathbf{z}) \mid \text{vac} > \\ &\quad = \Big(\frac{1}{\sqrt{2\omega_{\mathbf{z}_1}2\omega_{\mathbf{z}_2}}}\Big)(3)(2)\big[(\cosh\hat{f}\ \sinh\hat{f})(\mathbf{z}_1,\mathbf{z}_2) + (\sinh^2\hat{f})(\mathbf{z}_1,\mathbf{z}_2)\big]\Big|_{\mathbf{z}_1=\mathbf{z}_2=\mathbf{z};\ \mathbf{x}_2=\mathbf{z}} \\ &\quad = \Big(\frac{1}{\sqrt{2\omega_{\mathbf{z}_1}2\omega_{\mathbf{z}_2}}}\Big)3\big[e^{2\hat{f}} - \hat{I}\big](\mathbf{z}_1,\mathbf{z}_2)\Big|_{\mathbf{z}_1=\mathbf{z}_2=\mathbf{z};\ \mathbf{x}_2=\mathbf{z}} . \end{aligned} \tag{6.7}$$

The above can be simplified by using equation (5.23) and equations (5.8) and (5.9). In fact, from equation (5.8) we have

$$\frac{1}{\sqrt{2\omega_{\mathbf{z}_1}2\omega_{\mathbf{z}_2}}}f(\mathbf{z}_1,\mathbf{z}_2)\Big|_{\mathbf{z}_1=\mathbf{z}_2=\mathbf{z};\ \mathbf{x}_2=\mathbf{z}} = a\Big[\Big(\frac{1}{\sqrt{2\omega_z}}f_1(\mathbf{x}_1-\mathbf{z})\Big)^2 + \Big(\frac{1}{\sqrt{2\omega_z}}f_1(\mathbf{x}_2-\mathbf{z})\Big)^2\Big]_{\mathbf{x}_2=\mathbf{z}} \tag{6.8}$$

$$= a\big[F_1(\mathbf{r},f)^2 + c_1^2\big]. \tag{6.9}$$

In the above we have substituted $\mathbf{r} = \mathbf{x}_1 - \mathbf{z}$ and used equation (3.21) for the definition of c_1 along with the substitution

$$F_1(\mathbf{r},f) = \frac{1}{\sqrt{2\omega_{\mathbf{z}}}}f_1(\mathbf{x}_1-\mathbf{z}) = \frac{1}{(2\pi)^{3/2}}\int \tilde{f}_1(\mathbf{k}_1)\frac{d\mathbf{k}_1}{\sqrt{2\omega(\mathbf{k}_1)}}e^{i\mathbf{k}_1\mathbf{r}}, \tag{6.10}$$

which is a function of $\mathbf{r}$ and a functional of meson dressing f. Similarly we also note that from equation (5.9)

$$\frac{1}{\sqrt{2\omega_{\mathbf{z}_1}2\omega_{\mathbf{z}_2}}}f^{(e)}(\mathbf{z}_1,\mathbf{z}_2)\Big|_{\mathbf{z}_1=\mathbf{z}_2=\mathbf{z};\ \mathbf{x}_2=\mathbf{z}} = a\Big[2\Big(\frac{1}{\sqrt{2\omega_{\mathbf{z}}}}f_1(\mathbf{x}_1-\mathbf{z})\Big)\Big(\frac{1}{\sqrt{2\omega_{\mathbf{z}}}}f_1(\mathbf{x}_2-\mathbf{z})\Big)\Big]_{\mathbf{x}_2=\mathbf{z}} \tag{6.11}$$

$$= 2ac_1F_1(\mathbf{r},f). \tag{6.12}$$

Also, by equations (4.1), (5.11), and (5.15), b of equation (5.23) as in equation (6.6) is given as, with $r = |\mathbf{x}_1 - \mathbf{z}|$,

$$b = b(\mathbf{r}) = \Big(1 + \frac{r^2}{15R_M^2} + \frac{r^4}{60R_M^4}\Big)\exp\Big(-\frac{r^2}{4R_M^2}\Big). \tag{6.13}$$

