Kaonic hydrogen atom and low energy $\overline{K}N$ interaction

Kaori Tanaka and Akira Suzuki*
Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1

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The apparent discrepancy between the K^-p scattering length obtained from the $1S$ level shift of the kaonic hydrogen atom and that from the $\overline{K}N$ scattering at low energies is reexamined. Two models, one proposed by Kumar and Nogami and the other by Schnick and Landau, are extended by including all relevant channels so that comparison with all data available at low energies is possible. A value of the K^-p scattering length consistent with the atomic data and a good overall fit to all the other low energy data have been obtained by means of the extended Schnick-Landau model. The resulting K^-p scattering amplitude is quite different from the one determined earlier by means of the K-matrix parametrization. Its sign around threshold is opposite to that indicated by the low energy extrapolation of the Coulomb-nuclear interference data. It is argued that the extrapolation is subject to some uncertainty.

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I. INTRODUCTION

The $\overline{K}N$ interaction at low energies has rich, complex dynamical aspects. The low energy $\overline{K}N$ system comprises K^-p and $\overline{K^0}n$, which are in turn coupled with inelastic channels of $\pi\Sigma$ and $\pi\Lambda$. Experimental data are available for the following: (a) the K^-p cross sections for the elastic and inelastic processes, (b) the branching ratios for the processes $K^-p \to \pi\Sigma$ and $K^-p \to \pi\Lambda$ at the K^-p threshold, (c) the $\pi\Sigma$ invariant mass distribution below the K^-p threshold, which exhibits the $\Lambda(1405)$ resonance, and (d) the $1S$ level shift of the K^-p atom determined through measurements of the x rays from the atom [1–3].

The data for (a)–(c), with varying degrees of accuracy, became available in the 1950s through the 1970s. They were analyzed and parametrized in terms of the K matrix with the assumption that its matrix elements are smooth functions of energy. The data on (a) were (and still are) available only for about 1440 MeV and above in the center-of-mass energy E of the K^-p system, but they could be smoothly related to the data on (b) and (c) at lower energies. Note that the K^-p threshold energy is 1432 MeV. No particular disharmony or irregularity was noticed among those data. The $\overline{K}N$ scattering lengths were determined in this way [4–6]. For example, Chao *et al.* found that the $\overline{K}N$ scattering lengths a_I for isospin $I = 0$ and 1 to be [4]

$$a_0 = (-1.60 + i0.75)\ \text{fm}, \qquad a_1 = (0.08 + i0.69)\ \text{fm}. \tag{1}$$

Then the K^-p scattering length is given by

$$a_{K^-p} = \frac{1}{2}(a_0 + a_1) = (-0.76 + i0.72)\ \text{fm}. \tag{2}$$

Note that $\mathrm{Re}\, a_{K^-p} < 0$; this is due to the large negative value of $\mathrm{Re}\, a_0$. It is interesting that the values of the a_I's of Eq.(1) are not very different from those of solution (b−), which Dalitz and Tuan found many years earlier [7]. Dalitz and Tuan then suggested that the negative sign of $\mathrm{Re}\, a_0$ would imply the existence of a quasibound state in the $I = 0$ channel. The $\Lambda(1405)$ resonance, which was later found experimentally, is consistent with this. In addition to the data on (a)–(c), there are data on the Coulomb-nuclear interference in the K^-p scattering at higher energies, i.e., for $E \gtrsim 1500$ MeV, about 70 MeV above the K^-p threshold. When extrapolated (over the energy range of about 70 MeV) to low energies, those Coulomb-nuclear interference data were found to be consistent with the negative sign of $\mathrm{Re}\, a_{K^-p}$ [8]. Martin incorporated those data in dispersion relation analysis and also obtained a value of the K^-p scattering length with a negative real part [5].

When the data on (d) were first reported in 1979, it was immediately noticed that the data on the atomic level shift were in direct contradiction with the K^-p scattering length that had been determined earlier. This contradiction is often referred to as the "kaonic hydrogen puzzle." The energy levels of the K^-p atom are shifted from the Coulombic ones due to the K^-p strong interaction. The level shift is expected to be appreciable only in the $1S$ state; this is because of the short range nature of the strong interaction. The $1S$ level shift is related to the K^-p scattering length a_{K^-p} by [9]

$$\epsilon + \frac{i}{2}\Gamma = 2\alpha^3\mu^2 a_{K^-p}, \tag{3}$$

where α is the fine structure constant, and μ the reduced mass of K^- and p. There have been three experimental reports on the level shift [1–3]. Although there is some disagreement among the three reports, they agree that $\mathrm{Re}\, a_{K^-p}$ is positive. In Table I, we list several values of a_{K^-p} obtained from the x-ray measurements and from scattering analyses.

Several attempts at resolving this puzzle or explain-

*Permanent address: Department of Physics, Science University of Tokyo, Tokyo 162, Japan.

TABLE I. The K^-p scattering lengths determined from x-ray measurements and those from scattering analyses. Equations (2) and (3) are used to obtain a_{K^-p}.

	Authors	a_{K^-p} (fm)
Atomic x ray	Davies *et al.* [1]	$(0.10 \pm 0.14) + i\,(0.00\,^{+0.28}_{-0.00})$
	Izycki *et al.* [2]	$(0.65 \pm 0.19) + i\,(0.68 \pm 0.31)$
	Bird *et al.* [3]	$(0.47 \pm 0.14) + i\,(0.10\,^{+0.27}_{-0.10})$
Scattering analysis	Chao *et al.* [4]	$-0.76 + i\,0.72$
	Martin *et al.* [5]	$-0.66 + i\,0.64$
	Dalitz *et al.* [6]	$-0.73 + i\,0.63$

ing the data (d) have been made so far. The purpose of this paper is to reexamine those attempts, which can be classified into three types. The first one, in chronological order, assumes that there is an anomalously large Coulomb correction in the scattering length a_{K^-p} [10,11]. As we will discuss in Sec. IV, there are some serious difficulties in this approach. Therefore we focus on the other two approaches. Throughout this paper we assume that the Coulomb correction is unimportant.

