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Quark and gluon condensates in nuclear matter

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Quark and gluon condensates in nuclear matter are studied. These in-medium condensates may be linked to a wide range of nuclear phenomena and are important inputs to QCD sum-rule calculations at finite density. The Hellmann-Feynman theorem yields a prediction of the quark condensate that is model independent to first order in the nucleon density. This linear density dependence, with slope determined by the nucleon σ term, implies that the quark condensate is reduced considerably at nuclear matter saturation density—it is roughly 25–50% smaller than the vacuum value. The trace anomaly and the Hellmann-Feynman theorem lead to a prediction of the gluon condensate that is model independent to first order in the nucleon density. At nuclear matter saturation density, the gluon condensate is about 5% smaller than the vacuum value. Contributions to the in-medium quark condensate that are of higher order in the nucleon density are estimated with mean-field quark-matter calculations using the Nambu–Jona-Lasinio and Gell-Mann–Lévy models. Treatments of nuclear matter based on hadronic degrees of freedom are also considered, and the uncertainties are discussed.

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I. INTRODUCTION

Since hadrons are excitations of the vacuum, hadronic properties are ultimately related to properties of the vacuum. The degree to which one must understand the nature of the vacuum is not clear; a full solution of quantum chromodynamics (QCD) may be needed for a complete understanding of the strong-interaction properties of hadrons. However, to determine the spectral properties of many hadrons, it may be sufficient to characterize the vacuum in terms of a small number of parameters—the quark and gluon condensates. These condensates are expectation values of local composite operators such as $\bar{q}q$ and $G^a_{\mu\nu}G^{a\mu\nu}$, where q is an up or down quark field and $G^a_{\mu\nu}$ is the gluon field-strength tensor. The QCD sum-rule approach [1] is based on this possibility, and it has proved to be a useful tool in understanding the properties of hadrons in free space [2]. The vacuum values of the lowest-dimensional quark and gluon condensates used in sum-rule calculations are [2,3]

$$\langle \bar{q}q \rangle_{\text{vac}} \simeq -(225\pm 25\ \text{MeV})^3 , \tag{1.1}$$

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a\mu\nu} \right\rangle_{\text{vac}} \simeq (360\pm 20\ \text{MeV})^4 . \tag{1.2}$$

There has been recent interest in describing the properties of hadrons in nuclear matter in terms of *in-medium* quark and gluon condensates, which are shifted from these vacuum values.

In this paper, we discuss the quark and gluon condensates in the ground state of nuclear matter. We develop simple expressions for these condensates that are model independent to first order in the nucleon density. These model-independent results are therefore valid at sufficiently low nucleon densities. The Hellmann-Feynman theorem relates the shift of the quark condensate from its vacuum value to the nucleon σ term and the up and down current quark masses. Similarly, the trace anomaly and the Hellmann-Feynman theorem relate the shift of the gluon condensate from its vacuum value to the nucleon mass, the σ term, and the strangeness content of the nucleon. Depending on the precise value of the σ term, we find that the quark condensate at nuclear matter saturation density is roughly 25–50% smaller than the vacuum value. Neglecting the strangeness content of the nucleon, we find that the gluon condensate at nuclear matter saturation density is approximately 6% smaller than the vacuum value. Calculations assuming a large strangeness content of the nucleon, which is estimated through the use of SU(3)-flavor symmetry and first-order chiral perturbation theory, indicate that the gluon condensate is about 3–6% smaller than the vacuum value at nuclear matter saturation density.

To estimate corrections to the quark condensate due to terms of higher order in the nucleon density, we use the Nambu–Jona-Lasinio (NJL) and Gell-Mann–Lévy (GML) models. These models are natural candidates for such a study since they are chiral quark models, and the key physics of the in-medium quark condensate is partial chiral restoration. At the mean-field level, these models suggest that corrections to the model-independent prediction of the quark condensate are fairly small ($\sim 10\%$) up to nuclear matter saturation density. We also consider the in-medium condensate using models of nuclear matter based on hadronic degrees of freedom. We first perform a simple nonrelativistic calculation in which uncorrelated nucleons interact through one-pion exchange.

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This model predicts deviations from the model-independent result that are small ($\sim 10\%$) at nuclear matter saturation density. We also discuss more complete nonrelativistic calculations that include correlation effects and relativistic mean-field calculations based on quantum hadrodynamics (QHD).

Medium-modified condensates may be linked to a diverse range of nuclear phenomena. Such work has primarily concentrated on partial chiral restoration in the nuclear medium, i.e., a medium-modified quark condensate. It has been suggested that partial chiral restoration could lead to a reduction of the proton-neutron mass difference, which might account for the Nolen-Schiffer anomaly [5–7]. In addition, it has been proposed that partial chiral restoration could lead to a reduction in vector-meson masses, which might account for "anomalies" in K^+-nucleus scattering data [8], the suppression of the electromagnetic longitudinal response functions in electron-scattering experiments [9], and the enhancement of the ρNN tensor coupling in the nuclear medium [10].

The modification of the gluon condensate in a hadron might be connected to quark confinement. In the color dielectric model of Nielson and Patkós [11], confinement is governed by a color dielectric function that vanishes far from the hadron. In the hadron, however, the dielectric function is nonvanishing, and this change might be associated with a change in the gluon condensate in the hadron. In order to use this model to study corrections to the model-independent prediction of the in-medium gluon condensate, one must have a quantitative understanding of the connection between the dielectric function and the change in the gluon condensate. Unfortunately, this connection is only understood qualitatively at best.

A natural and direct use for in-medium condensates is in QCD sum-rule calculations of hadronic properties in nuclear matter. This approach can be used to predict in-medium spectral properties (e.g., effective masses and self-energies) of baryons and mesons. Recent work on the spectral properties of nucleons in nuclear matter suggests a connection between QCD and the phenomenology of relativistic nuclear physics [12]. Elements of nuclear-structure physics such as nuclear matter saturation [13] and the Nolen-Schiffer anomaly [6,7] are also being explored. In the QCD sum rules, one considers a correlation function of interpolating fields, built from quark fields, that carry the quantum numbers of the hadron of interest. By applying an operator product expansion for large spacelike momentum transfer, the correlator can be expressed as a sum of Wilson coefficients, calculated in QCD perturbation theory, that multiply the expectation values of composite operators. In the vacuum, the lowest-dimensional nonvanishing expectation values are the simple quark and gluon condensates, $\langle \bar{q}q \rangle_{\text{vac}}$ and $\langle (\alpha_s/\pi) G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\text{vac}}$. In nuclear matter, these condensates still play a major role; therefore, the estimation of the in-medium condensates is central to the application of QCD sum rules to finite-density nuclear systems.

In Refs. [8–10], all mass scales in medium are assumed to change with density in the same manner. Accordingly, the in-medium quark condensate is assumed to be related to the in-medium nucleon mass by the following scaling law [14]:

$$\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} \simeq \left[\frac{M_N^*}{M_N} \right]^3 , \tag{1.3}$$

where $\langle \bar{q}q \rangle_{\text{vac}}$ and $\langle \bar{q}q \rangle_{\rho_N}$ are the vacuum and in-medium quark condensates, and M_N and M_N^* are the mass and the in-medium effective mass of the nucleon. Similar relations are assumed to hold for other hadrons. However, the connection between M_N^* and observables is not clear—identifying M_N^* with one of the many effective nucleon masses used in many-body physics [15] is hard to justify. Moreover, the proposed scaling law itself is questionable. If one defines the effective mass to be the Lorentz-scalar part of the effective nucleon self-energy, then recent QCD sum-rule calculations [12] suggest that the effective nucleon mass scales as $\langle \bar{q}q \rangle_{\rho_N}$ and not as $\langle \bar{q}q \rangle_{\rho_N}^{1/3}$. We prefer a more concrete approach. In this paper, we connect the in-medium quark condensate to the σ term and the current quark masses rather than to the effective mass of the nucleon in nuclear matter.

Some of our results have been discussed in previous work. Drukarev and Levin [13] have discussed the model-independent predictions for the in-medium quark and gluon condensates at low densities, although not in the context of the Hellmann-Feynman theorem. Corrections to the quark condensate were calculated in Ref. [13] based on Pauli blocking of the pionic contribution to the σ term; this is equivalent to the pion Fock term calculation considered here. The NJL model has also been utilized to study chiral restoration at finite density [16–18]; however, the NJL calculations presented here differ from those of Refs. [16–18] in the method of choosing the free parameters of the model. In Ref. [16], the free parameters are chosen to fix the pion-decay constant and the quark condensate in vacuum. In Ref. [17], explicit chiral-symmetry-breaking terms are included in the model, and the free parameters are chosen to fix the pion-decay constant and the pion mass. In Ref. [18], the free parameters are chosen to fix the pion-decay constant and the constituent quark mass. Fixing the free parameters by one of these prescriptions leads to an in-medium quark condensate that depends in detail on the physics of the NJL model.