Hence from equations (5.23), (6.5), and (6.6) we obtain that

$$h_{\rm int}(u,f) = \frac{1}{3}\int d\mathbf{r}\ |U^{mn}(\mathbf{r})|^2\big[v_{\rm int}(\mathbf{r},f) + v_{\rm int}^{(e)}(\mathbf{r},f)\big], \tag{6.14}$$

where, with $b = b(\mathbf{r})$ as in equation (6.12),

$$v_{\rm int}(\mathbf{r},f) = \frac{G^2}{M}\frac{3}{(1-b^2)}\big[(e^{2a}\cosh(2ab) - 1) - b\ e^{2a}\sinh(2ab)\big]\big[F_1(\mathbf{r},f)^2 + c_1^2\big] \tag{6.15}$$

and

$$v_{\rm int}^{(e)}(\mathbf{r},f) = \frac{G^2}{M}\frac{3c_1}{(1-b^2)}\big[e^{2a}\sinh(2ab) - b\big(e^{2a}\cosh(2ab) - 1\big)\big]2F_1(\mathbf{r},f). \tag{6.16}$$

In the above, the expression for F_1 is as in equation (6.9). We also have in equation (6.13)

$$\frac{1}{3}|U^{mn}(\mathbf{r})|^2 = \cos^2\beta\ |u_s(r)|^2 + \sin^2\beta\ |u_d(r)|^2 \tag{6.17}$$

We may note the highly nonlinear nature of the contribution from meson dressing.

C. Meson kinetic energy

We shall now consider the contribution from the meson kinetic term. We first note the parallel of equation (3.24) as

$$< \text{vac} \mid U(\mathbf{x}_1,\mathbf{x}_2)^\dagger a_i(\mathbf{z})^\dagger \omega_{\mathbf{z}}\ a_i(\mathbf{z}) U(\mathbf{x}_1,\mathbf{x}_2) \mid \text{vac} >= 3\omega_{\mathbf{z}_1}\big(\sinh\hat{f}\ \sinh\hat{f}\big)(\mathbf{z},\mathbf{z}_1)\Big|_{\mathbf{z}_1=\mathbf{z}}. \tag{6.18}$$

Hence we obtain that

$$\begin{aligned} h_M(u,f) &\equiv \frac{1}{3}(2\pi)^3 < D^m(\mathbf{0})|\mathcal{H}_M(\mathbf{z})|D^m(\mathbf{0}) > \\ &= \int d\mathbf{x}_1 d\mathbf{x}_2 \mid U^{mn}(\mathbf{x}_1-\mathbf{x}_2)\mid^2 \omega_{\mathbf{z}_1}\big(\sinh\hat{f}\ \sinh\hat{f}\big)(\mathbf{z},\mathbf{z}_1)\Big|_{\mathbf{z}_1=\mathbf{z}} \\ &= \frac{1}{4}\int d\mathbf{x}_1 d\mathbf{x}_2 \mid U^{mn}(\mathbf{x}_1-\mathbf{x}_2)\mid^2 \omega_{\mathbf{z}_1}(e^{2\hat{f}}+e^{-2\hat{f}}-2)(\mathbf{z},\mathbf{z}_1)\Big|_{\mathbf{z}_1=\mathbf{z}}. \end{aligned} \tag{6.19}$$

With the same substitution as in (5.19), parallel to equation (5.21) we now obtain that

$$(e^{2\hat{f}}+e^{-2\hat{f}}-2)(\mathbf{z},\mathbf{z}_1) = \big(A^{-1}(e^{2A}+e^{-2A}-2)\big)_{11} f(\mathbf{z},\mathbf{z}_1) + \big(A^{-1}(e^{2A}+e^{-2A}-2)\big)_{21} f^{(e)}(\mathbf{z},\mathbf{z}_1). \tag{6.20}$$