The second approach is the one proposed by Kumar and Nogami [12]. They assumed that there is an "elementary" baryon Λ_0 which has the same quantum numbers as those of $\Lambda(1405)$ and its mass is near the K^-p threshold. The Λ_0 could be a three-quark state which is as elementary as the nucleon. The Λ_0 is a "bare" particle which cannot be observed directly, but it causes the observed resonance $\Lambda(1405)$. This distinguishes the Kumar-Nogami model from other models, like that of Dalitz and Tuan, in which $\Lambda(1405)$ is a two-body composite system. The presence of Λ_0 leads to a rapid variation of the K^-p scattering amplitude around threshold, invalidating smooth extrapolation of the amplitude to zero energy. Kumar and Nogami illustrated the idea by means of a rather simplistic model; they showed that the $\bar{K}N$ scattering amplitude can vary rapidly near threshold such that the apparent contradiction between the atomic level shift and the scattering data could be avoided. However, they did not attempt to fit all available data, such as the branching ratios at threshold. In this sense, the model has not been tested in detail. We extend the Kumar-Nogami model by incorporating all the relevant channels and examine whether or not it can accommodate all of the aspects of (a)–(d).

The third approach is typified by the model proposed by Schnick and Landau [13]. They claimed that the K^-p scattering length consistent with the atomic data could be obtained by using relativistic kinematics in a coupled channel model. Unlike the Kumar-Nogami model, $\Lambda(1405)$ is a purely two-body system. They obtained the K^-p scattering length with a positive real part by directly fitting the low energy scattering data. In the original calculation by Schnick and Landau, however, the branching ratio data were not fitted. We extend the Schnick-Landau model so that all the relevant channels are included; then we examine the fit to the branching ratio data as well. We will also investigate whether or not relativistic kinematics is really crucial for explaining the atomic data. In the next section the models are formulated. The results of the calculations are presented in Sec. III. A discussion of the results is given in Sec. IV.

II. MODELS

We consider only S-wave states. There are six relevant particle channels which are, in descending order of the threshold, $\bar{K}^0n$, K^-p, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$, and $\pi^0\Lambda^0$. We refer to these as particle channels 1–6 in this order. Each of these particle channels can be expressed as a linear combination of isospin channels, i.e., $I = 0$ and 1 of $\bar{K}N$, $I = 0$, 1, and 2 of $\pi\Sigma$, and $I = 1$ of $\pi\Lambda$. For the interaction we assume isospin symmetry and we express the interaction referring to the isospin channels. In reality, however, there are mass differences among the particles of the same isospin multiplet, and hence isospin symmetry is broken. These mass differences play important roles at low energies. In setting up the dynamical equations for the models, therefore, we use the particle channels so that the mass differences can be easily handled.

For the interaction, we assume the separable form,

$$V_{ij}^{(I)}(k_i,k_j) = \frac{C_{ij}^{(I)}}{4\pi} v_i(k_i) v_j(k_j), \tag{4}$$

where $C_{ij}^{(I)}$ is the coupling constant between isospin channels i and j when the total isospin of the two-body system is I. We regard $C_{ij}^{(I)}$ as free parameters except that they conform to isospin symmetry. We take the form factor as

$$v_i(k_i) = \frac{\beta_i}{\beta_i^2 + k_i^2}, \tag{5}$$

where k_i is the relative momentum in the center-of-mass system of channel i. We assume that the range parameters β_i take a common value in all the channels considered, $\beta_i = \beta$, and hence $v_i = v$. The Coulomb interaction is ignored throughout this paper.

In the Kumar-Nogami model, it is assumed that there is an "elementary" baryon Λ_0 which has the same quantum numbers as $\Lambda(1405)$. Kumar and Nogami assumed that Λ_0 is only coupled with $\bar{K}N$. However, we assume that all the $I = 0$ channels are coupled with Λ_0. These additional interactions through Λ_0 can be incorporated by modifying $C_{ij}^{(0)}$ as [12]

$$C_{ij}^{(0)} \rightarrow C_{ij}^{(0)} + g_i \frac{M_0}{E - M_0} g_j, \tag{6}$$

where M_0 is the mass of Λ_0, g_i is the coupling constant between Λ_0 and isosinglet channel i, and E is the center-of-mass total energy. To reduce the number of parameters, we use an SU(3) relation, $g_{\pi\Sigma} = \frac{3}{2} g_{\overline{K}N}$ [14,15]. The presence of Λ_0 causes the $I = 0$ $\overline{K}N$ scattering amplitude to have a Castillejo-Dalitz-Dyson (CDD) pole [16]. The "CDD pole" is actually a pole in the inverse scattering amplitude; there the scattering amplitude vanishes. In the Schnick-Landau model, there is no Λ_0, and hence $g_i = 0$. This is the only essential difference between the two models.

We solve the coupled-channel Lippmann-Schwinger equation in the particle channel basis. The transition matrix element from channel i to channel j satisfies

$$T_{ij}(k_i, k_j; E) = V_{ij}(k_i, k_j) + \sum_l \int d^3k_l V_{il}(k_i, k_l) G_l(k_l; E) T_{lj}(k_l, k_j; E), \tag{7}$$

where l is summed over the six particle channels, and k_i's are now evaluated in the particle channel basis. Note that, in the Kumar-Nogami model, V_{ij} depends on E when it is associated with $I = 0$ states. The propagator is given by

$$G_i(k) = [E - \omega_i(k) - \epsilon_i(k) + i\epsilon]^{-1}, \tag{8}$$

where ω_i and ϵ_i are relativistic energies for the meson and baryon in channel i, respectively. The propagator with nonrelativistic kinematics is obtained by substituting the relativistic energies with the nonrelativistic energies. Differences between relativistic and nonrelativistic kinematics will be discussed later. An important advantage of assuming the separable form for the interaction is that Eq. (7) can be solved in a closed form,

$$T_{ij}(k_i, k_j; E) = \frac{1}{4\pi} v(k_i) \{\mathbf{C}[\mathbf{1} - \mathbf{J}(E)\mathbf{C}]^{-1}\}_{ij} v(k_j), \tag{9}$$

where $\mathbf{C}$ is the 6×6 matrix whose ij element, C_{ij}, is the coupling constant between particle channels i and j. The $\mathbf{J}$ is a diagonal matrix; its ith component is defined by

$$J_i(E) = \int_0^\infty dk \frac{k^2 v^2(k)}{E - \omega_i(k) - \epsilon_i(k) + i\epsilon}. \tag{10}$$

The C_{ij} can be expressed in terms of $C_{ij}^{(I)}$ as

$$C_{ij} = \sum_I a_{ij}^{(I)} C_{ij}^{(I)}. \tag{11}$$

The coefficients $a_{ij}^{(I)}$ are given in Table II. It turns out that the admixture of the $I = 2$ state in the $\pi\Sigma$ channel is unimportant, and this is why $a_{\pi\Sigma,\pi\Sigma}^{(2)}$ are not shown in Table II. The elastic and inelastic cross sections initiated in the K^-p channel ($i = 2$) can be obtained from Eq. (9).