Our goals are more modest—we take advantage of the model-independent prediction of the in-medium quark condensate, and constrain the NJL model so that it reproduces this prediction at low densities. Thus we use the NJL model only to estimate *corrections* to the linear result. The model-independent prediction gives the rate of chiral restoration at low densities, which depends on a combination of the pion-decay constant, the pion mass, and the σ term. It is not obvious that the physics of the NJL model is sufficiently realistic to reproduce this combination of parameters based on fits to other observables; therefore, we choose the free parameters of the NJL model in order to fix the rate of chiral restoration directly. We choose the free parameters in the GML model in a like manner.

This paper is organized as follows. In Secs. II and III, we consider model-independent calculations of the in-medium quark and gluon condensates, respectively. The model independence extends to first order in the nucleon density; thus these results should be valid for sufficiently low nucleon densities. In Sec. IV, we consider corrections to the model-independent description of the in-medium quark condensate using quark-model calculations, and in Sec. V we consider these same corrections with hadronic models. Section VI is a summary.

II. IN-MEDIUM QUARK CONDENSATE: MODEL-INDEPENDENT CALCULATIONS

We first consider the in-medium quark condensate. We develop an expression for this condensate that is model independent to first order in the nucleon density by applying the Hellmann-Feynman theorem. If $H(\lambda)$ is a Hermitian operator that depends on a real parameter λ, and $|\psi(\lambda)\rangle$ is a normalized eigenvector of $H(\lambda)$ with eigenvalue $E(\lambda)$, i.e.,

$$H(\lambda)|\psi(\lambda)\rangle = E(\lambda)|\psi(\lambda)\rangle, \quad \langle\psi(\lambda)|\psi(\lambda)\rangle = 1 , \tag{2.1}$$

then, according to the Hellmann-Feynman theorem [19,20],

$$\langle\psi(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi(\lambda)\rangle = \frac{d}{d\lambda}E(\lambda) . \tag{2.2}$$

Alternatively, one can write Eq. (2.2) as

$$\langle\psi(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi(\lambda)\rangle = \frac{d}{d\lambda}\langle\psi(\lambda)|H(\lambda)|\psi(\lambda)\rangle . \tag{2.3}$$

Throughout this paper, we will use state vectors that are normalized to unity.

In the QCD Hamiltonian density $\mathcal{H}_{\rm QCD}$, chiral symmetry is explicitly broken by the current quark mass terms. We denote this part of the Hamiltonian $\mathcal{H}_{\rm mass}$, which is given by

$$\mathcal{H}_{\rm mass} \equiv m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \cdots , \tag{2.4}$$

where u, d, and s denote the up-, down-, and strange-quark fields with current quark masses m_u, m_d, and m_s, and $\cdots$ denotes similar contributions from heavier quarks. It is useful to reorganize the up- and down-quark contributions to $\mathcal{H}_{\rm mass}$ in order to isolate the isospin-breaking effects. Defining $\bar{q}q \equiv \frac{1}{2}(\bar{u}u + \bar{d}d)$, $m_q \equiv \frac{1}{2}(m_u + m_d)$, and $\delta m_q \equiv m_d - m_u$, Eq. (2.4) can be written as

$$\mathcal{H}_{\rm mass} = 2m_q\bar{q}q - \tfrac{1}{2}\delta m_q(\bar{u}u - \bar{d}d) + m_s\bar{s}s + \cdots . \tag{2.5}$$

Making the identifications $H \to \int d^3x\, \mathcal{H}_{\rm QCD}$ and $\lambda \to m_q$ in the Hellmann-Feynman theorem [Eq. (2.3)], one obtains

$$2m_q\langle\psi(m_q)|\int d^3x\, \bar{q}q\,|\psi(m_q)\rangle = m_q\frac{d}{dm_q}\langle\psi(m_q)|\int d^3x\, \mathcal{H}_{\rm QCD}|\psi(m_q)\rangle , \tag{2.6}$$

where we have multiplied both sides by m_q to obtain renormalization-group invariant quantities [21,22]. Note that it is not necessary to neglect isospin-breaking terms in the derivation of Eq. (2.6); however, in the calculations that follow we assume good isospin and neglect isospin-breaking terms.

Now consider the cases in which $|\psi(m_q)\rangle = |\rho_N\rangle$ and $|\psi(m_q)\rangle = |\text{vac}\rangle$ in Eq. (2.6), where $|\rho_N\rangle$ denotes the ground state of nuclear matter at rest with nucleon density ρ_N and $|\text{vac}\rangle$ denotes the vacuum state. Taking the difference of these two cases, and taking into account the uniformity of the system yields

$$2m_q(\langle\bar{q}q\rangle_{\rho_N} - \langle\bar{q}q\rangle_{\rm vac}) = m_q\frac{d\mathcal{E}}{dm_q} , \tag{2.7}$$

where the derivative is taken at fixed density. We have introduced the notation $\langle\Omega\rangle_{\rho_N} \equiv \langle\rho_N|\Omega|\rho_N\rangle$ and $\langle\Omega\rangle_{\rm vac} \equiv \langle\text{vac}|\Omega|\text{vac}\rangle$ for an arbitrary operator Ω. The energy density of nuclear matter $\mathcal{E}$ is given by

$$\mathcal{E} = M_N\rho_N + \delta\mathcal{E} , \tag{2.8}$$

where $\delta\mathcal{E}$ is the contribution to the energy density from the nucleon kinetic energy and N-N interactions. $\delta\mathcal{E}$ is of higher order in the nucleon density and is empirically small at low densities—the binding energy per nucleon at nuclear matter saturation density is less than 2% of the nucleon mass. We neglect $\delta\mathcal{E}$ in this section, and consider its effects in Secs. IV and V.

The quark condensate at low densities can be related to the nucleon σ term σ_N, which can be defined by [23]

$$\sigma_N \equiv \frac{1}{3}\sum_{a=1}^{3}\left(\langle N|[Q_A^a,[Q_A^a,H_{\rm QCD}]]|N\rangle - \langle\text{vac}|[Q_A^a,[Q_A^a,H_{\rm QCD}]]|\text{vac}\rangle\right) , \tag{2.9}$$

where Q_A^a is the axial charge, $H_{\rm QCD}$ is the QCD Hamiltonian, and $|N\rangle$ is the state vector for a nucleon at rest. Alternatively, the σ term can be expressed as [23]

$$\sigma_N = 2m_q\int d^3x(\langle N|\bar{q}q|N\rangle - \langle\text{vac}|\bar{q}q|\text{vac}\rangle) , \tag{2.10}$$

which, upon application of Eq. (2.6) [with $|\psi(m_q)\rangle = |N\rangle$ and $|\psi(m_q)\rangle = |\text{vac}\rangle$], becomes

$$\sigma_N = m_q\frac{dM_N}{dm_q} . \tag{2.11}$$

Combining Eqs. (2.7), (2.8), and (2.11) yields the following model-independent result:

$$2m_q(\langle\bar{q}q\rangle_{\rho_N} - \langle\bar{q}q\rangle_{\rm vac}) = \sigma_N\rho_N + \cdots . \tag{2.12}$$

We now consider the ratio of the in-medium condensate to its vacuum value. This ratio, which is also renormalization-group invariant, is given by

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{\rho_N}{\rho_N^\chi} + \cdots , \tag{2.13}$$

where

$$\rho_N^\chi \equiv \frac{m_\pi^2 f_\pi^2}{\sigma_N} , \tag{2.14}$$

TABLE I. Model-independent predictions of $\langle\bar{q}q\rangle_{\rho_N}/\langle\bar{q}q\rangle_{\rm vac}$ extrapolated to saturation density for selected values of σ_N.

