Semiclassical approximation to the two-step direct nucleon-nucleus reaction

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We extend the semiclassical approach which was implemented successfully for the one-step direct reaction to the two-step process. Using essentially the same approximations as in a previous paper by Luo and Kawai, we obtain a closed parameter-free formula for the two-step cross section. This formula contains experimental nucleon-nucleon cross sections and distorted-wave intensities, and is physically interpreted.

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I. INTRODUCTION AND SUMMARY

In a previous paper [1], hereafter referred to as I, Luo and Kawai (one of the present authors) developed a semiclassical distorted-wave approximation for the one-step nucleon-induced reaction leading to a continuum final state. The closed expression for the inclusive double-differential cross section for the $(p,p'x)$ and $(n,n'x)$ reactions obtained in this way allowed for both a simple interpretation and a parameter-free calculation. The results were in good agreement with data at 62–164 MeV bombarding energy.

The two-step direct process is almost as important as the one-step direct process for a quantitative description of the direct part of nucleon inelastic scattering. In fact, at large scattering angles at an energy loss in excess of several MeV it can even dominate. Motivated by this fact, and encouraged by the success of I, we extend the approach of I in the present paper to the two-step process.

The starting point for our derivation is the general theory of precompound reactions and the modern distinction between multistep direct and multistep compound processes [2,3]. Especially for the first few steps of the reaction, we believe this distinction to be superior to the older exciton models.

Specifically, we use the result of Ref. [3], hereafter referred to as II, for the two-step direct reaction cross section. This result is based on statistical properties of the residual nucleus, and on the sudden approximation: After the first collision of the incident nucleon with the target, the intermediate state of the target is described as a particle-hole excitation on the ground state of the target ("sudden approximation") rather than as an eigenstate of the target nucleus ("adiabatic approximation" as used in Refs. [2,4]). (The nature of the statistical assumptions used in various theories [2–4] was recently discussed in Ref. [5].) The sudden approximation was justified in II in terms of the characteristic nuclear time scales. Qualitatively, the cross section derived in II is the square of a second-order distorted wave Born approximation (DWBA) amplitude multiplied with the density of two-particle–two-hole excitations in the target nucleus.

In the spirit of I, we reduce this expression to a product of probabilities. We use the same approximations as in I: The slow variation in space of the density and the single-particle potentials for nucleons, the short range of the nucleon-nucleon interaction, and the summation over initial and final single-particle orbits allow us to introduce a local-density (or Thomas-Fermi) approximation for the nuclear states and a local semiclassical approximation for the distorted waves, and to neglect all interference terms and antisymmetrization effects. An additional entity not encountered in I is the Green function for the fast particle in the intermediate state. We observe that the optical potential varies slowly in space, and we therefore use for the Green function the Eikonal approximation, consistent with the other approximations mentioned above. As in I, we neglect the spin dependence of the nucleon-nucleon interaction and of the distorting potentials.

The central result of our work is the expression (3.19) for the double-differential two-step cross section. As is the case for the result obtained in I, this expression does not contain any adjustable parameters if the cross sections on the right-hand side (rhs) are taken to be experimental nucleon-nucleon cross sections, and if the distorted waves and the damping factor for the Green function are calculated from standard optical-model potentials. The cross-section formula (3.19) permits a straightforward intuitive interpretation and is in line with (albeit a generalization of) Goldberger's intranuclear cascade model [6].

In view of the above-mentioned approximations, our final result is not expected to be quantitatively reliable at low bombarding energies. All our approximations become more reliable with increasing bombarding energy. Still, it is difficult to predict precisely when our approach becomes adequate. We believe that at a few 100 MeV the error incurred should be in the 10% range. This, however,

may well be too pessimistic a view. It is quite possible that the present approach is useful already below 100 MeV. This will have to be checked by a comparison of results obtained from Eq. (3.19) with an evaluation of the two-step formula of II. In addition, there are several corrections at lower energies that need to be examined, such as the use of the t or G matrix for the nucleon-nucleon interaction, the nonlocality of the distorting potentials, and exchange effects between the incident and the target nucleons.

In Sec. II, we give a simple derivation of the two-step DWBA formula for the cross-section, referring to II for a more detailed and complete argument. This brief section is included to make our paper self-contained. The case of a channel-dependent optical potential, not envisaged in II but potentially important for practical calculations, is treated in Appendix A where we show that the two-step DWBA formula remains unchanged under this modification. The main argument is developed in Sec. III which ends with a physical interpretation of the central result, Eq. (3.19). One of the main approximations, the neglect of interference terms and antisymmetry, is investigated and justified in Appendix B.

