Quantal and thermal zero point motion formulae of barrier transmission probability

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A Green's function method is developed to derive quantal zero point motion formulae for the barrier transmission probability in heavy ion fusion reactions corresponding to various nuclear intrinsic degrees of freedom. In order to apply to the decay of a hot nucleus, the formulae are then generalized to the case where the intrinsic degrees of freedom are in thermal equilibrium with a heat bath. A thermal zero point motion formula for vibrational coupling previously obtained through the use of influence functional methods naturally follows, and the effects of rotational coupling are found to be independent of temperature if the deformation is rigid.

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I. INTRODUCTION

The observed fusion cross section of heavy ion collisions at energies near and below the Coulomb barrier is much larger than the prediction of a one-dimensional potential model [1]. A possible explanation is offered by a zero point motion formula of the barrier transmission probability, which takes the effects of the surface vibration of the target nucleus into account [2]. In this formula, the fusion cross section is expressed as a Gaussian average over the fusion cross sections which correspond to various values of the vibrational amplitude. More generally, the inclusive barrier transmission probability leading to fusion for each partial wave can be written as a suitable average of a transmission probability through a fluctuating potential barrier [3,4]. Experimental fusion cross sections were sometimes analyzed based on this idea. Recently, Stelson [5] and Rowley *et al.* [6] have tried to determine the distribution of the potential barriers directly from the experimental fusion cross section by an inversion method. Similar ideas have been used to discuss the effects of the nuclear intrinsic degrees of freedom on elastic scattering [7,8], where the zero point motion formula holds for the S matrix.

A finite-temperature adiabatic formula, which takes into account thermal fluctuations in the nuclear quadrupole degrees of freedom, was developed to explain the observed giant dipole resonance in hot rotating nuclei [9,10]. Fluctuations in the intrinsic shape degrees of freedom explain the broadening of the resonance absorption cross section at finite temperatures [9]. Fluctuations in the nuclear orientation with respect to its rotation axis explain the observed attenuation in the angular anisotropy of the gamma rays emitted in the resonance decay [10].

In Ref. [3] the influence functional method was used to express the effect of the coupling between the relative motion and the nuclear surface vibration on the fusion probability. In the adiabatic limit (i.e., when the surface vibration is very slow on the time scale of the fusion) this expression reduces to the zero point motion formula. This method has been extended in Ref. [11] to include thermal fluctuations for the case of vibrational coupling. The resulting formula has then been used to discuss the effects of surface vibrations of the residual nucleus on the spectral distribution of particles emitted from a hot compound nucleus.

The influence functional method is, however, quite intricate and cumbersome. The purpose of this paper is to show that, in the adiabatic limit and the no-Coriolis approximation [12,13], a zero point motion formula can be directly derived using Green's function methods. These methods are much simpler than the influence functional techniques, and can be easily applied to various intrinsic degrees of freedom of the target nucleus. Moreover, the relation between the weight function and the quantum mechanical distribution function becomes transparent. Generalizations of this formula at zero and finite temperature to any model Hamiltonian, which describes the internal degrees of freedom and their coupling to the relative motion, are derived elsewhere [14].

The paper is organized as follows. In Sec. II, we derive the zero point motion formula for the barrier transmission probability for a given partial wave in heavy ion fusion reactions corresponding to three different models of the intrinsic degrees of freedom: a vibrator, a rotor, and a system with finite number of equidistant energy levels. In Sec. III, we extend the formula to the case where the intrinsic degrees of freedom are in thermal equilibrium with a heat bath. We summarize the paper in Sec. IV.

II. ZERO POINT MOTION FORMULAE FOR FUSION

A. Harmonic oscillator linearly coupled to the translational motion

Let us first derive the zero point motion formula of Refs. [2] and [3] by assuming that the relative motion between heavy ions is linearly coupled to a λ-pole vibrational mode of excitation of the target nucleus. The Green's function for this system reads

$$G^{(+)}(E)=\frac{1}{E^{+}-\hat{H}_t-\hat{H}_v-f(r)\hat{\alpha}_{\lambda\mu}Y^{*}_{\lambda\mu}(\Omega_r)} , \tag{1}$$

where

$$\hat{H}_t=-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu r^2}\hat{l}^2+U(r) . \tag{2}$$