σ_N (MeV)	$\langle\bar{q}q\rangle_{\rho_N}/\langle\bar{q}q\rangle_{\rm vac}$
30	0.758
45	0.636
60	0.515

and m_π and f_π are the pion mass and pion-decay constant, respectively. To derive Eq. (2.14), we have used the Gell-Mann–Oakes–Renner relation,

$$2m_q\langle\bar{q}q\rangle_{\rm vac}=-m_\pi^2 f_\pi^2 \ . \tag{2.15}$$

Equation (2.13) gives the in-medium quark condensate to first order in the nucleon density. There are higher-order corrections that result from retaining $\delta\mathcal{E}$ in Eq. (2.8); therefore, the model-independent prediction of Eq. (2.13) is only valid at sufficiently low densities. Note that ρ_N^χ is the value of the chiral restoration density obtained by extrapolating Eq. (2.13).

To first order in the nucleon density, the ratio of the in-medium quark condensate to its vacuum value depends solely on the values of the pion mass, the pion-decay constant, and the σ term. We take $m_\pi=138$ MeV and $f_\pi=93$ MeV. A recent analysis of the σ term [24] yields a value of approximately 45 MeV with an uncertainty of order 7–10 MeV. Previous analyses arrived at values that were somewhat higher (for example, $\sigma_N=56.9\pm6.0$ MeV [25]) or somewhat lower (for example, $\sigma_N\simeq 25$–26 MeV [26]). To reflect this range of values, we consider three different values of the σ term in our analyses: $\sigma_N=30$, 45, and 60 MeV.

Although Eq. (2.13) is, thus far, only justified at low density, it is interesting to evaluate this expression at nuclear matter saturation density $\rho_N^{\rm sat}$. Here, and throughout this paper, we take the nucleon density at saturation to be $\rho_N^{\rm sat}=(110\ {\rm MeV})^3\simeq 0.173\ {\rm fm}^{-3}$ (which corresponds to a Fermi momentum of $1.37\ {\rm fm}^{-1}$). In Table I, we give numerical values for the in-medium quark condensate at saturation density. Depending on the precise value of the σ term, the extrapolation of the model-independent result suggests that the in-medium quark condensate is roughly 25–50 % smaller than the vacuum value. The essential physics of Eq. (2.13) is clear: Unless there is some conspiracy in the terms of higher order in the nucleon density, the quark condensate in medium is significantly altered from its vacuum value at nuclear matter saturation density.

III. IN-MEDIUM GLUON CONDENSATE: MODEL-INDEPENDENT CALCULATIONS

We now consider the in-medium gluon condensate. We develop a model-independent prediction that is valid to first order in the nucleon density through an application of the trace anomaly and the Hellmann-Feynman theorem. We first consider the nature of the trace anomaly, following the discussion of Refs. [27] and [28]. Consider the dilatation transformation

$$x_\mu\to\lambda^{-1}x_\mu,\quad A_\mu(x)\to\lambda A_\mu(\lambda x),\quad \psi\to\lambda^{3/2}\psi(\lambda x)\ , \tag{3.1}$$

where λ is an arbitrary dimensionless real parameter, and A_μ and ψ are the gluon and quark fields. In the limit of vanishing quark masses, the *classical* chromodynamics action is scale invariant; thus there is a conserved dilatation current $J_\mu^{\rm dil}$, which is given by

$$J_\mu^{\rm dil}=T_{\mu\nu}x^\nu\ , \tag{3.2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor. Conservation of the dilatation current implies $\partial^\mu J_\mu^{\rm dil}=T_\mu{}^\mu=0$ at the classical level. Quantum corrections, however, break scale invariance through the regularization of the theory, which introduces the QCD scale $\Lambda_{\rm QCD}$ and leads to the trace anomaly. Reinstating finite current quark masses, the trace of the energy-momentum tensor, including the anomaly, is

$$T_\mu{}^\mu=-\frac{1}{8}\left[\frac{11}{3}N_c-\frac{2}{3}N_f\right]\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}+m_u\bar{u}u+m_d\bar{d}d$$
$$+m_s\bar{s}s+m_c\bar{c}c+m_b\bar{b}b+m_t\bar{t}t+\cdots\ , \tag{3.3}$$

where we have neglected higher-order α_s corrections to each term [21,28,29]. N_c and N_f denote the number of quark colors and flavors; we take $N_c=3$ and $N_f=6$. The masses of the charm, bottom, and top quarks are large compared to the energy scales of interest; therefore, we eliminate their contributions via the inverse mass expansion,

$$\bar{Q}Q=-\frac{1}{12m_Q}\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}+O(m_Q^{-3})\ , \tag{3.4}$$

which is valid for large quark masses. Here we use Q to denote a generic heavy quark; note that the $O(m_Q^{-3})$ contribution to Eq. (3.4) is also of higher order in α_s [28]. With this simplification, the trace of the energy-momentum tensor is

$$T_\mu{}^\mu=-\frac{9\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu}+m_u\bar{u}u+m_d\bar{d}d+m_s\bar{s}s\ . \tag{3.5}$$

For nuclear matter in equilibrium, the ground-state expectation value of the trace of the energy-momentum tensor is

$$\langle T_\mu{}^\mu\rangle_{\rho_N}=\langle T_\mu{}^\mu\rangle_{\rm vac}+\mathcal{E}\ , \tag{3.6}$$

where $\mathcal{E}$ is the energy density of the nuclear matter. Note that pressure contributions vanish in Eq. (2.6) since we are considering nuclear matter in equilibrium. Combining Eq. (3.6) with the difference between the right-hand side of Eq. (3.5) evaluated in the nuclear-matter ground state and the vacuum yields the following result for the change in the gluon condensate:

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} - \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\text{vac}} = -\frac{8}{9}[\mathcal{E} - m_u(\langle \bar{u}u \rangle_{\rho_N} - \langle \bar{u}u \rangle_{\text{vac}}) - m_d(\langle \bar{d}d \rangle_{\rho_N} - \langle \bar{d}d \rangle_{\text{vac}}) - m_s(\langle \bar{s}s \rangle_{\rho_N} - \langle \bar{s}s \rangle_{\text{vac}})] \ . \quad (3.7)$$

The change in the up- and down-quark condensates can be written in terms of the σ term. Neglecting isospin breaking and using the results of Sec. II [Eq. (2.12)], we obtain

$$m_u(\langle \bar{u}u \rangle_{\rho_N} - \langle \bar{u}u \rangle_{\text{vac}}) + m_d(\langle \bar{d}d \rangle_{\rho_N} - \langle \bar{d}d \rangle_{\text{vac}}) = \sigma_N \rho_N + \cdots \ . \quad (3.8)$$

Similarly, one can write the change in the strange-quark condensate as

$$m_s(\langle \bar{s}s \rangle_{\rho_N} - \langle \bar{s}s \rangle_{\text{vac}}) = S\rho_N + \cdots \ , \quad (3.9)$$

where the strangeness content of the nucleon S is defined by

$$S \equiv m_s \int d^3x (\langle N|\bar{s}s|N \rangle - \langle \text{vac}|\bar{s}s|\text{vac} \rangle) \ . \quad (3.10)$$

Combining Eqs. (2.8) and (3.7)–(3.9), we obtain

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} - \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\text{vac}} = -\frac{8}{9}(M_N - \sigma_N - S)\rho_N + \cdots \ . \quad (3.11)$$

The precise value of the strangeness content of the nucleon is the subject of some controversy. It is commonly specified by the dimensionless quantity y defined by

$$y \equiv \frac{\int d^3x (\langle N|\bar{s}s|N \rangle - \langle \text{vac}|\bar{s}s|\text{vac} \rangle)}{\int d^3x (\langle N|\bar{q}q|N \rangle - \langle \text{vac}|\bar{q}q|\text{vac} \rangle)} \ , \quad (3.12)$$

which leads to

$$S = \frac{1}{2}\left[\frac{m_s}{m_q}\right]\sigma_N y \ . \quad (3.13)$$

In constituent quark models, there are no strange quarks in the nucleon; thus the strangeness content vanishes. On the other hand, calculations that analyze the baryonic spectrum in the context of SU(3)-flavor symmetry suggest that the strangeness content y is related to the σ term in the following manner [30,24]:

$$\sigma_N = \frac{\sigma_N^0}{1-y} \ , \quad (3.14)$$

where σ_N^0 is the σ term in the limit of the vanishing strangeness. The strangeness content S can then be parametrized as

$$S = \frac{1}{2}\left[\frac{m_s}{m_q}\right](\sigma_N - \sigma_N^0) \ . \quad (3.15)$$

TABLE II. Model-independent predictions of $\langle G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\rho_N} / \langle G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\text{vac}}$ extrapolated to saturation density for selected values of σ_N, assuming no strangeness content of the nucleon and a large strangeness content.