II. THE TWO-STEP CROSS SECTION

The starting point for our approximation scheme is the formula for the quantum-mechanical two-step cross section. We present a brief and simple derivation of this formula. We also indicate why this formula holds more generally than the derivation suggests.

We consider a target nucleus with A nucleons governed by the Hamiltonian $H^{(A)}$. This target is hit by a nucleon. The Hamiltonian for the $(A+1)$ particle system has the form $H = H^{(A)}+t+V$. Here, t is the operator of kinetic energy of relative motion and V the (effective) nucleon-nucleon interaction between the target and the nucleon. To account for absorption effects on the incident nucleon, we introduce a complex optical-model potential U, consisting of the mean field, the imaginary potential, and the dispersion integral giving a correction to the mean field. We write H in the form

$$H = (H^{(A)} + t + U) + \mathcal{V} \tag{2.1}$$

where $\mathcal{V} = (V - U)$, and calculate the cross section to second order in $\mathcal{V}$. Let $\varphi_i^{(+)}$ and $\varphi_f^{(-)}$ be the scattering eigenstates of $(H^{(A)}+t+U)$ with A nucleons in the initial and the final state of the target, respectively, and subject to the usual incoming or outgoing wave boundary conditions. They are normalized to a delta function in energy and carry normalization factors $(\mu k_i/2\pi\hbar^2)^{1/2}$ and $(\mu k_f/2\pi\hbar^2)^{1/2}$, respectively. Then, the two-step cross-section has the form

$$\sigma_{fi}^{(2)} = k_i^{-2}\sum_f \left|\left\langle \varphi_f^{(-)}\middle|\mathcal{V}\left(E_i^+ - H^{(A)} - t - U\right)^{-1}\mathcal{V}\middle|\varphi_i^{(+)}\right\rangle\right|^2 \delta(E_f - E_i). \tag{2.2}$$

Here, μ is the reduced mass, $\hbar k_i$ and $\hbar k_f$ are the initial and final momenta, and the delta function guarantees energy conservation.

Equation (2.2) can be simplified by assuming that the states of the target can be approximately described by the shell model, that the ground state is doubly magic, that in the collision of the incident nucleon with the target an ever-increasing number of particle-hole pairs is created, and that the collision is "sudden" in the sense that in the denominator of Eq. (2.2) $H^{(A)}$ can be replaced by the mean-field approximation. The last assumption was justified in II, the rest is standard in the field. We denote by Φ_i, Φ_f the initial and final states of the target, given by

$$|\Phi_i\rangle = \left(\prod_{j<k_F}|\alpha_j\rangle\right)_{\mathcal{A}}, \quad |\Phi_f\rangle = \left(|\beta_1\rangle\,|\beta_2\rangle\prod_{\substack{j<k_F\\ j\neq 1,2}}|\alpha_j\rangle\right)_{\mathcal{A}}. \tag{2.3}$$

Here, $\mathcal{A}$ is the antisymmetrisation operator, α_j denotes the states below the fermi surface, k_F, and β_1, β_2 (α_1, α_2) are the particle states (the hole states) generated by the twofold action of $\mathcal{V}$. We have

$$\left|\varphi_i^{(+)}\right\rangle = \left|\chi_i^{(+)}\Phi_i\right\rangle, \quad \left|\varphi_f^{(-)}\right\rangle = \left|\chi_f^{(-)}\Phi_f\right\rangle, \tag{2.4}$$

where $\chi_i^{(+)}, \chi_f^{(-)}$ are scattering eigenstates of the one-body operator $(t+U)$. Using the above-mentioned assumptions and Eqs. (2.3), (2.4), we find that Eq. (2.2) can be simplified as follows:

$$\sigma_{fi}^{(2)} = k_i^{-2}\sum_f \left|\sum_m \langle\chi_f^{(-)}\Phi_f|v|\Phi_m\rangle G_m(E_i - \epsilon_m)\langle\Phi_m^*|v|\chi_i^{(+)}\Phi_i\rangle\right|^2 \delta(E_f - E_i), \tag{2.5}$$

where v denotes the two-body effective interaction, the round bracket an integration over A nucleons, and Φ_m is one of the four possibilities

$$|\Phi_m\rangle = \left(|\beta_b\rangle \prod_{\substack{j<k_F \\ j\neq a}} |\alpha_j\rangle \right)_{\mathcal{A}}, \quad a \text{ and } b \text{ equal to 1 or 2.} \tag{2.6}$$

Moreover, $G_m(E_i - \epsilon_m)$ is the operator $(E_i^+ - \epsilon_m - t - U)^{-1}$, and ϵ_m the sum of the energies of the particle state β_b and of the hole state α_a in $|\Phi_m\rangle$. In the approximation scheme leading to Eq. (2.5), the delta function is now interpreted as the two-particle–two-hole level density in the Fermi-gas model at the excitation energy ΔE defined by the energy loss of the incident nucleon.