$U(r)$ is the bare potential as a function of the relative distance r between the heavy ions, and Ω_r specifies the direction of $\mathbf{r}$. $\hat{H}_v$ is the Hamiltonian of the λ-pole vibration, whose coordinate is denoted by $\hat{\alpha}_{\lambda\mu}$, and $\hat{l}$ is the angular momentum operator of the relative motion. The function $f(r)$ represents the coupling form factor. We now make a rotation from the coordinate system in which the z axis is along the beam direction, to the coordinate system in which the z axis is along the direction of the radial vector $\mathbf{r}$ [15]. If we denote the corresponding rotation operator in the target's space by $\hat{R}(\Omega)$ and the angle of the radial vector in the new coordinate system by Ω'_r, then

$$G^{(+)}(E)=\hat{R}(\Omega)\frac{1}{E^{+}-\hat{H}'_t-\hat{H}_v-f(r)\hat{\alpha}_{\lambda\mu}Y^{*}_{\lambda\mu}(\Omega'_r)}\hat{R}^{\dagger}(\Omega) , \tag{3}$$

where $\hat{H}'_t$ is $\hat{H}_t$ in the new coordinate system. Note that

$$Y_{\lambda\mu}(\Omega'_r)=\delta_{\mu 0}\sqrt{(2\lambda+1)/4\pi} . \tag{4}$$

We now introduce the no-Coriolis approximation [12], i.e., we replace $\hat{l}^2$ in Eq. (2) by $J(J+1)$, where J is the total angular momentum. Then $\hat{H}'_t=\hat{H}^{(0)}_t$ with

$$\hat{H}^{(0)}_t=-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu r^2}J(J+1)+U(r) . \tag{5}$$

The Green's function of Eq. (3) can be written as

$$G^{(+)}(E)=\hat{R}(\Omega)G^{(+)}_R(E)\hat{R}^{\dagger}(\Omega) , \tag{6}$$

where $G^{(+)}_R(E)$ is the Green's function in the rotating frame,

$$G^{(+)}_R(E)=\frac{1}{E^{+}-\hat{H}^{(0)}_t-\hat{H}_v-\sqrt{[(2\lambda+1)/4\pi]}\hat{\alpha}_{\lambda 0}f(r)} . \tag{7}$$

It is remarkable that the angular momentum algebra associated with the finite multipolarity of the intrinsic vibrational mode of excitation is removed in this approximation [12]. The finite multipolarity of the intrinsic excitation enters through the scaling factor of the coupling strength $\sqrt{(2\lambda+1)/4\pi}$.

For an inclusive process, one can show that $\hat{R}(\Omega)$ and $\hat{R}^{\dagger}(\Omega)$ in Eq. (6) can be ignored if the angular momentum of the initial state is zero, or more generally, if the initial state is not polarized (see Appendix). Thus in the following we will use the expression (7) for $G^{(+)}(E)$.

Now let us consider a problem, where the oscillator makes a transition from a state with quantum number n_i to a state with quantum number n_f. The associated reduced Green's function in the subspace of the coordinate r is given by

$$G^{(+)}_{n_f n_i}(E)=\langle n_f|\frac{1}{E^{+}-\hat{H}^{(0)}_t-\hat{H}_v-\sqrt{[(2\lambda+1)/4\pi]}\hat{\alpha}_{\lambda 0}f(r)}|n_i\rangle . \tag{8}$$

We next introduce the adiabatic approximation. In this limit since the energy scale associated with $\hat{H}_v$ is smaller than E, we discard it in Eq. (8). The quantum numbers n_i and n_f refer to all azimuthal components of the vibrational mode of excitation. Under the adiabatic and the no-Coriolis approximations, only the $\mu=0$ component is active. We therefore proceed in the following as if we are dealing with a monopole vibration. We will explicitly pay attention to the other components in Sec. III when we derive the thermal zero point motion formula. The important consequence of the adiabatic approximation is that the resultant Green's function operator is diagonal in the coordinate representation of the vibrator. Defining a dimensionless vibrational coordinate x by $\hat{\alpha}_{\lambda 0}=\alpha_0 x$ with

$$\alpha_0=\sqrt{\hbar/m\omega} , \tag{9}$$

we obtain

$$G^{(+)}_{n_f n_i}(E)=\int_{-\infty}^{\infty}dx\,\langle n_f|x\rangle\langle x|n_i\rangle G^{(+)}_0\left[E;U(r)+\left[\frac{2\lambda+1}{4\pi}\right]^{1/2}\alpha_0 x f(r)\right] . \tag{10}$$

