Transport theory of relativistic heavy-ion collisions with chiral symmetry

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(Received 24 June 1991)

A transport theory with chiral symmetry is developed from the quark level to describe the chiral dynamics in high-energy heavy-ion collisions. The strong interaction is treated effectively by the Nambu–Jona-Lasinio model. A set of generalized Boltzmann equations of constituent quarks and mesons is derived by using the closed time-path Green's function technique and a loop expansion approach. Chiral symmetry, energy spectrum and dissipation, and density distributions of constituent quarks and mesons can then be studied self-consistently in the nonequilibrium dynamical processes. In particular, the discussion for exploring chiral symmetry breaking and its restoration and for studying dynamics of meson production (as collective $q\bar{q}$ excitations) in heavy-ion collisions is presented.

PACS number(s): 25.75.+r, 24.10.−i, 05.60.+w

I. INTRODUCTION

Since particles produced during heavy-ion collisions, such as pions, kaons, dileptons, photons, etc., probe the state of the nucleus, a microscopic study of the dynamical processes of particle emission in collisions is the central topic of heavy-ion collisions in nuclear physics [1]. Meanwhile, at extreme high density and/or temperature, the restoration of chiral symmetry is expected to play an important role in high-energy heavy-ion collisions. Recently, by using Green's-function techniques [2,3], the microscopic dynamical theory of relativistic heavy-ion collisions [4–7] has been discussed extensively based on the Walecka-type model [8]. A hadronic transport theory which includes N, Δ, σ, π, ω, and ρ as elementary particles has also been studied [9,10]. These attempts have had some success in interpreting particle production. However, there is a consistency problem in derivation of the transport equations for baryon-meson coupled systems. For example, in order to obtain the collision integrals which describe particle production, one must go beyond the mean-field approximation. The transport equations that are currently used are obtained by using an approximation that the nucleon kinematic dynamics is described up to mean-field approximation, the collision integrals up to the next order of mean-field approximation and the meson dynamics is in the zeroth-order approximation. This may be incorrect since the higher-order correlations in a strong interaction theory may totally dismantle mean-field dynamics [11] and meson structures [12]. Therefore, a self-consistent and practically useful microscopic picture of the particle production at high-energy collisions, say, from a few hundreds of MeV to a few tens of GeV per nucleon, is still not very clear [13]. Furthermore, although recently one has begun to seek evidence of chiral symmetry restoration in heavy-ion collisions [14], the relativistic dynamical equations for describing production *with chiral symmetry* have not been explored at all. The purpose of this paper is to attempt to obtain a transport theory with chiral symmetry for exploring meson production and the effect of chiral dynamics in heavy-ion collisions.

In principle, a nonequilibrium dynamical theory of heavy-ion collisions should be described by quantum chromodynamics (QCD). This is because QCD is regarded as the underlying theory of strong interaction, and chiral symmetry is embedded in the theory as a fundamental symmetry. Particle production in heavy-ion collisions is then described by the collective excitations of the basic building blocks (quarks and gluons) of the system. Unfortunately, heavy-ion collisions involve low- and intermediate-energy scales of QCD, in which the theory is nonperturbative. Nonperturbative equilibrium QCD computations, such as lattice calculations, are extremely difficult and are far from practical at present. Furthermore, no practical formalism for nonequilibrium QCD processes has been derived except for some mean-field approximations which do not yield hadrons [15].

On the other hand, in the last few years, effective theories of strong interaction have shown certain progress in understanding hadronic physics. For examples, the quark version of the Nambu–Jona-Lasinio (NJL) model [16], a four-point quark interaction theory, can manifest chiral symmetry and consistently describe mesons (particularly pions, but also kaons by extending the flavor number to $N_f=3$) as Goldstone bosons of chiral symmetry breaking. It has become a popular model to describe chiral symmetry and meson dynamics in the last few years [17–20]. In contrast, the Skyrme model, which is a meson theory of baryons and is equivalent to QCD in the large N_c limit, can be used to address baryon properties [21]. Since particle production in high-energy heavy-ion collisions is dominated by pions, our attention will be directed to the dynamical process of meson production and signatures of chiral symmetry restoration. Thus, in this paper we take the NJL model as the starting point to develop a transport theory for studying chiral symmetry and meson dynamics.

For simplicity, we confine ourselves to the two-flavor ($N_f=2$) case in this paper. The model Lagrangian is then taken as follows [16]:

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$$\mathcal{L}=\bar{q}(i\gamma\cdot\partial-\hat{m})q+\frac{g^2}{2}[(\bar{q}q)^2+(\bar{q}i\gamma_5\tau q)^2] \quad (1.1)$$

where $q=\binom{u}{d}$ is the current quark field with the mass matrix $\hat{m}=\mathrm{diag}(m_u,m_d)$ and we assume that $m_u=m_d=m$. The operators $\tau=(\tau_1,\tau_2,\tau_3)$ are the usual $SU_f(2)$ generators, while $g^2/2$ is a dimensional four-point interaction coupling constant. In the absence of gluon degrees of freedom, the quark fields are globally symmetric in the color space and we take here the color number $N_c=3$. It is also straightforward to explicitly include the exchange terms (the vector interaction term and others) in (1.1) by considering a Fierz transformation [16]. The vector meson degrees of freedom can then be addressed. The Lagrangian (1.1) is Lorentz invariant and chiral symmetry [here $SU_L(2)\otimes SU_R(2)$] can be realized as it occurs in QCD. The four-point fermion interaction terms can be regarded as a local approximation to the instanton-induced effective interaction and related to the $U_A(1)$ anomaly [22]. Hence, it is an effective theory of interacting quarks, in which gluon degrees of freedom are eliminated and from which dynamical breaking of chiral symmetry is realized naturally via quark condensation. The accompanying Goldstone bosons and the associated massive meson fields (obtained by a small current quark mass m which explicitly breaks chiral symmetry) can then be described consistently and can be identified with the physical mesonic degrees of freedom. Extension to $N_f=3$ (so that the kaons can be included as Goldstone bosons as well) is straightforward. By integrating out the fermion degrees of freedom, one can obtain a chiral Lagrangian containing the Skyrme and the Wess-Zumino terms [23]. Thus, the model serves as an approximation to QCD in the low-energy long-wavelength limit, and is expected to work well in the region of intermediate length between asymptotic freedom and confinement regions.

It should be pointed out that the NJL model has shortcomings. The main problem is the lack of color confinement. Indeed there are no explicit color degrees of freedom and in principle one cannot directly describe baryon properties. However, recent results have shown that there are some localized soliton solutions with baryon number $B=1$ in the NJL model [24]. It then seems to be possible to get three quarks in a bound state by putting color in "by hand." Here, we use the naive quark model assumption of identifying three constituent quarks with a nucleon, and therefore connecting nuclear matter to quark matter [25,18,19]. Another defect is that the NJL model is unrenormalizable and a momentum cutoff has to be used in order to get finite results. Actually, such a cutoff results in the effective coupling constants vanishing at high momentum and may incorporate aspects of the asymptotic freedom of QCD. Nevertheless, in this model, chiral symmetry, which is an important aspect of strong interaction and is believed to be restored in the intermediate stage of high-energy heavy-ion collisions, is well addressed, and pions, which are the main particles produced in heavy-ion collisions, are described consistently as elementary collective $q\bar{q}$ excitations. It is, therefore, quite interesting to use the NJL model to study the dynamical *mechanism* of meson production in heavy-ion collisions and the effects on the chiral phase transition. We must emphasize here that the model may not be expected to quantitatively describe the data of heavy-ion collisions. But in the case of lacking a clear signature of chiral symmetry restoration *in heavy-ion collisions,* a transport theory of this model may be helpful to provide some insight about what effects on chiral symmetry restoration can be observed. Meanwhile, the approach we will use to develop the transport theory is model independent and may be applied to other Lagrangian systems.

To address explicitly the meson degrees of freedom in this model, we introduce the collective (mesonic) operators,

$$\hat{\phi}_i[\bar{q},q]\equiv g\bar{q}\Gamma_i q\to\begin{cases}\hat{\sigma}, & i=1\ ,\\ \hat{\pi}, & i=2\ ,\end{cases} \quad (1.2)$$

where

$$\Gamma_1=1,\quad \Gamma_2=i\gamma_5\tau\ . \quad (1.3)$$

One can then derive an equivalent action from (1.1) as

$$I_{\mathrm{eff}}[\bar{q},q,\phi]=\int d^4x d^4y\left[\bar{q}(x)S^{-1}(x,y)q(y)+\frac{1}{2}\sum_{i=1}^{2}\phi_i(x)D^{-1}(x,y)\phi_i(y)\right]+g\int d^4x\bar{q}(x)[\Gamma_1\phi_1(x)+\Gamma_2\phi_2(x)]q(x)\ , \quad (1.4)$$

where

$$S^{-1}(x,y)=(i\gamma\cdot\partial-\hat{m})\delta^4(x-y)\ , \quad (1.5)$$

$$D^{-1}(x,y)=-\delta^4(x-y)\ . \quad (1.6)$$

Equation (1.4) possesses the same symmetry as that of (1.1). The ϕ (i.e., σ and π) fields in (1.4) appear like dynamical variables. However, Eq. (1.6) indicates that the corresponding "free" mesons cannot propagate and their dynamical properties are actually determined by quark-quark interactions in (1.1). The mesonic variables in this formalism serve as order parameters and characterize the chiral symmetry breaking. Their Green's functions, as we will study later, describe two-particle correlations which determine dynamics of collective $q\bar{q}$ models. For our goal, it is convenient to formulate our theory directly from (1.4).

After an introduction to the NJL model and our motivation for this paper, we present in Sec. II a general description of the kinetic theory of the NJL model by using the functional integral formalism of Schwinger and Keldysh's closed time-path Green's functions (CTPGF) [26,27] and Cornwall, Jackiw, and Tomboulis' (CJT) loop expansion technique of the functional integral approach

[28]. It is known that the CTPGF is a basic formalism for describing nonequilibrium phenomena [29–31] and the CJT loop expansion technique allows us to consider systematically the quantum correlations to any order in $\hbar$ and to describe naturally the phase transition of chiral symmetry breaking. In Sec. III, we discuss briefly the equilibrium limit obtained from the general kinetic theory of Sec. II. The result completely coincides with that obtained from Matsubara's temperature Green's function for the NJL mode. In our framework, the higher-order contributions can also be considered. In Sec. IV, we derive in detail the transport theory. The generalized relativistic Boltzmann equation with collision integrals is obtained from a quasiclassical approximation, and is consistent with the loop expansion approximation. The resulting Boltzmann equations of constituent quark distributions have the usual form of the Vlasov-Uehling-Uhlenbeck (VUU) equation, and determine the dynamical processes of collisions and the phase transition of chiral symmetry breaking in transverse dynamics of heavy-ion collisions. The Boltzmann equations for meson distributions describe the fluctuation properties of quark field and the mechanism of meson production during heavy-ion collisions. It is also a microscopic equation to study particularly the pion collective flow and pion spectrum, and the signature of chiral symmetry restoration. In Sec. V, we address the problem of seeking the signature of chiral symmetry and the mechanism of pion yield. Finally, summary and discussion are given in Sec. VI.

II. DYNAMICAL EQUATIONS OF MOTION

In this section, by using the CJT loop expansion approach [28] and the closed time-path Green's functions [26,27] (see [32] for technical details), we derive a set of self-consistent dynamical equations of motion for describing the time evolution of quark condensate and its collective excitations (i.e., mesons).