We can considerably improve on this brief derivation without changing the form of the result, Eq. (2.2). One such improvement removes the assumption that the target is in a pure shell-model state, and that the effect of the residual interaction on nucleons in the orbitals $|\alpha_j\rangle, |\beta_1\rangle$, and $|\beta_2\rangle$ is altogether negligible. The derivation is given in II. Using the "sudden approximation" (which it justifies) and statistical assumptions on the eigenstates of the final nucleus, this paper yields an equation like (2.2), with $\delta(E_f - E_i)$ replaced by the full (non-Fermi-gas) two-particle–two-hole density at energy E_f. If that level density can be well approximated by the Fermi gas expression, we retrieve Eq. (2.2). We expect such an approximation to hold for excitation energies ΔE of the residual nucleus which are in excess of several MeV. Incidentally, the derivation of II shows why there is no interference between one-step and two-step processes. A second improvement relates to the fact that the optical model depends upon energy; this fact was not taken into account in our derivation. In Appendix A, we show that Eq. (2.2) can be derived even when U depends on the channel.

III. THE APPROXIMATIONS

We now introduce the central approximations needed to write Eq. (2.5) in semiclassical form. We proceed in close analogy to I. To make the argument transparent, we defer to Appendix B the justification of the main approximation (the neglect of interference terms).

Writing the absolute square in Eq. (2.5) as a double sum over the four values of m and m' each, we neglect the interference terms with $m \neq m'$. We likewise neglect the antisymmetry between the incident nucleon and the nucleons in the target. We use the identity, valid for any function $F(E)$,

$$F(E - \epsilon_m) = \int dE_m \delta(E_m - E + \epsilon_m) F(E_m). \tag{3.1}$$

With $b' \neq b$ and $a' \neq a$, we also have, in coordinate representation,

$$\begin{aligned} (\Phi_m | v | \Phi_i) &= \int d^3r_1 \beta_b^*(\mathbf{r}_1) v(\mathbf{r}_1 - \mathbf{r}_i) \alpha_a(\mathbf{r}_1), \\ (\Phi_f | v | \Phi_m) &= \int d^3r_2 \beta_{b'}^*(\mathbf{r}_2) v(\mathbf{r}_2 - \mathbf{r}_f) \alpha_{a'}(\mathbf{r}_2). \end{aligned} \tag{3.2}$$

Using the coordinate representation also for the Green function G_m, we obtain

$$\sigma_{fi}^{(2)} = k_i^{-2} \int dE_m \prod_{l=i,f,1,2} \int d^3r_l \int d^3r_{l'} \chi_f^{(-)*}(\mathbf{r}_f) \chi_f^{(-)}(\mathbf{r}_f') \chi_i^{(+)}(\mathbf{r}_i) \chi_i^{(+)*}(\mathbf{r}_i') G(\mathbf{r}_f, \mathbf{r}_i; E_m) G^*(\mathbf{r}_f', \mathbf{r}_i'; E_m)$$
$$\times v(\mathbf{r}_1 - \mathbf{r}_i) v(\mathbf{r}_1' - \mathbf{r}_i') v(\mathbf{r}_2 - \mathbf{r}_f) v(\mathbf{r}_2' - \mathbf{r}_f') K_1(\mathbf{r}_1, \mathbf{r}_1') K_2(\mathbf{r}_2, \mathbf{r}_2'). \tag{3.3}$$

We have defined

$$K_1(\mathbf{r}_1; \mathbf{r}_1') = \sum_{\alpha_1, \beta_1} \alpha_1(\mathbf{r}_1) \alpha_1^*(\mathbf{r}_1') \beta_1^*(\mathbf{r}_1) \beta_1(\mathbf{r}_1') \delta(E_m + \epsilon_{\beta_1} - \epsilon_{\alpha_1} - E_i), \tag{3.4}$$

and similarly for K_2. The sum in Eq. (3.4) over α_1 (β_1) is over occupied (empty) states.