σ_N (MeV)	No strangeness content	Large strangeness content
30	0.936	0.940
45	0.937	0.955
60	0.938	0.969

The value of σ_N^0 can be estimated from an analysis of the mass spectrum of the baryon octet, with violations of SU(3)-flavor symmetry treated perturbatively. First-order calculations [30,31,24] lead to $\sigma_N^0 \simeq 25$ MeV, while second-order corrections [30,24] raise this to $\sigma_N^0 = 35 \pm 5$ MeV. In the interest of obtaining an upper estimate of the effects of strangeness, we take $\sigma_N^0 = 25$ MeV. Consistent with the first-order calculations from which this value is derived, we estimate the strange-quark mass from the following relation [32,33]:

$$\frac{2m_q}{m_q + m_s} = \frac{m_\pi^2}{m_K^2} \ . \quad (3.16)$$

This yields $m_s/m_q = 25$, which is consistent with the results of Ref. [30].

We now consider the change in the gluon condensate at nuclear matter saturation density. In Table II, we give numerical values of the in-medium gluon condensate for selected values of the σ term. We take $M_N = 939$ MeV and assume that the gluon condensate in vacuum is given exactly by the central value in Eq. (1.2). We consider two cases—one assuming a vanishing strangeness content of the nucleon ($S = 0$) and one assuming a large strangeness content [S is estimated by Eq. (3.15)]. One notices immediately that the finite-density effects modify the gluon condensate to a much smaller extent than they do the quark condensate. The decrease in the gluon condensate at nuclear matter saturation density is in the neighborhood of 5%.

IV. IN-MEDIUM QUARK CONDENSATE: QUARK-MODEL CALCULATIONS

We now consider corrections to the model-independent behavior of the quark condensate given by Eq. (2.13). One might anticipate that these corrections are small since the shift in the condensate is proportional to $d\mathcal{E}/dm_q$ and, at nuclear matter saturation density, the energy density $\mathcal{E}$ is dominated by the contribution from the nucleon mass. In fact, at nuclear matter saturation density, the nucleon mass of 939 MeV is nearly two orders of magnitude larger than the 16-MeV binding energy. However, since it is the *derivative* of the interaction energy with respect to the current quark mass that is significant, and not simply the total interaction energy, it is not obvious that the model-independent results of Sec. II are valid near nuclear matter saturation density.

The model-independent prediction of the in-medium quark condensate is valid to first order in the nucleon density; in this section we estimate higher-order corrections with model calculations. It is natural to consider relativistic models with quark degrees of freedom since these models give direct access to a scalar quark density or quark condensate. Since the essential physics of the quark condensate is that of chiral symmetry breaking, we consider chiral quark-meson models—the Nambu–Jona-Lasinio and Gell-Mann–Lévy models. At finite density, the NJL and GML models describe *quark* matter rather than *nuclear* matter. In order for the NJL and GML descriptions to be physically meaningful, one must have in mind a scenario in which a nucleon is composed of N_c weakly interacting constituent quarks (where N_c is the number of colors) with masses generated by the spontaneous breaking of chiral symmetry. For this description to give a sensible energy density in the limit of low nucleon density, we must insist that the constituent quark mass is given by

$$M_q = \frac{M_N}{N_c} . \tag{4.1}$$

We take $N_c = 3$; therefore, the constituent quark mass is $M_q = 313$ MeV. In order to simulate symmetric nuclear matter, we consider models with two quark flavors, i.e., we take $N_f = 2$.

Using the NJL and GML models, we solve for the relative change in the quark condensate using a mean-field (Hartree) approximation—quark fields are treated at the one-loop level and meson fields are treated at tree level. This approximation gives the leading-order behavior of the models in a $1/N_c$ expansion. Recall that the NJL and GML models are designed such that, in the large-N_c limit, quantities calculated with the models depend on N_c in the same way as analogous quantities in QCD. Fock contributions are of lower order in N_c, and are therefore not included in this simple treatment. We note that these mean-field quark-matter calculations are radically different from mean-field calculations with a hedgehog ansatz for the meson fields, which yield soliton configurations that describe a single nucleon or Δ isobar. The latter technique has been used for the GML model [34–36], and also, more recently, for the NJL model [37–43].

For convenience, we do not include explicit chiral-symmetry-breaking terms in the NJL and GML Lagrangians; thus we work in the chiral limit. This approximation should be good since explicit chiral-symmetry-breaking terms in the QCD Lagrangian are small, resulting in a pion mass that is small compared to other hadron masses. We choose the free parameters of the models to match the model-independent result [Eq. (2.13)] at low densities; i.e., we fix the quantity $\rho_N^\chi \equiv m_\pi^2 f_\pi^2/\sigma_N$. Note that the σ term and the pion mass both vanish in the chiral limit; however, the ratio of the σ term to the square of the pion mass remains finite. It is easy to show that the value of ρ_N^χ based on empirical data is equal to ρ_N^χ in the chiral limit to leading order in chiral perturbation theory. One might expect corrections of a few percent due to the finite current quark masses. We can therefore satisfy the low-density result of Eq. (2.13) in these chiral models by calculating ρ_N^χ in the context of these models and setting it equal to the value obtained by substituting the empirical values of m_π, f_π, and σ_N in the definition of ρ_N^χ.

We now estimate the in-medium quark condensate with the NJL model. The Lagrangian for this model is

$$\mathcal{L}_{\rm NJL} = i\bar\psi\gamma^\mu\partial_\mu\psi + \tfrac{1}{2}G[(\bar\psi\psi)^2 + (\bar\psi i\gamma^5\boldsymbol{\tau}\psi)^2] , \tag{4.2}$$

where ψ is a quark field with Dirac, color, and flavor indices suppressed [44]. The Lagrangian can be cast into a form in which the four-quark couplings are represented by the exchange of auxiliary σ and π mesons, which are nondynamical at tree level [45]:

$$\mathcal{L}_{\rm NJL} = i\bar\psi\gamma^\mu\partial_\mu\psi + g\bar\psi(\sigma + i\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi - \tfrac{1}{2}\mu^2(\sigma^2 + \boldsymbol{\pi}^2) , \tag{4.3}$$

where

$$\mu^2 \equiv \frac{g^2}{G} . \tag{4.4}$$

The auxiliary meson fields and couplings are not normalized, i.e., σ and π are not necessarily canonical fields; thus we will eventually express everything in terms of the original parameters and fields of the model. We solve the model using standard mean-field techniques. In the vacuum and in nuclear matter, the π field vanishes; the σ field, however, remains finite. In the vacuum, the nonvanishing σ field generates a constituent quark mass $M_q = -g\langle\sigma\rangle_{\rm vac}$; at finite density, the nonvanishing σ field generates a density-dependent effective constituent quark mass $M_q^* = -g\langle\sigma\rangle_{\rho_N}$. Using the "equation of motion" for the σ field, the quark masses can be written in terms of the original parameters and fields in the NJL model:

$$M_q = -G\langle\bar\psi\psi\rangle_{\rm vac}, \quad M_q^* = -G\langle\bar\psi\psi\rangle_{\rho_N} . \tag{4.5}$$

One can determine the effective constituent quark mass by minimizing the energy density, excluding the trivial solution $M_q^* = 0$. The value of the effective quark mass is thus given by the following finite-density NJL gap equation:

$$1 = 4N_c N_f G\left[\int_0^\Lambda \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2 + M_q^{*2}} - \frac{1}{2}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{1}{(\mathbf{k}^2 + M_q^{*2})^{1/2}}\right] , \tag{4.6}$$

where N_c and N_f are the number of quark colors and flavors in the NJL model, respectively. In the Dirac sea, we use a sharp cutoff in the Euclidean momentum at $k_E^2 = \Lambda^2$. Alternative Euclidean cutoff schemes can be incorporated by using $\int d^4k_E f(k_E^2,\Lambda^2)$ in place of $\int_0^\Lambda d^4k_E$ in the gap equation, where $f(k_E^2,\Lambda^2)$ is some other appropriate cutoff function that guarantees convergence of the integral. Noncovariant three-momentum cutoffs can also be used. The contribution to the gap equation from the Fermi sea is cut off at the Fermi momentum k_F, which is related to the "nucleon" density by $\rho_N = N_f k_F^3/3\pi^2$.

