Partial restoration of chiral symmetry in nuclear matter

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Recent work of Cohen, Furnstahl, and Griegel has advanced our understanding of the behavior of quark and gluon condensates in nuclear matter. We make use of their analysis to discuss the role of chiral condensates as they appear in relativistic Brueckner-Hartree-Fock theory. We find some support for assumptions we used to discuss the properties of nuclear matter in our earlier work. We also find that a rather consistent picture emerges from these studies, when we relate the parameters of the boson-exchange model of nuclear forces to an underlying field-theoretic description of nuclear matter.

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I. INTRODUCTION

Some time ago, we discussed the role of quantum field theory in understanding the properties of nuclear matter [1]. In particular, we associated the large scalar fields, found in the phenomenological analysis of nucleon-nucleus scattering and in studies of nuclear matter, with a partial restoration of chiral symmetry at finite baryon density. To create a theoretical framework to discuss the role of chiral symmetry breaking in the nuclear many-body problem, we made use the the Gell-Mann–Levy sigma model [2], with nucleon and meson degress of freedom. (The sigma field of that model is identified as an order parameter describing chiral symmetry breaking.) Our approximations led to the expression

$$\sigma=f_\pi-\frac{G_{\sigma NN}\rho_S}{m_\sigma^2}\ , \tag{1.1}$$

thereby setting up a *linear* relation between the sigma field in matter and the value of the nuclear scalar density, ρ_S. (Note that ρ_S depends on the value of the medium-modified nucleon mass, $\tilde{m}_N$.) Further, $f_\pi=93$ MeV is the pion decay constant and $G_{\sigma NN}$ is the sigma-nucleon coupling constant. Values of about 10 for $G_{\sigma NN}$ are usually used in the boson-exchange model of the nucleon-nucleon interaction [3]. If we take the Gell-Mann–Levy model seriously, and put $G_{\sigma NN}\simeq 10$, we have $m_N=G_{\sigma NN}f_\pi\simeq 930$ MeV in vacuum. Also, if we use $m_\sigma=600$ MeV, $\rho_S=0.16\ \text{fm}^{-3}$, and $G_{\sigma NN}=10$ in Eq. (1.1), we find a 44% reduction of the vacuum value of the sigma field and of the nucleon mass in nuclear matter.

Since the work of Cohen, Furnstahl, and Griegel [4] provides some support for the use of a *linear* relation between the order parameter and the scalar density, we review some aspects of their analysis in Sec. II of this work. In Sec. III, we discuss the role played by the sigma field in relativistic Brueckner-Hartree-Fock theory (RBHF) [5] since the authors of Ref. [4] only presented a rather schematic discussion of how their results might be used in a field-theoretic description of nuclear matter. (For example, in our earlier work, we were able to discuss the role of exchange diagrams and short-range correlations in determining the properties of nuclear matter. In addition, we included the full array of mesons used in the boson-exchange model of the nucleon-nucleon interaction in our calculations [5].) In Sec. IV we provide some further discussion and summarize our conclusions.

II. CHIRAL SYMMETRY ORDER PARAMETERS AT FINITE BARYON DENSITY

In this discussion, we will often follow the notation used in [4]. We denote the average current quark mass as m_q [$m_q=(m_u+m_d)/2$]. The *constituent* quark mass will be M_q in vacuum and $\tilde{M}_q$ in nuclear matter. In Ref. [4] the assumption is made that $m_N=3M_q$ and $\tilde{m}_N=3\tilde{M}_q$, so that the coupling constant of a quark to the sigma field, g, is seen to be one-third of $G_{\sigma NN}$, if we use the relations $m_N=G_{\sigma NN}f_\pi$ and $M_q=gf_\pi\approx 310$ MeV.

First, we quote a model-independent relation, which is valid at low density [4],

$$\frac{\langle\rho_N|\bar{q}q|\rho_N\rangle}{\langle 0|\bar{q}q|0\rangle}=1-\frac{\rho_N}{\rho_N^\chi}+\cdots\ , \tag{2.1a}$$

$$(\rho_N^\chi)^{-1}=\frac{\sigma_N}{m_\pi^2 f_\pi^2}\ . \tag{2.1b}$$

Here, $\sigma_N=2m_q\langle N|\bar{q}q|N\rangle$ is the nucleon sigma term and ρ_N is the baryon density. The matrix element $\langle\rho_N|\bar{q}q|\rho_N\rangle$ denotes the value of the scalar density in nuclear matter. We have

$$\langle\rho_N|\bar{q}q|\rho_N\rangle\simeq\langle 0|\bar{q}q|0\rangle+\rho_N\langle N|\bar{q}q|N\rangle\ .$$

One question that arises relates to the range of validity of Eq. (2.1). That issue is addressed in Ref. [4] via the study of some simple models and a study of QCD sum rules in matter.