There are three branching ratios for the K^-p decay at rest which we are going to fit. They are defined in terms of cross sections by

$$\gamma = \lim_{k \to 0} \frac{\sigma(K^-p \to \pi^+\Sigma^-)}{\sigma(K^-p \to \pi^-\Sigma^+)}, \tag{12}$$

$$R_c = \lim_{k \to 0} \frac{\sigma(K^-p \to \text{charged particles})}{\sigma(K^-p \to \text{all final states})}, \tag{13}$$

$$R_n = \lim_{k \to 0} \frac{\sigma(K^-p \to \pi^0\Lambda^0)}{\sigma(K^-p \to \text{all neutral states})}. \tag{14}$$

The invariant mass distribution of $\pi\Sigma$ is given, apart from a constant factor, by

$$W_{\pi\Sigma}(E) \propto k_4 |T_{44}(k_4, k_4; E)|^2. \tag{15}$$

By varying M_0, β, $C_{ij}^{(I)}$, and g_i, we try to fit the data (a) the K^-p elastic and inelastic cross sections [17–19], (b) the K^-p decay branching ratios at rest, which are [20–22]

$$\gamma = 2.36 \pm 0.04, \quad R_c = 0.664 \pm 0.011, \quad R_n = 0.189 \pm 0.015, \tag{16}$$

TABLE II. Coefficients $a_{ij}^{(I)}$ of Eq. (11), where i refers to any of $\overline{K^0}n$, K^-p, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$, and $\pi^0\Lambda$; and similarly for j. The first values in parentheses are for $I = 0$, and the second ones for $I = 1$.

	$\overline{K^0}n$	K^-p	$\pi^+\Sigma^-$	$\pi^-\Sigma^+$	$\pi^0\Sigma^0$	$\pi^0\Lambda^0$
$\overline{K^0}n$	$(\frac{1}{2}, \frac{1}{2})$	$(-\frac{1}{2}, \frac{1}{2})$	$(-\sqrt{\frac{1}{6}}, -\frac{1}{2})$	$(-\sqrt{\frac{1}{6}}, \frac{1}{2})$	$(\sqrt{\frac{1}{6}}, 0)$	$(0, \sqrt{\frac{1}{2}})$
K^-p	$(-\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(\sqrt{\frac{1}{6}}, -\frac{1}{2})$	$(\sqrt{\frac{1}{6}}, \frac{1}{2})$	$(-\sqrt{\frac{1}{6}}, 0)$	$(0, \sqrt{\frac{1}{2}})$
$\pi^+\Sigma^-$	$(-\sqrt{\frac{1}{6}}, -\frac{1}{2})$	$(\sqrt{\frac{1}{6}}, -\frac{1}{2})$	$(\frac{1}{3}, \frac{1}{2})$	$(\frac{1}{3}, -\frac{1}{2})$	$(-\frac{1}{3}, 0)$	$(0, -\sqrt{\frac{1}{2}})$
$\pi^-\Sigma^+$	$(-\sqrt{\frac{1}{6}}, \frac{1}{2})$	$(\sqrt{\frac{1}{6}}, \frac{1}{2})$	$(\frac{1}{3}, -\frac{1}{2})$	$(\frac{1}{3}, \frac{1}{2})$	$(-\frac{1}{3}, 0)$	$(0, \sqrt{\frac{1}{2}})$
$\pi^0\Sigma^0$	$(\sqrt{\frac{1}{6}}, 0)$	$(-\sqrt{\frac{1}{6}}, 0)$	$(-\frac{1}{3}, 0)$	$(-\frac{1}{3}, 0)$	$(\frac{1}{3}, 0)$	$(0, 0)$
$\pi^0\Lambda$	$(0, \sqrt{\frac{1}{2}})$	$(0, \sqrt{\frac{1}{2}})$	$(0, -\sqrt{\frac{1}{2}})$	$(0, \sqrt{\frac{1}{2}})$	$(0, 0)$	$(0, 1)$

and (c) the $\pi\Sigma$ invariant mass distribution for $1350 \lesssim E \lesssim 1450$ MeV [23]. In fitting the cross-section data of (a), we confine ourselves to the low energy region of $k_{\text{lab}} \lesssim 200$ MeV/c, namely, $E \lesssim 1460$ MeV. This is because we have only S-wave interactions. In addition to data (a)–(c), we are of course interested in explaining the K^-p atom data [1–3]. Therefore we try to find a parameter set which leads to $\text{Re}\, a_{K^-p} > 0$. We do not try to fit a_0 and a_1, like those of Eq. (1), which have been determined by the K-matrix analysis [4–6] and which are in disagreement with the atomic data.

III. CALCULATIONS

We first examine the Kumar-Nogami model [12]. When the model was conceived, only the data of Davies *et al.* [1] were available. Their data suggested that the K^-p scattering amplitude was almost zero at threshold. Kumar and Nogami assumed that the K^-p amplitude is dominated by the $I = 0$ component, and they designed the model such that the $I = 0$ $\overline{K}N$ scattering amplitude f_0 satisfies the following conditions: (i) $\text{Re}\, f_0$ vanishes at $E \simeq 1405$ MeV, implying an unstable bound state, and $\text{Im}\, f_0$ has a peak there, (ii) $\text{Re}\, f_0 \approx 0$ at threshold, and (iii) $|\text{Re}\, f_0|$ and $\text{Im}\, f_0$ are about the same in magnitude as those obtained in Ref. [4] for $E \gtrsim 1460$ MeV. In order to meet conditions (i)–(iii), they assumed that f_0 has a rapid energy dependence related to a CDD pole around threshold, caused by the presence of Λ_0.

In Ref. [12], only the $\overline{K}N$ channel was considered explicitly; Λ_0 was only coupled with the $\overline{K}N$ channel. The $\pi\Sigma$ channel was taken into account by assuming a $\overline{K}N$ two-body interaction with a complex coupling constant. In our extended version of the Kumar-Nogami model, all inelastic channels are explicitly included and all coupling constants are real. This extended version, with the constraint that Λ_0 is only coupled with the $\overline{K}N$ channel ($g_{\pi\Sigma} = 0$), should have a solution similar to that obtained in Ref. [12], namely, an f_0 that satisfies the above conditions (i)–(iii). We indeed found such an f_0, which is shown in Fig. 1 [24]. The $\text{Re}\, f_0$ vanishes at $E \simeq 1405$ MeV, which implies an unstable bound state. If we artificially switch off the coupling between Λ_0 and $\overline{K}N$, keeping all other parameters fixed, the unstable bound state disappears. This means that the unstable bound state, which corresponds to $\Lambda(1405)$, is caused by the existence of Λ_0. The width of the unstable bound state is much narrower than the observed one. This is probably because Λ_0 is only coupled with the $\overline{K}N$ channel. Kumar and Nogami anticipated that the K^-p scattering data at low energies could be reproduced by making f_0 satisfy condition (iii) and by adjusting the $I = 1$ amplitude f_1. With f_0 fixed as aforementioned, we tried to fit the K^-p elastic cross-section data for $E \lesssim 1460$ MeV by varying the $I = 1$ parameters only. Note that, in the $I = 1$ channels, there is no "elementary" particle assumed but only two-body interactions. We were not able to adjust f_1 such that the K^-p elastic cross-section data for $E \lesssim 1460$ MeV were well reproduced.

