Extraction of nuclear spin response functions from spin observables of nucleon quasifree scattering

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Extraction of spin-longitudinal and -transverse response functions from polarization transfer measurements of nucleon-nucleus quasifree scatterings is discussed. The method proposed by Carey *et al.* is reconsidered and more general formulas are presented. Spin-longitudinal and -transverse interactions are well defined in the nucleon-nucleon scattering t matrix in the nucleon-nucleon center-of-mass frame. However, observed data are given in the nucleon-nucleus laboratory frame and theoretical analysis based on the distorted-wave and plane-wave impulse approximations is carried out in the nucleon-nucleus center-of-mass system, in which the t matrix in a certain optimum frame of the nucleon-nucleon system is used. Careful consideration is paid for transformations among these reference frames relativistically.

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I. INTRODUCTION

Extraction of the nuclear spin response functions by measuring polarization transfers, D_{ij}, of $(\vec{p},\vec{p}\,')$, $(\vec{p},\vec{n})$ scatterings, etc., in the quasifree scattering region is a current exciting subject.

For this purpose Carey *et al.* [1,2] introduced "longitudinal and transverse spin flip probabilities," S_L and S_T, as

$$IS_L \equiv \frac{I}{4}[1-D_{NN}+(D_{SS'}-D_{LL'})\sec\theta_{\rm lab}] ,$$
$$IS_T \equiv \frac{I}{4}[1-D_{NN}-(D_{SS'}-D_{LL'})\sec\theta_{\rm lab}] , \qquad (1.1)$$

where I is the unpolarized differential cross section and $\theta_{\rm lab}$ is the scattering angle in the laboratory frame. The quantities D_{ij} are the parity-allowed polarization transfers in the laboratory frame where the left (right) subscript denotes the initial (final) polarization direction. N denotes the direction normal to the reaction plane and $L(L')$ and $S(S')$ denote that of the momentum of the incident (outgoing) nucleon and the corresponding transverse one in the reaction plane, respectively. Recent measurements [1,2] of S_L and S_T provoked a very interesting question since it contradicted the theoretical prediction [3].

Theoretically it is predicted that, for the relatively large momentum q $(=1.5-2.5\ {\rm fm}^{-1})$ region, the spin-longitudinal response function R_L is enhanced and softened (energy spectrum being shifted downwards) due to pionic correlation in the nucleus while the spin-transverse response function R_T is quenched and hardened (energy spectrum being shifted upwards) due to the short-range correlation and therefore the ratio R_L/R_T should be larger than unity.

Experimentally the ratio was evaluated in Refs. [1,2] on the basis of the ansatz of Bertsch, Scholten, and Esbensen [4],

$$IS_L = I^{NN}S_L^{NN}N_eR_L(q,\omega) ,$$
$$IS_T = I^{NN}S_T^{NN}N_eR_T(q,\omega) , \qquad (1.2)$$

where the quantities with the superscript NN mean those for the NN scattering and N_e is the effective number of participating nucleons. Rather than calculate S_L^{NN} and S_T^{NN} from the phase shift, they assumed that those for the averaged values of the pp and pn observables can be replaced by those of the deuteron, S_L^d and S_T^d, as

$$S_L^d = S_L^{NN},\quad S_T^d = S_T^{NN} . \qquad (1.3)$$

Then, the ratio of the response functions is given by

$$\frac{R_L(q,\omega)}{R_T(q,\omega)} = \frac{S_L/S_L^d}{S_T/S_T^d} . \qquad (1.4)$$

The ratio thus experimentally obtained was found to be less than unity [1,2].

After this contradiction was found, many theoretical refinements [5] were performed, such as inclusion of distortion effects, surface effects, two-particle–two-hole configuration mixing, different choices of effective interactions, etc.

In contrast to these works, here we want to reconsider the validity of the formulas (1.1) themselves. As we will show in Sec. II, for the free nucleon-nucleon (NN) scattering the proposed linear combinations of D_{ij}, i.e., IS_L and IS_T, of Eq. (1.1), exclusively extract the contribution from the spin-longitudinal part, $E'\sigma_{0q}\sigma_{1q}$, and the spin-transverse part, $F'\sigma_{0p}\sigma_{1p}$, respectively, of the NN t matrix in the center-of-mass (c.m.) frame shown in Eq. (2.10).

