Contribution of the induced tensor form factor to the A=8 β-ν-α angular correlation

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Experimental limits on the second class induced tensor weak current are reviewed. A method, involving the measurement of the β-ν-α angular correlations in ^{8}Li and ^{8}B decays, is proposed to test for this current with improved precision.

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I. INTRODUCTION

The most general matrix elements of the weak neutron $\rightarrow$ proton current, constrained only by Lorentz covariance, are [1]

$$(p|V_\lambda|n) = i(u_p|f_v\gamma_\lambda + f_{wm}\sigma_{\lambda\nu}q_\nu + if_s q_\lambda|u_n), \qquad (1)$$

$$(p|A_\lambda|n) = i(u_p| - f_a\gamma_\lambda\gamma_5 - f_t\sigma_{\lambda\nu}q_\nu\gamma_5 + if_{ps}\gamma_5 q_\lambda|u_n). \qquad (2)$$

The six form factors must be relatively real to ensure invariance under time reversal, while charge symmetry of weak interactions requires f_v, f_a, f_{wm}, f_{ps} (the vector, axial, weak magnetism, and induced pseudoscalar form factors, respectively) to be real and f_t, f_s (the induced tensor and induced scalar form factors) to be purely imaginary [1]. The f_{wm}, f_{ps}, f_t, f_s form factors are induced by the strong interaction [2] in absence of which one would have only f_v and f_a. Because the strong interaction is essentially invariant under charge symmetry, one expects the real parts of f_t and f_s to vanish. Because f_s is also forbidden by the conservation of the vector current (CVC), experiments testing the charge symmetry of the weak interactions are designed to search for the f_t current [3].

The goals of this paper are threefold: (1) To review critically the experimental data that constrain the f_t current, (2) to emphasize the limiting factors of the present experiments, and (3) to propose a new method of probing this current with improved precision.

In Sec. II, we review the results obtained from the β angular correlation of aligned A=12 nuclei which provide the tightest constraint on f_t. These experiments determine the *sum* of the weak magnetism and induced tensor form factors: $f_{wm} + f_t$. We discuss three different methods for independently determining f_{wm} and use the results of each of the three methods to infer a value for f_t. The best result [3] is $f_t < 0.6 f_a$. We show in Sec. II that the separation of the weak magnetism and induced tensor form factors is an important source of the uncertainty in f_t. In fact, due to the expected values of $f_{wm} \approx 4.0$ and $f_t \approx 0.0$, 10% measurements of both $f_{wm} + f_t$ and f_{wm} provide a relatively poor upper limit $f_t < 0.6$. (Note that 10% determinations of $f_{wm} + f_t$ and f_{wm} require measurements at the 10^{-3} level.)

In Sec. III, we propose a novel observable in ^{8}Li and ^{8}B β decays that can isolate experimentally the contribution of f_t: *the average electron-neutrino angular correlation measured in coincidence with the alpha particles emitted in the plane perpendicular to the electron momentum.* This particular correlation depends only on f_t/f_a:

$$W(\theta_{e\nu}) = 1 + (E_0/M)(f_t/f_a)\cos\theta_{e\nu} \ . \qquad (3)$$

Thus the question of second class current can be investigated without any assumption about the weak magnetism form factor.

II. PRESENT EXPERIMENTAL LIMITS ON f_t

The best upper limit on the f_t form factor comes from the measurement of β angular correlations in $A = 12$ nuclei. In order to interpret these data, a formalism [4] has been developed in which nuclei are considered as elementary particles described by various form factors: F_{wm}, F_a, F_t, F_{ps}. The nuclear weak currents are written in complete analogy with the nucleon ones. However, F_t has now both a first- and a second-class part, $F_t^{\mp} = F_t^1 \mp F_t^2$. If one assumes the validity of the impulse approximation, a simple relation [5, 4, 6, 24] can be derived between the nucleon and the nucleus-induced tensor form factors: $F_t^2/F_a = f_t/f_a$.