Here $G^{(+)}_0(E;U(r))$ denotes the Green's function for a one-dimensional problem with potential $U(r)$ and energy E,

$$G^{(+)}_0(E;U(r))=\frac{1}{E^{+}-\hat{H}^{(0)}_t} . \tag{11}$$

Note that the bare potential $U(r)$ is replaced by an effective potential $U(r)+\sqrt{[(2\lambda+1)/4\pi]}\alpha_0 x f(r)$ (where x is a pa-

rameter) in the Green's function $G_0^{(+)}$ that appears on the right-hand side (r.h.s.) of Eq. (10). Using the completeness relation

$$\sum_{n_f} \langle x_1|n_f\rangle\langle n_f|x_2\rangle=\delta(x_1-x_2)\ , \tag{12}$$

the inclusive transmission probability from a point r_i on one side of the potential barrier to a point r_f on the other side of the barrier is found to be

$$\begin{aligned} P_{\text{inclus}}(n_i;r_f,r_i)&=\sum_{n_f} P(n_f,r_f;n_i,r_i) \\ &=\int_{-\infty}^{\infty} dx\,|\langle n_i|x\rangle|^2\left|\langle r_f|G_0^{(+)}\left[E;U(r)+\left[\frac{2\lambda+1}{4\pi}\right]^{1/2}\alpha_0 x f(r)\right]|r_i\rangle\right|^2 . \end{aligned} \tag{13}$$

The second factor in the integrand of Eq. (13) is the barrier transmission probability through a potential barrier $U(r)+\sqrt{[(2\lambda+1)/4\pi]}\alpha_0 x f(r)$. If $|n_i\rangle$ is the vibrator's ground state, as is the case for heavy ion fusion reactions, then

$$\langle n_i|x\rangle=\pi^{-1/4}e^{-x^2/2}\ . \tag{14}$$

Equation (13) then becomes

$$P_{\text{inclus}}(n_i=0;r_f,r_i)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} dx\ e^{-x^2}P^{(0)}\left[E;U(r)+\left[\frac{2\lambda+1}{4\pi}\right]^{1/2}\alpha_0 x f(r)\right] . \tag{15}$$

In Eq. (15), $P^{(0)}(E;U(r))$ is the transmission probability of a particle with energy E across a potential barrier $U(r)$. This equation is nothing but the zero point motion formula obtained in Ref. [3] except that in Ref. [3] $\alpha_0/\sqrt{2}$ was used instead of α_0 and $x/\sqrt{2}$ instead of x. This method clearly exhibits that the weight function of the averaging procedure is given by the square of ground state wave function of the harmonic oscillator.

B. Rotational motion coupled to the translational motion

We next consider the effects of coupling the relative motion to the rotational motion of a deformed target. We assume a static quadrupole deformation with axial symmetry, and make in Eq. (1) the well known transformation [16] from $\hat{\alpha}_{\lambda\mu}$ to the intrinsic deformation parameter β and the Euler angles. The properties of the Wigner D function and similar steps to those used to obtain Eq. (7) lead to

$$G^{(+)}(E)=\frac{1}{E^+-\hat{H}_t^{(0)}-\hat{H}_R-\sqrt{(5/4\pi)}\beta P_2(\cos\hat{\theta})f(r)}\ , \tag{16}$$

where $\hat{H}_R$ is the Hamiltonian of a rotor, and $\hat{\theta}$ is the angle between the symmetry axis of the deformed nucleus and the beam direction. In the adiabatic limit we discard $\hat{H}_R$ in Eq. (16). The reduced Green's function that corresponds to a rotational transition from $|I_iM_i\rangle$ to $|I_fM_f\rangle$ is then given by

$$G^{(+)}_{I_fM_fI_iM_i}(E)=\int_{-1}^{1} d\cos\theta\int_0^{2\pi} d\phi\langle I_fM_f|\theta\phi\rangle\langle\theta\phi|I_iM_i\rangle G_0^{(+)}\left[E;U(r)+\left[\frac{5}{4\pi}\right]^{1/2}\beta P_2(\cos\theta)f(r)\right] . \tag{17}$$