We start from the two-point source connected generating functional $W_p[J,K,M]$ of $I_{\rm eff}[\bar{q},q,\phi]$ which is defined as follows:

$$e^{iW_p[J,K,M]/\hbar}\equiv\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}(I_{\rm eff}[\bar{q},q,\phi]+J\phi+\bar{q}Kq+\tfrac{1}{2}\phi M\phi)\right]. \qquad (2.1)$$

Here, we retain $\hbar$ explicitly in our formalism. The subscript p represents a closed-time path (cf. Fig. 1). The

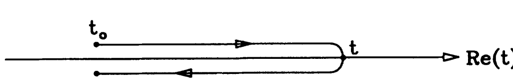

FIG. 1. The closed time-path contour for the evolution of operator expectation values in an arbitrary initial state. The time path goes from $-\infty$ to $+\infty$ (positive branch) and then returns back from $+\infty$ to $-\infty$ (negative branch). Any point at the negative branch is considered as a later instant than any time at the positive branch.

quantities $J_i(x)$, $K(x,y)$, and $M_i(x,y)$ are the one- and two-point external sources and are required to vanish at the final step of the procedure. Note that the source terms in Eq. (2.1) are also required to be Lorentz invariant and chiral symmetric in path integral formalism [33]. The generating functional, and so $W_p[J,K,M]$, possess then the same symmetries as the original theory. Furthermore, the summation of time-space degrees of freedom in the exponential of the right-hand side of (2.1) (and most of formula hereafter) is suppressed in the functional sense. They are defined, for example,

$$J\phi\equiv\sum_{i=1}^{2}\int_p d^4xJ_i(x)\phi_i(x)\,, \qquad (2.2)$$

$$\bar{q}Kq\equiv\int_p d^4xd^4y\bar{q}(x)K(x,y)q(y)\,, \qquad (2.3)$$

$$\phi M\phi\equiv\sum_{i=1}^{2}\int_p d^4xd^4y\phi_i(x)M_i(x,y)\phi_i(y)\,. \qquad (2.4)$$

The notation $\int_p d^4x$ denotes a four-dimensional time-space integration with the closed-time path p. In Eq. (2.1), we do not include one-point fermion source term since the condensation of fermion field is forbidden in the absence of the external source.

The vertex generating functional (or effective action) of irreducible Green's functions is then a functional Legendre transformation of $W_p[J,K,M]$ which is defined

$$\Gamma_p[\phi_c,G,\Delta]\equiv W_p[J,K,M]-J\phi_c+i\hbar{\rm Tr}[KG_p]-\tfrac{1}{2}{\rm Tr}[\phi_cM\phi_c+i\hbar M\Delta_p]\,, \qquad (2.5)$$

where the symbol Tr denotes trace over spin, flavor, and color; ϕ_{ic} is the expectation value of meson fields,

$$\phi_{ic}(x)=\frac{\delta W_p}{\delta J_i(x)}\,, \qquad (2.6)$$

and G_p and Δ_{ip} are the quark and meson CTPGF,

$$G_p(x,y)\equiv-\frac{i}{\hbar}\langle T_p(q(x)\bar{q}(y))\rangle\,, \qquad (2.7)$$

$$\Delta_{ip}(x,y)\equiv-\frac{i}{\hbar}[\langle T_p(\phi_i(x)\phi_i(y))\rangle-\phi_{ic}(x)\phi_{ic}(y)]\,, \qquad (2.8)$$

respectively. The notation T_p in (2.7)–(2.8) is the time-ordering operator along the path p. It is identical to the standard T operator on the positive branch $(-\infty,+\infty)$ and represents $\tilde{T}$, an anti-time-ordering operator, on the negative branch $(+\infty,-\infty)$. It is worth pointing out that the effective action retains, by definition of (2.5), all symmetries of the original NJL Lagrangian.

Equation (2.5) is in fact the generating functional in ϕ for the two-particle irreducible Green's function expressed in terms of G and Δ_i. A series expansion of Γ_p in orders of $\hbar$, namely, the loop expansion, is given as follows (details see the derivation given in the appendix or Ref. [28]):

$$\Gamma_p[\phi_c,G,\Delta]=I_{\rm eff}[\phi_c]+i\hbar{\rm Tr}[\ln[S_p^{-1}G_p]-G_{0p}^{-1}G_p+1]-\frac{i\hbar}{2}{\rm Tr}[\ln[D_p^{-1}\Delta_p]-\Delta_{0p}^{-1}\Delta_p+1]+\Gamma_{2p}[\phi_c,G,\Delta]\ , \tag{2.9}$$

where the first term is a classical action defined by

$$I_{\rm eff}[\phi_c]=I_{\rm eff}[\bar{q},q,\phi]|_{\bar{q}=q=0,\ \phi_i=\phi_{ic}}\ , \tag{2.10}$$

the second and third terms correspond to quark and meson one-loop contributions in which G_{0p} and Δ_{0p} are the quark and meson CTPGF modified by the mean field,

$$G_{0p}^{-1}(x,y)=S_p^{-1}(x,y)+g\Gamma\phi_c(x)\delta_p^4(x-y)\ , \tag{2.11}$$

$$\Delta_{0p}^{-1}(x,y)=D_p^{-1}(x,y) \tag{2.12}$$

and the last term $\Gamma_{2p}[\phi_c,G,\Delta]$ is, as shown in [28], the sum of all the two-particle-irreducible vacuum graphs of a theory with propagators G_p, Δ_p and vertices $g\Gamma_i$. It is of order $\hbar^2$ and can be easily evaluated by the usual Feynman-Dyson-Wick expansion. In the two-loop approximation (cf. Fig. 2), we find that

$$\Gamma_{2p}[\phi_c,G,\Delta]=-\frac{\hbar^2g^2}{2}\sum_{i=1}^{2}{\rm Tr}[\Gamma_iG_p\Gamma_iG_p\Delta_{ip}]\ . \tag{2.13}$$

The self-consistent dynamical equations of motion for describing quarks and their collective excitations are finally obtained by the variation of Γ_p with respect to its variables ϕ_c, G_p, and Δ_p and by switching off the external sources,

$$\frac{\delta\Gamma_p}{\delta\phi_{ic}(x)}=0\ , \tag{2.14}$$

$$-\frac{i}{\hbar}\frac{\delta\Gamma_p}{\delta G_p(y,x)}=G_p^{-1}(x,y)-G_{0p}^{-1}(x,y)+\Sigma_p(x,y)=0\ , \tag{2.15}$$

$$\frac{2i}{\hbar}\frac{\delta\Gamma_p}{\delta\Delta_{ip}(y,x)}=\Delta_{ip}^{-1}(x,y)-\Delta_{i0p}^{-1}(x,y)+\Pi_{ip}(x,y)=0\ , \tag{2.16}$$

where Σ_p and Π_{ip} are defined as proper self-energy parts of constituent quarks and mesons. We see that Eq. (2.14) describes the time evolution of the collective $q\bar{q}$ excited (mesonic) fields, and (2.15) and (2.16) are the familiar Dyson-Schwinger equations for quark and meson CTPGF. In the two-loop approximation, the quark and meson self-energies are given as follows:

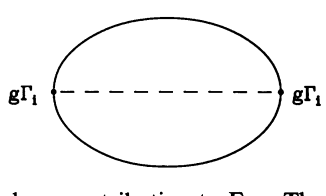

FIG. 2. Two-loop contribution to Γ_{2p}. The solid line is the quark propagator $\hbar G$, the dashed line the meson propagator $\hbar\Delta_i$, and $g\Gamma_i$ represents the vertex ($i=1,2$).

$$\Sigma_p(x,y)=ig^2\hbar\sum_{i=1}^{2}\Gamma_iG_p(x,y)\Gamma_i\Delta_{ip}(y,x)\ , \tag{2.17}$$

$$\Pi_{ip}(x,y)=-ig^2\hbar{\rm Tr}[\Gamma_iG_p(x,y)\Gamma_iG_p(y,x)]\ . \tag{2.18}$$

The above approach provides not only a succinct derivation of the quantum dynamical equations of field theory, but also the best way to describe critical dynamics of spontaneously symmetry breaking. The reason is that the order parameters which characterize the phase transition of spontaneously symmetry breaking are dynamical variables in the effective action.

For practical applications, it is convenient to reexpress the dynamical equations of motion, (2.14)–(2.16), in terms of advanced, retarded, and correlation Green's functions of the physical representation. To this end, we rewrite the CTPGF in terms of 2×2 matrices [32] inasmuch as x,y can assume values on either positive or negative time branches:

$$G_p(x,y)\to\hat{G}(x,y)=\begin{bmatrix}G^+ & G^<\\ G^> & G^-\end{bmatrix}\ , \tag{2.19}$$

$$\Delta_{ip}(x,y)\to\hat{\Delta}_i(x,y)=\begin{bmatrix}\Delta_i^+ & \Delta_i^<\\ \Delta_i^> & \Delta_i^-\end{bmatrix}\ . \tag{2.20}$$

The two-point functional matrix elements in Eqs. (2.19)–(2.20), denoted simply by $A^i(x,y)$ for both quarks and mesons, are defined in the single time axis [32]:

$$A^+(x,y)=-\frac{i}{\hbar}\langle T(\psi(x)\psi^\dagger(y))\rangle\ , \tag{2.21}$$

$$A^<(x,y)=\mp\frac{i}{\hbar}\langle\psi^\dagger(y)\psi(x)\rangle\ , \tag{2.22}$$

$$A^>(x,y)=-\frac{i}{\hbar}\langle\psi(x)\psi^\dagger(y)\rangle\ , \tag{2.23}$$

$$A^-(x,y)=-\frac{i}{\hbar}\langle\tilde{T}(\psi(x)\psi^\dagger(y))\rangle\ , \tag{2.24}$$

where A^+ is the usual Feynman causal propagator, A^- is called anticausal propagator, which is defined as expectation value of anti-time-ordering product, and $A^<$ and $A^>$ are correlation functions for $t_x<t_y$ and $t_x>t_y$, respectively. The upper (lower) sign in Eq. (2.22) is for mesons (quarks). In general, only three of these four two-point functions are independent. Thus, it turns out that the independent Green's functions, which are called physically the advanced, retarded, and correlation Green's functions, are obtained by the following transformation [32]:

$$\underline{A}(x,y)=\mathcal{T}\hat{A}(x,y)\mathcal{T}^{-1}\ , \tag{2.25}$$

where

$$\underline{A}(x,y)=\begin{bmatrix}0 & A_a(x,y)\\ A_r(x,y) & A_c(x,y)\end{bmatrix},\quad \mathcal{T}=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\ . \tag{2.26}$$

Note that the relation between $\underline{A}$ and $\hat{A}$ given by (2.25) is valid for any two-point function, such as self-energy functions $\Sigma(x,y)$ and $\Pi_i(x,y)$.

By using the above physical representation and the closed-time path Feynman rule given in Ref. [32], Eqs. (2.14)–(2.16) become:

$$\phi_{ic}(x)=-\frac{ig\hbar}{2}\mathrm{Tr}[\Gamma_i G_c(x,x)] , \quad (2.27)$$

$$G_r^{-1}(x,y)=G_{0r}^{-1}(x,y)-\Sigma_r(x,y) , \quad (2.28)$$

$$G_c(x,y)=-G_r(x,x_1)[G_{0c}^{-1}(x_1,y_1) -\Sigma_c(x_1,y_1)]G_a(y_1,y) , \quad (2.29)$$

$$\Delta_{ir}^{-1}(x,y)=\Delta_{i0r}^{-1}(x,y)-\Pi_{ir}(x,y) , \quad (2.30)$$

$$\Delta_{ic}(x,y)=-\Delta_{ir}(x,x_1)[\Delta_{i0c}^{-1}(x_1,y_1) -\Pi_{ic}(x_1,y_1)]\Delta_{ia}(y_1,y) , \quad (2.31)$$

and the equations for G_a and Δ_{ia} are the Hermitian conjugation of (2.28) and (2.30). This is a set of self-consistent dynamical equations of motion which describes various nonequilibrium processes and the effects on chiral symmetry in the NJL model. In particular, Eq. (2.27), called the generalized Landau-Ginzburg (LG) equation, determines the quark condensation of chiral symmetry breaking. Equations (2.28) and (2.30) describe the energy spectrum of quasiparticles (quarks and mesons) and their dissipation. Equations (2.29) and (2.31) are the quantum kinetic equations of particle density distributions which result in a transport theory of the quasiparticles in the quasiclassical approximation, as we will discuss in detail in Sec. IV.