The $\beta^{\mp}$ angular correlation [4] from an oriented nucleus, for the particular case of a $1^+ \rightarrow 0^+$ spin sequence, is

$$1 \mp P(1 + E\alpha_{\mp})\mathcal{P}_1 + AE\alpha_{\mp}\mathcal{P}_2, \qquad (4)$$

where $P = p_1 - p_{-1}$ is the nuclear polarization, $A = 1 - 3p_0$ is the nuclear alignment, p_i are the populations of the magnetic sublevels, $\mathcal{P}_i$ are the Legendre polynomials, E is the β energy, and $\alpha_{\mp} = (\tilde{F}_{wm}^{\mp} - \tilde{F}_t)/3M$ with $\tilde{F}_i = F_i/F_a$. It is widely recognized that the determination of $\alpha_{\mp}$ through the polarization term is an *indirect* procedure (i.e., one has to subtract the dominant term 1 from the measured quantity). On the other hand, the determination of $\alpha_{\mp}$ through the alignment term is a *direct* procedure and thus provides the best information about F_{wm} and F_t. The data obtained in the A=12 system by two different groups in Refs. [7–10] are, respectively,

$$\alpha_- = -(0.07 \pm 0.20),$$
$$\alpha_- = +(0.06 \pm 0.18),$$
$$\alpha_+ = -(2.77 \pm 0.52),$$
$$\alpha_+ = -(2.73 \pm 0.39).$$

The quantity $\Delta\alpha = \alpha_- - \alpha_+$ is proportional to the second-class part of F_t,

$$\Delta\alpha = \frac{1}{3M}(\tilde{F}_{wm}^- - \tilde{F}_{wm}^+ + 2\tilde{F}_t^2). \tag{5}$$

Averaging the results of Refs. [7–10], one obtains a value

$$< \Delta\alpha >= 2.74 \pm 0.34 \tag{6}$$

that seems to be firmly established.

However, before reaching any conclusion about the second class current, F_{wm}^- and F_{wm}^+ must be known from an independent experiment. Three possibilities exist: (1) Determining F_{wm}^- and F_{wm}^+ via β spectrum shape measurements in ^{12}B and ^{12}N; (2) determining F_{wm}^- in μ + ^{12}C capture and assuming the equality $F_{wm}^- = F_{wm}^+$ predicted by the conservation of the vector current (CVC); and (3) measuring Γ_{M1}, the width of the analog electromagnetic transition and inferring the values of F_{wm}^- and F_{wm}^+ implied by CVC. We now discuss these three possibilities separately.

A. β spectrum measurements

The $\beta^\mp$ spectrum [11] of an allowed $1^+ \to 0^+$ transition is

$$S^\mp(Z, E, E_0)\ (1 + a_\mp E), \tag{7}$$

where $S^\mp$ is the Fermi function, $a_\mp = \tilde{a}_\mp \mp \delta_\mp(\alpha Z)$, $\tilde{a}_\mp = \pm(4/3M)\tilde{F}_{wm}^\mp$, and the radiative corrections, $\delta_\mp$, have been calculated by Behrens [11]

$$\delta_- = -1.3\ \text{GeV}^{-1} \text{ and } \delta_+ = +2.7\ \text{GeV}^{-1}.$$

The determination of F_{wm} by the a β shape measurement is a *indirect* procedure; a change of only 3×10^{-3} in the first excited state branching ratio of the β^+ decay would modify the inferred $\tilde{F}_{wm}^+$ by 25%. The results measured in the A=12 system are

$$a_- = 18.2 \pm 0.9,\ a_+ = +6.0 \pm 0.8\ [12],$$
$$a_- = 19.2 \pm 0.9,\ a_+ = +10.7 \pm 0.8\ [13],$$
$$a_- = 5.5 \pm 1.0,\ a_+ = -5.2 \pm 0.6\ [14],$$
$$a_- = 4.2 \pm 1.0,\ a_+ = -1.7 \pm 0.6\ [13],$$
$$a_- = 4.1 \pm 1.0,\ a_+ = -1.7 \pm 0.9\ [15],$$
$$a_- = 9.1 \pm 1.1,\ a_+ = -0.7 \pm 0.9\ [16].$$