We further extended the calculation by relaxing the constraint on Λ_0 and allowed it to couple to the $\pi\Sigma$ channel ($g_{\pi\Sigma} \neq 0$). Once again, we required that the K^-p elastic cross-section data for $E \lesssim 1460$ MeV and the $\pi\Sigma$ invariant mass distribution be fitted. Again we were able to find an f_0 that satisfies conditions (i)–(iii). In this case, however, the binding mechanism of the $\Lambda(1405)$ turned out to be different from the one described in the preceding paragraph. The unstable bound state remains, although at a different energy, when we switch off all the couplings with Λ_0. This means that the binding is due to the $\overline{K}N$ two-body interaction rather than to the coupling with Λ_0.

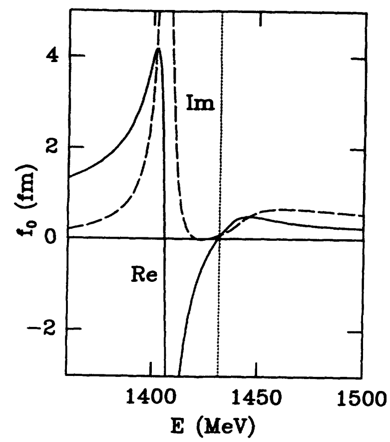

FIG. 1. The $\overline{K}N$ isosinglet amplitude f_0 obtained with the constraint that Λ_0 is only coupled with $\overline{K}N$ ($g_{\pi\Sigma} = 0$). The solid and dashed lines represent the real and imaginary parts, respectively. The dotted vertical line shows the K^-p threshold.

So far, only the K^-p elastic cross sections and the $\pi\Sigma$ mass distribution have been fitted. Taking those parameters that yielded the f_0 obtained above as starting values, we tried to fit the various K^-p inelastic cross sections by varying both of the $I = 0$ and $I = 1$ parameters. In so doing, we replaced conditions (i) and (iii) for f_0 with the condition that the $\pi\Sigma$ invariant mass distribution be fitted and the condition that the K^-p elastic and inelastic cross sections be reproduced, respectively. For f_0 we kept only condition (ii). We found it difficult to meet all of these three conditions as some of the inelastic cross sections were only poorly reproduced.

Kumar and Nogami intended to fit $\text{Re}\, a_{K^-p} \approx 0$, which was indicated by the data of Davies *et al.* In view of other atomic data, however, the value of $\text{Re}\, a_{K^-p}$ is quite uncertain. Therefore we further examined the Kumar-Nogami model by replacing condition (ii) for f_0 with the weaker condition that $\text{Re}\, a_{K^-p} \gtrsim 0$. We varied the $I = 0$ and $I = 1$ parameters and tried to fit all the available data at low energies, namely, the cross sections, the $\pi\Sigma$ mass distribution, and the branching ratios at threshold. We limited our consideration to the cases where M_0 is close to the K^-p threshold value. However, we found it difficult to fit the branching ratios. When we relaxed the fit to the branching ratios, a marginal fit to all the cross sections and the $\pi\Sigma$ invariant mass distribution was obtained, as shown in Fig. 2. The resulting branching ratios in this case were

$$\gamma = 0.013, \quad R_c = 0.317, \quad R_n = 0.889. \tag{17}$$

These are very different from the experimental values of Eq. (16). The obtained K^-p scattering amplitude f_{K^-p} is shown in Fig. 2(c). The f_{K^-p} is influenced by the presence of Λ_0 over a wide energy range around threshold. We also tried to fit all the data, by making the couplings with Λ_0 small and thereby confining the influence of a CDD pole on f_{K^-p} in a narrower energy region around threshold. In this case, however, the reaction cross-section data for the $\pi\Lambda$ channel were only poorly reproduced, and again we found it difficult to fit the branching ratios.

One may wonder why the presence of Λ_0 of $I = 0$ disturbs the $\pi\Lambda$ channel with $I = 1$ so much. This can be understood as follows. Suppose one fits the data for