The point we want to investigate is whether one can apply the same relations to the nucleon-nucleus (NA) scatterings as Carey *et al.* did. We will show that their formulas (1.1) work when the target nucleus is assumed simply as an ensemble of free nucleons at rest and free

NN scattering occurs only once between the incident nucleon and a nucleon in the nucleus. However, even in the simple plane-wave impulse approximation (PWIA), applicability of the formulas to the NA scattering is questionable and must be reconsidered when we take account of Fermi motion, binding effects, recoil of residual nucleus, etc.

A question is the following. Theoretical analysis is usually carried out in the c.m. frame of the NA system, and calculated results are then to be expressed by observed quantities in the NA laboratory frame or vice versa. The relations between D_{ij}'s in the NA c.m. and in the NA laboratory frame are not the same as those between D_{ij}'s in the NN c.m. and in the NN laboratory frame in general. The difference gives rise to questions about the applicability of the formulas (1.1). We derive the general formulas valid for the transformation from the NA laboratory to the NA c.m. frame, and consider some limiting cases in Sec. II.

Another question is as follows. In impulse approximation, the driving force that excites the nucleus is the NN scattering t matrix. It can be on shell as well as off shell and depends on the momentum of the collided nucleon in the nucleus, over which one must integrate. In the distorted-wave impulse approximation (DWIA) more complicated integrals are involved. It is a general problem to approximate the t matrix by the known on-shell free NN t matrix in the NN c.m. frame and to carry out the integral.

For this purpose, an optimum factorization procedure is usually adopted, by which the t matrix is replaced by the on-shell t matrix in a certain special frame (the optimal frame) and is factored out from the integral. For instance, the t matrix in the Breit frame is often used for the elastic scattering [6]. For inelastic and charge-exchange reactions with large energy transfer, it has been pointed out that the optimal frame could be much different from the NN c.m. frame and the Breit frame and that more elaborate consideration is required [7,8].

If the t matrix in the NN c.m. frame or in the NN Breit frame is used, one knows what linear combinations of D_{ij} in the NA c.m. frame isolates the spin-longitudinal and -transverse parts in PWIA. However, one must ask what kinds of combinations should be utilized for inelastic and charge-exchange reactions when one takes account of the proper choice of the optimal frame. This problem is discussed in Sec. III. We treat the frame transformations relativistically and thus the relativistic spin rotations (Wigner rotations) are taken into account. A summary is given in Sec. IV.

II. KINEMATICAL CONSIDERATION

Let us consider elastic, inelastic, or charge-exchange nucleon scatterings with a target X which can be a nucleon or a nucleus,

$$N+X\rightarrow N+X^* .$$

First we discus the scattering in the c.m. frame of the NX system. In this frame we represent the momenta of the incident and outgoing nucleons by $\mathbf{k}$ and $\mathbf{k}'$, respectively, and the momentum and energy transfer to N by

$$\mathbf{q}=\mathbf{k}'-\mathbf{k},\ \ \Delta=E(\mathbf{k}')-E(\mathbf{k}) , \tag{2.1}$$

with $E(\mathbf{k})=\sqrt{k^2+m^2}$ where m is the nucleon mass. We also use the notation ω ($=-\Delta$) for the energy transfer to X. We then introduce the unit vectors

$$\hat{\mathbf{q}}=\frac{\mathbf{q}}{|\mathbf{q}|},\ \ \hat{\mathbf{n}}=\frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|} ,\ \ \hat{\mathbf{p}}=\hat{\mathbf{q}}\times\hat{\mathbf{n}} , \tag{2.2}$$

which form a Cartesian frame. The directions of them are denoted by q, n, and p, respectively.