We are interested in seeing what relation might exist between Eqs. (1.1) and (2.1). Such a relation may be developed following the study made in [4]. For example, we consider the analysis made there for the Gell-Mann–Levy sigma model. The assumption is made that

$$m_\pi^2 f_\pi \sigma = -2m_q \langle \rho_N | \bar{q}q | \rho_N \rangle \ . \tag{2.2}$$

This equation is consistent with the relation in vacuum,

$$m_\pi^2 f_\pi^2 = -2m_q \langle 0 | \bar{q}q | 0 \rangle \ . \tag{2.3}$$

[If $\langle \bar{q}q \rangle_{\text{vac}} = -(250 \text{ MeV})^3$, $m_\pi = 138$ MeV, and $f_\pi = 93$ MeV, we obtain $m_q = 5.36$ MeV.] Thus, we have

$$\frac{\sigma}{f_\pi} = \frac{\langle \rho_N | \bar{q}q | \rho_N \rangle}{\langle 0 | \bar{q}q | 0 \rangle} \tag{2.4}$$

$$= \frac{\tilde{M}_q}{M_q} \tag{2.5}$$

$$= \frac{\tilde{m}_N}{m_N} \ . \tag{2.6}$$

Then, for densities where $\rho_S \approx \rho_N$, one has

$$\frac{\sigma}{f_\pi} = 1 - \frac{\rho_S}{\rho_N^\chi} + \cdots \ , \tag{2.7}$$

upon using Eqs. (2.1) and (2.4).

We note that the Gell-Mann–Levy model is used for a system of *quarks* and meson fields in Ref. [4]. The Langrangian of the model is

$$\mathcal{L}(x) = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - g\bar{\Psi}(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\Psi + \tfrac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \tfrac{1}{2}\partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi} - U_0(\sigma, \boldsymbol{\pi}) - U_{\text{eff}}(\sigma) \ , \tag{2.8}$$

with

$$U_0(\sigma, \boldsymbol{\pi}) = \frac{m_\sigma^2}{8f_\pi^2}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 \tag{2.9}$$

and

$$U_{\text{eff}}(\sigma) = -\frac{N_c N_f}{16\pi^2}\left[g^4\sigma^4 \ln\left[\frac{\sigma^2}{f_\pi^2}\right] - g^4 f_\pi^2(\sigma^2 - f_\pi^2) - \tfrac{3}{2}g^4(\sigma^2 - f_\pi^2)^2 \right] . \tag{2.10}$$

Here N_c is the number of the colors and N_f is the number of flavors. Further, $U_{\text{eff}}(\sigma)$ is a correction to the potential calculated at the one-loop level [4]. We quote the equation for the σ field obtained by minimizing the energy [4],

$$\frac{m_\sigma^2}{2f_\pi^2}(\sigma^2 - f_\pi^2)\sigma - \frac{N_c}{2\pi^2}g^2\sigma\left[g^2\sigma^2 \ln\left[\frac{\sigma^2}{f_\pi^2}\right] - g^2(\sigma^2 - f_\pi^2)\right] = \frac{-gN_c(g\sigma)}{\pi^2}\left[k_F(k_F^2 + g^2\sigma^2)^{1/2} - g^2\sigma^2 \ln\left[\frac{k_F + (k_F^2 + g^2\sigma^2)^{1/2}}{g\sigma}\right]\right] . \tag{2.11}$$

The second term on the left-hand side of Eq. (2.11) arises from U_{eff}, while the term on the right-hand side is proportional to the quark scalar density, which is calculated for a quark *gas* with a constituent quark mass, $\tilde{M}_q = g\sigma$, and with the Fermi momentum of nuclear matter, such that $\rho_N = (N_f/3\pi^2)k_F^3$ with $N_f = 2$. (Note that there are three quark distributions, one for each color.) While the calculation made for the quark scalar density is questionable, we proceed with a discussion of this model. Equation (2.11) is used in Ref. [4] to derive an equation such as Eq. (2.7), except that ρ_N^χ is now given by [4]

$$(\rho_N^\chi)^{-1} = \frac{N_c M_q}{m_\sigma^2 f_\pi^2} \ . \tag{2.12}$$

We may compare Eq. (2.12) with the model-independent result [Eq. (2.1)] and find

$$\sigma_N = \frac{m_\pi^2 m_N}{m_\sigma^2} \tag{2.13}$$

if $m_N = N_c M_q$. Thus, upon using Eqs. (2.1), (2.4), and (2.13), we have

$$\sigma = f_\pi - \frac{G_{\sigma NN}\rho_S}{m_\sigma^2} \ . \tag{2.14}$$

Here, we have put $m_N = G_{\sigma NN} f_\pi$ and have replaced ρ_N by ρ_S, the nucleon scalar density. (If we are interested in a theory with *hadronic* degrees of freedom, the source for the σ field has to be the *nucleon* scalar density. To lowest order in k_F, we have $\rho_S \approx \rho_N$.)