We further note that

$$\omega_{\mathbf{z}_1} f(\mathbf{z}_1,\mathbf{z}_2)\Big|_{\mathbf{z}_1=\mathbf{z}} = a\big[F_2(\mathbf{x}_1-\mathbf{z})f_1(\mathbf{x}_1-\mathbf{z}) + F_2(\mathbf{x}_2-\mathbf{z})f_1(\mathbf{x}_2-\mathbf{z})\big], \tag{6.21}$$

where

$$F_2(\mathbf{x}-\mathbf{z}) = \omega_{\mathbf{z}} f_1(\mathbf{x}-\mathbf{z}) = \frac{1}{(2\pi)^{3/2}}\int \tilde{f}_1(\mathbf{k})\omega(\mathbf{k})e^{i\mathbf{k}(\mathbf{x}-\mathbf{z})}\,d\mathbf{k}. \tag{6.22}$$

Parallel to equation (6.20) we also obtain that

$$\omega_{\mathbf{z}_1} f^{(e)}(\mathbf{z}_1,\mathbf{z}_2)\Big|_{\mathbf{z}_1=\mathbf{z}} = a\big[F_2(\mathbf{x}_1-\mathbf{z})f_1(\mathbf{x}_2-\mathbf{z}) + F_2(\mathbf{x}_2-\mathbf{z})f_1(\mathbf{x}_1-\mathbf{z})\big]. \tag{6.23}$$

For the evaluation of the expression in equation (6.18), we now simplify the expressions involving the matrix A in equation (6.19). For this purpose we note that from equation (5.19) we have

$$\begin{aligned} A^{-1}(e^{2A}+e^{-2A}-2) = \frac{2}{a(1-b^2)}\Big[&\big((\cosh(2a)\ \cosh(2ab)-1) - b\ \sinh(2a)\ \sinh(2ab)\big) \\ &+\tau_1\big(\ \sinh(2a)\ \sinh(2ab) - b(\cosh(2a)\ \cosh(2ab)-1)\big)\Big]. \end{aligned} \tag{6.24}$$

Clearly in the above b is a function of $\mathbf{x}_1-\mathbf{x}_2$. Let us now substitute [2]

$$\mathbf{x}_1-\mathbf{z} = \mathbf{X}+\frac{1}{2}\mathbf{r}, \tag{6.25a}$$

$$\mathbf{x}_2-\mathbf{z} = \mathbf{X}-\frac{1}{2}\mathbf{r}, \tag{6.25b}$$

and in equation (6.18) change the integration variables to $\mathbf{X}$ and $\mathbf{r}$. In that case, except for the terms arising from $f(\mathbf{z}_1,\mathbf{z}_2)$ and $f^{(e)}(\mathbf{z}_1,\mathbf{z}_2)$ as in equations (6.20) and (6.22), all the remaining terms are independent of $\mathbf{X}$, which allows us easily to evaluate the integration over $\mathbf{X}$. In fact equations (6.20) and (6.22) with the above change of variables yield that [2]

$$\int \omega_{\mathbf{z}_1} f(\mathbf{z},\mathbf{z}_1)\Big|_{\mathbf{z}_1=\mathbf{z}} d\mathbf{X} = 2ac_M \tag{6.26}$$

and

$$\int \omega_{\mathbf{z}_1} f^{(e)}(\mathbf{z},\mathbf{z}_1)\Big|_{\mathbf{z}_1=\mathbf{z}} d\mathbf{X} = 2aF_M(\mathbf{r},f), \tag{6.27}$$

where

$$F_M(\mathbf{r},f) = \int \omega(\mathbf{k})|\tilde{f}_1(\mathbf{k})|^2\cos(\mathbf{k}\cdot\mathbf{r})d\mathbf{k}. \tag{6.28}$$

We may note that in the above we have used equation (3.26′) for c_M, and that $F_M(\mathbf{0}) = c_M$. On using equation (6.16), equation (6.18) then gives that

$$h_M(u,f) = \int d\mathbf{r}\ \left(\cos^2\beta\ |u_s(r)|^2 + \sin^2\beta\ |u_d(r)|^2\right) \times \left[v_M(\mathbf{r},f) + v_M^{(e)}(\mathbf{r},f)\right], \qquad (6.29)$$

where with b as in equation (6.12),

$$v_M(\mathbf{r},f) = \frac{3}{1-b^2}\left[\ (\cosh(2a)\ \cosh(2ab) - 1) - b\ \sinh(2a)\ \sinh(2ab)\right] c_M, \qquad (6.30)$$

and

$$v_M^{(e)}(\mathbf{r},f) = \frac{3}{1-b^2}\left[\ \sinh(2a)\ \sinh(2ab) - b(cosh(2a)\ cosh(2ab) - 1)\right] \times F_M(\mathbf{r},f). \qquad (6.31)$$

Equation (6.28) is clearly the parallel of equation (6.13) for the interaction term and F_M is given by equation (6.27). As before, the nonlinear nature of the contribution for meson dressing may be noted.