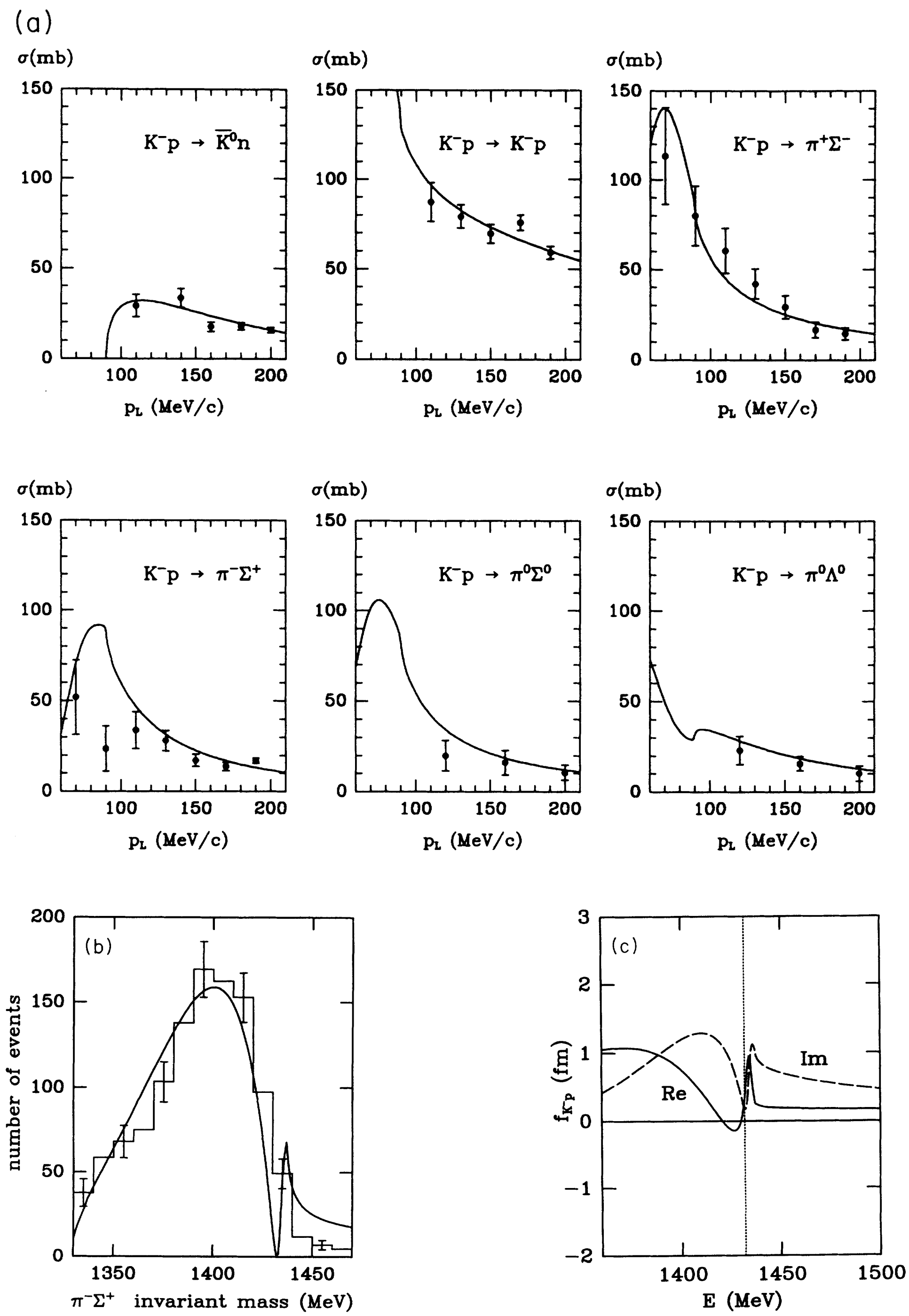

FIG. 2. (a) The K^-p elastic and inelastic scattering cross sections and (b) the $\pi\Sigma$ invariant mass distribution fitted by the Kumar-Nogami model. (c) The K^-p elastic scattering amplitude; the solid and dashed lines represent the real and imaginary parts of the amplitude, respectively.

$K^-p \rightarrow K^-p$, of which the amplitude is given by $(f_0 + f_1)/2$. If f_0 is modified by the presence of Λ_0, then f_1 has to be modified such that $(f_0 + f_1)/2$ remains the same. In this way, the presence of Λ_0 affects f_1, and this is essentially how the effect of Λ_0 can show up in the $\pi\Lambda$ channel. Alternatively, the data for the $\pi\Lambda$ channel play an important role in determining the $I = 0$ parameters.

In summary, although the Kumar-Nogami model can fit the level shift of the kaonic hydrogen atom and the energy of $\Lambda(1405)$, it fails to reproduce other data well, such as the branching ratios at threshold and the K^-p reaction cross sections. Let us add that, for the coupling between Λ_0 and the $I = 0$ channels, we imposed an SU(3) relation. If we relax this restriction, the fit will improve. However, we did not pursue this, because, as we show below, one can obtain a good overall fit, without Λ_0, by means of the extended Schnick-Landau model.

Next we examine the Schnick-Landau model [13], which differs from the Kumar-Nogami model only in that it does not assume the existence of Λ_0. In Schnick and Landau's original calculations, some of the channels were not explicitly treated. The three $\pi\Sigma$ channels were represented by the $\pi^0\Sigma^0$ channel as one effective $I = 0$ channel, and the $I = 1$ channels of $\pi\Sigma$ and $\pi\Lambda$ were taken into account altogether by assuming a complex coupling constant of an $I = 1$ $\bar{K}N$ interaction. They made no attempt to fit the branching ratio data. We extend the Schnick-Landau model by including all relevant channels explicitly, and examine whether or not the branching ratios together with the atomic data can be fitted. We also examine whether or not relativistic kinematics is really crucial as they claimed.

Let us first see if we can get results similar to those of Ref. [13] in our extended version, under the same conditions as those of Ref. [13]. A particularly interesting feature of the results of Ref. [13] is that $\mathrm{Re}\, f_0$ is positive in the whole energy region considered. This f_0, in combination with f_1, leads to $\mathrm{Re}\, a_{K^-p} > 0$, which is consistent with the atomic data. We tried to fit all the available data, except for the branching ratios, with the requirement that $\mathrm{Re}\, f_0$ be positive at threshold. With a reasonable fit to the K^-p cross sections and the $\pi\Sigma$ mass distribution, we indeed obtained f_0 and f_1 similar to those of Ref. [13]; $\mathrm{Re}\, f_0$ was found to be positive at threshold. The K^-p scattering length obtained is

$$\begin{aligned} a_{K^-p} &= \frac{1}{2}[(0.16 + i0.81)_0 + (1.12 + i0.18)_1]\ \mathrm{fm} \\ &= (0.64 + i0.50)\ \mathrm{fm}, \end{aligned} \tag{18}$$

which is consistent with the atomic value. However, the resulting branching ratios are

$$\gamma = 1.012, \quad R_c = 0.626, \quad R_n = 0.145, \tag{19}$$

which are far from their experimental values.

We also tried to fit all the data including the branching ratios. When we kept the condition that $\mathrm{Re} f_0$ be positive at threshold, we found it difficult to reproduce the branching ratios well. Therefore, we replaced the condition for f_0 with the weaker condition that $\mathrm{Re}\, a_{K^-p}$ be positive. In this way we obtained a very good fit to all the data. Figures 3 (a) and (b) show the K^-p cross sections and the $\pi\Sigma$ invariant mass distribution (solid lines). The branching ratios thus obtained are

$$\gamma = 2.354, \quad R_c = 0.646, \quad R_n = 0.187, \tag{20}$$

which are in good agreement with experiment. The a_{K^-p} has a positive real part;

$$\begin{aligned} a_{K^-p} &= \frac{1}{2}[(-0.43 + i1.10)_0 + (1.11 + i0.44)_1]\ \mathrm{fm} \\ &= (0.34 + i0.77)\ \mathrm{fm}. \end{aligned} \tag{21}$$

This is consistent with the atomic data. The obtained f_{K^-p} is shown in Fig. 3(c); this is similar to the f_{K^-p} of Ref. [13]. Thus the extended Schnick-Landau model successfully incorporates the branching ratios as well as all the other data, and the K^-p scattering length obtained is consistent with the atomic data.