In the two-loop expansion, the self-energy functions in (2.28)–(2.31) are obtained from Eqs. (2.17)–(2.18),

$$\Sigma_r(x,y)=\frac{ig^2\hbar}{2}[\Gamma_i G_r(x,y)\Gamma_i\Delta_{ic}(y,x)+\Gamma_i G_c(x,y)\Gamma_i\Delta_{ia}(y,x)] , \quad (2.32)$$

$$\Sigma_c(x,y)=\frac{ig^2\hbar}{2}\{\Gamma_i[G_r(x,y)-G_a(x,y)]\Gamma_i[\Delta_{ia}(y,x)-\Delta_{ir}(y,x)]+\Gamma_i G_c(x,y)\Gamma_i\Delta_{ic}(y,x)\} , \quad (2.33)$$

$$\Pi_{ir}(x,y)=-\frac{ig^2\hbar}{2}\mathrm{Tr}[\Gamma_i G_r(x,y)\Gamma_i G_c(y,x)+\Gamma_i G_c(x,y)\Gamma_i G_a(y,x)] , \quad (2.34)$$

$$\Pi_{ic}(x,y)=-\frac{ig^2\hbar}{2}\mathrm{Tr}\{\Gamma_i[G_r(x,y)-G_a(x,y)]\Gamma_i[G_a(y,x)-G_r(y,x)]+\Gamma_i G_c(x,y)\Gamma_i G_c(y,x)\} . \quad (2.35)$$

The physical implication of the above formulation is also very transparent. The Green's function G_{0p} [see the definition of (2.11)] represents the constituent quark Green's function. The relation between $G_{0p}(x,y)$ and $S_p(x,y)$, the free current quark Green's function, is given by

$$S_p^{-1}(x,y)=G_{0p}^{-1}(x,y)|_{\phi_{ic}=0} . \quad (2.36)$$

In the present case, $\phi_{2c}=0$ and ϕ_{1c} is proportional to the quark condensate (see the discussion in Sec. III). When $\phi_{1c}=0$, chiral symmetry is restored and the constituent quark propagator is reduced to the current quark propagator. This offers a picture of the phase transition from the constituent quark phase to the current quark phase in terms of chiral symmetry breaking and its restoration. The dressed propagator of (2.28) corresponds to the quasiconstituent quark, which includes also the higher order correlations, and we call it the "dynamical quark" hereafter. Furthermore, the quantity Δ_{0i} (being a δ function) indicates that the meson degrees of freedom are completely determined via the quark dynamics. From (1.2), we see that $\Delta_i(x,y)$ describes indeed two-quark Green's function,

$$\Delta_i(x,y)\equiv-\frac{i}{\hbar}[\langle T(\phi_i(x)\phi_i(y))\rangle-\phi_{ic}(x)\phi_{ic}(y)]$$
$$=-\frac{ig^2}{\hbar}[\langle T(\bar{q}(x)\Gamma_i q(x)\bar{q}(y)\Gamma_{iq}(y))\rangle -\langle\bar{q}(x)\Gamma_i q(x)\rangle\langle\bar{q}(y)\Gamma_i q(y)\rangle] . \quad (2.37)$$

It characterizes two-particle correlation via meson dynamics and therefore offers a consistent description of meson structure.

III. EQUILIBRIUM LIMIT

The equilibrium limit of the system can be obtained simply from the general formulation derived in the previous section. This is because the excitation densities are completely determined by the spectrum (propagator), temperature, and chemical potential. We perform the explicit calculation in momentum space and for convenience set $\hbar=1$ in the equilibrium calculation.

From (1.5), (1.6), (2.11), and (2.12), one can find that

$$G_{0r}^{-1}(p)=(\gamma\cdot p-M_H) , \quad (3.1)$$

$$G_{0c}(p)=-2\pi i[1-2n_q(p)](\gamma\cdot p+M_H)\delta(p^2-M_H^2) , \quad (3.2)$$

$$\Delta_{i0r}^{-1}(p)=-1 , \quad (3.3)$$

$$\Delta_{i0c}(p)=0 , \quad (3.4)$$

where

$$M_H=m-g\Gamma_i\phi_{ic} \quad (3.5)$$

is the constituent quark mass in the Hartree approximation and

$$\phi_{ic}=-\frac{ig}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\Gamma_i G_{0c}(p)] \quad (3.6)$$

is the LG equation. Chiral symmetry breaking and the critical point of phase transition are determined from this equation. In fact, Eqs. (3.1)–(3.6) are the self-consistent equations describing the constituent quark properties as a dynamical consequence of chiral symmetry breaking.

Using the equilibrium particle and antiparticle density distributions

$$n_q(E_{\rm p})=[e^{(E_{\rm p}-\mu)/T}+1]^{-1}\,,$$
$$\bar{n}_q(E_{\rm p})=[e^{(E_{\rm p}+\mu)/T}+1]^{-1}\,, \tag{3.7}$$

where T represents temperature, μ the chemical potential, and $E_{\rm p}=\sqrt{{\bf p}^2+M_H^2}$, the LG equation can be expressed explicitly as follows:

$$\phi_{1c}=-\frac{N_f N_c g}{\pi^2}M_H\int_0^{\Lambda}{\rm p}^2 d|{\rm p}|\frac{F(E_{\rm p})}{E_{\rm p}}\,, \tag{3.8}$$

$$\phi_{2c}=0\,. \tag{3.9}$$

In Eq. (3.8), we have defined $F(E_{\rm p})\equiv 1-n_q(E_{\rm p})-\bar{n}_q(E_{\rm p})$ and have introduced a noncovariant cutoff Λ in the momentum integration. It is also possible to introduce other cutoff schemes [18,19]. The LG equation clearly indicates why the quark condensate, $\langle\bar{q}q\rangle=\phi_{1c}/g$, is an order parameter which characterizes the chiral phase transition. The constituent quark mass is determined by the gap equation (3.5):

$$M_H=m-g\phi_{1c}$$
$$=m+\frac{N_f N_c g^2}{\pi^2}M_H\int_0^{\Lambda}{\rm p}^2 d|{\rm p}|\frac{F(E_{\rm p})}{E_{\rm p}}\,. \tag{3.10}$$

In the loop expansion, the above result is the relativistic Hartree solution (one-loop approximation) in which all higher-order quantum correlations are ignored by setting $\Gamma_{2p}=0$. Up to this order of approximation, we can only address the critical point of chiral phase transition. In order to describe collective excitations of quark field fluctuations which are identified with mesons, we must consider the next order, namely, at least the two-loop approximation for Γ_{2p}. It is then shown that a self-consistent description of dynamical quark and meson properties should be treated by solving simultaneously (2.28) and (2.30). In momentum space, they are

$$G_r^{-1}(p)=G_{0r}^{-1}(p)-\Sigma_r(p)\,, \tag{3.11}$$

$$\Delta_{ir}^{-1}(k)=-1-\Pi_{ir}(k)\,, \tag{3.12}$$

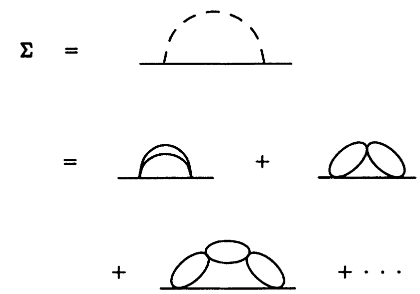

FIG. 3. The Feynman diagrams of quark propagators with higher-order contributions. The solid line is given by G (the dressed propagator).

where $\Sigma(p)$ and $\Pi_i(p)$ are given by

$$\Sigma_r(p)=\frac{ig^2}{2}\int\frac{d^4p'}{(2\pi)^4}\Gamma_i[G_r(p')\Gamma_i\Delta_{ic}(p'-p)$$
$$+G_c(p')\Gamma_i\Delta_{ia}(p'-p)]\,, \tag{3.13}$$

$$\Pi_{ir}(k)=-\frac{ig^2}{2}\int\frac{d^4p}{(2\pi)^4}{\rm Tr}[\Gamma_i G_r(p)\Gamma_i G_c(p-k)$$
$$+\Gamma_i G_c(p)\Gamma_i G_a(p-k)]\,, \tag{3.14}$$

and

$$G_c(p)=-2\pi i[1-2n_q(E_p)](\gamma\cdot p+M)\delta(p^2-M^2)\,, \tag{3.15}$$

$$\Delta_{ic}(k)=-2\pi i[1+2n_i(k)]\delta(k^2-m_i^2)\,. \tag{3.16}$$

The quantities M and m_i are the dynamical quark and meson masses determined by the poles of (3.11) and (3.12), and $n_i(k)(i=1,2)$ is the meson (σ and π) density distribution in the momentum space. Equations (3.11)–(3.16) allows one to estimate the contribution of higher-order correlations to the quark condensation and meson structure (see Fig. 3). However, to focus on the main goal of this paper, we address the solutions of these self-consistent equations elsewhere.

As an intermediate step between the one-loop and two-loop approximations, one may use the constituent quark Green's functions of (3.1)–(3.2) to describe meson dynamics. This corresponds to a random phase approximation (RPA), and yields an analytical solution of meson structure. By substituting (3.1) and (3.2) into (3.14), the real parts of (3.12) are then reduced to

$${\rm Re}\Delta_\sigma^{-1}(k)=-1+\frac{N_f N_c g^2}{4\pi^3}\int_0^{\Lambda}d^3{\rm p}\frac{F(E_{\rm p})}{E_{\rm p}}[1-(k^2-4M_R^2)I(k,{\rm p})]\,, \tag{3.17}$$

$${\rm Re}\Delta_\pi^{-1}(k)=-1+\frac{N_f N_c g^2}{4\pi^3}\int_0^{\Lambda}d^3{\rm p}\frac{F(E_{\rm p})}{E_{\rm p}}[1-k^2 I(k,{\rm p})]\,, \tag{3.18}$$

where

$$I(k,{\rm p})=\frac{1}{2E_{\rm p+k}}\left[\frac{E_{\rm p}+E_{\rm p+k}}{k_0^2-(E_{\rm p}+E_{\rm p+k})^2}-\frac{E_{\rm p}-E_{\rm p+k}}{k_0^2-(E_{\rm p}-E_{\rm p+k})^2}\right]\,. \tag{3.19}$$

The poles and residues of (3.17) and (3.18) determine the σ and π masses and their coupling constants to quarks, respectively. In the chiral limit ($m=0$), for example, combining with the gap equation, the pole of Eq. (3.17) is located at $k^2=4M_H^2$ so that $m_\sigma^2=4M_H^2$. The poles of the pseudoscalar meson (3.18) is located at $k^2=0$, which results in massless pions, i.e., Goldstone bosons. This is a standard solution of the Goldstone theorem via the spontaneously breaking of the SU(2)⊗SU(2) chiral symmetry to SU(2) symmetry. With a finite current quark mass, the chiral symmetry is explicitly broken and these pions become massive. The σ and π masses are then determined by the poles of (3.17) and (3.18) at the limit $\mathbf{k}\to 0$,

$$0=1-\frac{N_f N_c g^2}{\pi^2}\int_0^\Lambda \mathbf{p}^2 d|\mathbf{p}|\frac{F(E_\mathbf{p})}{E_\mathbf{p}}\left[1-\frac{m_\sigma^2-4M_\mathrm{H}^2}{m_\sigma^2-4E_\mathbf{p}^2}\right], \tag{3.20}$$

$$0=1-\frac{N_f N_c g^2}{\pi^2}\int_0^\Lambda \mathbf{p}^2 d|\mathbf{p}|\frac{F(E_\mathbf{p})}{E_\mathbf{p}}\left[1-\frac{m_\pi^2}{m_\pi^2-4E_\mathbf{p}^2}\right], \tag{3.21}$$

while the meson-quark coupling constants are the pole residues,

$$g_{\sigma qq}^2\equiv g^2(k^2-m_\sigma^2)\Delta_\sigma(k)\overset{\text{on shell}}{\longrightarrow} g^2\left[\frac{\partial\Delta_\sigma^{-1}}{\partial k^2}\right]^{-1}\Bigg|_{k^2=m_\sigma}, \tag{3.22}$$

$$g_{\pi qq}^2\equiv g^2(k^2-m_\pi^2)\Delta_\pi(k)\overset{\text{on shell}}{\longrightarrow} g^2\left[\frac{\partial\Delta_\pi^{-1}}{\partial k^2}\right]^{-1}\Bigg|_{k^2=m_\pi}. \tag{3.23}$$

The above discussion shows that the real-time calculation of the CTPGF reproduces exactly the finite temperature result of the NJL model obtained from Matsubara's temperature Green's functions in the equilibrium limit [17].