The bare potential $U(r)$ is now replaced by an effective potential $U(r)+\sqrt{(5/4\pi)}\beta P_2(\cos\theta)f(r)$ in the Green's function $G_0^{(+)}$ that appears on the r.h.s. of Eq. (17). Using the completeness relation of the spherical harmonics, the inclusive transmission probability for an unpolarized target is given by

$$\begin{aligned} P_{\text{inclus}}(I_i;r_f,r_i)&=\frac{1}{2I_i+1}\sum_{M_i}\sum_{I_f}\sum_{M_f} P(I_fM_f,r_f;I_iM_i,r_i) \\ &=\int_{-1}^{1} d\cos\theta\int_0^{2\pi} d\phi\frac{1}{2I_i+1}\sum_{M_i}|Y_{I_iM_i}(\theta,\phi)|^2\left|\langle r_f|G_0^{(+)}\left[E;U(r)+\left[\frac{5}{4\pi}\right]^{1/2}\beta P_2(\cos\theta)f(r)\right]|r_i\rangle\right|^2 . \end{aligned} \tag{18}$$

Using the addition theorem for the spherical harmonics, Eq. (18) becomes

$$\begin{aligned} P_{\text{inclus}}(I_i;r_f,r_i)&=\int_0^1 d\cos\theta\left|\langle r_f|G_0^{+}\left[E;U(r)+\left[\frac{5}{4\pi}\right]^{1/2}\beta P_2(\cos\theta)f(r)\right]|r_i\rangle\right|^2 \\ &=\int_0^1 d\cos\theta\,P^{(0)}\left[E;U(r)+\left[\frac{5}{4\pi}\right]^{1/2}\beta P_2(\cos\theta)f(r)\right] , \end{aligned} \tag{19}$$

and gives an inclusive transmission probability which is independent of the initial spin I_i of the rotor. Equation (19) corresponds to the zero point motion formula discussed in the previous section for a vibrational coupling. Note that the Hermite and the Gauss integrals in Eqs. (15) and (19) reduce to the Hermite and the Gauss finite points' quadratures, respectively, if the vibrational and the rotational modes of excitation are truncated at some quantum number [4]. Note also that the contribution of high-lying states converges fairly quickly [12].

C. Linear coupling to a system with finite number of equidistant energy levels

We now consider a simple system with equidistant $(2j+1)$-energy levels

$$\hat{H}_s=\omega_0\hat{J}_z \ , \tag{20}$$

coupled to a translational degree of freedom r as

$$\hat{H}_{\rm int}=2\hat{J}_x f(r) \ , \tag{21}$$

where $\hat{J}_x$ and $\hat{J}_z$ are the SU(2) generators in the $(2j+1)$-dimensional representation and $f(r)$ is a real function of r. One can think of this system, for example, as a two-level Lipkin-Meshkov-Glick system with no self-interactions, where transitions between the two levels are caused only by the coupling to the external degree of freedom. The system with $j=\frac{1}{2}$ can be used as a simple model to discuss the effects of nucleon transfer on the fusion cross section. The Green's function of this system is given by

$$G^{(+)}(E)=\frac{1}{E^+-\hat{H}_t^{(0)}-\hat{H}_s-2\hat{J}_x f(r)} \ . \tag{22}$$

We discard $\hat{H}_s$ in the adiabatic limit. It is convenient to make the following transformation to the resultant Green's function before we consider the reduced Green's function (see the Appendix of Ref. [3]):

$$G^{(+)}(E)=e^{i\hat{J}_y\pi/2}\frac{1}{E^+-\hat{H}_t^{(0)}-2\hat{J}_z f(r)}e^{-i\hat{J}_y\pi/2} \ . \tag{23}$$

The reduced Green's function corresponding to the transition from the $|jm_i\rangle$ state to the $|jm_f\rangle$ state is now easily found to be

$$G^{(+)}_{jm_f,jm_i}(E)=\sum_m d^{j*}_{mm_f}\left[\frac{\pi}{2}\right]\frac{1}{E^+-\hat{H}_t^{(0)}-2mf(r)}d^{j}_{mm_i}\left[\frac{\pi}{2}\right] \ . \tag{24}$$

The initial quantum number m_i equals $-j$ in the fusion process. Changing the notation of the quantum number of the intermediate state from m to k according to $m=j-k$, the inclusive transmission probability is given by

$$P_{\rm inclus}(m_i=-j;r_f,r_i)=\frac{1}{4^j}\sum_{k=0}^{2j}\frac{(2j)!}{k!(2j-k)!}P^{(0)}(E;U(r)+2(j-k)f(r)) \ . \tag{25}$$

This is the same equation obtained in Ref. [3] through the influence functional method. Note that the weight of each potential barrier in Eq. (25) is given by

$$\left|d^{j}_{m-j}\left[\frac{\pi}{2}\right]\right|^2=\frac{1}{4^j}\frac{(2j)!}{(j+m)!(j-m)!} \ . \tag{26}$$

III. THERMAL ZERO POINT MOTION FORMULA

We now extend the zero point motion formula to the case where the relative motion couples to a nuclear intrinsic degree of freedom which is in thermal equilibrium with a heat bath at temperature T. This applies, e.g., to the problem of a particle emission from a hot compound nucleus. In that case r refers to the relative motion between the emitted particle and the residual nucleus. The intrinsic degrees of freedom are those either of the emitted particle or of the residual nucleus.