There is, however, an interesting difference regarding a_{K^-p} between this fit and that of Ref. [13]. As seen in Eq. (21), the positive $\mathrm{Re}\, a_{K^-p}$ resulted from cancellation between a_0 with a smaller *negative* real part and a_1 with a larger *positive* real part. On the other hand, in Ref. [13], a positive or almost zero $\mathrm{Re}\, a_0$, together with a positive $\mathrm{Re}\, a_1$, yielded the positive $\mathrm{Re}\, a_{K^-p}$. As we said earlier, we obtained f_0 and f_1 similar to those of Ref. [13], without fitting the branching ratios. In that case, $\mathrm{Re}\, a_0 > 0$ as shown in Eq. (18). The difference between the two solutions, one related to Eq. (21) and the other to Eq. (18), stems from the difference regarding the branching ratios, i.e., whether or not the branching ratios are fitted. Thus the branching ratios are very important for determining the $\bar{K}N$ elastic amplitudes at threshold.

Next let us turn to the question: Is relativistic kinematics really crucial in obtaining a_{K^-p} consistent with the atomic data? The answer that we found is negative. We repeated calculations with nonrelativistic propagators, readjusting all the parameters, and obtained a fit to all the data which was as good as the one obtained with relativistic kinematics. The cross sections and the $\pi\Sigma$ mass distribution which we obtained are shown with dashed lines in Figs. 3(a) and (b). The two fits are almost indistinguishable. The branching ratios obtained are

$$\gamma = 2.353, \quad R_c = 0.646, \quad R_n = 0.187. \tag{22}$$

The a_{K^-p} again is the sum:

$$\begin{aligned} a_{K^-p} &= \frac{1}{2}[(-0.50 + i1.15)_0 + (1.13 + i0.47)_1]\ \mathrm{fm} \\ &= (0.32 + i0.81)\ \mathrm{fm}. \end{aligned} \tag{23}$$

Note that Eqs. (20) and (21) are almost identical with Eqs. (22) and (23), respectively. The f_{K^-p} also has an energy dependence very similar to the one obtained with relativistic kinematics. Therefore, kinematics is not essential for fitting the data.

In Table III we list the coupling constants and the range parameters for the above two calculations, one by using relativistic kinematics and the other by using nonrelativistic kinematics. The parameters are very different between the two calculations. For example, the range parameter is $\beta = 2050$ MeV for the relativistic calculation and $\beta = 1230$ MeV for the nonrelativistic one. Hence different kinematics requires very different values of the parameters. Neither of the sets of the coupling constants with relativistic and with nonrelativistic kinematics nearly conform to the flavor SU(3) symmetry. Regarding the importance of kinematics in the low energy K^-p problem, Siegel and Weise [15] also reached the conclusion that the difference between relativistic and nonrelativistic kinematics is not essential. They calculated $J_i(E)$ defined by Eq. (10) with relativistic and with nonrelativistic kinematics, varying the range parameter in each channel. They then stated that for a set of nonrelativistic ranges there exists a set of relativistic ranges which will give similar results. They seem to imply that the difference in kinematics has little influence on the coupling constants. However, this is not true as we men-

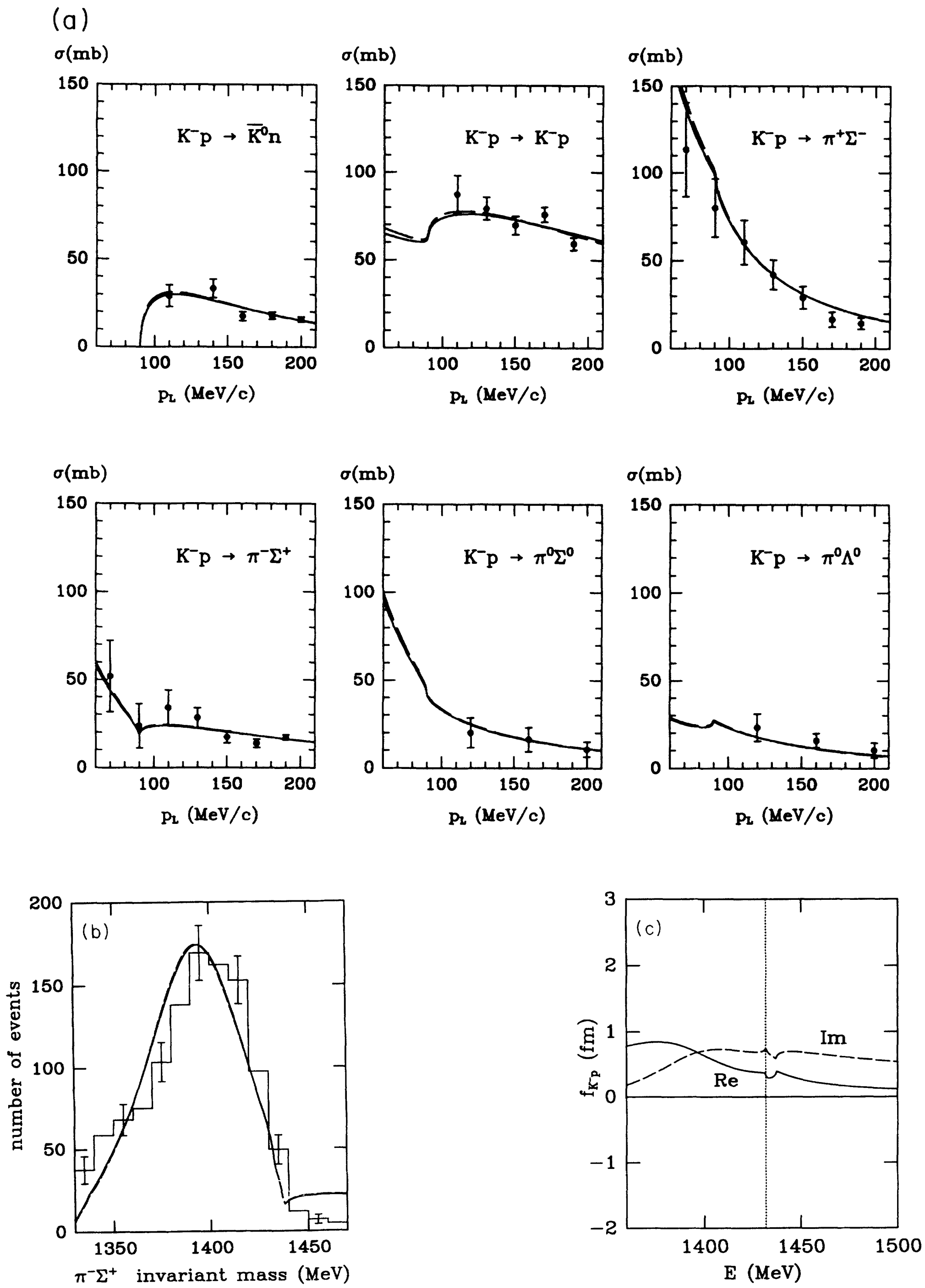

FIG. 3. (a) The K^-p elastic and inelastic scattering cross sections and (b) the $\pi\Sigma$ invariant mass distribution fitted by the extended Schnick-Landau model, by using relativistic (solid lines) and nonrelativistic (dashed lines) kinematics. (c) The K^-p elastic scattering amplitude; the solid and dashed lines represent the real and imaginary parts of the amplitude, respectively.