IV. TRANSPORT THEORY

We are now in the position to derive the transport theory of the NJL model. When the system lies in nonequilibrium states (e.g., in the dynamical processes of heavy-ion collisions), the propagation of quasiparticles (dynamical quarks and mesons) will dissipate. Furthermore, the density cannot be expressed *a priori* in terms of propagators as in the equilibrium case. We have to determine the energy spectra together with the kinetic equations of (2.29) and (2.31). In the quasiclassical limit discussed next, these kinetic equations are reduced to the generalized Boltzmann equations of transport theory.

A. Transport equations

The derivation of the transport equations in our study will be carried out by using the approach of Chou *et al.* [32].

First, we need to distinguish microscopic (relative) and macroscopic (center of mass) time-space scales by a Wigner transformation of any two-point function, $A(x,y)$, with respect to the relative coordinates

$$A(X,p)=\int d^4x' e^{ipx'/\hbar}A(X+x'/2,X-x'/2) \tag{4.1}$$

where $X=\frac{1}{2}(x+y)$, $x'=x-y$. We *assume* that the macroscopic variable is sufficiently slowly varying so that these two-point functions can be treated quasiclassically with the variation of X on the scale of the microscopic variable x'. This assumption is defined as a quasiclassical limit.

The Green's functions $\{G_{r(a)},\Delta_{ir(x)}\}$ can be separated into dispersive parts $\{D,d_i\}$ and dissipative parts $\{A,a_i\}$:

$$G_r^{-1}(X,p)=D(X,p)+iA(X,p),\quad G_a=G_r^*, \tag{4.2}$$

$$\Delta_{ir}^{-1}(X,k)=d_i(X,k)+ia_i(X,k),\quad \Delta_{ia}=\Delta_{ir}^*, \tag{4.3}$$

and $D(d_i)$ and $A(a_i)$ determine actually the kinematics and collision processes of the quark (mesons) in the transport theory, respectively. This will become clear if we rewrite the quantum kinetic equations (2.29) and (2.31) as

$$G_c(X,p)=(G_rN_q-N_qG_a)(X,p), \tag{4.4}$$

$$\Delta_{ic}(X,k)=(\Delta_{ir}N_i-N_i\Delta_{ia})(X,k), \tag{4.5}$$

where $N_{q,i}$ ($i=1,2$), which are related to quasiparticle distributions, must satisfy the following equations:

$$(N_qD-DN_q)(X,p)=i(N_qA+AN_q)(X,p)-(G_{0c}^{-1}-\Sigma_c)(X,p), \tag{4.6}$$

$$(N_id_i-d_iN_i)(X,k)=i(N_ia_i+a_iN_i)(X,k)-(\Delta_{i0c}^{-1}-\Pi_{ic})(X,k). \tag{4.7}$$

Note that the Wigner transformation of a product of any two-point functions, $\int A(x,z)B(z,y)dz$, is defined by the identity

$$(AB)(X,p)\equiv\int dx' e^{ipx'/\hbar}\int dz\,A(x,z)B(z,y)=A(X,p)e^{-i\hbar\overleftrightarrow{D}/2}B(X,p), \tag{4.8}$$

where

$$\overleftrightarrow{D}=\frac{\overleftarrow{\partial}}{\partial X}\frac{\overrightarrow{\partial}}{\partial p}-\frac{\overleftarrow{\partial}}{\partial p}\frac{\overrightarrow{\partial}}{\partial X}$$

[34]. Equations (4.6) and (4.7) are the quantum transport equations of constituent quarks and mesons, as we will see later. Explicitly, the lhs of (4.6) and (4.7) corresponds to the transport kinematics while the rhs determines the collision processes.

To find the explicit functions $N_{q,i}$, we further introduce the so-called quasiparticle limit, namely, that the dissipation of quasiparticles must be relatively small. In such a limit, the poles of G_r and Δ_{ir} are determined by the zeros of D and d_i,

$$D(X,p)=\gamma\cdot p-m+g\Gamma_i\phi_i(X)-\mathrm{Re}\Sigma_r(X,p)=0, \tag{4.9}$$

$$d_i(X,k)=-[1+\mathrm{Re}\Pi_{ir}(X,k)]=0, \tag{4.10}$$

respectively. The quasiparticle energy spectra of quark and meson are then obtained from the above equations as $p_0=p_0(X,\mathbf{p})$ and $k_{i0}=k_{i0}(X,\mathbf{k}_i)$.

The quasiparticle density distributions are directly related to the single-time Green's functions $G^{\lessgtr}$ and $\Delta_i^{\lessgtr}$ defined by (2.22) and (2.23). At the poles of G_r and Δ_{ir}, if the regularization of the quasiparticle wave functions is ignored [35], these Green's functions can be expressed as follows:

$$G^{\lessgtr}(X,p)=-2\pi i(\gamma\cdot p+M)\times[\theta(\mp p_0)-f_q(X,p)]\delta(p^2-M^2)\ , \tag{4.11}$$

$$\Delta_i^{\lessgtr}(X,k)=-2\pi i[\theta(\mp k_0)+f_i(X,k)]\delta(k^2-m_i^2)\ , \tag{4.12}$$

where $f_q(X,p)$ and $f_i(X,k)$ are the Wigner density distributions of the dynamical quark and mesons which are energy-dependent and especially spin-dependent for the quark field. Explicitly,

$$f_q(X,p)\delta(p^2-M^2)\equiv\frac{1}{2E}[f_q(X,\mathbf{p})\delta(p_0-E)+\tilde{f}_q(X,-\mathbf{p})\delta(p_0+E)]\ ,$$

where $E=\sqrt{\mathbf{p}^2+M^2}$, $f_q(X,\mathbf{p})$ denotes the quasiparticle distribution function and $\tilde{f}_q(X,\mathbf{p})$ the antiparticle distribution function. There is a similar relationship for $f_i(X,k)$. It should be pointed out that in Eqs. (4.11) and (4.12) we have ignored the quantum interference between particles and antiparticles and the mixing of spin components for quark field, as a consequence of slow variation of the macroscopic variables in the quasiclassical limit [36]. To simplify the derivation, we have also used the spin symmetric assumption so that the spinor product in the formulation is reduced to the projection operator $\gamma\cdot p+M$, and $f_q(X,p)$ becomes a scale function. This implies the loss of spin polarization effects. Strictly speaking, a scalar Wigner distribution function for the fermion field does not indeed exist and it is at least an 8×8 matrix representation as shown by Elze *et al.* [37].

Based on the above consideration, it follows that G_c and Δ_{ic} with the on-shell condition can be expressed as follows:

$$G_c(X,p)=(G_r[1-2f_q]-[1-2f_q]G_a)(X,p)\ , \tag{4.13}$$

$$\Delta_{ic}(X,k)=(\Delta_{ir}[1+2f_i]-[1+2f_i]\Delta_{ia})(X,k)\ . \tag{4.14}$$

This set of relationships is called the nonequilibrium stationary state fluctuation-dissipation theorem [32]. Comparing with (4.4) and (4.5) and (4.14) and (4.14), we find that

$$N_{q,i}=1\mp2f_{q,i}\ , \tag{4.15}$$

where the upper (lower) sign is for quarks (mesons). Furthermore, the restriction to on-mass-shell results in the vanishing of G_{0c} and Δ_{i0c} when quantum correlations (Σ and Π) are considered [38]. Thus, by substituting (4.15) into (4.6) and (4.7) and using the fact that $A(X,p)=(1/2i)[\Sigma^<(X,p)-\Sigma^>(X,p)]$ and similarly for a_i, it turns out that

$$(f_qD-Df_q)(X,p)=-\tfrac{1}{2}((1-f_q)\Sigma^<+\Sigma^<(1-f_q)+f_q\Sigma^>+\Sigma^>f_q)(X,p)\ , \tag{4.16}$$

$$(f_id_i-d_if_i)(X,k)=\tfrac{1}{2}((1+f_i)\Pi_i^<+\Pi_i^<(1+f_i)-f_i\Pi_i^>-\Pi_i^>f_i)(X,k)\ . \tag{4.17}$$

Equations (4.16) and (4.17) are the *quasiparticle transport equations.* In the two-loop approximation, the self-energies are determined by Eqs. (2.17) and (2.18):

$$\Sigma^{\lessgtr}(X,p)=ig^2\hbar\int\frac{d^4p'd^4k}{(2\pi)^8}(2\pi)^4\delta^4(p'-k-p)[\Sigma_iG^{\lessgtr}(X,p')\Gamma_i\Delta_i^{\gtrless}(X,k)]\ , \tag{4.18}$$

$$\Pi_i^{\lessgtr}(X,k)=-ig^2\hbar\int\frac{d^4p_1d^4p_2}{(2\pi)^8}(2\pi)^4\delta^4(p_1-p_2-k)\mathrm{Tr}[\Gamma_iG^{\lessgtr}(X,p_1)\Gamma_iG^{\gtrless}(X,p_2)]\ . \tag{4.19}$$

Finally, the separation of the microscopic and macroscopic time-space scales ensures that we can handle the macroscopic variables quasiclassically. By expanding both sides of the transport equation in terms of $\hbar$, the lowest-order nonzero terms on the rhs of (4.16) and (4.17) correspond to collision terms which are order of $\hbar$ [see Eqs. (4.18) and (4.19)]

$$-\{[1-f_q(X,p)]\Sigma^<(X,p)+f_q(X,p)\Sigma^>(X,p)\}+O(\hbar^3)\ , \tag{4.20}$$

$$\{[1+f_i(X,k)]\Pi_i^<(X,k)-f_i(X,k)\Pi_i^>(X,k)\}+O(\hbar^3)\ , \tag{4.21}$$

while the lowest-order nonzero terms on the lhs result in the classical Poisson brackets, which are also proportional to $\hbar$ [34],

$$(f_q D - D f_q)(X,p) = -i\hbar \left[\frac{\partial f(X,p)}{\partial X_\mu}\frac{\partial D(X,p)}{\partial p^\mu} - \frac{\partial f(X,p)}{\partial p_\mu}\frac{\partial D(X,p)}{\partial X^\mu} \right] + O(\hbar^3)$$

$$= -i\hbar \frac{\partial D}{\partial p_0}\bigg|_{\text{on shell}} \left[\frac{\partial f_q}{\partial t} + \mathbf{v}_q \cdot \nabla_X f_q + \frac{\partial p_0}{\partial x_\mu}\frac{\partial f_q}{\partial p^\mu} \right] + O(\hbar^3) , \tag{4.22}$$

$$(f_i d_i - d_i f_i)(X,k) = -i\hbar \left[\frac{\partial f_i(X,k)}{\partial X_\mu}\frac{\partial d_i(X,k)}{\partial k^\mu} - \frac{\partial f_i(X,k)}{\partial k_\mu}\frac{\partial d_i(X,k)}{\partial X^\mu} \right] + O(\hbar^3)$$

$$= -i\hbar \frac{\partial d_i}{\partial k_0}\bigg|_{\text{on shell}} \left[\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_X f_i + \frac{\partial k_{i0}}{\partial X_\mu}\frac{\partial f_i}{\partial k^\mu} \right] + O(\hbar^3) , \tag{4.23}$$

In Eqs. (4.22)–(4.13), the constituent quark and meson velocities are given by

$$\mathbf{v}_q = \frac{\partial p_o}{\partial \mathbf{p}}, \quad \mathbf{v}_i = \frac{\partial k_{i0}}{\partial \mathbf{k}} , \tag{4.24}$$

where the spectrum p_0 and k_{i0} are determined by Eqs. (4.9) and (4.10). Thus, in the quasiclassical limit, the lowest-order transport equations are the generalized *relativistic Boltzmann equations*

$$\frac{\partial f_q}{\partial t} + \mathbf{v}_q \cdot \nabla_X f_q + \frac{\partial p_0}{\partial X_\mu}\frac{\partial f_q}{\partial p^\mu} = I^q_{\text{colli}} , \tag{4.25}$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_X f_i + \frac{\partial k_{i0}}{\partial X_\mu}\frac{\partial f_i}{\partial k^\mu} = I^i_{\text{colli}} . \tag{4.26}$$

In Eqs. (4.25) and (4.26), the collision integrals $I^{i(q)}_{\text{colli}}$ are determined as follows: I^q_{colli} is a trace over spin of (4.20) multiplied by D and divided by $(i/\hbar)\mathrm{Tr}_s[D\cdot(\partial D/\partial p_0)]$, while I^i_{colli} is more simple, being just (4.21) divided by $(i/\hbar)(\partial d_i/\partial k_{i0})$.