We choose the free parameters of the NJL model, G

and Λ, to satisfy the conditions of Eqs. (2.13) and (4.1). In determining the cutoff Λ, we first consider the low-density behavior of $\langle \bar{q}q \rangle_{\rho_N}/\langle \bar{q}q \rangle_{\text{vac}}$ as given by Eq. (2.13). From Eq. (4.5), one can trivially solve for the vacuum and in-medium quark condensates,

$$\langle \bar{q}q \rangle_{\text{vac}} = -\frac{M_q}{N_f G}, \quad \langle \bar{q}q \rangle_{\rho_N} = -\frac{M_q^*}{N_f G} ; \tag{4.7}$$

however, the connection between these quark condensates, which are calculated in the NJL model, and the QCD quark condensates is not clear. The quark condensate of QCD is a scale-dependent quantity, i.e., it is not renormalization-group invariant. The quark condensate of the NJL model depends on the cutoff of the model, but it is not clear how this cutoff is related to a renormalization scale in QCD. It seems reasonable to assume, however, that the ratio of the in-medium condensate to the vacuum condensate, which is renormalization-group invariant, is equivalent to this ratio as predicted by the NJL model:

$$\left[\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} \right]_{\text{QCD}} = \left[\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} \right]_{\text{NJL}} . \tag{4.8}$$

From Eq. (4.7), this implies

$$\left[\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} \right]_{\text{QCD}} = \frac{M_q^*}{M_q} . \tag{4.9}$$

Given that the constituent quark mass is one-third of the nucleon mass [see Eq. (4.1)], the ratio in Eq. (4.9) is consistent with the behavior suggested by QCD sum rules at finite density [12]. Note, however, that Eq. (4.9) does not appear to be consistent with the effective-mass scaling law used in Refs. [8–10] [see Eq. (1.3)]. Combining Eqs. (2.13) and (4.9), one obtains

$$\frac{M_q^*}{M_q} = 1 - \frac{\rho_N}{\rho_N^\chi} + \cdots , \tag{4.10}$$

where $\cdots$ denotes terms of higher order in ρ_N. Taking the derivative of Eq. (4.10) with respect to ρ_N and evaluating this derivative in the vacuum yields the following *exact* result:

$$\frac{1}{M_q} \left[\frac{dM_q^*}{d\rho_N} \right]_{\text{vac}} = -\frac{1}{\rho_N^\chi} . \tag{4.11}$$

By differentiating both sides of the gap equation [Eq. (4.6)] with respect to ρ_N, one can easily solve for $(dM_q^*/d\rho_N)_{\text{vac}}$. One thus obtains the relation

$$\rho_N^\chi = 8 N_f M_q^3 \int_0^\Lambda \frac{d^4 k_E}{(2\pi)^4} \frac{1}{(k_E^2 + M_q^2)^2} , \tag{4.12}$$

and from this, along with Eq. (4.1), one can readily determine the cutoff Λ. With Λ and M_q known, it is a simple matter to extract the coupling constant G from the gap equation. Values of the free parameters are shown in Table III for selected values of the σ term.

Given the free parameters, we now consider finite-density matter. The value of the effective quark mass at finite density is given by the gap equation [Eq. (4.6)]. In Fig. 1, we plot $\langle \bar{q}q \rangle_{\rho_N}/\langle \bar{q}q \rangle_{\text{vac}} = M_q^*/M_q$ vs the "nucleon" density for selected values of the σ term. We see that the deviation from linearity is fairly small. In Table IV, we give numerical values for the in-medium condensate at nuclear matter saturation density. With a σ term less than 45 MeV, the corrections to the model-independent prediction are fairly small: The in-medium condensate predicted by the NJL model is less than 6% smaller than the model-independent prediction at saturation density. With a large σ term of 60 MeV, the effects of interactions are enhanced—here the NJL model predicts an in-medium condensate that is 21% smaller than the model-independent prediction. Preliminary calculations indicate that the deviation of the NJL condensate from the model-independent condensate is significantly reduced if finite quark masses are included in the NJL Lagrangian. Unfortunately, the connection between the quark mass in the NJL model and the scale-dependent current quark mass in QCD is not clear.

It is useful to calculate the in-medium quark condensate from a chiral model other than the NJL model to test the model dependence of the NJL results. The GML model describes physics in a manner similar to that of the NJL model; however, the effective potentials of these models can differ radically—the NJL model is nonrenormalizable and hence uses a momentum cutoff in the effective potential. The GML model, in contrast, *is* renormalizable, and the effective potential therefore includes counterterm subtractions to yield a finite result. The GML Lagrangian is

$$\mathcal{L}_{\text{GML}} = i\bar{\psi}\gamma^\mu \partial_\mu \psi + g\bar{\psi}(\sigma + i\gamma^5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - U(\sigma,\boldsymbol{\pi}) , \tag{4.13}$$

where ψ, σ, and $\boldsymbol{\pi}$ denote fields for the quark and σ and π mesons [46]. The tree-level potential is

$$U(\sigma,\boldsymbol{\pi}) = \frac{m_\sigma^2}{8 f_\pi^2} (\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 , \tag{4.14}$$

where $f_\pi = 93$ MeV is the usual pion-decay constant. We

TABLE III. NJL free parameters for selected values of σ_N.

σ_N (MeV)	G (10^{-5} MeV^{-2})	Λ (MeV)
30	0.601	1166
45	1.425	814
60	2.417	662

TABLE IV. Quark-model predictions of $\langle \bar{q}q \rangle_{\rho_N}/\langle \bar{q}q \rangle_{\text{vac}}$ at nuclear matter saturation density for selected values of σ_N.

σ_N (MeV)	NJL	GML
30	0.753	0.757
45	0.598	0.626
60	0.405	0.501

solve the model using standard mean-field techniques. As in the NJL model, the π field vanishes in the vacuum and in nuclear matter, and the σ field remains finite. The nonvanishing σ field again generates the constituent quark mass and the density-dependent effective constituent quark mass,

$$M_q=-g\langle\sigma\rangle_{\rm vac}=gf_\pi,\quad M_q^*=-g\langle\sigma\rangle_{\rho_N}\ . \tag{4.15}$$

In the expression for M_q, we have used the fact that $\langle\sigma\rangle_{\rm vac}=-f_\pi$ in the GML model. At the one-quark-loop level, the effective potential is given by

$$U_{\rm eff}=-\frac{N_cN_f}{16\pi^2}\left[M_q^{*4}\ln\left[\frac{M_q^{*2}}{M_q^2}\right]-M_q^2(M_q^{*2}-M_q^2)-\frac{3}{2}(M_q^{*2}-M_q^2)^2\right]\ . \tag{4.16}$$

The effective potential is renormalized in the vacuum so as to preserve chiral symmetry and fix the values of f_π and m_σ [47]. One can determine the effective constituent quark mass by minimizing the energy density, excluding the trivial solution $M_q^*=0$. This yields the GML analog of the NJL gap equation,

$$\frac{m_\sigma^2f_\pi^2}{2M_q^4}(M_q^{*2}-M_q^2)-\frac{N_cN_f}{4\pi^2}\left[M_q^{*2}\ln\left[\frac{M_q^{*2}}{M_q^2}\right]-(M_q^{*2}-M_q^2)\right]+2N_cN_f\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{1}{(\mathbf{k}^2+M_q^{*2})^{1/2}}=0\ , \tag{4.17}$$

where N_c and N_f are the number of quark colors and flavors in the GML model, respectively. The contribu-