TABLE III. $C_{ij}^{(I)}$ obtained from the best fit of the data with relativistic and nonrelativistic kinematics. For each kinematics, the upper row corresponds to $I = 0$ and the lower row to $I = 1$. The range parameter is $\beta = 2050$ MeV for relativistic, and $\beta = 1230$ MeV for nonrelativistic kinematics.

	$C^{(I)}_{\bar{K}N,\bar{K}N}$	$C^{(I)}_{\bar{K}N,\pi\Sigma}$	$C^{(I)}_{\bar{K}N,\pi\Lambda}$	$C^{(I)}_{\pi\Sigma,\pi\Sigma}$	$C^{(I)}_{\pi\Sigma,\pi\Lambda}$	$C^{(I)}_{\pi\Lambda,\pi\Lambda}$
Relativistic	-2.1181	-0.3154		-2.3954		
	5.6898	-19.1365	3.6610	44.2425	-8.5451	-0.3010
Nonrelativistic	-2.1608	-1.1055		-3.4656		
	-1.7223	-1.1252	0.4884	0.8286	-0.4719	-1.3493

tioned above: Not only the range parameters, but also all the coupling constants have to be readjusted to a large extent if similar transition matrix elements are to be obtained.

In summary, the extended Schnick-Landau model fits the level shift of the kaonic hydrogen atom and all the other available data at low energies. Contrary to Schnick and Landau's claim, however, relativistic kinematics is not essential.

IV. DISCUSSION

We have extended and reexamined the Kumar-Nogami and Schnick-Landau models, which had been proposed to explain the $1S$ level shift of the kaonic hydrogen atom. An essential feature of the Kumar-Nogami model is that the K^-p amplitude exhibits a rapid energy dependence around threshold; this is due to the assumed "elementary" baryon Λ_0 which underlies the $\Lambda(1405)$ resonance. The Schnick-Landau model assumes no such Λ_0, and the interactions are all in the form of the two-body (separable) potential. We have extended both models in such a way that all relevant channels are explicitly included.

We found that, although it can be made consistent with the atomic data and $\Lambda(1405)$, the Kumar-Nogami model does not fit the inelastic processes well. This is with a certain constraint on the coupling of Λ_0. On the other hand, the extended Schnick-Landau model can fit all the available data, including those of the atomic level shift and the branching ratios. Therefore, there is no need to introduce Λ_0 as was done in the Kumar-Nogami model. We also found that, contrary to Schnick and Landau's claim, the difference between relativistic and nonrelativistic kinematics is not essential. Either of them can be used to obtain essentially the same results, provided that the coupling constants and the range parameters are appropriately readjusted.

The structure of $\Lambda(1405)$ has been a subject of considerable interest [14,25]. In the Kumar-Nogami model, $\Lambda(1405)$ is essentially caused by the presence of the elementary Λ_0. As we said above, however, this model does not work as well as the Schnick-Landau model [26]. In the latter model, $\Lambda(1405)$ is a two-body composite system. We are then led to ask whether or not the $\Lambda(1405)$ is a $\bar{K}N$ bound state rather than a $\pi\Sigma$ resonance? It is of course conceivable that the $\bar{K}N$ and $\pi\Sigma$ channels are coupled so strongly that there is no clear answer to this question. However, our analysis provides some insight into this aspect. As seen in Fig. 3(c), the real part of the f_{K^-p} of the extended Schnick-Landau model is positive over the whole energy region considered. The $\mathrm{Re}\, f_0$ that we obtained vanishes and changes its sign below the $\bar{K}N$ threshold. This implies that there is an unstable bound state of $\bar{K}$ and N in the isosinglet state. In this sense, $\Lambda(1405)$ is an unstable bound state of $\bar{K}$ and N.

Siegel and Weise studied the $\bar{K}N$ system at low energies with practically the same models as those we have used [15]. It is interesting that, when all the coupling constants were required to conform to the flavor SU(3) symmetry, they found Λ_0 necessary in fitting the branching ratios well. In their calculation, the Λ_0 induced a resonance near the $\bar{K}N$ threshold in the $\pi\Sigma$ channel. This is in addition to the $\Lambda(1405)$ resonance. They said that, so as not to spoil the $\pi\Sigma$ mass spectrum, the mass of Λ_0 needs to be close to the K^-p threshold and the coupling very weak. A main difference between their calculation and ours is that they did not try to fit the K^-p atomic level shift. Their results are indeed incompatible with the atomic data. In our calculation, we regarded all the coupling constants as free parameters, and were able to obtain, without assuming Λ_0, a good fit to the atomic data as well as the branching ratios. We should note that Siegel and Weise also obtained a fit to all the data except the atomic shift without Λ_0, varying the coupling-constant ratios within a deviation of 25% from the SU(3) values. Let us also comment on the SU(3) ratios of the coupling constants. Siegel and Weise used nonrelativistic kinematics. However, as we pointed out towards the end of Sec. III, different kinematics requires very different values of the coupling constants. Therefore, if their calculation is repeated by using relativistic kinematics, the coupling constants may substantially deviate from the SU(3) values.

In addition to the two models we have examined, there has been another model proposed in relation to the kaonic hydrogen problem [10,11]. This model assumes that the Coulomb correction in the K^-p system is anomalously large. Law *et al.* constructed an explicit model in this line; they used the Klein-Gordon equation for the $\bar{K}N$ system, and assumed a very strong, singular potential of the Lorentz vector nature [11]. This model illustrates a situation in which the Coulomb-nuclear interference effect can be very large. They adjusted the parameters in the model such that the level shift of kaonic hydrogen and the conventional values of the scattering lengths a_0 and a_1 were fitted. This, however, leads to the following difficulties. If the Coulomb correction is very large, the a_0 and a_1 no longer determine the K^-p scattering length. In fact, the notion of "isospin" becomes useless. Also, $\Lambda(1405)$ would obtain a significant admixture of the $I = 1$ component such that it could decay into $\pi\Lambda$. No such $\pi\Lambda$ decay has been observed. These difficulties associated with the anomalous Coulomb effect were discussed earlier by Kumar *et al.* [27]. They also constructed a model which exhibits an anomalous Coulomb effect, and pointed out that the effect cannot be confined within a narrow energy region around threshold. The effect will show up in the K^-p scattering well above threshold and in $\Lambda(1405)$ below threshold. The model of Law *et al.* was also criticized on the grounds that the Klein-Gordon equation is not really a legitimate equation for a two-body system [28]. However, the difficulties that we noted above would not depend on any specific choice of the dynamical equation used.