Several remarks should be clarified for (4.25) and (4.26) before we explicitly derive the collision integrals.

(1) If we multiply by p_0 and k_{i0} to the two sides of (4.25) and (4.26), respectively, the transport equations become covariant. However, the form of (4.25) and (4.26) is practically convenient.

(2) The above derivation of the Boltzmann equations with collision integrals is obtained self-consistently from the quasiclassical approach *plus* the loop expansion method. The inconsistency in the usual derivation of the quantum transport equations with collision terms, namely, the inconsistency between the quasiclassical approximation to order $\hbar$ in the kinetic term and the perturbative approximation to order of the coupling constants in the collision terms, is eliminated. The quasiparticle energy spectra (p_0,k_{i0}) are determined from (4.9) and (4.10) by using the same order approximation as that of determining the collision terms. The transport equations now do not depend explicitly on $\hbar$ to this order. Without any loss of generality, we can reset $\hbar=1$ hereafter.

(3) Once the collision terms are ignored, the Poisson brackets must be zero in the same sense of the quasiclassical limit. This leads to a relativistic quark Vlasov equation in quantum kinetic theory and coincides with the exact derivation from the relativistic Hartree approximation. In this case, the spectra (p_0,k_{i0}) are determined by the one-loop approximation in which meson structure disappears. This can be seen directly from Eqs. (4.10) and (4.24) that when we set $\Pi_{ir}=0$ the meson velocity $\mathbf{v}_i=0$, as we mentioned in the introduction. Thus, the meson dynamics in heavy-ion collisions, such as pion collective flows, pion spectrum, and pion production, are only manifested beyond the Hartree approximation in the NJL model.

To explore the dynamical mechanism of particle production, we need further to derive the explicit forms of the collision integrals.

B. Collision integrals

Collision integrals come from the dissipation of quasiparticles with vacuum polarization. To study the physical processes in these collision terms it is useful to find their explicit forms by substituting (4.18) and (4.19) into (4.20) and (4.21).

First, we calculate the meson collision terms. From (2.22) and (2.23) and (4.12), we find that $\bar{f}_i(X,\mathbf{k})=f_i(X,\mathbf{k})$ $(i=1,2,3)$. Thus, we need only to consider the case of $k_0>0$. It involves two physical processes: one is particle-hole excitations and the other corresponds to creation and annihilation of dynamical quark-antiquark pairs (see Fig. 4). It is well known that these processes cannot occur in vacuum under the on-shell condition. However, as we will discuss in the next section, some of the processes take place indeed in medium. The contributions of such processes to the meson collision terms are

$$I^i_{\text{colli}}(X,\mathbf{k}) = I^i_{qq\omega}(X,\mathbf{k}) + I^i_{q\bar{q}\omega}(X,\mathbf{k}) , \tag{4.27}$$

where

$$I^i_{qq\omega}(X,\mathbf{k})=\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6} N_f N_c M^i_{qq\omega}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k})(2\pi)^3\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{k}) \times([1+f_i(X,\mathbf{k})][f_q(X,\mathbf{p}_1)(1-f_q(X,\mathbf{p}_2)]+\bar{f}_q(X,\mathbf{p}_1)[1-\bar{f}_q(X,\mathbf{p}_2)] -f_i(X,\mathbf{k})\{[1-f_q(X,\mathbf{p}_1)]f_q(X,\mathbf{p}_2)+[1-\bar{f}_q(X,\mathbf{p}_1)]\bar{f}_q(X,\mathbf{p}_2)\}) , \quad (4.28)$$

$$I^i_{q\bar{q}\omega}(X,\mathbf{k})=\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6} N_f N_c M^i_{q\bar{q}\omega}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k})(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}) \times\{[1+f_i(X,\mathbf{k})]f_q(X,\mathbf{p}_1)\bar{f}_q(X,\mathbf{p}_2)-f_i(X,\mathbf{k})[1-f_q(X,\mathbf{p}_1)][1-\bar{f}_q(X,\mathbf{p}_2)]\} , \quad (4.29)$$

and

$$M^i_{qq\omega}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k})=\frac{2\pi g^2}{8\omega_i E_1 E_2}\delta(E_1-E_2-\omega_i)\mathrm{Tr}_s[\Gamma_i(\gamma\cdot p_1+M)\Gamma_i(\gamma\cdot p_2+M)] =\frac{2\pi g^2 M^2}{2\omega_i E_1 E_2}\delta(E_1-E_2-\omega_i)\sum_{s_1,s_2}\bar{u}(p_1,s_1)\Gamma_i u(p_2,s_2)\bar{u}(p_2,s_2)\Gamma_i u(p_1,s_1) , \quad (4.30)$$

$$M^i_{q\bar{q}\omega}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k})=\frac{2\pi g^2}{8\omega_i E_1 E_2}\delta(E_1+E_2-\omega_i)\mathrm{Tr}_s[\Gamma_i(\gamma\cdot p_1+M)\Gamma_i(\gamma\cdot p_2-M)] =\frac{2\pi g^2 M^2}{2\omega_i E_1 E_2}\delta(E_1+E_2-\omega_i)\sum_{s_1,s_2}\bar{u}(p_1,s_1)\Gamma_i v(p_2,s_2)\bar{v}(p_2,s_2)\Gamma_i u(p_1,s_1) , \quad (4.31)$$

are the probabilities of these two quasiparticle scattering processes. Here, we have taken the absolute values of quark and meson energy spectra, $E=|p_0(X,\mathbf{p})|$ and $\omega_i=|k_{i0}(X,\mathbf{k})|$. We have also rewritten the projection operator in terms of products of the quasiparticle Dirac's spinors, u and v. Note that in order to recover the regularization of quasiparticle wave functions, the bare coupling constant in (4.30) and (4.31), g^2, should be replaced by the effective on-shell coupling constants of (3.22) and (3.23), $g^2_{iqq}(k^2=m_i^2)$.

The collision integral for the quark transport equation is derived in the same way. For particle distributions ($p_0>0$), the quark collision term contains the similar processes as shown in Fig. 4:

$$I^q_{\text{colli}}(X,\mathbf{p})=I^q_{qq\omega}(X,\mathbf{p})+I^q_{q\bar{q}\omega}(X,\mathbf{p}) , \quad (4.32)$$

where

$$I^q_{qq\omega}(X,\mathbf{p})=\sum_{i=1}^{2}\int \frac{d^3\mathbf{p}' d^3\mathbf{k}}{(2\pi)^6}\frac{1}{2}M^i_{qq\omega}(\mathbf{p},\mathbf{p}',\mathbf{k})(2\pi)^3\delta(\mathbf{p}'-\mathbf{k}-\mathbf{p}) \times\{[1-f_q(X,\mathbf{p})]f_q(X,\mathbf{p}')-f_q(X,\mathbf{p})[1-f_q(X,\mathbf{p}')]\}[1+f_i(X,\mathbf{k})+f_i(X,-\mathbf{k})] , \quad (4.33)$$

$$I^q_{q\bar{q}\omega}(X,\mathbf{p})=\sum_{i=1}^{2}\int \frac{d^3\mathbf{p}' d^3\mathbf{k}}{(2\pi)^6}\frac{1}{2}M^i_{q\bar{q}\omega}(\mathbf{p},\mathbf{p}',\mathbf{k})(2\pi)^3\delta(\mathbf{k}-\mathbf{p}'-\mathbf{p}) \times\{[1-f_q(X,\mathbf{p})][1-\bar{f}_q(X,\mathbf{p}')]f_i(X,\mathbf{k})-f_q(X,\mathbf{p})\bar{f}_q(X,\mathbf{p}')[1+f_i(X,\mathbf{k})]\} . \quad (4.34)$$

The collision term for antiparticles ($p_0<0$) can also be obtained easily in this way.

In fact, the above collision integrals involve various basic collision contributions, i.e., quark-quark and quark-antiquark collisions in the NJL model. To see clearly the elementary collision processes, we eliminate the meson field so that the quark transport equation includes only quark degrees of freedom.

Using the kinetic equation of (2.31), one can find

$$\Delta_i^{\lessgtr}=(1+\Pi_{ir})^{-1}\Pi_i^{\lessgtr}(1+\Pi_{ia})^{-1}=\Pi_i^{\lessgtr}+\cdots . \quad (4.35)$$

The first term in the second equality of (4.35) represents the Born approximation. Ignoring higher-order correlation terms in (4.33), one immediately obtains from (4.16) that

$$\Sigma^{\lessgtr}(X,p)=g^4\int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{12}}(2\pi)^4\delta(p_1+p_2-p_3-p)\{\Gamma_i G^{\lessgtr}(X,p_1)\Gamma_i \mathrm{Tr}[\Gamma_i G^{\gtrless}(X,p_3)\Gamma_i G^{\lessgtr}(X,p_2)]\} . \quad (4.36)$$

Hence, the quark collision term in the Born approximation becomes

$$I^q_{\text{colli}}(X,\mathbf{p})=I_{qq}(X,\mathbf{p})+I_{q\bar{q}}(X,\mathbf{p}) , \quad (4.37)$$

where the quark-quark collision terms is given by

$$I_{qq}(X,\mathbf{p})=\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}N_fN_c|M_{qq}|^2(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p})$$
$$\times\{[1-f_q(X,\mathbf{p})]f_q(X,\mathbf{p}_1)[1-f_q(X,\mathbf{p}_3)]f_q(X,\mathbf{p}_2)-f_q(X,\mathbf{p})[1-f_q(X,\mathbf{p}_1)]f_q(X,\mathbf{p}_3)[1-f_q(X,\mathbf{p}_2)]\}\quad(4.38)$$

and the quark-antiquark collision term is

$$I_{q\bar{q}}(X,\mathbf{p})=\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}2N_fN_c|M_{q\bar{q}}|^2(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p})$$
$$\times\{[1-f_q(X,\mathbf{p})]f_q(X,\mathbf{p}_1)[1-\bar{f}_q(X,\mathbf{p}_3)]\bar{f}_q(X,\mathbf{p}_2)-f_q(X,\mathbf{p})[1-f_q(X,\mathbf{p}_1)]\bar{f}_q(X,\mathbf{p}_3)[1-\bar{f}_q(X,\mathbf{p}_2)]\}\;.\quad(4.39)$$

The quantities M_{qq} and $M_{q\bar{q}}$ are quark-quark and quark-antiquark scattering amplitudes:

$$M_{qq}=2\pi\delta(E_1+E_2-E_3-E)\frac{g^2}{2}\sqrt{M^4/E_1E_2E_3E}$$
$$\times\sum_{i,s_1,s_2,s_3,s}[\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p},s)\bar{u}(\mathbf{p}_2,s_2)\Gamma_iu(\mathbf{p}_3,s_3)-\bar{u}(\mathbf{p}_2,s_2)\Gamma_iu(\mathbf{p},s)\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p}_3,s_3)]\quad(4.40)$$

$$M_{q\bar{q}}=2\pi\delta(E_1+E_2-E_3-E)\frac{g^2}{2}\sqrt{M^4/E_1E_2E_3E}$$
$$\times\sum_{i,s_1,s_2,s_3,s}[\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p},s)\bar{v}(\mathbf{p}_3,s_3)\Gamma_iv(\mathbf{p}_2,s_2)-\bar{u}(\mathbf{p}_1,s_1)\Gamma_iv(\mathbf{p}_2,s_2)\bar{v}(\mathbf{p}_3,s_3)\Gamma_iu(\mathbf{p},s)]\;.\quad(4.41)$$