D. Meson nucleon repulsion

We next proceed to evaluate the contribution from the repulsion term as in equation (4.14) taken phenomenologically as due to the composite nature of the nucleon and the meson. As before, here we have

$$\begin{aligned} h_{nmr}(u,f) &\equiv \frac{1}{3}(2\pi)^3 < D^m(0)|\mathcal{H}_{nmr}(\mathbf{x})|D^m(0) > \\ &= 2\int d\mathbf{x}_1 d\mathbf{z}\ g(\mathbf{x}-\mathbf{z})\ |\ U^{mn}(\mathbf{x}_1-\mathbf{x})\ |^2\ (\sinh^2\hat{f})(\mathbf{z},\mathbf{z}) \\ &= \frac{1}{2}\int d\mathbf{x}_1 d\mathbf{z}\ g(\mathbf{x}-\mathbf{z})\ |\ U^{mn}(\mathbf{x}_1-\mathbf{x})\ |^2\ (e^{2\hat{f}} + e^{-2\hat{f}} - 2)(\mathbf{z},\mathbf{z}). \end{aligned} \qquad (6.32)$$

The simplification of the above easily follows from equation (6.19). For this purpose we may do the $\mathbf{z}$ integration first to obtain that

$$\begin{aligned} &\int d\mathbf{z}\ g(\mathbf{x}-\mathbf{z}) f(\mathbf{z},\mathbf{z})\big|_{\mathbf{x}_2=\mathbf{x}} \\ &\quad = a\int d\mathbf{z}\ g(\mathbf{x}-\mathbf{z})\left[\left(f_1(\mathbf{x}_1-\mathbf{z})\right)^2 + \left(f_1(\mathbf{x}-\mathbf{z})\right)^2\right] \\ &\quad = a\left[F_{nmr}(\mathbf{x}_1-\mathbf{x}) + c_{nmr}\right], \end{aligned} \qquad (6.33)$$

where c_{nmr} is given by equation (4.20) and

$$F_{nmr}(\mathbf{x}_1-\mathbf{x}) = \int d\mathbf{z}\ g(\mathbf{x}-\mathbf{z})\left(f_1(\mathbf{x}_1-\mathbf{z})\right)^2. \qquad (6.34)$$

Similarly we have

$$\int d\mathbf{z}\ g(\mathbf{x}-\mathbf{z}) f^{(e)}(\mathbf{z},\mathbf{z})\big|_{\mathbf{x}_2=\mathbf{x}}$$

$$= 2a\int d\mathbf{z}\ g(\mathbf{x}-\mathbf{z}) f_1(\mathbf{x}_1-\mathbf{z}) f_1(\mathbf{x}-\mathbf{z}) \qquad (6.35)$$

$$\equiv 2a\ F_{nmr}^{(e)}(\mathbf{x}_1-\mathbf{x}). \qquad (6.36)$$

Hence from equation (6.31) we obtain that, with equation (6.16),

$$h_{nmr}(u,f) = \int d\mathbf{r}\ \left(\cos^2\beta|u_s(r)|^2 + \sin^2\beta|u_d(r)|^2\right) \times \left[v_{nmr}(\mathbf{r},f) + v_{nmr}^{(e)}(\mathbf{r},f)\right], \qquad (6.37)$$

where, with $b = b(\mathbf{r})$ as in equation (6.12),

$$v_{nmr}(\mathbf{r},f) = \frac{3}{(1-b^2)}\left[\ (\cosh(2a)\ \cosh(2ab) - 1) - b\ \sinh(2a)\ \sinh(2ab)\right] \times\left[F_{nmr}(\mathbf{r}) + c_{nmr}\right] \qquad (6.38)$$

and

$$v_{nmr}^{(e)}(\mathbf{r},f) = \frac{3}{(1-b^2)}\left\{\ \sinh(2a)\ \sinh(2ab) - b\left[\cosh(2a)\ \cosh(2ab) - 1\right]\right\} \times 2F_{nmr}^{(e)}(\mathbf{r}). \qquad (6.39)$$