The scattering T matrix in this frame is generally written [9] as

$$T(\mathbf{k},\mathbf{k}')=\hat{T}_0+\hat{T}_n\sigma_{0n}+\hat{T}_q\sigma_{0q}+\hat{T}_p\sigma_{0p} , \tag{2.3}$$

where σ_{0i} is the projection to direction i of the Pauli matrix of the scattering nucleon and $\hat{T}_i$'s are the operators on the target X. In this paper we suppress the isospin degree of freedom. It can be included straightforwardly if necessary.

The unpolarized differential cross section I and the polarization transfer D_{ij} are given by

$$I=K\,\mathrm{Tr}\,\mathrm{Tr}'[TT^\dagger],\ \ D_{ij}=\frac{\mathrm{Tr}\,\mathrm{Tr}'[T\sigma_{0i}T^\dagger\sigma_{0j}]}{\mathrm{Tr}\,\mathrm{Tr}'[TT^\dagger]} , \tag{2.4}$$

with the kinematical factor

$$K=\frac{\mu_i\mu_f}{(2\pi)^2}\frac{k'}{k}\frac{1}{2(2J_X+1)} , \tag{2.5}$$

where J_X is the target spin. The relativistic reduced masses, μ_i and μ_f, are given by

$$\mu_i=\frac{E(\mathbf{k})E_X^i}{E(\mathbf{k})+E_X^i},\ \ \mu_f=\frac{E(\mathbf{k}')E_X^f}{E(\mathbf{k}')+E_X^f} , \tag{2.6}$$

where E_X^i (E_X^f) is the energy of the initial (final) state of X. The symbol Tr means the trace of the spin substates of the scattering nucleon and Tr$'$ symbolically denotes the summation of all allowed initial and final states of X,

$$\mathrm{Tr}'[TT^\dagger]=\sum_{i,f}\langle f|T|i\rangle\langle i|T^\dagger|f\rangle\delta(\omega-(E_X^f-E_X^i)) . \tag{2.7}$$

One then obtains

$$\begin{aligned}
I&=2K\,\mathrm{Tr}'[\hat{T}_0\hat{T}_0^\dagger+\hat{T}_n\hat{T}_n^\dagger+\hat{T}_q\hat{T}_q^\dagger+\hat{T}_p\hat{T}_p^\dagger] ,\\
ID_{nn}&=2K\,\mathrm{Tr}'[\hat{T}_0\hat{T}_0^\dagger+\hat{T}_n\hat{T}_n^\dagger-\hat{T}_q\hat{T}_q^\dagger-\hat{T}_p\hat{T}_p^\dagger] ,\\
ID_{qq}&=2K\,\mathrm{Tr}'[\hat{T}_0\hat{T}_0^\dagger-\hat{T}_n\hat{T}_n^\dagger+\hat{T}_q\hat{T}_q^\dagger-\hat{T}_p\hat{T}_p^\dagger] ,\\
ID_{pp}&=2K\,\mathrm{Tr}'[\hat{T}_0\hat{T}_0^\dagger-\hat{T}_n\hat{T}_n^\dagger-\hat{T}_q\hat{T}_q^\dagger+\hat{T}_p\hat{T}_p^\dagger] .
\end{aligned} \tag{2.8}$$

Bleszynski *et al.* [9] introduced the observables D_i, which isolate the strength $\mathrm{Tr}'[T_iT_i^\dagger]$, as

$$ID_0=\frac{I}{4}[1+D_{nn}+D_{qq}+D_{pp}]=2K\,\mathrm{Tr}'[\hat{T}_0\hat{T}_0^\dagger]\;,$$
$$ID_n=\frac{I}{4}[1+D_{nn}-D_{qq}-D_{pp}]=2K\,\mathrm{Tr}'[\hat{T}_n\hat{T}_n^\dagger]\;,$$
$$ID_q=\frac{I}{4}[1-D_{nn}+D_{qq}-D_{pp}]=2K\,\mathrm{Tr}'[\hat{T}_q\hat{T}_q^\dagger]\;,\qquad(2.9)$$
$$ID_p=\frac{I}{4}[1-D_{nn}-D_{qq}+D_{pp}]=2K\,\mathrm{Tr}'[\hat{T}_p\hat{T}_p^\dagger]\;.$$