One of the central approximations of this paper, kin to the local density approximation or the Thomas-Fermi approximation, is based on the following approximation, exploited already in I: The potential v has a short range of about 1 fm. And the function K_1 is a short-ranged function of $|\mathbf{r}_1 - \mathbf{r}_1'|$ with approximate range $\kappa_F^{-1}(\mathbf{r}_1)$, where $\hbar\kappa_F$ is the local Fermi momentum. Only the region of $\mathbf{r}_1$ values with sufficiently high nucleon density contributes significantly to $\sigma_{fi}^{(2)}$. Here, $\kappa_F(\mathbf{r}_1)$ does not differ significantly from the nuclear matter value $k_F \cong 1$ fm^{-1}. The same argument applies to K_2. This shows that the integrand in Eq. (3.3) is significantly different from zero only when $\mathbf{r}_1 \cong \mathbf{r}_1' \cong \mathbf{r}_i \cong \mathbf{r}_i'$ and $\mathbf{r}_2 \cong \mathbf{r}_2' \cong \mathbf{r}_f \cong \mathbf{r}_f'$. We therefore approximate the expression (3.3) by locally expanding the integrand around the points of coincidence of each of the two sets of vectors $(\mathbf{r}_1, \mathbf{r}_1', \mathbf{r}_i, \mathbf{r}_i')$ and $(\mathbf{r}_2, \mathbf{r}_2', \mathbf{r}_f, \mathbf{r}_f')$. This approximation is expected to be valid for heavy nuclei with radii large in comparison with 1 fm. In implementing the local expansion, we assume that *locally* both single-particle wave functions and the distorted waves can be approximated by plane waves. We denote the local momenta appearing in the exponents by the symbol κ, with κ real or complex (as the case may be) and in any case depending on the position. The integrals appearing on the rhs of Eq. (3.2) take the form ($\mathbf{x} = \mathbf{r}_1 - \mathbf{r}_i$)

$$\int d^3r_1\, \beta_b^*(\mathbf{r}_1) v(\mathbf{r}_1 - \mathbf{r}_i)\alpha_a(\mathbf{r}_1)$$

$$\cong \exp\left[i(\boldsymbol{\kappa}_a - \boldsymbol{\kappa}_b)\cdot \mathbf{r}_i\right] \int d^3x\, \beta_b^*(\mathbf{x}) v(\mathbf{x}) \alpha_a(\mathbf{x}) \quad (3.5)$$

with $\boldsymbol{\kappa}_a, \boldsymbol{\kappa}_b$ real. With $\mathbf{r}_i' = \mathbf{r}_i + \mathbf{s}_i$ and $\boldsymbol{\kappa}_i$ complex, we have similarly for $\chi_i^{(+)}(\mathbf{r}_i')$,

$$\chi_i^{(+)}(\mathbf{r}_i') \cong \chi_i^{(+)}(\mathbf{r}_i) \exp\left(i\boldsymbol{\kappa}_i \cdot \mathbf{s}_i,\right), \quad (3.6)$$

where $\boldsymbol{\kappa}_i$ has the direction of the local flux and the magnitude

$$\kappa_i^2 = k_i^2 - \frac{2\mu}{\hbar^2} U_i(\mathbf{r}_i). \quad (3.7)$$

We proceed analogously for $\chi_f^{(-)}(\mathbf{r}_f')$, introducing $\boldsymbol{\kappa}_f$. For the Green function, we use the eikonal approximation [7]

$$G(\mathbf{r}_f, \mathbf{r}_i; E_m) = -\frac{\mu}{2\pi\hbar^2} \frac{\exp\left[iS_m(\mathbf{r}_f, \mathbf{r}_i)\right]}{|\mathbf{r}_f - \mathbf{r}_i|}, \quad (3.8)$$

where the complex phase S_m is given by

$$S_m(\mathbf{r}_f, \mathbf{r}_i) = \int_{\mathbf{r}_i}^{\mathbf{r}_f} ds\, \boldsymbol{\kappa}_m(\mathbf{s}). \quad (3.9)$$

The integration is taken along the classical path, with the direction of $\boldsymbol{\kappa}_m$ given by the tangent to the path or, equivalently, by the flux at s, and the magnitude of $\boldsymbol{\kappa}_m$ is given by

$$\kappa_m^2 = \frac{2\mu}{\hbar^2}\left[E_m - U_m(\mathbf{r})\right]. \quad (3.10)$$