Now let us check that the above analysis makes sense if we use *empirical* values of m_σ and $G_{\sigma NN}$. We have already noted that, for boson-exchange models, $G_{\sigma NN} \approx 10$, so that $m_N = G_{\sigma NN} f_\pi \approx 930$ MeV. Now consider the value of σ_N obtained from Eq. (2.13). With the commonly used value of $m_\sigma = 550$ MeV [3], we have $\sigma_N = 59$ MeV and with $m_\sigma = 600$ MeV, we have $\sigma_N = 50$ MeV. (See Fig. 1.) The most recent evaluation of σ_N [6] yields a value of 45 ± 8 MeV and, if Eq. (2.13) is valid, we have $m_\sigma \simeq 630$ MeV. We see that the theoretical picture obtained is in reasonable accord with phenomenological models. One may also take the attitude that Eq. (2.13) is correct for conventional values of m_σ and for values of σ_N within the range of the values suggested in the literature [6,7]. That means that Eq. (2.1) may be rewritten as Eq. (1.1). What is then needed is an argument for using the resulting linear relation for values of the nucleon density as large as that of nuclear matter ($\rho^{\text{NM}} = 0.17 \text{ fm}^{-3}$). As noted above, this issue is discussed in Ref. [4], where some justification for using the linear relation over a range of values of ρ_N is given on the basis of the behavior

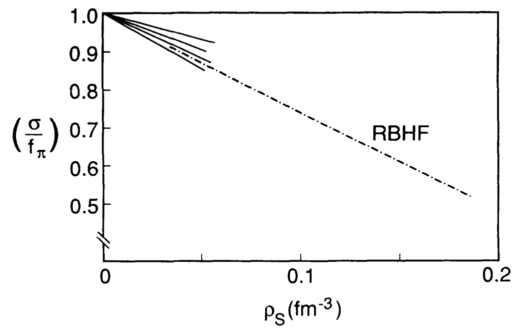

FIG. 1. The dash-dotted line shows the dependence of σ/f_π on the nuclear scalar density in the RBHF theory [5]. We have used the relation $\sigma/f_\pi=1-G_{\sigma NN}\rho_S/m_\sigma^2$, with $G_{\sigma NN}=9.45$ and $m_\sigma=550$ MeV. The short lines represent the model-independent result evaluated for $\sigma_N=30$, 40, 50, and 60 MeV with the uppermost line corresponding to $\sigma_N=30$ MeV, etc. The slope of the short line would be equal to the slope of the dash-dotted line if $\sigma_N=m_\pi^2 f_\pi G_{\sigma NN}/m_\sigma^2$. For the parameters, $G_{\sigma NN}=9.45$ and $m_\sigma=550$ MeV, we have $\sigma_N=55.3$ MeV, while the latest theoretical analysis yields $\sigma_N=45\pm8$ MeV [6]. [The range of validity of the model-independent result (short lines) is not known, although the work reported in [4] suggests that a linear extrapolation to higher densities may be a good approximation.]

of some simple models. Probably, a more compelling argument can be made by considering QCD sum rules in nuclear matter. For example, in the thesis of Griegel [4] it is found that the nucleon mass is *linearly* related to the value of the quark condensate. To our mind, that result provides the best justification for the introduction of Eq. (2.2). Finally, we note that in his discussion of hot and dense QCD matter, Hatsuda assumes that a linear relation may exist over a broad range of densities, without attempting further justification [8].

In Ref. [4], the relation of these results for the chiral order parameter to a relativistic many-body theory that uses nucleons and mesons as the appropriate degrees of freedom is only discussed for a very simple model containing nucleons, sigmas, and pions. In this work we wish to discuss how these results may be used in relativistic Brueckner-Hartree-Fock theory [5]. That issue is taken up in the next section.