For the case of nucleon-nucleon scattering, namely, X being a nucleon, the NN-scattering T matrix is written as

$$\begin{aligned}T(\mathbf{k},\mathbf{k}')&=\langle\mathbf{k}',-\mathbf{k}'|t|\mathbf{k},-\mathbf{k}\rangle\\&=A'+B'\sigma_{1n}\sigma_{0n}+C'(\sigma_{1n}+\sigma_{0n})\\&\quad+E'\sigma_{1q}\sigma_{0q}+F'\sigma_{1p}\sigma_{0p}\;,\end{aligned}\qquad(2.10)$$

where σ_{1i} is the projection to direction i of the Pauli matrix of the target nucleon 1. We denote the NN t matrix in general by $\langle\mathbf{k}_0',\mathbf{k}_1'|t|\mathbf{k}_0,\mathbf{k}_1\rangle$ for the NN scattering with momentum change $\mathbf{k}_0,\mathbf{k}_1\rightarrow\mathbf{k}_0',\mathbf{k}_1'$. Then one gets

$$\hat{T}_0=A'+C'\sigma_{1n},\quad \hat{T}_n=B'\sigma_{1n}+C'\;,$$
$$\hat{T}_q=E'\sigma_{1q},\quad \hat{T}_p=F'\sigma_{1p}\;.\qquad(2.11)$$

The t matrix in the NN c.m. frame is related with the standard form [10] of the NN scattering amplitude $M(\mathbf{k},\mathbf{k}')$ as

$$\begin{aligned}M(\mathbf{k},\mathbf{k}')&=-\frac{E(\mathbf{k})}{4\pi}\langle\mathbf{k}',-\mathbf{k}'|t|\mathbf{k},-\mathbf{k}\rangle\\&=A+B\sigma_{1n}\sigma_{0n}+C(\sigma_{1n}+\sigma_{0n})\\&\quad+E\sigma_{1q}\sigma_{0q}+F\sigma_{1p}\sigma_{0p}\;,\end{aligned}\qquad(2.12)$$

and thus Eqs. (2.9) and (2.11) give the well-known formulas

$$I^{NN}D_0^{NN}=|A|^2+|C|^2,\quad I^{NN}D_n^{NN}=|B|^2+|C|^2\;,$$
$$I^{NN}D_q^{NN}=|E|^2,\quad I^{NN}D_p^{NN}=|F|^2\;.\qquad(2.13)$$

Here we see that $I^{NN}D_q^{NN}$ and $I^{NN}D_p^{NN}$ exclusively extract the contribution from the spin-longitudinal part, $E\sigma_{1q}\sigma_{0q}$, and that from the spin-transverse part, $F\sigma_{1p}\sigma_{0p}$, of the NN c.m. amplitude, respectively. To separate $|A|^2$, $|B|^2$, and $|C|^2$, one needs to observe another spin observable such as

$$C_{pppp}=\frac{\mathrm{Tr}\,\mathrm{Tr}'[T\sigma_{1p}\sigma_{0p}T^\dagger\sigma_{1p}\sigma_{0p}]}{\mathrm{Tr}\,\mathrm{Tr}'[TT^\dagger]}\;,\qquad(2.14)$$

by which $|C|^2$ is evaluated as [11]

$$|C|^2=\frac{I}{4}[1-C_{pppp}]\;.\qquad(2.15)$$

To observe C_{pppp}, one needs the polarized target as well as the polarized beam and has to measure the polarization of the recoil nucleon as well as that of the scattered nucleon.

Even for an arbitrary target X, we may expect ID_q and ID_p to exclusively represent the spin-longitudinal and spin-transverse responses if we adopt PWIA. This expectation will be investigated in the next section. From the experimental difficulty of separating C terms as was mentioned above, we will henceforth only consider the D_q and D_p observables.