The consistency between these data is quite poor. Using the results of the most recent measurement, one obtains

$$\tilde{F}_{wm}^- = 5.44 \pm 0.76 \text{ and } \tilde{F}_{wm}^+ = -2.37 \pm 0.63.$$

The equality predicted by CVC, $\tilde{F}_{wm}^- = -\tilde{F}_{wm}^+$, is not verified, probably due to systematic experimental errors. On the other hand, the quantity $\tilde{F}_{wm}^- - \tilde{F}_{wm}^+ = 7.81 \pm 0.98$ is expected to be more reliable due to partial cancelation of systematic uncertainties. [Note that this result is in excellent agreement with the CVC prediction based on the measured lifetime of the electromagnetic analog transition: $\tilde{F}_{wm}^-(\text{CVC}) - \tilde{F}_{wm}^+(\text{CVC}) = 8.08 \pm 0.11$.] Substituting these measured values of $\tilde{F}_{wm}^-$ and $\tilde{F}_{wm}^+$ and $\Delta\alpha$ given above into Eq. (5), one finds

$$\tilde{F}_t^2 = -0.09 \pm 0.68\ . \tag{8}$$

This result provides an upper limit on the second-class induced tensor current.

B. Muon capture on ^{12}C

The $\tilde{F}_{wm}^-$ form factor can be determined via the three observables accessible to μ capture experiments [3] and appropriate scaling of the form factor: (1) The capture rate, (2) the polarization of the ^{12}B in the recoil frame (the so-called longitudinal polarization), and (3) the polarization of the ^{12}B in the μ^- frame (the so-called average polarization).

By virtue of the μ-e universality, the capture rate [4] Γ_c can be expressed in term of the ^{12}B β^- decay rate Γ^-:

$$\Gamma_c = \mathcal{K}\ [F_a(q^2)/F_a(0)]^2\ (1 + \tilde{F}_{wm}\ E_\nu/2\,M)^2\ (2 + x^2), \tag{9}$$

where $q^2 = (p_\mu - p_\nu)^2 = 0.740\, m_\mu^2$,

$$\mathcal{K} = \pi E_\nu^2 \left(1 - \frac{E_\nu}{m_\mu + M_\text{B}}\right) C(^{12}\text{C}) \left(\frac{Z(^{12}\text{C})}{137} \frac{m_\mu M_\text{C}}{m_\mu + M_\text{C}}\right) \left(\frac{\Gamma^-}{m_e^5\ f^-}\right), \tag{10}$$

$$x = \left(1 + \tilde{F}_{ps}\ \frac{m_\mu\ E_\nu}{m_\pi^2} - \tilde{F}_t\ \frac{E_\nu}{2m}\right) \Big/ \left(1 + \tilde{F}_{wm}\ \frac{E_\nu}{2m}\right). \tag{11}$$

M_B and M_C are the ^{12}B and ^{12}C masses, f^- is the integrated Fermi function, and C is a correction for the finite-size charge distribution. After inserting the values

$$\Gamma_c = (6.18 \pm 0.26) \times 10^3\ \text{s}^{-1}\ [17, 18],$$

$$\mathcal{K} = 3.63 \times 10^3\ \text{s}^{-1}\ [3],$$

$$[F_a(q^2)/F_a(0)]^2 = 0.555 \pm 0.019\ [6],$$

and

$x = 0.25 \pm 0.6$ [6],

one obtains $\tilde{F}_{wm} = 4.5 \pm 0.5$.

Finally, combining this result with the average value of $\Delta\alpha$, one finds

$$\tilde{F}_t^2 = -0.68 \pm 0.68. \quad (12)$$

Clearly, the measurement of the capture rate is the main source of uncertainty and because of the slow rate of the μ capture with respect to the μ lifetime and the quite high branching ratio to excited states (10%), we do not anticipate significant improvement on this result.