A. Harmonic oscillator linearly coupled to the translational motion

Let us first consider the case where the intrinsic degree of freedom is vibrational. We generalize Eq. (15) to incorporate the thermal fluctuations of the vibration. In order to show how the case with a finite multipolarity of the vibration is treated, we specify the full quantum numbers of the vibrational motion by introducing a suffix μ that specifies the azimuthal quantum number. The inclusive transmission probability which generalizes Eq. (13) is then defined by

$$P_{\rm inclus}(T;r_f,r_i)=\frac{\left[\prod_\mu\sum_{n_i^{(\mu)}}\sum_{n_f^{(\mu)}}\right]P\left[\prod_\mu n_f^{(\mu)},r_f;\prod_\mu n_i^{(\mu)},r_i\right]\exp\left[-\hbar\omega\left[\sum_{n_i^{(\mu)}}n_i^{(\mu)}+\frac{5}{2}\right]/T\right]}{\left[\prod_{\mu=-2,-1,0,1,2}\sum_{n_i^{(\mu)}}\sum_{n_f^{(\mu)}}\right]\exp\left[-\hbar\omega(n_i^{(-2)}+n_i^{(-1)}+n_i^{(0)}+n_i^{(1)}+n_i^{(2)}+\frac{5}{2})/T\right]} \ , \tag{27}$$

where we have used a shorthand notation for the azimuthal components μ, which run from -2 to 2 as suggested in the denominator. Since $P(n_f^{(\mu)},\mu=-2,-1,0,1,2,r_f;n_i^{(\mu)},\mu=-2,-1,0,1,2,r_i)$ involves only the $\mu=0$ component [see Eq. (7)], the other components cancel out in the denominator and the numerator. We thus obtain

$$P_{\rm inclus}(T;r_f,r_i)=\int_{-\infty}^{\infty}dx\ W(T;x)P^{(0)}\left[E;U(r)+\left[\frac{2\lambda+1}{4\pi}\right]^{1/2}\alpha_0 x f(r)\right] \ , \tag{28}$$

where

$$W(T;x)=\frac{\sum_{n_i}|\langle n_i|x\rangle|^2\exp[-\hbar\omega(n_i+\frac{1}{2})/T]}{\sum_{n_i}\exp[-\hbar\omega(n_i+\frac{1}{2})/T]} . \tag{29}$$

Here we have used n_i to denote $n_i^{(0)}$. The distribution W is known to be given by [17]

$$W(T;x)=\left[\frac{1}{\pi}\tanh\left[\frac{\hbar\omega}{2T}\right]\right]^{1/2}\exp\left[-x^2\tanh\left[\frac{\hbar\omega}{2T}\right]\right] . \tag{30}$$

Making the coordinate transformation from x to y according to,

$$x^2\tanh\left[\frac{\hbar\omega}{2T}\right]=y^2 , \tag{31}$$

and using x for y, we have

$$P_{\text{inclus}}(T;r_f,r_i)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dx\,e^{-x^2}P^{(0)}\left[E;U(r)+\left[\frac{2\lambda+1}{4\pi}\right]^{1/2}\alpha_{\text{eff}}(T)xf(r)\right] , \tag{32}$$

where

$$\alpha_{\text{eff}}(T)=\alpha_0\frac{1}{\left[\tanh\left[\frac{\hbar\omega}{2T}\right]\right]^{1/2}} . \tag{33}$$

Equation (32) together with Eq. (33) agrees with the thermal zero point motion formula that has been obtained in Ref. [11] through the influence functional method. Note that the limit of $\omega\to 0$ had to be taken in an indirect way in order to obtain Eq. (32) by the influence functional methods. The present method offers a much more straightforward derivation and a transparent interpretation of the result.