Equation (4.23) with Eqs. (4.35)–(4.37) is the relativistic VUU equation for the dynamical quark distribution. The collision integrals correspond to the usual lowest order two-particle collisions. The further consideration of higher-order terms in (4.33) will provide more realistic processes which include the quark-quark and quark-antiquark T matrix. If we neglect the same order (i.e., the correlation terms) in (4.33) as we have done in (4.18), the collision integrals keep the same form as (4.36) and (4.37) but the scattering amplitudes M_{qq} and $M_{q\bar{q}}$ are changed,

$$M_{qq}=2\pi\delta(E_1+E_2-E_3-E)\frac{g^2}{2}\sqrt{M^4/E_1E_2E_3E}$$
$$\times\sum_{i,s_1,s_2,s_3,s}[\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p},s)\Delta_{ir}(X,p-p_1)\bar{u}(\mathbf{p}_2,s_2)\Gamma_iu(\mathbf{p}_3,s_3)$$
$$-\bar{u}(\mathbf{p}_2,s_2)\Gamma_iu(\mathbf{p},s)\Delta_{ir}(X,p-p_1)\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p}_3,s_3)]\quad(4.42)$$

$$M_{q\bar{q}}=2\pi\delta(E_1+E_2-E_3-E)\frac{g^2}{2}\sqrt{M^4/E_1E_2E_3E}$$
$$\times\sum_{i,s_1,s_2,s_3,s}[\bar{u}(\mathbf{p}_1,s_1)\Gamma_iu(\mathbf{p},s)\Delta_{ir}(X,p+p_1)\bar{v}(\mathbf{p}_3,s_3)\Gamma_iv(\mathbf{p}_2,s_2)$$
$$-\bar{u}(\mathbf{p}_1,s_1)\Gamma_iv(\mathbf{p}_2,s_2)\Delta_{ir}(X,p+p_1)\bar{v}(\mathbf{p}_3,s_3)\Gamma_iu(\mathbf{p},s)]\;,\quad(4.43)$$

where Δ_{ir} is given by a form similar to (3.17) and (3.18) in the RPA calculation. They correspond to the processes of Fig. 5(a) and (b) and represent the long-wavelength effect of quark field fluctuations. The above discussion indicates that the VUU-type relativistic equations can be obtained only for models in which the meson degrees of freedom are not the fundamental variables. Meanwhile, meson dynamics is manifested from the fermion loop effect.

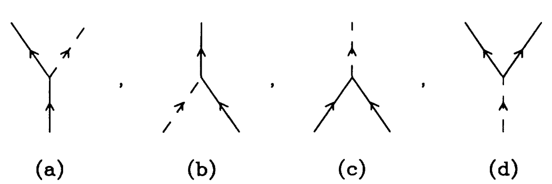

FIG. 4. An illustration of particle producing processes in heavy-ion collisions based on the two-loop approximation of the NJL model. (a) and (b) are the particle-hole excitations and (c) and (d) the $q\bar{q}$ pair annihilation and production.

C. A practical calculation scheme

Now, we can outline a practical calculation scheme for solving the transport equations of the NJL model.

(1) The separation of quark transport equations from the collective excitation (meson) transport equations is practically useful. It will allow us to solve the VUU-type transport equation solely for the quark density distribution. One can then use such quark distributions to solve the meson transport equation and to explore the dynam-

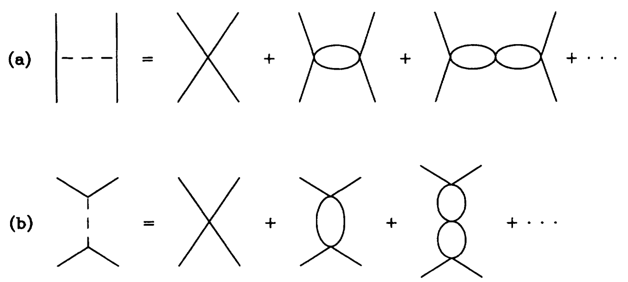

FIG. 5. Within the NJL model, by eliminating the collective meson degrees of freedom, the processes of Fig. 4(a)–(d) can be reduced to the collision processes given by the quark-quark and quark-antiquark collisions (a) and (b) here, respectively.

ics of meson production and meson spectrum. Of course, such a separation is not necessary if one could solve numerically the set of coupled equations (4.23) and (3.24) [39].

(2) We also need to specify the relation between the transition matrices (or cross sections) of the processes of Fig. 5 and the data of NN and $N\bar{N}$ scattering to parametrize the transition matrices in the collision integrals. The transport equations can then be solved numerically utilizing any of several approaches, e.g., the test particle approach [40], the Chapman-Enskog expansion, and various polynomial expansion methods [41], etc.

(3) As a first approximation, the quark energy spectrum may be determined in the relativistic Hartree approximation. The meson energy spectrum is determined by the RPA. The particles in collision terms are given by the same approximation. Such a consideration is not self-consistent in terms of the two-loop approximation but it is consistent in the sense that the Goldstone theorem is rigorously satisfied in the chiral limit ($m \to 0$) in this model. The advantage is that it is easy to perform numerical calculations and to explore the essence of the phenomena.

(4) A complete calculation within two-loop approximation is as follows. One should first solve the energy spectrum p_0 from the self-energy diagrams of Fig. 3. Then one can solve numerically the quark transport equation (4.23) with collision terms (4.34)–(4.37). Finally, one uses p_0 and the quark distribution to determine k_{i0} and to solve the meson transport equation (4.24). In this last step (4), although we have obtained the formulation, much work needs to be done before the numerical calculation can be performed.

V. APPLICATION TO HEAVY-ION COLLISIONS

The transport theory derived in the previous section is valid for a variety of nonequilibrium quark systems, e.g., the early Universe, the interior of neutron stars, and heavy-ion collisions for certain energy regimes in the quasiclassical limit. As a first application, we consider high-energy heavy-ion collisions. A nucleus with A nucleons is regarded as a $3A$ constituent quark system. Thus, the nuclear density is one-third of the quark density,

$$\rho_q = \frac{N_f N_c}{3\pi^2} k_F^3 = 3\rho_B \ , \tag{5.1}$$

where k_F is the Fermi momentum in the cold ground state. With this assumption, we now apply the theory to nucleus-nucleus collisions.

Two of the most important properties for studying the nuclear equation of state via heavy-ion collisions are pion dynamics and signature of chiral symmetry. These are the main topics we shall discuss in this section.

A. Effect of chiral symmetry in transport dynamics

In Sec. III, it is shown that in the equilibrium case, chiral symmetry breaking is characterized by the quark condensate, $\langle \bar{q}q \rangle = \phi_{1c}/g$. Numerical calculations [17–19] indicate that there is a phase transition related to the chiral symmetry breaking and its restoration. In terms of nuclear density and/or temperature, one finds that chiral symmetry is restored in the NJL model when the density is larger than several (over three) times the normal nuclear matter density or a temperature $T \sim 200$ MeV. The critical point depends on high-momentum cutoff. Consider the chiral limit ($m=0$). The critical density or temperature is then obtained by the gap equation (3.10) with the constituent quark mass $M=0$. Two parameters, the coupling constant $g^2/2$ and the cutoff Λ must be fixed before we perform a practical calculation. These two parameters can be determined to reproduce the pion mass ($m_\pi = 139$ MeV) and the pion decay constant ($f_\pi = 93$ MeV) in the vacuum with zero temperature. For instance, for a noninvariant cutoff ($|\mathbf{p}| = \Lambda$), we find that $\Lambda \approx 820$ MeV and $g^2\Lambda^2 \approx 4$ with current quark mass $m_u = m_d = m \approx 4$ MeV.

To look for the effect of this chiral phase transition in heavy-ion collisions, we first apply the Hartree approximation, the same approximation for determining quark condensate from (3.8) or the gap equation from (3.10), to the transport equations. The generalized Boltzmann equations are then reduced to a quark Vlasov equation,

$$\frac{\partial f_q}{\partial t} + \mathbf{v}_q \cdot \nabla_X f_q - \frac{\partial E}{\partial \mathbf{X}} \cdot \frac{\partial f_q}{\partial \mathbf{p}} = 0 \ , \tag{5.2}$$

where

$$\mathbf{v}_q = \mathbf{p}/E, \quad E = \sqrt{\mathbf{p}^2 + M_H^2} \ , \tag{5.3}$$

and M_H is determined by the gap equation

$$M_H(X) = m + \frac{N_f N_c g^2}{4\pi^3} M_H(X) \times \int_0^\Lambda d^3\mathbf{p} \frac{1}{E} [1 - f_q(X,\mathbf{p}) - \bar{f}_q(X,\mathbf{p})] = m - g^2 \langle \bar{q}q \rangle \ . \tag{5.4}$$

The Vlasov equation provides a connection between heavy-ion collision dynamics and the equation of state through, e.g., transverse flow. It can be solved usually by the test-particle method [40]. In this approach, quarks satisfy the classical equations of motion

$$\dot{\mathbf{X}}=\mathbf{v}_q=\mathbf{p}/E \ , \tag{5.5}$$

$$\dot{\mathbf{p}}=-\nabla_{\mathbf{X}}E=g^2(M_H/E)\nabla_{\mathbf{x}}\langle\bar{q}q\rangle \ . \tag{5.6}$$

These equations of motion show that the Vlasov dynamics is controlled by the quark condensate, and therefore the possible signature of chiral phase transition may be manifested in the transverse flow.

However, for a more realistic calculation in the Hartree limit, one may also need to include vector mesons to reproduce nuclear matter properties. There is no difficulty to include explicitly such an effect within the NJL model by considering a Fierz transformation, as we have mentioned in the Introduction. An effective way is to directly add an exchange interaction term with chiral symmetry into (1.1) [18,19],

$$\mathcal{L}_v=-\frac{g_v^2}{2}[(\bar{q}\gamma_\mu q)^2+(\bar{q}\gamma_\mu\tau q)^2+(\bar{q}\gamma_5\gamma_\mu\tau q)^2] \ , \tag{5.7}$$

where g_v is another coupling constant. With the Lagrangian $\mathcal{L}'=\mathcal{L}+\mathcal{L}_v$, the formulation obtained in the previous sections is kept in the same form except for a shift of the constituent quark momentum-energy vector,

$$\mathbf{p}\to\mathbf{p}^*=\mathbf{p}-g_v^2\frac{N_fN_c}{4\pi^3}\int d^3\mathbf{p}\frac{\mathbf{p}^*}{E^*}[f(X,\mathbf{p}^*)-\bar{f}(X,\mathbf{p}^*)] \ , \tag{5.8}$$

$$\begin{aligned}E\to E^*&=\sqrt{\mathbf{p}^{*2}+M_H^2}\\&=E-g_v^2\frac{N_fN_c}{4\pi^3}\int d^3\mathbf{p}[f(X,\mathbf{p}^*)-\bar{f}(X,\mathbf{p}^*)] \ ,\end{aligned} \tag{5.9}$$

and an extension of the index to $i=1,\ldots,5$ with

$$\Gamma_3^\mu=\gamma_\mu, \quad \Gamma_4^\mu=\gamma_\mu\tau, \quad \Gamma_5^\mu=\gamma_5\gamma_\mu\tau \ . \tag{5.10}$$

By including explicitly the exchange term in the NJL model, we see that the Vlasov equation has a form similar to the equation obtained in the Walecka model [4,8]. The difference is that in the NJL model, the constituent quark mass is a dynamical consequence of chiral symmetry breaking. Thus, the Vlasov dynamics in this formalism provides a way to explore the transverse dynamics from the fundamental symmetry. Of course, it is worth warning that when the exchange interaction is included, the chiral symmetry effect may become ambiguous due to the competition in dominating the Vlasov dynamics between the scalar and vector $q\bar{q}$ condensates, where the latter is only a control parameter which does not characterize the chiral phase transition.

On the other hand, the Vlasov dynamics corresponds to the mean-field limit which only determines the critical point of phase transition but cannot describe the critical dynamics. The dynamical effect of the chiral phase transition must come from higher-order quark correlations. In our theory, the meson dynamics describes the fluctuations of the mean field as an effect of two-particle correlation. Thus, a further exploration of chiral symmetry effect is needed to study meson production and its transportation in heavy-ion collisions.