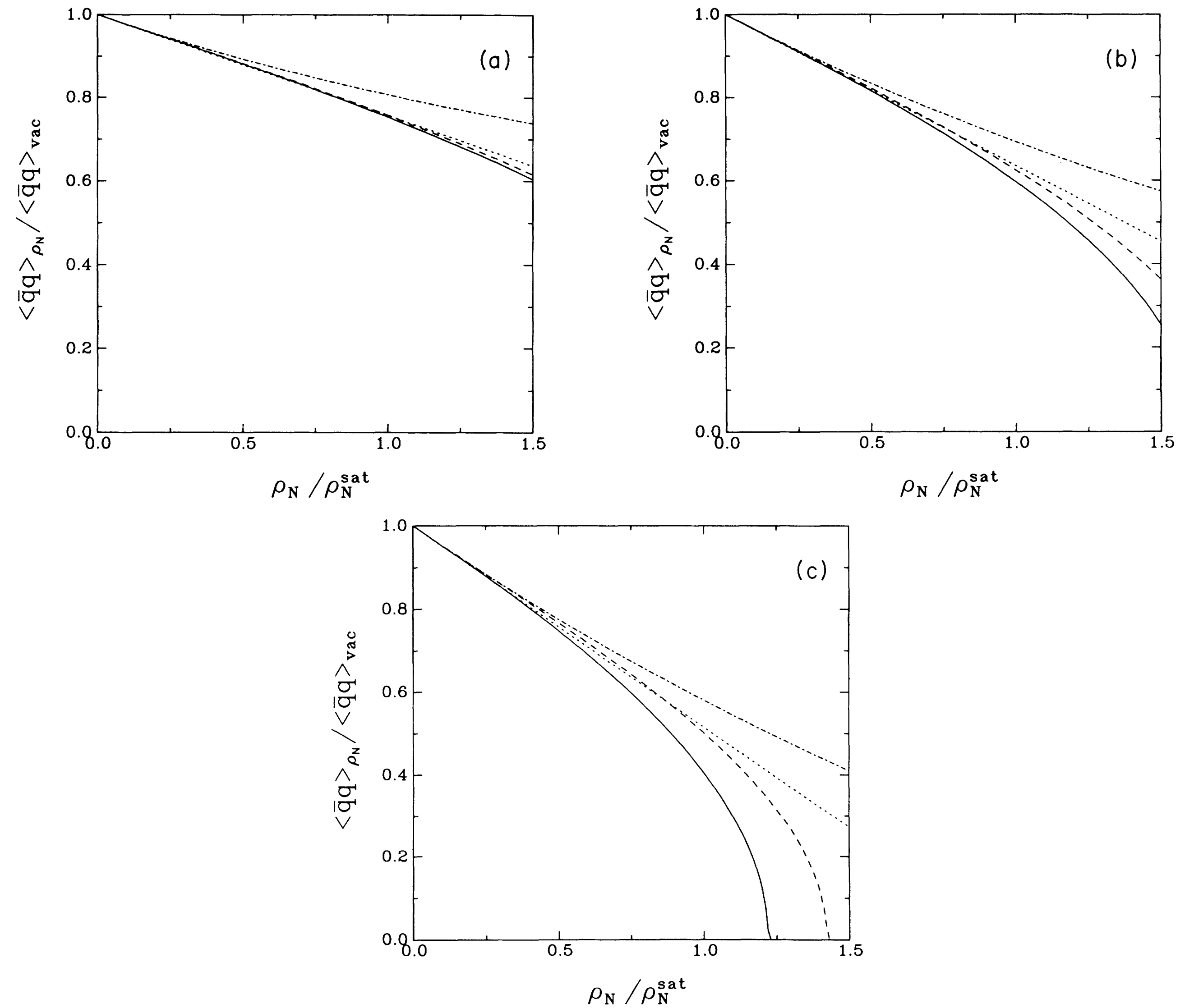

FIG. 1. In-medium quark condensate vs the nucleon density for (a) $\sigma_N=30$ MeV, (b) $\sigma_N=45$ MeV, and (c) $\sigma_N=60$ MeV. The solid lines are for the NJL calculation, the dashed lines are for the GML calculation, the dot-dashed lines are for the π-N calculation, and the dotted lines are for the extrapolation of the model-independent result.

tion to the "gap" equation from the Fermi sea is cut off at the Fermi momentum k_F, which is again related to the "nucleon" density by $\rho_N = N_f k_F^3/3\pi^2$.

As with the NJL model, we choose the free parameters of the GML model, g and m_σ, to satisfy the conditions of Eqs. (2.13) and (4.1). From Eqs. (4.1) and (4.15), we obtain the value of the coupling constant

$$g = \frac{M_q}{f_\pi} = \frac{M_N}{N_c f_\pi} . \tag{4.18}$$

The value of m_σ, on the other hand, is determined by the constraint of Eq. (2.13). Since the mechanism for generating the constituent quark mass is essentially the same in the NJL and GML models, we assume [as in Eq. (4.9)]

$$\left[\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\rm vac}} \right]_{\rm QCD} = \frac{M_q^*}{M_q} , \tag{4.19}$$

where M_q and M_q^* are now the constituent quark masses generated in the GML model. This behavior is also consistent with the partial conservation of axial current (PCAC) relation,

$$2 m_q \bar{q}q = m_\pi^2 f_\pi \sigma . \tag{4.20}$$

Motivated by these considerations, we rewrite Eq. (2.13) as

$$\frac{M_q^*}{M_q} = 1 - \frac{\rho_N}{\rho_N^\chi} + \cdots , \tag{4.21}$$

where $\cdots$ denotes terms of higher order in ρ_N. We proceed as with the NJL model. Taking the derivative of Eq. (4.21) with respect to ρ_N and evaluating this derivative in the vacuum yields the following *exact* result:

$$\frac{1}{M_q} \left[\frac{dM_q^*}{d\rho_N} \right]_{\rm vac} = - \frac{1}{\rho_N^\chi} . \tag{4.22}$$

By differentiating both sides of the "gap" equation [Eq. (4.17)] with respect to ρ_N, one can easily solve for $(dM_q^*/d\rho_N)_{\rm vac}$. One thus obtains the relation

$$\rho_N^\chi = \frac{m_\sigma^2 f_\pi^2}{N_c M_q} , \tag{4.23}$$

and from this, along with Eq. (4.1), one can readily determine the mass of the σ meson: $m_\sigma = m_\pi \sqrt{M_N/\sigma_N}$.

Given the free parameters, we now consider finite-density matter. The effective quark mass is given by the "gap" equation [Eq. (4.17)]. In Fig. 1, we plot $\langle \bar{q}q \rangle_{\rho_N} / \langle \bar{q}q \rangle_{\rm vac} = M_q^*/M_q$ vs the "nucleon" density for selected values of the σ term. The deviation of the quark condensate from linearity is even smaller than seen with the NJL model. In Table IV, we give numerical values of the in-medium quark condensate at nuclear matter saturation density. The GML results are extremely close to the model-independent results of Sec. II. Even with a σ term of 60 MeV, the in-medium quark condensate predicted by the GML model is less than 3% smaller than the model-independent prediction at saturation density.

Thus the NJL and GML models give similar results, which indicates that the in-medium quark condensate, as predicted by a chiral quark model, is not very sensitive to the detailed form of the effective potential. In addition, the NJL and GML models suggest that the deviation of the condensate from the model-independent prediction of Eq. (2.13) is small ($\sim 10\%$) up to nuclear matter saturation density.

V. IN-MEDIUM QUARK CONDENSATE: HADRONIC-MODEL CALCULATIONS

While the quark models considered in Sec. IV include the physics of chiral symmetry breaking—physics that is essential to the concept of a quark condensate—they do not account for the tendency of quarks to cluster into nucleons. Thus the NJL and GML models may give misleading results. Therefore, in this section, we consider the in-medium quark condensate using models with hadronic degrees of freedom. We first estimate the in-medium condensate using a nonrelativistic model of nuclear matter with pion and nucleon degrees of freedom. We calculate the effects of π-N interactions from the pion Fock term; this is equivalent to the Pauli-blocking correction of Ref. [13]. We briefly discuss the effects of correlations on the result. We also estimate the in-medium quark condensate with quantum hadrodynamics.