B. Rotational motion coupled to the translational motion

Let us next consider the case when the translational motion is coupled to a rotational mode of excitation, which is in thermal equilibrium with a heat bath of temperature T. The inclusive transmission probability is given by

$$P_{\text{inclus}}(T;r_f,r_i)=\frac{\sum_{I_i}\exp(-I_i^2/2\mathcal{I}T)P_{\text{inclus}}(I_i;r_f,r_i)}{\sum_{i_i}\exp(-I_i^2/2\mathcal{I}T)} . \tag{34}$$

Since $P_{\text{inclus}}(I_i;r_f,r_i)$ is independent of I_i [see Eq. (19)], we obtain

$$P_{\text{inclus}}(T;r_f,r_i)=\int_0^1 d\cos\theta\,P^{(0)}\left[E;U(r)+\left[\frac{5}{4\pi}\right]^{1/2}\beta P_2(\cos\theta)f(r)\right] , \tag{35}$$

which is identical to the zero point motion formula at $T=0$. Thus the effects of rotational coupling are independent of the temperature. Note that we are discussing here the transmission probability at a given energy. This is different from the question of how the dominant decay mode of a metastable state changes from a quantum tunneling to a thermal hopping as the temperature of the system is raised [18,19].

IV. SUMMARY

We have used the properties of the reduced Green's function in the limit of the adiabatic and the no-Coriolis approximations to derive the zero point motion formula for the barrier transmission probability when the relative motion, i.e., the coordinate of the fusion or the decay process, is coupled to intrinsic vibrational or rotational degrees of freedom. We can easily handle not only the case where the internal degrees of freedom are initially in their ground state, but also the case when they are in thermal equilibrium with a heat bath. Another advantage of this method is that the weight function used in the averaging procedure of the zero point motion formula is clearly related to the quantum mechanical wave functions of the relevant intrinsic degrees of freedom. We note that Eqs. (7) and (16) can be used to derive similar zero point motion formulae for the S-matrix elements of elastic and inelastic scattering [14]. This follows from the relation of the S-matrix to the Green's function [20].

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APPENDIX: INCLUSIVE TRANSMISSION PROBABILITY FOR UNPOLARIZED INITIAL STATE

In this appendix we show that for an unpolarized initial state, the inclusive transmission probability can be calculated using only the Green's function in the rotating frame $G_R^{(+)}(E)$, i.e., we can neglect $\hat{R}$ and $\hat{R}^\dagger$ in Eq. (6).

Starting from Eq. (6) we have

$$|G^{(+)}_{n_f,n_i}(E)|^2=\langle n_i|\hat{R}(\Omega)[G_R^{(+)}(E)]^\dagger\hat{R}^\dagger(\Omega)|n_f\rangle \times\langle n_f|\hat{R}(\Omega)G_R^{(+)}(E)\hat{R}^\dagger(\Omega)|n_i\rangle \ . \quad \text{(A1)}$$

Using the completeness relation for the final states, we obtain

$$\sum_{n_f}|G^{(+)}_{n_f,n_i}(E)|^2=\langle n_i|\hat{R}(\Omega)[G_R^{(+)}(E)]^\dagger G_R^{(+)}(E)\hat{R}^\dagger(\Omega)|n_i\rangle . \quad \text{(A2)}$$

Assuming that the initial state has spin I and projection M along the beam direction, we get

$$\hat{R}^\dagger(\Omega)|IM\rangle=\sum_{M'}D^{I*}_{MM'}|IM'\rangle \ . \quad \text{(A3)}$$

Substituting in (A2) leads to

$$\sum_{n_f}|G^{(+)}_{n_f,n_i}(E)|^2 = \sum_{M'M''}D^{I}_{MM''}(\Omega)\langle IM''|(G_R^{(+)})^\dagger G_R^{(+)}|IM'\rangle D^{I*}_{MM'}(\Omega) \ . \quad \text{(A4)}$$

Since $G_R^{(+)}(E)$ contains only the $\mu=0$ component, $M'=M''$. Averaging over the initial M (for an unpolarized nucleus) and using the orthogonality of Wigner's D functions, we obtain

$$\frac{1}{2I+1}\sum_M\sum_{n_f}|G^{(+)}_{n_f,n_i}(E)|^2 = \frac{1}{2I+1}\sum_{M'}\langle IM'|(G_R^{(+)})^\dagger(E)G_R^{(+)}(E)|IM'\rangle \ . \quad \text{(A5)}$$

Equation (A5) shows that if the initial state is unpolarized we can neglect R and $R^\dagger$ in Eq. (6).

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