We note that c_{nmr} is as in equation (4.20), and, with g as given in equations (4.17), (4.19) and (4.20), we have from equations (6.33) and (6.35)

$$F_{nmr}(\mathbf{r}) = c_{nmr}\Big(1 - \frac{6r^2}{R_M^2} + \frac{183r^4}{30R_M^4} - \frac{26r^6}{15R_M^6} + \frac{2r^8}{15R_M^8}\Big) \times \exp\big(-\frac{r^2}{R_M^2}\big) \qquad (6.40)$$

and

$$F_{nmr}^{(e)}(\mathbf{r}) = c_{nmr}\Big(\ 1 - \frac{13r^2}{12R_M^2} - \frac{57r^4}{120R_M^4} + \frac{2r^6}{15R_M^6} - \frac{r^8}{480R_M^8}\Big)\exp\Big(-\frac{r^2}{2R_M^2}\Big) \qquad (6.41)$$

E. One pion exchange contribution

In the calculations as above, we have included only scalar isospin singlet pion pairs for the construction of the off-mass-shell pions. However, we know that one pion exchange contribution is quite important for the two nucleon forces, which, for example, generates the tensor force for nuclear binding. We wish to simulate the off-shell pion content of the two nucleon system in a realistic environment. For this purpose we shall consider the solution of the problem for the deuteron with the above picture of pion pairs, as well add to it the single pion exchange contribution. This does not involve any double counting, and in fact in a heuristic way includes both the effects of even and odd number of pions. Thus, we take into account now the effect of the first term of equation (2.3b) for the conventional pionic interaction by adding to the Hamiltonian density the one-pion exchange contribution

$$\mathcal{V}_{ope}(\mathbf{x}) = \frac{G^2}{4\pi}\frac{m^2}{4M^2}\frac{m}{3}\int d\mathbf{y} : \big[-\psi_I(\mathbf{x})^\dagger\sigma_i\tau_a\psi_I(\mathbf{x})\psi_I(\mathbf{y})^\dagger\sigma_i\tau_a\psi_I(\mathbf{y})\big(y_0(r)+y_2(r)\big) + 3y_2(r)\psi_I(\mathbf{x})^\dagger(\boldsymbol{\sigma}\cdot\hat{r})\tau_a\psi_I(\mathbf{x})\psi_I(\mathbf{y})^\dagger(\boldsymbol{\sigma}\cdot\hat{r})\tau_a\psi_I(\mathbf{y})\big] : . \tag{6.42}$$

In the above, : : indicates normal ordering $\mathbf{r}=\mathbf{x}-\mathbf{y}$ and $r=|\mathbf{r}|$, and $y_0(r)$ and $y_2(r)$ are given as

$$y_0(r) = -\frac{e^{-mr}}{mr} \tag{6.43}$$

and

$$y_2(r) = \Big(1+\frac{3}{mr}+\frac{3}{(mr)^2}\Big)\frac{e^{-mr}}{mr}. \tag{6.44}$$

This corresponds to the familiar pion exchange potential given as [11]

$$V = \frac{G^2}{4\pi}\frac{m^2}{4M^2}\frac{m}{3}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\big[-(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)y_0(r)+S_{12}y_2(r)\big], \tag{6.45}$$

where the tensor force is

$$S_{12} = 3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2). \tag{6.43'}$$

We can then obtain the energy expectation value for one pion exchange as

$$h_{\rm OPE}(u) \equiv \frac{1}{3}(2\pi)^3 < D^m(0)|\mathcal{V}_{\rm OPE}(\mathbf{x})|D^m(0)> = \frac{1}{3}\int U^{mn}(\mathbf{r})v^{nn'}(\mathbf{r})U^{n'm}(\mathbf{r})d\mathbf{r}, \tag{6.46}$$

where

$$v^{nn'}(\mathbf{r}) = \frac{G^2}{4\pi}\frac{m^3}{4M^2}\big[\delta^{nn'}y_0(r)+2\sqrt{2}P^{nn'}(\hat{r})y_2(r)\big]. \tag{6.47}$$