III. CHIRAL SYMMETRY RESTORATION AND THE RELATIVISTIC BRUECKNER-HARTREE-FOCK MODEL

One aim in this work was to show how the relation given in Eq. (2.14) is to be understood within the context of relativistic Brueckner-Hartree-Fock theory. To that end, we write the self-energy of a nucleon of momentum **p**, $\Sigma=A+B\gamma^0+\boldsymbol{\gamma}\cdot\mathbf{p}C/m_N$, and suggest that the appropriate relation is

$$A_\sigma^H=G_{\sigma NN}\sigma' \tag{3.1}$$

$$=-\frac{G_{\sigma NN}^2\rho_S}{m_\sigma^2}\ . \tag{3.2}$$

Here A_σ^H is the contribution of the fluctuating part of the sigma field, $\sigma'\equiv\sigma-f_\pi$ (with $\sigma'<0$), to the evaluation of the quantity A in the *Hartree approximation.* We note that A_σ^H of Eq. (3.2) is only one element of a complex (RHBF) calculation that includes the effects of exchange and of short-range correlations [5]. Also, in our calculations we have included the contributions of *all* the bosons of the boson-exchange model $(\sigma,\pi,\omega,\rho,\ldots)$ [3]. The details of such calculations may be found in Ref. [5]. For example, we see that the Fock terms arising from omega exchange contribute about -135 MeV to the value of A, the scalar term in the self-energy. Short-range correlations also affect the value obtained for A. Once all these effects are taken into account, as in Ref. [5], we then use the relation

$$\tilde{m}(\mathbf{p})=m_N+\tfrac{1}{4}\mathrm{Tr}\Sigma(\mathbf{p}) \tag{3.3}$$

$$=m_N+A(\mathbf{p})\ . \tag{3.4}$$

Although, many terms contribute to the value of $A(\mathbf{p})$, one finds $A(0)\simeq-400$ MeV, which happens to be rather close to the value obtained in the Hartree approximation, $A_\sigma^H=-391$ MeV. (The fact that the result of rather complex calculations of A and B yield values that are not too different from the values calculated in the Hartree approximation could, in part, account for the generally successful phenomenology built on the type of model studied by Walecka and collaborators [9].)

IV. DISCUSSION

We have followed the work presented in Ref. [4] and assumed that a *linear* relation exists between the order parameter $\langle\bar{q}q\rangle$ and the field σ. It then follows that the model-independent relation given in Eq. (2.1) can also be used for σ/f_π. The rate of change of σ/f_π with density depends on $(\rho_N^\chi)^{-1}$, which, in turn, depends on σ_N. [See Eq. (2.1).] We have noted that Eq. (1.1) follows from Eq. (2.4), if σ_N is related to m_σ as in Eq. (2.13).

It has been our goal in this work to relate some aspects of the material presented in [4] to the RBHF formalism and we have described how such a relation may be obtained. We have also shown how the modification of the sigma field in matter leads to a large scalar potential for the nucleon. In the RBHF theory, the medium-modified nucleon mass is then given by $\tilde{m}_N=m_N+A$, where A is made up of contributions that include the Hartree term arising from the sigma field, as well as significant contributions from the Fock terms associated with the exchange of *all* mesons. In addition, short-range correlations also affect the value of A.

Finally, we remark that this analysis may be considered from another point of view. We may argue that there is a body of empirical evidence [5] that supports Eq. (1.1) and, therefore, chiral models whose dynamics may be approximated by Eq. (1.1) should be preferred. As an example, we may note that the solution of Eq. (2.11) is well approximated by Eq. (1.1) over the entire range $0<\rho_N<\rho^{\mathrm{NM}}$. However, if we drop U_{eff}, the linear range for the solution is approximately $0<\rho_N<0.4\rho^{\rm NM}$. Whether that result implies that $U_{\rm eff}$ has been calculated correctly remains to be seen. Clearly, more work is needed to clarify such issues.

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[1] L. S. Celenza, B. Goulard, and C. M. Shakin, Phys. Rev. D **24**, 912 (1981).

[2] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).

[3] R. Machleidt, in *Advances in Nuclear Physics,* edited by J. W. Negele and E. Vogt (Plenum, New York, 1989), Vol. 19.

[4] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **45**, 1881 (1992), this issue. See also D. K. Griegel, Ph.D. thesis, University of Maryland, 1991.

[5] L. S. Celenza and C. M. Shakin, *Relativistic Nuclear Physics: Theories of Structure and Scattering* (World Scientific, Singapore, 1986).

[6] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B **253**, 252 (1991).

[7] R. Jaffee and C. L. Korpa, Comments Nucl. Part. Phys. **17**, 163 (1987); P. M. Gensini, Nuovo Cimento **60A**, 221 (1980).

[8] T. Hatsuda, University of Washington Report DOE/ER/40561-08-INT91-00-05, 1991.

[9] B. D. Serot and J. D. Walecka, in *Advances in Nuclear Physics,* edited by J. W. Negele and E. Vogt (Plenum, New York, 1985), Vol. 16.