As we mentioned in Sec. I, the "kaonic hydrogen puzzle" refers to the discrepancy regarding the sign of $\mathrm{Re}\, a_{K^-p}$. The atomic data indicated that $\mathrm{Re}\, a_{K^-p}$ is positive, in contradiction to the traditional values which had been determined by means of the K-matrix analysis. However, we have been able to fit all relevant low energy data, including the atomic level shift. The obtained values of a_0 and a_1 of Eq. (21), which yield a

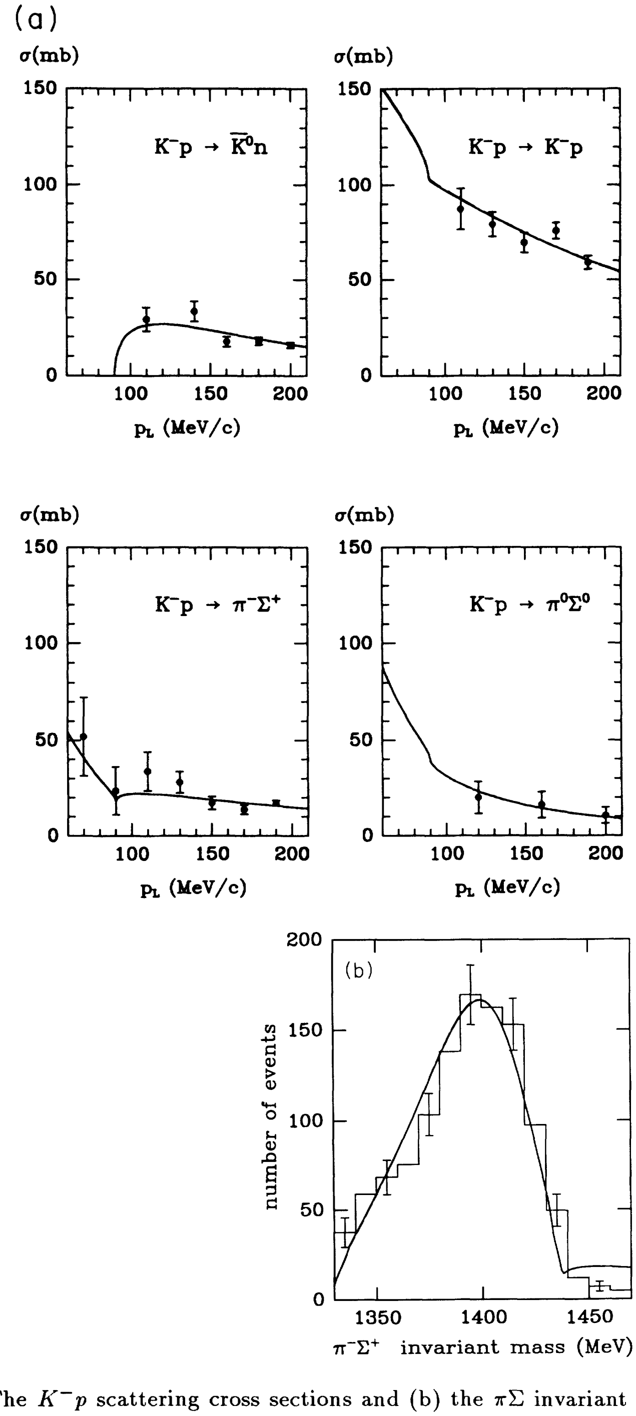

FIG. 4. (a) The K^-p scattering cross sections and (b) the $\pi\Sigma$ invariant mass distribution (b) fitted, by disregarding the K^-p atomic data, with relativistic (solid lines) and nonrelativistic (dashed lines) kinematics.

positive $\mathrm{Re}\, a_{K^-p}$, should be equally credible, if not more so, as the earlier values. In this sense, the puzzle has been resolved. However, a question remains regarding the sign of $\mathrm{Re}\, f_{K^-p}$. This sign can be determined from the Coulomb-nuclear interference in the K^-p scattering. As we mentioned in Sec. I, however, experimental data for the Coulomb-nuclear interference have been available only at higher energies, $E \gtrsim 1500$ MeV. These data indicate that $\mathrm{Re}\, f_{K^-p}$ is positive in that energy region. Several analyses have deduced, by extrapolating these data to threshold ($E = 1432$ MeV), that $\mathrm{Re}\, f_{K^-p}$ is negative around threshold [8]. This sign is opposite to that of the f_{K^-p} we obtained. However, we suspect that there is considerable uncertainty in the extrapolation over the energy interval of about 70 MeV. Indeed, several extrapolated values in those analyses are spread over a rather large range [8].

About the K^-p atom data, the statistics seem to be rather poor, and the error bars are large [29]. It may well turn out that $\mathrm{Re}\, a_{K^-p}$ is even negative. Therefore we also tried calculations in which the atomic data are disregarded. We found a set of parameters which produces very good fit to all the data (other than those for the K^-p atom). The obtained cross sections and the $\pi\Sigma$ mass distribution are shown in Fig. 4. Again for comparison, the fit with relativistic kinematics (solid lines) and the one with nonrelativistic kinematics (dashed lines) are shown together; they are practically indistinguishable. The branching ratios are, with either kinematics,

$$\gamma = 2.354, \quad R_c = 0.651, \quad R_n = 0.188. \tag{24}$$

The scattering length so obtained is

$$a_{K^-p} = (-1.11 + i0.70)\ \mathrm{fm}. \tag{25}$$

This a_{K^-p} is consistent with those obtained by K-matrix analyses, for example, that of Ref. [5], when the correction due to the threshold difference between K^-p and $\overline{K^0}n$ is taken into account,

$$a_{K^-p} = (-1.03 + i0.75)\ \mathrm{fm}. \tag{26}$$

Finally, in order to settle the issue concerning the behavior of f_{K^-p} near the threshold, measurements of the Coulomb-nuclear interference at lower energies as well as further x-ray measurements of the kaonic hydrogen atom are urged. If it should so happen that f_{K^-p} behaves in a strange manner, then we might indeed need Λ_0 as in the Kumar-Nogami model.

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