In the local approximation, we assume $U_m(\mathbf{r})$ to vary so little that the path in (3.9) is straight and, with $\mathbf{r} = \frac{1}{2}(\mathbf{r}_f + \mathbf{r}_i)$,

$$S_m(\mathbf{r}_f, \mathbf{r}_i) \cong \boldsymbol{\kappa}_m(\mathbf{r}) |\mathbf{r}_f - \mathbf{r}_i|. \quad (3.11)$$

We note that Eq. (3.11) goes beyond the previous local approximation. For $G^*(\mathbf{r}_f', \mathbf{r}_i'; E_m)$ we proceed analogously, writing $\mathbf{r}_i' = \mathbf{r}_i + \mathbf{s}_i$, etc. as before. Then, with $\boldsymbol{\kappa}_m$ in the direction of $(\mathbf{r}_f - \mathbf{r}_i)$,

$$G^*(\mathbf{r}_f', \mathbf{r}_i'; E_m) \cong -\frac{\mu}{2\pi\hbar^2} \frac{e^{-i\kappa_m^* |\mathbf{r}_f - \mathbf{r}_i|}}{|\mathbf{r}_f - \mathbf{r}_i|} \exp\left[-i\boldsymbol{\kappa}_m \cdot (\mathbf{s}_f - \mathbf{s}_i)\right]. \quad (3.12)$$

In the spirit of these local approximations, the summations in Eq. (3.4), when written in terms of local momenta, are constrained by the condition $\kappa_a < \kappa_F < \kappa_b$.

To further simplify the resulting expressions, it is necessary to assume that $U_i \cong U_m \cong U_f$. More precisely, we assume that in the integration over $\mathbf{s}_i$, with an integrand given by the exponential of $i\mathbf{s}_i \cdot (\boldsymbol{\kappa}_i + \boldsymbol{\kappa}_a - \boldsymbol{\kappa}_m - \boldsymbol{\kappa}_b)$, the imaginary parts of $\boldsymbol{\kappa}_i$ and $\boldsymbol{\kappa}_m$ cancel to the extent that the result in good approximation is $(2\pi)^3$ times a delta function of $(\boldsymbol{\kappa}_i + \boldsymbol{\kappa}_a - \boldsymbol{\kappa}_m - \boldsymbol{\kappa}_b)$, and similarly for $\mathbf{s}_f$. By the same token, we replace the argument of the delta function in Eq. (3.4) by its local equivalent, i. e., by $(\hbar^2/2\mu)\left(\kappa_f^2 - \kappa_m^2 + \kappa_b^2 - \kappa_a^2\right)$. Using all this in Eq. (3.3), we obtain

$$\sigma_{fi}^{(2)} = k_i^{-2} \int dE_m \int d^3r_i \int d^3r_f \left|\chi_f^{(-)}(\mathbf{r}_f)\right|^2 \left|\chi_i^{(+)}(\mathbf{r}_i)\right|^2 \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{\left|\exp\left[i\kappa_m(\mathbf{r})\,|\mathbf{r}_f - \mathbf{r}_i|\right]\right|^2}{|\mathbf{r}_i - \mathbf{r}_f|^2}$$
$$\times \left\{ \sum_{\kappa_{a_1} < \kappa_F(\mathbf{r}_i) < \kappa_{b_1}} |(\beta_1|\, v\, |\alpha_1)|^2 (2\pi)^3 \delta^3(\boldsymbol{\kappa}_{b_1} + \boldsymbol{\kappa}_m(\mathbf{r}) - \boldsymbol{\kappa}_{a_1} - \boldsymbol{\kappa}_i) \right.$$
$$\left. \times\ \delta\left(\frac{\hbar^2}{2\mu}\left[\kappa_{b_1}^2 + \kappa_m^2(\mathbf{r}) - \kappa_{a_1}^2 - \kappa_i^2\right]\right) \right\}$$
$$\times \left\{ \sum_{\kappa_{a_2} < \kappa_F(\mathbf{r}_f) < \kappa_{b_2}} |(\beta_2|\, v\, |\alpha_2)|^2 (2\pi)^3 \delta^3(\boldsymbol{\kappa}_{b_2} + \boldsymbol{\kappa}_f - \boldsymbol{\kappa}_{a_2} - \boldsymbol{\kappa}_m(\mathbf{r})) \right.$$
$$\left. \times\ \delta\left(\frac{\hbar^2}{2\mu}\left[\kappa_{b_2}^2 + \kappa_f^2 - \kappa_{a_2}^2 - \kappa_m(\mathbf{r})\right]\right) \right\}. \quad (3.13)$$