We can easily simplify equation (6.44) when we note that

$$P^{mn}P^{nk} = \frac{1}{\sqrt{2}}P^{mk}+\delta_{mk}. \tag{6.48}$$

We then obtain that

$$h_{\rm OPE}(u) = \frac{G^2}{4\pi}\frac{m^3}{4M^2}\int d\mathbf{r}\big[\cos^2\beta\,|u_s(r)|^2y_0(r)+\sin^2\beta\,|u_d(r)|^2y_0(r) + 4\sqrt{2}\cos\beta\,\sin\beta\,u_s(r)u_d(r)y_2(r)+2\,\sin^2\beta\,|u_d(r)|^2y_2(r)\big]. \tag{6.49}$$

F. Nucleon nucleon repulsion

We shall also include the nucleon nucleon repulsion term due to ω exchange given as

$$\mathcal{V}_\omega(\mathbf{x}) = \frac{1}{2}\frac{g_\omega^2}{4\pi}\int :\psi_I(\mathbf{x})^\dagger\psi_I(\mathbf{x})\frac{e^{-m_\omega|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{y}|}\times\psi_I(\mathbf{y})^\dagger\psi_I(\mathbf{y}):d\mathbf{y}. \tag{6.50}$$

We then have the energy expectation value given as

$$h_\omega(u) \equiv \frac{1}{3}(2\pi)^3 < D^m(0)|\mathcal{V}_\omega(\mathbf{x})|D^m(0)> = \frac{1}{3}\int U^{mn}(\mathbf{r})v_\omega(\mathbf{r})U^{nm}(\mathbf{r})d\mathbf{r}, \tag{6.51}$$

where

$$v_\omega(\mathbf{r}) = \frac{g_\omega^2}{4\pi}\frac{e^{-m_\omega r}}{r}. \tag{6.52}$$

The value of $h_\omega(u)$ for nucleon nucleon repulsion can thus be obtained by using equation (6.16).

G. Average meson number

In the above we have taken a dressing of the two nucleon system with off-mass-shell pions. It shall be desirable to have an intuitive picture for this dressing. The simplest object for this purpose shall be the average number of off-shell pions, which we now proceed to calculate. We shall now consider the average number of mesons corresponding to a particular dressing for a given two nucleon wave function. Similar to equation (2.11), this number is given as

$$n_M(u,f) \equiv \frac{1}{3}(2\pi)^3 < D^m(0) \mid a_i(\mathbf{z})^\dagger a_i(\mathbf{z}) \mid D^m(0) >$$
$$= \frac{1}{3}\int d\mathbf{x}_1 d\mathbf{x}_2\, |U^{mn}(\mathbf{x}_1 - \mathbf{x}_2)|^2 < \text{vac} \mid U(\mathbf{x}_1,\mathbf{x}_2)^\dagger a_i(\mathbf{z})^\dagger a_i(\mathbf{z}) U(\mathbf{x}_1,\mathbf{x}_2) \mid \text{vac} > . \quad (6.53)$$

The simplification of the above is very similar to the simplification of the meson kinetic term in equation (6.18). We thus obtain, parallel to equation (6.28) that

$$n_M(u,f) = \int d\mathbf{r}\, \left(\cos^2\beta |u_s(r)|^2 + \sin^2\beta |u_d(r)|^2\right)\left[\rho_M(\mathbf{r},f) + \rho_M^{(e)}(\mathbf{r},f)\right], \quad (6.54)$$

where

$$\rho_M(\mathbf{r},f) = \frac{3}{(1-b^2)}\left[\left(\cosh(2a)\ \cosh(2ab) - 1\right) - b\ \sinh(2a)\ \sinh(2ab)\right] \quad (6.55)$$

and

$$\rho_M^{(e)}(\mathbf{r},f) = \frac{3}{(1-b^2)}\left[\sinh(2a)\ \sinh(2ab) - b\left(\cosh(2a)\ \cosh(2ab) - 1\right)\right] F_M^{(n)}(\mathbf{r},f), \quad (6.56)$$

with

$$F_M^{(n)}(\mathbf{r},f) = \int |\tilde{f}_1(\mathbf{k})|^2 \cos(\mathbf{k}\cdot\mathbf{r}) d\mathbf{k}, \quad (6.57)$$

as parallel to equation (6.27). We shall see later that the above meson content of the two nucleon system has a basic conceptual difference from the idea of a potential and may have testable conclusions.