B. Dynamics of meson production

The current study of meson production in hadronic descriptions is based on the process $NN\to N\Delta\to NN\pi$. In the low-energy region, it is certainly the main process of pion yield. However, it is hard to justify that such a process is still dominant in high-energy collisions. Theoretically, however, a recent derivation of transport equation [13] shows that if one does not introduce explicitly the Δ as an elementary particle [9,10], the collision integrals do not include such a process in the quasiparticle and quasiclassical approximations. In our study, the collision integrals derived in the previous section show that meson production is an effect of $q\bar{q}$ collective excitations. They come from the dissipation of quasiparticles (i.e., the imaginary part of the quasiparticle retarded Green's functions). In our two-loop approximation, the particle production processes are shown in Figs. 4(a)–(d). These processes correspond to simple particle-hole excitations, and pair production and annihilation. Neither of these processes can occur *in vacuum* due to the restriction of on-mass-shell energy and momentum conservation in the approximation. However, as we shall show, some of these processes can take place at high density (and/or high temperature). This is because in the NJL model all the mesons, which are consistently described as collective $q\bar{q}$ excitations of two-particle correlations, depend intimately on density and/or temperature. Hence, the kinematically forbidden ones will be removed in high density and/or high temperature, and these processes become in fact very important in high energy heavy-ion collisions.

As an illustration we focus on pion dynamics. In order to understand pion dynamics in heavy-ion collisions, it is useful to study the structure of pion propagation in the medium [42]. Using RPA in the NJL model, the pion dispersion relation (energy spectrum) is determined by the equation

$$\Delta_\pi^{-1}(X,k)=-1+\frac{N_fN_cg^2}{4\pi^3}\int_0^\Lambda d^3\mathbf{p}\frac{1}{E_\mathbf{p}}[1-f_q(X,\mathbf{p})-\bar{f}_q(X,\mathbf{p})][1-k^2I(k,\mathbf{p})] \ , \tag{5.11}$$

which is the same equation as (3.18) and $I(k,\mathbf{p})$ is given by (3.19), except that the density distribution is position dependence due to the finiteness of the nuclei. In principle, a self-consistent numerical study must be performed together with the transport equation of quark distribution functions. To understand pion propagation in medium in the NJL model, however, it may be helpful to consider a limiting case, i.e., the uniform matter. The numerical solution of the pion dispersion relation determined by (5.11) is shown in Fig. 6.

The δ functions in the collision terms (4.26)–(4.29), namely, energy-momentum conservation, indicate that $q\bar{q}$ pair annihilation occurs in the timelike region ($k^2=k_0^2-\mathbf{k}^2>0$) and the particle-hole excitations take place in the spacelike region ($k^2<0$). One can see from Fig. 6 that in the above approximation, all the dispersion curves lie in the timelike region. This implies that particle-hole excitations cannot occur, even at high density. Pair annihilation is also forbidden in the region bounded by $E_{2q}=\sqrt{\mathbf{k}^2+4M_H^2}$ and $E=|\mathbf{k}|$. Thus, in low density, all processes in Fig. 4 are forbidden, as is well known. However, with increasing density, the effective quark mass decreases and the pion mass increases as a consequence of $q\bar{q}$ condensation (see Fig. 7). The pion dispersion curve then moves out from the forbidden region and pion production via $q\bar{q}$ annihilation becomes possible [e.g., Fig. 6(c)]. Furthermore, due to the Pauli principle, the energy threshold of pion production is given by $E_c=2E_F$, where $E_F=\sqrt{k_F^2+M_H^2}$ and k_F is the Fermi momentum. Therefore, such a process is only possible at quite high density (close to the critical point of chiral symmetry restoration) with certain momentum transfers. Furthermore, pions produced at high density via pair annihilation are different from the physical (vacuum) mesons. They are indeed the resonances embedded in the $q\bar{q}$ continuum. Physically, they are the main source for producing other particles in heavy-ion collisions, such as dileptons and photons, as a decay process ($\pi^+\pi^-\to e^+e^-$, $\pi^0\to 2\gamma$) [43].

In fact, the particles produced from the processes of Fig. 4 can only be observed indirectly. That is, with the above mechanism, a huge number of $q\bar{q}$ states, which correspond generally to various mesoniclike modes, are excited when the collision system reaches a high matter density. Most of them will decay back to quarks and antiquarks during the collisions. However, after collisions the density in the collision region rapidly decreases, but still many $q\bar{q}$ resonances remain. Due to the nonobservability of quarks, these $q\bar{q}$ resonances cannot decay back to

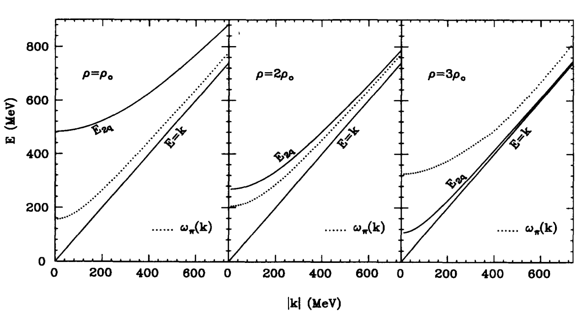

FIG. 6. The pion dispersion curves in the medium with different densities, where $\rho_0=1.4\ \mathrm{fm}^{-1}$ is the normal nuclear matter density. The line $E=|\mathbf{k}|$ is the boundary of spacelike and timelike regions. $E_{2q}=\sqrt{\mathbf{k}^2+4M_H^2}$. In this calculation, the model parameters are taken as follows, $\Lambda=820$ MeV, $g^2\Lambda^2=4$ and $m=4$ MeV.

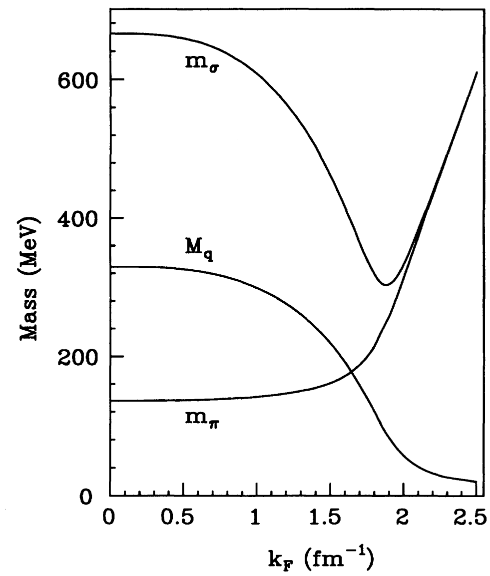

FIG. 7. The effective constituent quark and σ and π mass as a function of nuclear density calculated by the RPA. The parameters Λ, g^2, and m are taken as the same as in Fig. 6. The critical density in the case of nonzero current quark mass is defined as the point at which the σ and π become degenerate.

quarks and antiquarks within this low density collision region after the collisions. Thus, they have to decay to various physical mesons. The decay channels include

$$q\bar{q}\ \text{excitations}\to\begin{cases}\pi^+\pi^-\to e^+e^-\ ,\\ \pi^0\to 2\gamma\ ,\\ \rho\to\pi\pi\ ,\\ \sigma\to\pi\pi\ ,\end{cases}\tag{5.12}$$

etc. The decay rates are determined by the imaginary part of the retarded meson Green's functions of (4.3) which is density dependent via $\langle\bar{q}q\rangle$ condensate or constituent quark mass. This is a way of manifesting chiral dynamics from particle production in high-energy heavy-ion collisions.

The above mechanism of particle production is based on the NJL model in the lowest-order approximation of two-particle correlations. A direct process of meson production may correspond to the diagram as shown in Fig. 8 [44] which can be obtained from the next order of the factorization of (4.35). A quantitative description of this process can be found in [45]. Nevertheless, the picture of particle production via $q\bar{q}$ excitations presented here in

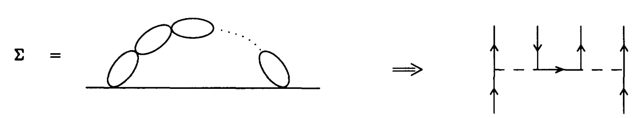

FIG. 8. A higher-order collision process that is embedded in Fig. (3) and is related to $q\bar{q}$ pair production. Note that this process is obtained from Fig. 3 by breaking two quark lines.

the NJL model is consistent with the spirit of QCD. The cardinal point of this mechanism comes from the fact that the negative-energy sea has been regularized through a high-momentum cutoff in the NJL model, and the medium dependence of quark and meson dynamics has been considered consistently in this formalism. These produced particles manifest the fluctuations of quark dynamics around the critical point of chiral symmetry breaking. Since the critical density (about four times the normal nuclear matter density in our calculation) is much lower than the corresponding density of momentum cutoff (about 25 to 30 times the normal nuclear matter density for a cutoff of about 1 GeV), the uncertainty in the NJL model calculations caused by the cutoff should still be small near the chiral critical density. Meanwhile, one expects that deconfinement (if its exists) should occur later than the chiral symmetry restoration in the cold matter [46]. Thus, around the chiral critical point, gluon degrees of freedom may still be frozen and the NJL model may still work well in the intermediate region between confinement and asymptotic freedom regions. Furthermore, the medium modification of quark and meson dispersion relations discussed in this paper is also consistent with the conclusion of the sigma model [44] and the recent calculations of the QCD sum rule [47]. This indicates that the medium dependence of $q\bar{q}$ excitations may be a model-independent consequence of chiral dynamics. The above mechanism therefore could provide an effective way to explore the signature of chiral symmetry restoration. In the realistic world, whether color deconfinement may destroy the above picture is questionable. Further theoretical and experimental justifications are needed. However, since one does not know at the moment how to formulate the confinement mechanism in any $(3+1)$-dimensional field theory and thereby define a clear signature of deconfinement in experiments, the mechanism discussed here can be helpful in understanding the underlying hadronic structure from heavy-ion collisions.

VI. SUMMARY AND DISCUSSIONS

We have developed a microscopic transport theory for heavy-ion collisions from the quark version of the NJL model. Medium effects of chiral symmetry are considered consistently in the Vlasov dynamics and collision processes. The theory describes dynamically chiral symmetry and meson production, the two most important features for studying strong interactions and the nuclear equation of state via high-energy heavy-ion collisions. In the two-loop approximation, the formalism has been reduced to a form amenable to present numerical methods for heavy-ion collisions. Of course, we do not expect that the theory can be applied directly to study the dynamics of the quark-gluon plasma at ultrarelativistic energies. This is because the NJL model is not suitable in this case.

The framework used is a real-time Green's function calculation with closed time-path (CTPGF). This offers a unified description of equilibrium and nonequilibrium systems. It is particularly useful to study a system with spontaneous symmetry breaking and phase transition because the order parameters enter the theory as dynamical variables. The formulation also provides a clear picture to describe hadronic dynamics in terms of constituent quarks and mesons. The loop-expansion approach is used to derive the transport equations. In such a derivation, the generalized Boltzmann equations with collision terms are obtained. Both the quasiparticle energy spectra and the collision integrals are determined at the same order of approximation.

The problem of the chiral symmetry signature and pion production in heavy-ion collisions have been addressed specifically. As we have seen, the quark condensate, which measures the chiral symmetry breaking, plays a very important role in the Vlasov dynamics and thus in the transverse flow. Although the behavior of transverse flow in certain energy regimes in heavy-ion collisions can be described quite well by some phenomenological models, such as density-dependent potential models or the transport equation of the Walecka model, our result can help us to understand the underlying features of these medium-dependent effects of strong interactions. Pion production is described by $q\bar{q}$ excitations. Such a description offers a consistent way to study pion dynamics in heavy-ion collisions. We find that pion production via particle-hole excitations cannot take place. The possible mechanism of pion production in high energy heavy-ion collisions is antiquark (or $q\bar{q}$ pair) excitations. These excitations correspond to various mesonic modes. Physically, they finally decay to pions, dileptons, and photons. Thus, relativistic effects change not only the Vlasov dynamics but also the mechanism of particle production in our theory. Such a picture has not been described in current hadronic descriptions. All quantitative investigations of pion production are based on the process of $NN \rightarrow N\Delta \rightarrow NN\pi$ in which the theory must explicitly include the Δ as an elementary particle [9,10].