A problem we have to solve is to express the spin observables in the c.m. frame by those in the laboratory frame for arbitrary X, including the effect of relativistic spin rotation. In the laboratory frame, we denote the unit vector to the beam direction by $\hat{\mathbf{L}}$ and that of the outgoing nucleon by $\hat{\mathbf{L}}'$. The unit vector normal to the reaction plane is written as $\hat{\mathbf{N}}$ $(=\hat{\mathbf{L}}\times\hat{\mathbf{L}}'/|\hat{\mathbf{L}}\times\hat{\mathbf{L}}'|)$ and those of the sideway direction by $\hat{\mathbf{S}}=\hat{\mathbf{N}}\times\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}'=\hat{\mathbf{N}}\times\hat{\mathbf{L}}'$. In the relativistic kinematics, the spin in the $\hat{\mathbf{L}}'$ direction in the laboratory frame corresponds to that in the $\hat{\mathbf{L}}_R'$ direction in the c.m. frame, due to the spin rotation by angle Ω around the axis $\hat{\mathbf{n}}$. Figure 1 illustrates relevant directions, where the scattering angles in the c.m. and laboratory frames are denoted by θ and $\theta_{\rm lab}$, respectively, and θ_p represents the angle between incident beam direction and the unit vector $\hat{\mathbf{p}}$ defined in Eq. (2.2).

The relativistic spin rotation angle Ω is given by [12]

$$\tan(\theta-\theta_{\rm lab}-\Omega)=\frac{\sin\theta}{\gamma(\cos\theta+\beta/\beta_{\rm c.m.})}\;,\qquad(2.16)$$

in contrast with the well-known relation between θ and $\theta_{\rm lab}$:

$$\tan\theta_{\rm lab}=\frac{\sin\theta}{\gamma_{\rm c.m.}(\cos\theta+\beta_{\rm c.m.}/\beta)}\;,\qquad(2.17)$$

where $\beta_{\rm c.m.}$ is the velocity of the c.m. frame relative to the laboratory frame, $\gamma_{\rm c.m.}=(1-\beta_{\rm c.m.}^2)^{-1/2}$, and β is the velocity of the outgoing nucleon in the c.m. frame and $\gamma=(1-\beta^2)^{-1/2}$. Note that when mass of X is equal to m one gets $\beta=\beta_{\rm c.m.}$ and thus $\Omega=\theta-2\theta_{\rm lab}$.

From Fig. 1 one sees

$$\begin{aligned}\hat{L}_R&=\hat{L}=\cos\theta_p\hat{\mathbf{p}}-\sin\theta_p\hat{\mathbf{q}}\;,\\\hat{S}_R&=\hat{S}=\sin\theta_p\hat{\mathbf{p}}+\cos\theta_p\hat{\mathbf{q}}\;,\\\hat{L}_R'&=\cos(\theta_p-\theta_{\rm lab}-\Omega)\hat{\mathbf{p}}-\sin(\theta_p-\theta_{\rm lab}-\Omega)\hat{\mathbf{q}}\;,\\\hat{S}_R'&=\sin(\theta_p-\theta_{\rm lab}-\Omega)\hat{\mathbf{p}}+\cos(\theta_p-\theta_{\rm lab}-\Omega)\hat{\mathbf{q}}\;;\end{aligned}\qquad(2.18)$$

hence one gets the relations between D_{ij} in the c.m. frame and those in the laboratory frame as

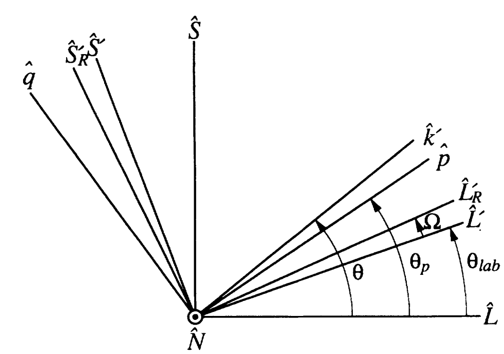

FIG. 1. NN scattering kinematics in the c.m. and laboratory frames.