One obtains the in-medium quark condensate from the Hellmann-Feynman theorem by calculating the energy density of nuclear matter. From Eq. (2.7), the in-medium condensate is

$$\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle_{\rm vac} + \frac{1}{2} \frac{d\mathcal{E}}{dm_q} , \tag{5.1}$$

where $\mathcal{E}$ is the energy density. By using the chain rule, the derivative can be reexpressed so that the in-medium condensate is

$$\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle_{\rm vac} + \frac{1}{2} \left[\frac{\partial \mathcal{E}}{\partial M_N} \frac{dM_N}{dm_q} + \frac{\partial \mathcal{E}}{\partial m_\pi} \frac{dm_\pi}{dm_q} + \frac{\partial \mathcal{E}}{\partial g_\pi} \frac{dg_\pi}{dm_q} + \cdots \right] , \tag{5.2}$$

where $\cdots$ denotes contributions from other hadron masses, coupling constants, etc. In general, one does not know the quark-mass derivatives in Eq. (5.2). In the case of the nucleon and pion masses, however, the derivatives can be obtained by applying the Hellmann-Feynman theorem and the Gell-Mann–Oakes–Renner relation. From Eqs. (2.11) and (2.15), these derivatives are

$$\frac{dM_N}{dm_q} = \frac{\sigma_N}{m_q}, \quad \frac{dm_\pi}{dm_q} = \frac{m_\pi}{2m_q} , \tag{5.3}$$

where the derivative of the pion mass is valid to leading order in chiral perturbation theory. We neglect all other derivatives; therefore, we obtain

$$\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\rm vac}} \simeq 1 - \frac{1}{\rho_N^\chi} \left[\frac{\partial \mathcal{E}}{\partial M_N} + \frac{\partial \mathcal{E}}{\partial m_\pi} \frac{m_\pi}{2\sigma_N} \right] . \tag{5.4}$$

We now consider a simple nonrelativistic model of nuclear matter in which uncorrelated nucleons interact through one-pion exchange. The energy density is

$$\mathcal{E}=\gamma\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\left[M_N+\frac{\mathbf{k}^2}{2M_N}\right]+\mathcal{E}_{\rm Fock}\ , \tag{5.5}$$

where $\gamma=4$ is the spin-isospin degeneracy and $\mathcal{E}_{\rm Fock}$ is the pion Fock (exchange) energy. Note that the Hartree (direct) energy vanishes for the pion. The Fock energy can be calculated by standard many-body techniques [48,49]. By performing a nonrelativistic reduction on the relativistic π-N interaction Lagrangian (with pseudoscalar or pseudovector coupling), one obtains the following nonrelativistic interaction Lagrangian:

$$\mathcal{L}_{\pi N}=-\frac{g_\pi}{2M_N}N^\dagger\sigma_i\tau_a N\partial_i\pi_a\ , \tag{5.6}$$

where $g_\pi=13.5$ is the πNN coupling constant, N and π_a are nonrelativistic field operators for the nucleon and pion, and i and a are spatial and isospin indices. Using this interaction Lagrangian, the one-pion-exchange contribution to the nucleon self-energy can be determined through the application of standard Feynman-rule techniques, resulting in

$$\Sigma_{\rm Fock}(k)=3i\left[\frac{g_\pi}{2M_N}\right]^2\int\frac{d^4k'}{(2\pi)^4}e^{ik_0'\eta}(\mathbf{k}-\mathbf{k}')^2\times G(k')\Delta(k-k')\ , \tag{5.7}$$

where $G(k)$ and $\Delta(k)$ are the noninteracting nucleon and pion propagators. We neglect correlations, therefore, we do not iterate the self-energy in Dyson's equation. From the self-energy, one can deduce the Fock energy; after performing a vacuum subtraction, one obtains

$$\mathcal{E}_{\rm Fock}=6\left[\frac{g_\pi}{2M_N}\right]^2\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}\frac{(\mathbf{k}-\mathbf{k}')^2}{(\mathbf{k}-\mathbf{k}')^2+m_\pi^2}\ . \tag{5.8}$$

Consistent with the nonrelativistic limit, we have neglected recoil, which introduces terms that are of higher order in $1/M_N$. The Fermi momentum k_F is related to the nucleon density by the usual relation, $\rho_N=2k_F^3/3\pi^2$.

Combining Eqs. (5.5) and (5.8) and inserting the result into Eq. (5.4), one obtains the in-medium quark condensate, which, for this simple model, is given by

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\rm vac}}=1-\frac{\rho_N}{\rho_N^\chi}\left[1-\frac{3}{10}\left[\frac{k_F}{M_N}\right]^2\right]+\frac{3g_\pi^2}{\rho_N^\chi M_N^3}I_1+\frac{3g_\pi^2}{2f_\pi^2M_N^2}I_2\ . \tag{5.9}$$

We have introduced the integrals

$$I_n\equiv\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}\frac{(\mathbf{k}-\mathbf{k}')^2}{[(\mathbf{k}-\mathbf{k}')^2+m_\pi^2]^n}\ . \tag{5.10}$$

It is instructive, although somewhat misleading (see below), to consider the density dependence of this result [13]. Expanding in powers of the Fermi momentum k_F, assuming $k_F\gg m_\pi$ due to the approximate chiral symmetry of QCD, one finds that

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\rm vac}}=1-\frac{\rho_N}{\rho_N^\chi}+\frac{3g_\pi^2k_F^4}{32\pi^4f_\pi^2M_N}+O(\rho_N^{5/3})\ . \tag{5.11}$$

Thus the contribution to the in-medium condensate from the pion Fock term, which is $O(\rho_N^{4/3})$, is the next term in the density expansion following the model-independent result. The contribution from the nucleon kinetic energy is $O(\rho_N^{5/3})$. Other contributions are of higher order in the nucleon density.

We show the in-medium condensate versus the nucleon density in Fig. 1 for selected values of the σ term. Note that the sign of the deviation is the opposite of that observed with the NJL and GML models. However, as with the NJL and GML calculations, the pion Fock term contribution leads to small deviations from the linear prediction for the in-medium quark condensate. In Table V, we give numerical values for the in-medium condensate at saturation density. For the range of the σ term under consideration, we find that the in-medium condensate predicted by this simple pion Fock term calculation is 6–13 % larger than the model-independent prediction at saturation density. Thus, in this approximation, the effects of the N-N interactions and kinetic-energy contributions are fairly modest. We have also considered relativistic corrections, which are numerically small.

In the preceding discussion, we have neglected dg_π/dm_q, which makes even this simple calculation ambiguous. Note, for example, that we might have chosen to neglect the derivative of the pseudovector coupling constant $g_\pi/2M_N$ instead. In this case, the I_1 term in Eq. (5.9) would be absent, which would slightly reduce the deviation from the model-independent result.

In any event, the simple pion Fock term calculation is clearly not a complete treatment of nuclear matter. A fundamental weakness of this calculation is the lack of tensor correlations, which, in the conventional nonrelativistic picture, are essential to the physics of saturation. Moreover, the fact that the pion Fock term contribution to the quark condensate is $O(\rho_N^{4/3})$ is an artifact of the overly simple choice of the nuclear wave function. At the Fock level, the maximum momentum carried by a pion is $2k_F$. Iterations of the potential, however, lead to a nontrivial fraction of the wave function at larger momenta, and thus k_F does not necessarily fix the density dependence of the pionic contributions in a simple fashion.

What can be learned about the in-medium quark condensate from more complete treatments of nuclear matter? The pion Fock term calculation can be refined

TABLE V. Hadronic-model predictions of $\langle\bar{q}q\rangle_{\rho_N}/\langle\bar{q}q\rangle_{\rm vac}$ at nuclear matter saturation density for selected values of σ_N.

σ_N (MeV)	π-N	QHD
30	0.807	0.761
45	0.694	0.642
60	0.580	0.523

by considering a more realistic treatment of N-N interactions. Using a model of nuclear matter based on N-N potentials, one can calculate the change in the energy density due to a change in the pion mass. One can estimate this effect using a correlated nuclear wave function based on the Bethe-Goldstone equation. Such calculations for finite nuclei have been done by Banerjee [50]. He finds fairly modest results: the pionic effects lead to an in-medium condensate that is larger than the model-independent prediction by a few percent. These results are consistent with those of the pion Fock term calculation discussed above.

It is worth noting, however, that even the most realistic treatment of nuclear matter does not automatically lead to a complete treatment of the condensate. As discussed above, one must calculate the quark-mass derivative of the nuclear-matter energy density to determine the in-medium condensate. We have *estimated* this derivative by using a truncated chain rule that only accounts for the quark-mass derivatives of the nucleon and pion masses. There are many other contributions to the quark-mass derivative, including those from other hadron masses, coupling constants, and form factors. State-of-the-art calculations determine nuclear-matter properties such as the energy density with models based on realistic N-N potentials. These potentials are extracted from two-body scattering in the physical world, with the quark masses fixed at their physical values. It is not clear how these potentials would change if the current quark masses were changed; unless this is known, the full quark-mass derivative of the energy density cannot be reliably calculated. Thus it is difficult to predict the in-medium quark condensate using a realistic treatment of nuclear matter.