C. The analog electromagnetic transition

The conservation of the vector current predicts a relation between the $\tilde{F}_{wm}^{\mp}$ form factors and the $M1$ isovector radiative decay of the analog state:

$$\tilde{F}_{wm}^{\mp}(\mathrm{CVC})/M = (1/F_a^{\mp})[(3/\alpha)\Gamma_\gamma/E_\gamma^3]^{1/2}, \quad (13)$$

where $E_\gamma = 15.109 \pm 0.004$ MeV. The axial-vector form factors $F_a^{\pm}$ deduced [4] from the $\beta^{\pm}$ decay rates of ^{12}B and ^{12}N are $F_a^- = 0.51$, $F_a^+ = 0.48$. The weighted average of six measurements of Γ_γ [19] using the technique of nuclear resonance fluorescence (after reevaluation with currently accepted branching ratios) is $\Gamma_\gamma = 40.4 \pm 2.0$ eV. Six measurements of Γ_γ using $^{12}\mathrm{C}(e,e')$ have been published. The combined result of the three first measurements is $\Gamma_\gamma = 32.6 \pm 3.5$ eV. Finally, the latest three experiments yield $\Gamma_\gamma = 37.0 \pm 1.1$ [19], 35.74 ± 0.86 eV [20], 38.5 ± 0.8 eV [21]. It is usually accepted that the resonance of fluoresence and the electron scattering methods agree at the level of 10%. We adopt the value $\Gamma_\gamma = 38.8 \pm 1.6$ eV which is an average of the combined data obtained by nuclear resonance fluorescence and the combined result of the last three electron scattering experiments; the error bar is chosen to make the two sets of data consistent with each other. Then, the predictions of CVC are $\tilde{F}_{wm}^- = 3.92 \pm 0.08$, $\tilde{F}_{wm}^+ = 4.16 \pm 0.08$. Combining this result with the average value of $\Delta\alpha$, one obtains

$$\tilde{F}_t^2 = -0.22 \pm 0.47\ \mathrm{GeV}^{-1}.$$

The results of the determination of F_t^2 from β decay, μ capture, and electron scattering experiments are consistent with each other and with the absence of second-class currents, but the experimental uncertainties are quite large.

III. THE β-ν-α ANGULAR CORRELATION IN ^{8}Li AND ^{8}B DECAYS

I now propose an observable, *the average e-ν angular correlation, in ^{8}Li and ^{8}B decays, measured in coincidence with the α's emitted in the plane perpendicular to the electron momentum,* that should improve significantly the uncertainty on the measurement of F_t^2. Thus the question of second-class current can be investigated without any assumption about the weak magnetism form factor.

The β decays of ^{8}Li and ^{8}B feed a very broad level of the ^{8}Be, which splits into two α particles. The leading terms of the electron-neutrino-alpha angular correlation are

$$W(\hat{e},\hat{\nu},\hat{\alpha}) = 1 - (\hat{e}\cdot\hat{\alpha})\ (\hat{\nu}\cdot\hat{\alpha}) \mp \frac{2}{3}\frac{E_e - E_\nu}{M}\ \tilde{F}_{wm}^{\mp}\ (\hat{e}\cdot\hat{\nu}) + \frac{2}{3}\frac{E_e + E_\nu}{M}\ \tilde{F}_t^{\mp}(\hat{e}\cdot\hat{\nu}). \quad (14)$$

The relations between $\tilde{F}_{wm}^{\mp}$, $\tilde{F}_t^{\mp}$, and the form factors c, b, d defined by Holstein [5] are $\tilde{F}_{wm}^{\mp} = b^{\mp}/c$ and $\tilde{F}_t^{\mp} = d^{\mp}/c$. A measurement of the momenta of the electron and both alpha particles overdetermines the neutrino momentum. With the conditions $< \cos\theta_{e\alpha} >= 0$ and $< E_e - E_\nu >= 0$, the e-ν correlation becomes