To arrive at the final formula of this section, we connect each of the terms in curly brackets in Eq. (3.13) with the nucleon-nucleon cross section. To this end, we write

$$\sum_{\kappa_a < \kappa_F < \kappa_b} = 2\left(\frac{\Omega}{(2\pi)^3}\right)^2 \left(\left[\underbrace{\int d^3\kappa_a \int d^3\kappa_b}_{\kappa_a < \kappa_F < \kappa_b} \right]_n + \left[\underbrace{\int d^3\kappa_a \int d^3\kappa_b}_{\kappa_a < \kappa_F < \kappa_b} \right]_p \right), \quad (3.14)$$

where the factor 2 is due to spin (we take v to be spin independent), where $\Omega/(2\pi)^3$ is a density-of-states factor and Ω the size of the Thomas-Fermi cell volume, and where the indices n (p) indicate that the struck nucleon is a neutron

(proton). Locally, we have

$$|(\beta|\, v\,|\alpha)|^2 \cong \Omega^{-1}\left|\int_{\Omega} d^3x \exp\left[i\,(\kappa_a - \kappa_b\cdot]\,\mathbf{x}\right) v(\mathbf{x})\right|^2 . \tag{3.15}$$

In the two-nucleon c.m. frame and in Born approximation, the nucleon-nucleon cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{NN} = \frac{(m/2)^2}{(2\pi\hbar^2)^2}\,\Omega^2\,|(\beta|\, v\,|\alpha)|^2 , \tag{3.16}$$

where v is isospin dependent. We introduce the average local nucleon-nucleus cross section in Born approximation by

$$\left(\frac{d^2\sigma}{dE'd\Omega'}\right)_{\mathbf{r}_i} = 4\,\frac{\kappa_m(\mathbf{r})}{\kappa_i(\mathbf{r}_i)}\left(\frac{4\pi}{3}\kappa_F^3(\mathbf{r}_i)\right)^{-1} \underbrace{\int d^3\kappa_a \int d^3\kappa_b}_{\kappa_a<\kappa_F(\mathbf{r}_i)<\kappa_b} \left(\frac{d\sigma}{d\Omega}\right)_{NN} \delta^3(\kappa_b + \kappa_m(\mathbf{r}) - \kappa_a - \kappa_i)$$
$$\times\,\delta\left(\frac{\hbar^2}{2\mu}\left[\kappa_b^2 + \kappa_m^2(\mathbf{r}) - \kappa_a^2 - \kappa_i^2\right]\right), \tag{3.17}$$

and similarly at point $\mathbf{r}_f$. It was shown in I that upon replacing $(d\sigma/d\Omega)_{NN}$ in Eq. (3.17) by an isotropic expression, $\sigma_t/(4\pi)$, with σ_t the experimental total nucleon-nucleon cross-section, the rhs of Eq. (3.17) becomes equal to the cross section of Kikuchi and Kawai [8]. Using Eqs. (3.16) and (3.17), we can express the curly brackets in Eq. (3.13) in terms of the local nucleon-nucleon cross section at $\mathbf{r}_i$ and $\mathbf{r}_f$. The normalization factors of $|\chi_i^{(+)}|^2$ and $|\chi_f^{(-)}|^2$ are $|\chi_i^{(+)}|^2 = (\mu k_i/2\pi\hbar^2)|\tilde{\chi}_i^{(+)}|^2$, where $\tilde{\chi}$ is asymptotically normalized to a plane wave of unit amplitude. We introduce the local density for protons and neutrons,

$$\rho_{n,p} = \frac{2}{(2\pi)^3}\frac{4\pi}{3}\kappa_{F\,n,p}^3 \tag{3.18}$$

and find

$$\sigma_{fi}^{(2)} = \left(\frac{A}{A+1}\right)^4 \int dE_m \int d^3r_i \int d^3r_f \frac{\kappa_i(\mathbf{r}_i)/k_i}{\kappa_f(\mathbf{r}_f)/k_f}\left|\tilde{\chi}_f^{(-)}(\mathbf{r}_f)\right|^2 \left(\frac{\partial^2\sigma}{\partial E_f\,\partial\Omega_f}\right)_{\mathbf{r}_f} \rho(\mathbf{r}_f)$$
$$\times\frac{\exp\left[-2\gamma_m(\mathbf{r})\,|\mathbf{r}_f - \mathbf{r}_i|\right]}{|\mathbf{r}_f - \mathbf{r}_i|^2}\left(\frac{\partial^2\sigma}{\partial E_m\,\partial\Omega_m}\right)_{\mathbf{r}_i} \rho(\mathbf{r}_i)\left|\tilde{\chi}_i^{(+)}(\mathbf{r}_i)\right|^2 . \tag{3.19}$$

Here, $\gamma_m(\mathbf{r})$ stands for the imaginary part of $\kappa_m(\mathbf{r})$, with $\mathbf{r} = \frac{1}{2}(\mathbf{r}_i + \mathbf{r}_f)$. The product of the double differential cross section times ρ appearing twice on the rhs of Eq. (3.19) is a shorthand notation for the sum of two terms relating to neutrons and protons in the target as in Eq. (3.14).