The conventional nonrelativistic treatment is not the only theoretical approach to nuclear matter. During the past decade, a relativistic field-theoretic description of nuclear matter based on nucleon, σ-meson, and ω-meson degrees of freedom has received considerable attention. This approach is known as quantum hadrodynamics (QHD). When solved at the mean-field level, QHD has been successful in explaining a wide range of nuclear phenomena [49]. The mean-field saturation mechanism in QHD is based on the interplay of Lorentz-scalar attraction and vector repulsion, and is characteristically different from the mechanism in the conventional nonrelativistic picture. Thus it is of particular interest to use this model to estimate the interaction effects on the in-medium condensate. The Lagrangian for this model is

$$\mathcal{L}_{\mathrm{QHD}}=\bar{\psi}[\gamma^\mu(i\partial_\mu-g_\omega\omega_\mu)-(M_N-g_\sigma\sigma)]\psi +\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\tfrac{1}{2}m_\sigma^2\sigma^2-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} +\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \,, \tag{5.12}$$

where ψ, σ, and ω are the fields for the nucleon, σ meson, and ω meson.

It is difficult to predict how the parameters of an effective hadronic model should vary as the quark masses of the underlying theory (QCD) are changed without understanding how the effective model is related to QCD. It is useful, however, to make plausible assumptions about how the QHD parameters vary to gain intuition about the problem. For example, we might expect that the dependence of the σ- and ω-meson masses on the current quark masses resembles that of the nucleon (rather than that of the pion, since the pion is an approximate Goldstone boson). To explore this possibility, we adopt the following ansatz, which is motivated by Eq. (5.3):

$$\frac{dm_\sigma}{dm_q}=\chi_\sigma\frac{\sigma_N}{m_q},\quad \frac{dm_\omega}{dm_q}=\chi_\omega\frac{\sigma_N}{m_q}\,. \tag{5.13}$$

These derivatives depend on the parameters χ_σ and χ_ω; constituent quark models suggest the values

$$\chi_\sigma\simeq\frac{m_\sigma}{M_N},\quad \chi_\omega\simeq\frac{m_\omega}{M_N}\,. \tag{5.14}$$

We will neglect any variation of the coupling constants with the current quark masses in this simple exploration.

Applying Eq. (5.2) to QHD, and using the expressions for the derivatives in Eqs. (5.3) and (5.13), one obtains the in-medium quark condensate:

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\mathrm{vac}}}\simeq 1-\frac{1}{\rho_N^\chi}\left[\frac{\partial\mathcal{E}}{\partial M_N}+\frac{\partial\mathcal{E}}{\partial m_\sigma}\chi_\sigma+\frac{\partial\mathcal{E}}{\partial m_\omega}\chi_\omega\right]. \tag{5.15}$$

The mean-field energy density is [49]

$$\mathcal{E}=\gamma\int_0^{k_F}\frac{d^3k}{(2\pi)^3}(\mathbf{k}^2+M_N^{*2})^{1/2} +\frac{m_\sigma^2}{2g_\sigma^2}(M_N-M_N^*)^2+\frac{g_\omega^2}{2m_\omega^2}\rho_N^2\,, \tag{5.16}$$

where we have defined the effective nucleon mass, $M_N^*\equiv M_N-g_\sigma\sigma$. The effective mass is determined by minimizing the energy density, which establishes a self-consistency relation:

$$M_N^*=M_N-\frac{g_\sigma^2}{m_\sigma^2}\gamma\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{M_N^*}{(k_F^2+M_N^{*2})^{1/2}}\,. \tag{5.17}$$

The necessary derivatives can now be computed, which yields

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\mathrm{vac}}}=1-\frac{1}{\rho_N^\chi}\left[\frac{m_\sigma^2}{g_\sigma^2}(M_N-M_N^*) +\chi_\sigma\frac{m_\sigma}{g_\sigma^2}(M_N-M_N^*)^2 -\chi_\omega\frac{g_\omega^2\rho_N^2}{m_\omega^3}\right]. \tag{5.18}$$

We take $m_\sigma=550$ MeV and $m_\omega=783$ MeV. Assuming that nuclear matter saturates at a density of (110 MeV)^3 and a binding energy of 16 MeV per nucleon, the coupling constants and effective nucleon mass are $g_\sigma=10.20$, $g_\omega=12.55$, and $M_N^*=514$ MeV. In Table V, we show the in-medium condensate at saturation density obtained using Eq. (5.18), with χ_σ and χ_ω given by Eq. (5.14). The contributions from the σ and ω mesons cancel to a large degree—the in-medium condensate predicted by QHD is only about 1% larger than the model-independent prediction at saturation density. This cancellation is fairly insensitive to the exact choice of χ_σ and χ_ω. For example, with a 45-MeV σ term, changing both χ_σ and χ_ω from the estimates in Eq. (5.14) by 30% in opposite directions leads to a 13% change in the condensate. Thus, based on the assumptions in Eqs. (5.13) and (5.14), the net effect of the σ and ω mesons is fairly small. Note that the large cancellation of the σ- and ω-meson contributions to the in-medium condensate is a consequence of the built-in cancellation of these contributions to the energy density. We remind the reader that these assumptions are unsubstantiated, and also that contributions due to variations of the coupling constants could, in principle, be large.

In this section, we have used the Hellmann-Feynman theorem to relate the in-medium quark condensate to the energy density of nuclear matter using models with hadronic degrees of freedom. We have considered both nonrelativistic and relativistic examples. In practice, this approach is limited because it requires knowledge of how all model parameters (masses, couplings, etc.) change as the current quark masses change. While we can reliably predict the quark-mass dependence of some parameters (M_N and m_π), the behavior of the other parameters is uncertain. Nevertheless, given plausible assumptions, we find no examples indicating large deviations from the linear model-independent prediction for the in-medium quark condensate at nuclear matter saturation density.

VI. SUMMARY

In this paper, we have studied the in-medium quark and gluon condensates, $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle (\alpha_s/\pi) G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\rho_N}$. First, we have used the Hellmann-Feynman theorem to derive expressions for the condensates in nuclear matter that are model independent to first order in the nucleon density. It is interesting to extrapolate these low-density results to nuclear matter saturation density. The shift in the quark condensate, which depends on the value of the σ term, is large—the condensate is roughly 25–50% smaller than its vacuum value. The shift in the gluon condensate is estimated from the trace anomaly. At saturation density, the gluon condensate is roughly 3–6% smaller than its vacuum value. This range is largely due to the uncertainty in the strangeness content of the nucleon.

For the quark condensate, we have estimated the scale of the deviations from the linear model-independent relation by studying a variety of simple models. In particular, we have considered quark-based Nambu–Jona-Lasinio and Gell-Mann–Lévy models at the mean-field level as well as models with hadronic degrees of freedom. It is possible that none of these models is sophisticated enough to give definitive predictions for the deviation. Moreover, these models do not agree among themselves—even the sign of the deviation differs from model to model. However, we observe that models based on very different physical assumptions all seem to indicate that the deviation from the linear model-independent prediction is small ($\sim 10\%$) up to saturation density. This suggests that the model-independent relation may be sufficient for studying ordinary nuclear phenomena.

Although these results are encouraging, ultimately we would hope to be able to extract the in-medium quark condensate without relying on simple models. Improving the models or the approximations used in Secs. IV and V is unhelpful in the absence of a detailed understanding of how the models relate to QCD. For example, applying the Hellmann-Feynman theorem to even the most sophisticated nuclear matter calculation is not conclusive unless one knows how *all* of the model parameters depend on the current quark masses [see Eq. (5.2)]. One possible alternative is a direct lattice calculation of the in-medium condensate; however, finite-density calculations are not available at present and are unlikely to be available in the foreseeable future. Another possible way to obtain information about the in-medium quark condensate is through the study of pionic atoms [51].

What do our results imply about nuclear phenomenology? As discussed in the Introduction, the in-medium quark condensate is an input to a large number of calculations that describe a variety of nuclear phenomena [5–10,12,13]. It is clear that the change in the quark condensate is *large* and therefore should play an important role in describing hadrons in nuclei. However, the in-medium quark condensate alone does not capture all of the physics associated with hadrons in the nuclear medium. For example, at finite density, the "vector quark condensate" $\langle q^\dagger q \rangle_{\rho_N}$ (which vanishes in the vacuum) can enter at the same level as the scalar quark condensate; nevertheless, $\langle q^\dagger q \rangle_{\rho_N}$ is often neglected. The importance of the vector quark condensate can be seen explicitly in QCD sum-rule calculations at finite density. The change in the quark condensate gives rise to a large attractive scalar self-energy for nucleons; on the other hand, the vector quark condensate implies a large repulsive vector self-energy [12]. The net effect is that the quasinucleon energy is only slightly charged from its free-space value, as expected from nuclear phenomenology. Finally, we observe that the density dependence of higher-dimensional condensates is also relevant and should be studied.

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