VII. ENERGY MINIMISATION AND DEUTERON

We illustrate the present method with the ground state of deuteron as an example. We have now completed the framework for dressing of the two nucleon system with scalar-isoscalar pion pairs. We have also discussed in Secs. IV E and IV F the one pion exchange contribution, as well as the nuclear repulsion, not included in pion dressing. With all these contributions, the situation for nuclear structure can be realistic. We now take a simple ansatz for the s and d wave function of the deuteron, and minimise energy to illustrate the method in a realistic environment.

We thus proceed to minimise the energy as defined in equations (6.4), (6.13), (6.28), (6.36), (6.47), and (6.49). With this in mind we take the ansatz

$$u_s = \begin{cases} (x - x_0)(a_1 + \frac{b_1}{x})e^{-x/R_n}, & x_0 < x < x_p \\ \frac{e^{-\alpha x}}{x}, & x > x_p, \end{cases} \quad (7.1)$$

and

$$u_d = \begin{cases} (x - x_0)(a_2 + \frac{b_2}{x})e^{-x/R_n}, & x_0 < x < x_p, \\ (1 + \frac{3}{\alpha x} + \frac{3}{(\alpha x)^2})\frac{e^{-\alpha x}}{x}, & x > x_p. \end{cases} \quad (7.2)$$

In the above, $\alpha = 0.231$, $x_0 = 0.3$, $x_p = 10.383$, and a_1, b_1, a_2, and b_2 are determined through the continuity of the two functions and R_n is a parameter to be determined through variation. Clearly from equations (6.4), (6.13), (6.28), (6.36), (6.47), and (6.49) we have

$$h(R_n,\beta) = h_s(R_n)\cos^2\beta + h_d(R_n)\sin^2\beta + h_{\text{mix}}(R_n)\cos\beta\,\sin\beta. \quad (7.3)$$

We take $a = -4.23 \times 10^{-3}$ and $R_M = 0.926$ fm as obtained from the single nucleon and minimise $h(R_n,\beta)$ over R_n and β. In the above, h_s, h_d, and h_{mix} are the contributions to the energy from s-wave part, d-wave part, and s-d mixed part, respectively. Now minimising β in equation (7.3) we have

$$h(R_n) = \frac{1}{2}\left[h_s(R_n) + h_d(R_n) - \sqrt{(h_s(R_n) - h_s(R_n))^2 + h^2_{\text{mix}}(R_n)}\right], \quad (7.4)$$

$$\sin(2\beta) = -\frac{h_{\text{mix}}(R_n)}{\sqrt{(h_s(R_n) - h_s(R_n))^2 + h^2_{\text{mix}}(R_n)}}. \quad (7.5)$$

The values of the pion nucleon coupling constant $G^2/4\pi$ is taken to be 14.6. The same value was also taken in the single nucleon case. We take $A_\pi = 0.1$ fm^{-6} as in the single nucleon case. The value of the coupling constant $g_\omega^2/4\pi$ is taken to be 4.5 as in Ref. [12]. Then the total energy given by equation (7.4) is minimised with respect to R_n. This minimisation gives the binding energy of deuteron to be 2.2603 MeV which is in good agreement with experimental value of 2.2 MeV. The contributions to energy from h_s, h_d, and h_{mix} are 3.975, 61.771, and 39.963 MeV respectively. We find from equation (7.5) the d-state probability to be 8.87 %. The value of the variational parameter, R_n, is 2.903 fm. The values of a_1, b_1, a_2, and b_2 determined through the continuty of the wave function at $x = x_p$ are 4.514×10^{-3}, 0.274, -5.845×10^{-2}, and 1.497, respectively. Using equation (6.52) the average meson number is found to be 1.157×10^{-4}. We next proceed to calculate magnetic moment, the rms radius, and electric quadrupole moment. We note that

$$\mu_d = 0.879 - 0.569\sin^2\beta, \quad (7.6)$$

$$< r^2 > = \frac{1}{4}\left[\cos^2\beta \int r^2 u_s^2(r) d\mathbf{r} + \sin^2\beta \int r^2 u_d^2(r) d\mathbf{r}\right] \quad (7.7)$$