Our result, Eq. (3.19), has a simple interpretation. The incident flux, weakened by absorption effects and refracted by the real potential, penetrates into the nuclear interior. At point $\mathbf{r}_i$ the first nucleon-nucleon collision occurs, described by the product of the double-differential cross section (a local quantity because it depends on the local momenta), and the local density of nucleons. Then the incident nucleon, having lost some of its energy, propagates to point $\mathbf{r}_f$. The propagation is described by the geometric factor $|\mathbf{r}_i - \mathbf{r}_f|^{-2}$ and by the factor $\exp(-2\gamma_m|\mathbf{r}_i - \mathbf{r}_f|)$ which accounts for loss of flux by absorption. At $\mathbf{r}_f$, the second collision takes place. Eventually, the flux leaves the nucleus in the direction of $\mathbf{k}_f$, after it was further deflected and absorbed as described by the factor $|\tilde{\chi}_f^{(-)}|^2$.

In applying Eq. (3.19) it is well to remember that after the first collision the incident nucleon leaves the point $\mathbf{r}_i$ with a local energy that corresponds asymptotically to the energy E_m; the corresponding local energy at point $\mathbf{r}_f$ defines the energy with which the second collision is initiated. And the direction in which the nucleon leaves the point $\mathbf{r}_i$ is that of the vector $\mathbf{r}_f - \mathbf{r}_i$.

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APPENDIX A: CHANNEL-DEPENDENT OPTICAL POTENTIAL

We allow for a channel dependence of the optical potential and denote by $U_c^{(+)}$ the optical potential in channel c, by $U_c^{(-)}$ its complex conjugate, and by $\mathcal{V}_c^{(\pm)} =$

$V - U_c^{(\pm)}$ the remaining interaction. Let ϕ_c be the continuum eigenfunctions of $H^{(A)} + t$, products of the target state Φ_c and a plane wave of relative motion for the nucleon in the continuum, orthonormalized in the usual way. The optical-model distorted wave states $\varphi_c^{(\pm)}$ obey the Lippmann-Schwinger equation

$$\left|\varphi_c^{(\pm)}\right\rangle = \left|\phi_c\right\rangle + \left[E^{\pm} - H^{(A)} - t\right]^{-1} U_c^{(\pm)} \left|\varphi_c^{(\pm)}\right\rangle . \tag{A1}$$

The full scattering solution $\Psi_c^{(\pm)}$ obeys both the equations

$$\left|\Psi_c^{(\pm)}\right\rangle = \left|\phi_c\right\rangle + \left[E^{\pm} - H\right]^{-1} V \left|\phi_c\right\rangle \tag{A2}$$

and

$$\left|\Psi_c^{(\pm)}\right\rangle = \left|\varphi_c^{(\pm)}\right\rangle + \left[E^{\pm} - H\right]^{-1} \mathcal{V}_c^{(\pm)} \left|\varphi_c^{(\pm)}\right\rangle . \tag{A3}$$

The transition amplitude is given by

$$T_{fi} = \left\langle \phi_f \left| V \right| \Psi_i^{(+)} \right\rangle . \tag{A4}$$

In Eq. (A4), we use Eq. (A3) for channel i, then Eq. (A2) for channel f, and finally Eq. (A3) for channel f. This yields (we drop the plus sign on U_c and $\mathcal{V}_c$)

$$\begin{aligned} T_{fi} = &\left\langle \phi_f \left| U_i \right| \varphi_i^{(+)} \right\rangle + \left\langle \varphi_f^{(-)} \left| \mathcal{V}_i \right| \varphi_i^{(+)} \right\rangle \\ &+ \left\langle \varphi_f^{(-)} \left| \mathcal{V}_f \left[E^{+} - H\right]^{-1} \mathcal{V}_i \right| \varphi_i^{(+)} \right\rangle . \end{aligned} \tag{A5}$$

Equation (A5) is still exact. We identify the first term on the r.h.s. with the optical-model contribution to elastic scattering ($i = f$), and the second term with the one-step DWBA amplitude. Only the last term can contribute to the two-step amplitude. In the sudden approximation, we neglect the interaction between nucleons in bound shell-model states and replace in each intermediate channel m the operator H by $(\epsilon_m + t + U_m)$. This yields the desired generalization of Eq. (2.2).