It has to be said that although the transport theory of the NJL model describes consistently the chiral phase transition and mesonic properties, we must assume that the nucleon is three constituent quarks when we apply the theory to high-energy heavy-ion collision dynamics. A careful test of the validity of the assumption in heavy-ion collisions is to study the nuclear matter properties within the NJL model, which is in progress in our two-loop approximation [48]. The dynamical interpretation of this assumption, namely confinement in QCD, is a cardinal problem in hadronic physics. An original motivation of relativistic heavy-ion collisions was to explore the mechanism of confinement/deconfinement. Lattice calculations indicate that deconfinement and chiral symmetry restoration may be related [49]. Hence, a dynamically self-consistent description of the chiral phase transition and meson production could be useful for our understanding of this fundamental problem.

It is also worth noting that the approach we used in this paper can be applied directly to nonrelativistic and relativistic hadronic models for low- and intermediate-energy heavy-ion collisions. It is particularly interesting to apply the present formalism to the sigma model in which the chiral symmetry can also be addressed [50]. However, from our systematic derivation one sees that the transport equations of relativistic hadronic models cannot be reduced to the form of the usual nonrelativistic VUU equation because the free meson propagators in these relativistic hadronic models cannot be simplified by δ functions. A similar conclusion has also been obtained recently by Davis and Perry [13], namely, that in order to have a relativistic VUU equation the meson fields should remain in local equilibrium states.

This paper clearly leaves many questions unanswered. An extension of the theory by including explicitly kaons (with $N_f=3$) is certainly of interest for studying kaon production in heavy-ion collisions; it is straightforward in our framework. A detailed numerical investigation of the present theory in heavy-ion collisions could provide some information from direct measurements about chiral phase transition and meson dynamics. Also, the application of the present theory to the early Universe and the interior of neutron stars in which a hot and/or dense quark matter may be formed is worth exploring. Furthermore, the formulation of the quark confinement and the incorporation of chiral symmetry in transport theory is mostly attractive. Using the present approach to derive the QCD transport theory and thereby explore the dynamics of the quark-gluon plasma at ultrarelativistic energies is the ultimate goal. These are several problems for the future.

ACKNOWLEDGMENTS

The authors thank Professor Che Ming Ko, Professor Berndt Mueller, Professor Tetsuo Hatsuda, Professor Ernest M. Henley, Professor P. J. Siemens, and Professor Da Hsuan Feng for useful discussions and valuable comments. This work is supported by the U.S. Department of Energy.

APPENDIX: CJT LOOP EXPANSION

In this appendix, we apply the CJT loop-expansion technique to the NJL model. As was pointed out in Ref. [32], the only difference between the CTPGF and the ordinary Green's functions is the range of time axis. Hence the loop-expansion technique for the vertex functional (effective action) developed by CJT [28] in quantum field theory can be extended directly to the CTPGF formalism by taking into account properly the difference in the definition of the time axis.

According to CJT, the required series of $\Gamma_p[\phi_c,G,\Delta]$ for the NJL model in the loop expansion should have the following form:

$$\begin{aligned}\Gamma_p[\phi_c,G,\Delta]=&I_{\rm eff}[\phi_c]+i\hbar\,{\rm Tr}\ln G_p-i\hbar\,{\rm Tr}G_{0p}^{-1}G_p\\&-\frac{i\hbar}{2}{\rm Tr}\ln\Delta_p+\frac{i\hbar}{2}\Delta_{0p}^{-1}\Delta_p\\&+\Gamma_{2p}[\phi_c,G,\Delta]+{\rm const}\ ,\end{aligned}\tag{A1}$$

which can reproduce the Dyson-Schwinger equations when one takes the variation of $\Gamma_p[\phi_c,G,\Delta]$ with respect to G_p and Δ_{ip}. In Eq. (A1),

$$\begin{aligned}G_{0p}^{-1}&=-\frac{\delta^2 I_{\rm eff}[\bar q,q,\phi]}{\delta\bar q\delta q}\bigg|_{\bar q=q=0,\phi_i=\phi_{ic}}\\&=S_p^{-1}(x,y)+g\Gamma\phi_c(x)\delta_p^4(x-y)\ ,\end{aligned}\tag{A2}$$

$$\Delta_{i0p}^{-1}=\frac{\delta^2 I_{\rm eff}[\bar q,Q,\phi]}{\delta^2\phi_i}\bigg|_{\bar q=q=0,\phi_i=\phi_{ic}}=D_p^{-1}(x,y)\ .\tag{A3}$$

On the other hand, $\Gamma_p[\phi_c,G,\Delta]$ should coincide with the conventional effective action [51] at $K=M=0$,

$$\begin{aligned}\Gamma_p[\phi_c]=\Gamma_p[\phi_c]&=\Gamma_p[\phi_c,G_{0p},\Delta_{0p}]\\&=I_{\rm eff}[\phi_c]+i\hbar\,{\rm Tr}\ln G_{0p}\\&\quad-\frac{i\hbar}{2}{\rm Tr}\ln\Delta_{0p}+O(\hbar^2)\\&=I_{\rm eff}[\phi_{cj}]+\Gamma_{1p}[\phi_c]\ .\end{aligned}\tag{A4}$$

where Γ_{1p} is of order $\hbar$. The constant in (A1) must then be $(i/2)\hbar\,{\rm Tr}1$. Furthermore, if the normalization factor is included explicitly in (2.1),

$$\begin{aligned}\mathcal{N}&=\int_p\mathcal{D}[\bar q,q,\phi]\exp\left[\frac{i}{\hbar}[\bar q S_p^{-1}q+\tfrac{1}{2}\phi D_p^{-1}\phi]\right]\\&=[\det S_p][\det D_p]^{-1/2}=\exp({\rm Tr}\ln S_p-\tfrac{1}{2}{\rm Tr}\ln D_p)\ ,\end{aligned}\tag{A5}$$

the vertex functional of (A1) finally becomes (2.9).

The cardinal point of the expansion of $\Gamma_p[\phi_c,G,\Delta]$ to all orders in $\hbar$ is to prove that $\Gamma_{2p}[\phi_c,G,\Delta]$ is the sum of all two-particle irreducible vacuum diagrams constructed by the theory governed by action $I_{\rm eff}[\bar q,q,\phi]$ with G and Δ as its propagators. The two-particle irreducible graphs are defined as those connected diagrams which remain connected when any two arbitrary internal lines are cut. Since $\Gamma_p[\phi_c,G,\Delta]$ is the generating functional in ϕ_c for two-particle irreducible n-point functions for $I_{\rm eff}[\bar q,q,\phi]$ with lines $i\hbar G$ and $i\hbar\Delta$, it follows that $\Gamma_p[0,G,\Delta]$ is the sum of two-particle irreducible vacuum graphs of the same theory. From the definition of (2.5), we have

$$K=\frac{1}{i\hbar}\frac{\delta\Gamma_p}{\delta G_p},\quad M=-\frac{2}{i\hbar}\frac{\delta\Gamma_p}{i\hbar\delta\Delta_p}\ ,\tag{A6}$$

and hence

$$\Gamma_p[0,G,\Delta]=\mathrm{Tr}G_p\frac{\delta\Gamma_p[0,G,\Delta]}{\delta G_p}+\mathrm{Tr}\Delta_p\frac{\delta\Gamma_p[0,G,\Delta]}{\delta\Delta_p}$$
$$-i\hbar\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}\left[\bar{q}S_p^{-1}q+\tfrac{1}{2}\phi D_p^{-1}\phi+g\bar{q}\Gamma\phi q+\phi J^0-\frac{i}{\hbar}\bar{q}\frac{\delta\Gamma_p[0,G,\Delta]}{\delta G_p}q+\frac{i}{\hbar}\phi\frac{\delta\Gamma_p[0,G,\Delta]}{\delta\Delta_p}\phi\right]\right]$$
$$+i\hbar\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}(\bar{q}S_p^{-1}q+\tfrac{1}{2}\phi D_p^{-1}\phi)\right] \quad \text{(A7)}$$

is equal to the sum of two-particle irreducible vacuum graphs of a theory governed by I_{eff} with lines $\hbar G$ and $\hbar\Delta$, where the last term in the first equality comes from the normalization factor of (A5) and J^0 is that value of J which makes Eq. (2.6) vanish. Now we can prove that $\Gamma_{2p}[\phi_c,G,\Delta]$ satisfies the same equation as (A5). We rewrite $\Gamma_p(\phi_c,G,\Delta)$ as

$$\Gamma_p[\phi_c,G,\Delta]=\Gamma_p^{K,M}[\phi_c]-\frac{1}{2}\phi_c M\phi_c+i\hbar\mathrm{Tr}GK-\frac{i\hbar}{2}\mathrm{Tr}\Delta M\ , \quad \text{(A8)}$$

where

$$\Gamma_p^{K,M}[\phi_c]=W_p[J,K,M]-J\phi \quad \text{(A9)}$$

is an effective action for a theory governed by classical action

$$I^{K,M}[\bar{q},q,\phi]=I_{\mathrm{eff}}[\bar{q},q,\phi]+\bar{q}Kq+\tfrac{1}{2}\phi M\phi\ . \quad \text{(A10)}$$

The conventional loop expansion [51] shows that

$$\Gamma_p^{K,M}[\phi_c]=I^{K,M}[\phi_c]+\Gamma_{1p}^{K,M}[\phi_c]\ , \quad \text{(A11)}$$

$$\Gamma_{1p}^{K,M}[\phi_c]=-i\hbar\ln\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}\left[\bar{q}(G_{0p}^{-1}+K)q+\frac{1}{2}\phi(\Delta_{0p}^{-1}+M)\phi+g\bar{q}\Gamma\phi q-\phi\frac{\delta\Gamma_{1p}[\phi_c]}{\delta\phi_c}\right]\right]\ . \quad \text{(A12)}$$

Substituting (A10) and (A11) into (A1),

$$\Gamma_{2p}[\phi_c,G,\Delta]+\mathrm{const}=-i\hbar\mathrm{Tr}\ln G_p+\frac{i\hbar}{2}\mathrm{Tr}\ln\Delta_p+i\hbar\mathrm{Tr}(G_{0p}^{-1}+K)G_p-\frac{i\hbar}{2}\mathrm{Tr}(\Delta_{0p}^{-1}+M)\Delta_p+\Gamma_{1p}^{K,M}[\phi_c]\ . \quad \text{(A13)}$$

By eliminating K and M in the above equation from the explicit expressions of (A6),

$$K=G_p^{-1}-G_{0p}^{-1}-\frac{i}{\hbar}\frac{\delta\Gamma_{2p}}{\delta G_p},\quad M=\Delta_p^{-1}-\Delta_{0p}^{-1}-\frac{i}{2\hbar}\frac{\delta\Gamma_{2p}}{\delta\Delta_p}\ , \quad \text{(A14)}$$

Eq. (A13) becomes

$$\Gamma_{2p}[\phi_c,G,\Delta]=\mathrm{Tr}G_p\frac{\delta\Gamma_p[\phi_c,G,\Delta]}{\delta G_p}+\mathrm{Tr}\Delta_p\frac{\delta\Gamma_p[\phi_c,G,\Delta]}{\delta\Delta_p}$$
$$-i\hbar\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}\left[\bar{q}G_p^{-1}q+\frac{1}{2}\phi\Delta_p^{-1}\phi+g\bar{q}\Gamma\phi q\right.\right.$$
$$\left.\left.+\phi\frac{\partial\Gamma_{2p}[\phi_c,G,\Delta]}{\partial\phi_c}-\frac{i}{\hbar}\bar{q}\frac{\delta\Gamma_{2p}[\phi_c,G,\Delta]}{\delta G_p}q+\frac{i}{\hbar}\phi\frac{\delta\Gamma_{2p}[\phi_c,G,\Delta]}{\delta\Delta_p}\phi\right]\right]$$
$$+i\hbar\int_p\mathcal{D}[\bar{q},q,\phi]\exp\left[\frac{i}{\hbar}\left[\bar{q}G_p^{-1}q+\frac{1}{2}\phi\Delta_p^{-1}\phi\right]\right]\ . \quad \text{(A15)}$$

Comparison with (A4) shows precisely that $\Gamma_{2p}[\phi_c,G,\Delta]$ is the sum of two-particle irreducible vacuum graphs of the theory governed by the classical action $I_{\mathrm{eff}}[\bar{q},q,\phi]$ with propagators G and Δ.

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