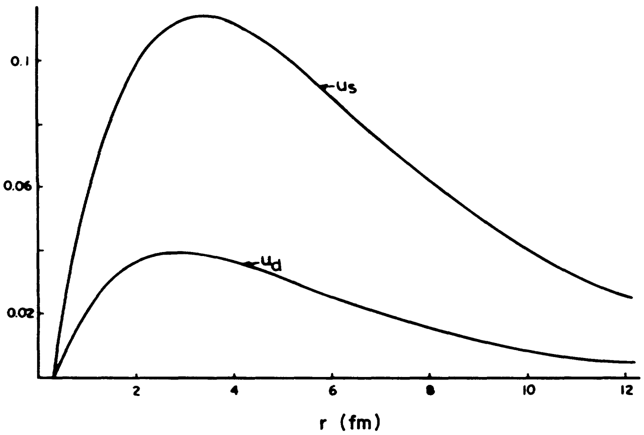

FIG. 1. Normalized s and d wave functions for deuteron are plotted as a function of r in fm, after multiplication by $\cos\beta$ and $\sin\beta$ respectively.

and

$$Q = \frac{4\pi}{10}\Big(\sqrt{2}\cos\beta\,\sin\beta\int u_s(r)u_d(r)r^4dr - \frac{1}{2}\sin^2\beta\int u_d^2r^4dr\Big). \tag{7.8}$$

Using the equations (7.6), (7.7), and (7.8) we get $\mu_d =$ 0.834 nm, $< r^2 >^{1/2} =$ 2.74 fm, and $Q = 9.2 \times 10^{-27}$ cm^2, compared with experimental values of $\mu_d = 0.857$ nm, $< r^2 >^{1/2} =$ 1.95 fm, and $Q = 2.82 \times 10^{-27}$ cm^2. We note that both $< r^2 >$ and Q are larger by a factor of 2 to 3 whereas the magnetic moment is properly reproduced.

The s and d wave functions are plotted in Fig.1. In our calculation we have assumed a hard core at $x = x_0$ so that the wave functions vanished for $x \leq x_0$. The wave function is similar to what one obtains otherwise [13, 14].

VIII. DISCUSSIONS

Let us consider what is achieved here. Firstly, we have dressed single nucleons by scalar isoscalar pion pairs by minimising the total energy. We next consider dressing for the two nucleon system. The algebra is straight forward but unfamiliar, and is aimed at including effects of arbitrary order through an infinity of off-shell pion quanta with construction similar to that of coherent states. It thus becomes a generalisation of mean field approximation of Walecka to a quantum level. We illustrate here how the method can be utilised for the case of deuteron, reproducing features of spectroscopy. We note that the method indicates the average meson content in deuteron to be as small as 10^{-4}. The meson cloud is off-shell, but when excited, the channel $\pi^+\pi^- \longrightarrow 2\gamma$ will be available due to electromagnetic interaction of pions, and the probability of two hard photon in coincidence, though extremely small as a signal, will not have any background. It may be possible to measure this.

The specific example of spectroscopy of deuteron however shall need additional considerations. It is likely that the Δ resonance may also have a significant contribution [3, 15]. Further, as it stands, the effective Hamiltonian taken here does not have chiral symmetry. We may visualise that the dynamical modes corresponding to pion pairs [2, 3] could yield chiral symmetry and generate the corresponding effective Hamiltonian. This will be similar to getting effective Ginzburg-Landau expression from Cooper pairs, but so far we are unable to implement this. Our objective is to show that replacing potential by quantum coherent states shall have, in principle, additional physics output not contained in earlier methods, and can be applied to strong interactions of nuclear physics.

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