APPENDIX B: NEGLECT OF INTERFERENCE TERMS

It is intuitively clear that the terms with $m \neq m'$ should become less important as the incident energy increases. In this appendix, we show that this is indeed the case, and we give a rough estimate of the bombarding energy E_i where neglect of these terms is justified.

By way of example, let us consider for m the sequence $|0\rangle \rightarrow |\alpha_1\beta_1\rangle \rightarrow |\alpha_1\alpha_2\beta_1\beta_2\rangle$ and for m' the sequence $|0\rangle \rightarrow |\alpha_2\beta_1\rangle \rightarrow |\alpha_1\alpha_2\beta_1\beta_2\rangle$, in obvious notation. Without introducing the E_m integration via Eq. (3.1), we follow the derivation of Sec. III until we arrive at a product of sums of the type displayed in Eq. (3.4). These now have the form

$$\sum_{\alpha_1 < k_f < \beta_1} \alpha_1(\mathbf{r}_1)\alpha_1^*(\mathbf{r}_2\,')\beta_1^*(\mathbf{r}_1)\beta_1(\mathbf{r}_1\,')$$

and (B1)

$$\sum_{\alpha_2 < k_f < \beta_2} \alpha_2(\mathbf{r}_2)\alpha_2^*(\mathbf{r}_1\,')\beta_2^*(\mathbf{r}_2)\beta_2(\mathbf{r}_2\,').$$

The terms in expression (B1) are small unless $\mathbf{r}_1 \cong \mathbf{r}_1\,' \cong \mathbf{r}_2\,'$ and $\mathbf{r}_2 \cong \mathbf{r}_2\,' \cong \mathbf{r}_1\,'$. Together with the conditions derived from the short range of the potential $v(\mathbf{r})$, these constraints amount to saying that the essential contribution to the interference term is due to the integration over the point where all vectors $\mathbf{r}_1, \mathbf{r}_1\,', \mathbf{r}_2, \mathbf{r}_2\,', \mathbf{r}_i, \mathbf{r}_i\,', \mathbf{r}_f, \mathbf{r}_f\,'$ (nearly) coincide. This is a stronger constraint than encountered for the terms with $m = m'$ and qualitatively accounts for the neglect of interference terms.

Semiquantitatively, we expect the cutoff to be given by k_F^{-1}. This has to be compared with the integration over $|\mathbf{r}_i - \mathbf{r}_f|$ in Eq. (3.19) which is limited by the exponential damping factor in the Green function. The ratio of both is $\gamma/k_F = (lk_F)^{-1}$, where l is the elastic mean free path. Now, l is a function of energy. Because of the exclusion principle, l reaches a minimum of about 4 fm at an energy of about 100 MeV, and increases slowly with increasing energy thereafter [8,9]. The product $(k_F l)$ is thus always larger than unity, especially at small bombarding energies. There, however, the interference effects considered below are important.

There is another type of interference term in which the integrand is significant only when $\mathbf{r}_1 \cong \mathbf{r}_2\,'$ and $\mathbf{r}_2 \cong \mathbf{r}_1\,'$. This constraint is much weaker than in the case discussed above. Nevertheless, the contribution of this type is small compared to the terms with $m = m'$. Indeed, the constraint just mentioned and the short range of v lead to the condition $\mathbf{r}_i \cong \mathbf{r}_f\,'$ and $\mathbf{r}_f \cong \mathbf{r}_i\,'$, and to the appearance of the factor $\chi_f^{(-)*}(\mathbf{r}_f)\chi_f^{(-)}(\mathbf{r}_i)\chi_i^{(+)}(\mathbf{r}_i)\chi_i^{(+)*}(\mathbf{r}_f)$ in the integrand of $\sigma_{fi}^{(2)}$. This factor is an oscillating function of both $\mathbf{r}_i$ and $\mathbf{r}_f$, especially at high incident energies and/or deep distorting potentials, in contrast to the positive definite factor appearing in Eq. (3.19).

In summary, the arguments presented above only indicate that the neglect of interference terms is a fair approximation at a few 100 MeV, and possibly useful even below 100 MeV, depending on the required accuracy.

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