$$1 + (E_0/M)\ \tilde{F}_t(\ \hat{e}\cdot\hat{\nu}). \quad (15)$$

The $W(\hat{e},\hat{\nu})$ observable is directly and only sensitive to $\tilde{F}_t$. Moreover, the A=8 system provides a unique opportunity to study the E_0 dependence of this correlation because the final state is a broad level. The possible presence of twice-forbidden vector form factors does not affect $W(\hat{e},\hat{\nu})$ because their contribution vanishes after averaging over the lepton energy. This is a consequence of a theorem due to Weinberg (see Appendix A). However, radiative corrections are expected to contribute to the asymmetry of the e-ν angular correlations. We used (see Appendix B) the modified spectral function described by Holstein [5] to make an estimate of the radiative correction. The expected asymmetry is $2E_0(\tilde{F}_t^2+0.04)$ GeV^{-1}. If this estimate is correct, the contribution of the radiative corrections to the e-ν angular correlation is well below the present upper limit on F_t^2. To the extent that isospin is a good symmetry, the twice-forbidden axial terms will contribute equally to the β^- and β^+ angular correlations. Due to the breaking of isospin symmetry in the real world, the twice-forbidden axial terms will give rise to an asymmetry between the β^- and β^+ correlations. This asymmetry is expected to mimic a non zero F_t^2 at a level 3 to 4 times below the present upper limit on F_t^2.

An experiment [22] to measure the β-α-α correlation in ^{8}Li and ^{8}B is currently under development at the University of Washington. This experiment is expected to improve the precision of the F_t^2 measurement by a factor 5.

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APPENDIX A: A THEOREM OF WEINBERG

Weinberg [23] has shown that *"if we perform any experiment with arbitrarily large momentum transfer, which does not distinguish between the electron and the neutrino (e.g., measurement of the total decay rate or the average electron-neutrino angular correlation, but not the* *measurement of the electron energy spectrum) and if we may neglect the charge and mass of the lepton, then there can be no interference between the vector and the axial vector current."*

The weak magnetism and the twice-forbidden vector form factors contribute to the e-ν angular correlation via an interference with the leading order form factor F_a of the axial current and, thus, their contributions are expected to vanish after averaging over the lepton energy.

It can be shown [5] that the twice-forbidden vector form factors contribute to the electron-neutrino angular correlation with the following kinematical dependence:

$$\frac{E_0 - 2E}{2M}, \tag{A1}$$

$$\frac{E_0^2 - 2E\,E_0 + m_e^2}{2M^2}. \tag{A2}$$

Clearly, after averaging over lepton energy, these contributions vanish as predicted by the theorem, except for the small $m_e^2/2M^2$ term which can be totally neglected.

APPENDIX B: RADIATIVE CORRECTIONS

Although the mass of the electron can be neglected, its charge cannot. This charge may well induce measurable radiative corrections. Using the spectral functions defined in Holstein [5], it is found that the allowed approximation term of e-ν angular correlation is the sum of two terms:

$$f_2\ (\hat{e}\cdot\hat{\nu}) + f_{12}\ [(\hat{e}\cdot\hat{\alpha})\ (\hat{\nu}\cdot\hat{\alpha}) - \tfrac{1}{3}(\hat{e}\cdot\hat{\nu})] \tag{B1}$$

with $f_2 = -\frac{1}{3}F_a^2 +$ (recoil order form factors) and $f_{12} = -F_a^2$ +(recoil order form factors). The sum of the leading-order contributions clearly vanishes. To take into account the radiative corrections, we consider the modified spectral functions defined by Holstein [5]

$$\tilde{f}_2 = f_2 + \Delta_2,$$

$$\text{with } \Delta_2 = \pm F_a^2\ (8\alpha Z/3\pi)[\tfrac{4}{3}E(2X+Y) - E_0 X], \tag{B2}$$

$$\tilde{f}_{12} = f_2 + \Delta_{12},$$

$$\text{with } \Delta_{12} = \pm F_a^2\ (8\alpha Z/3\pi)E(5X+4Y), \tag{B3}$$

where $X = Y = 9\pi R/140$ for uniform electrical and weak charge densities. Finally, the contribution $\delta^{\mp}$ of the radiative corrections to the average e-ν angular correlation is

$$\delta^{\mp} = \Delta_2 - \tfrac{1}{3}\Delta_{12} = \pm(8\alpha Z/3\pi)(E - E_0)X. \tag{B4}$$

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