$$D_{NN}=D_{nn}\ ,$$
$$\begin{bmatrix} D_{LL'} \\ D_{SS'} \\ D_{LS'} \\ D_{SL'} \end{bmatrix} = \begin{bmatrix} \cos\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & \sin\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & -\cos\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & -\sin\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) \\ \sin\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & \cos\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & \sin\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & \cos\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) \\ \cos\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & -\sin\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & \cos\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & -\sin\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) \\ \sin\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) & -\cos\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & -\sin\theta_p\sin(\theta_p-\theta_{\rm lab}-\Omega) & \cos\theta_p\cos(\theta_p-\theta_{\rm lab}-\Omega) \end{bmatrix}$$
$$\times \begin{bmatrix} D_{pp} \\ D_{qq} \\ D_{pq} \\ D_{qp} \end{bmatrix} . \tag{2.19}$$

From Eqs. (2.9) and (2.19), D_q and D_p are expressed as

$$\begin{aligned} D_q &= \tfrac{1}{4}[1-D_{NN}+(D_{SS'}-D_{LL'})\cos(2\theta_p-\theta_{\rm lab}-\Omega)-(D_{LS'}+D_{SL'})\sin(2\theta_p-\theta_{\rm lab}-\Omega)]\ , \\ D_p &= \tfrac{1}{4}[1-D_{NN}-(D_{SS'}-D_{LL'})\cos(2\theta_p-\theta_{\rm lab}-\Omega)+(D_{LS'}+D_{SL'})\sin(2\theta_p-\theta_{\rm lab}-\Omega)]\ . \end{aligned} \tag{2.20}$$

In the nonrelativistic limit, the recoil particle is emitted to the direction of $-\hat{\mathbf{q}}$ in the laboratory frame, therefore

$$\theta_p+\theta_r=\frac{\pi}{2}\ , \tag{2.21}$$

where θ_r is the recoil angle in the laboratory frame. Since $\Omega=0$ in this limit, the relations (2.19) become

$$D_{NN}=D_{nn}\ ,$$
$$\begin{bmatrix} D_{LL'} \\ D_{SS'} \\ D_{LS'} \\ D_{SL'} \end{bmatrix} = \begin{bmatrix} \sin\theta_r\sin(\theta_r+\theta_{\rm lab}) & \cos\theta_r\cos(\theta_r+\theta_{\rm lab}) & -\sin\theta_r\cos(\theta_r+\theta_{\rm lab}) & -\cos\theta_r\sin(\theta_r+\theta_{\rm lab}) \\ \cos\theta_r\cos(\theta_r+\theta_{\rm lab}) & \sin\theta_r\sin(\theta_r+\theta_{\rm lab}) & \cos\theta_r\sin(\theta_r+\theta_{\rm lab}) & \sin\theta_r\cos(\theta_r+\theta_{\rm lab}) \\ \sin\theta_r\cos(\theta_r+\theta_{\rm lab}) & -\cos\theta_r\sin(\theta_r+\theta_{\rm lab}) & \sin\theta_r\sin(\theta_r+\theta_{\rm lab}) & -\cos\theta_r\cos(\theta_r+\theta_{\rm lab}) \\ \cos\theta_r\sin(\theta_r+\theta_{\rm lab}) & -\sin\theta_r\cos(\theta_r+\theta_{\rm lab}) & -\cos\theta_r\cos(\theta_r+\theta_{\rm lab}) & \sin\theta_r\sin(\theta_r+\theta_{\rm lab}) \end{bmatrix} \begin{bmatrix} D_{pp} \\ D_{qq} \\ D_{pq} \\ D_{qp} \end{bmatrix} . \tag{2.22}$$

They coincide with the formulas derived by Moss [13] [see Eq. (7) of Ref. [13]], if one assumes $D_{pq}+D_{qp}=0$ which exactly hold [14] only for the elastic scattering. Then D_q and D_p are given by

$$\begin{aligned} D_q &= \tfrac{1}{4}[1-D_{NN}-(D_{SS'}-D_{LL'})\cos(2\theta_r+\theta_{\rm lab})-(D_{LS'}+D_{SL'})\sin(2\theta_r+\theta_{\rm lab})]\ , \\ D_p &= \tfrac{1}{4}[1-D_{NN}+(D_{SS'}-D_{LL'})\cos(2\theta_r+\theta_{\rm lab})+(D_{LS'}+D_{SL'})\sin(2\theta_r+\theta_{\rm lab})]\ . \end{aligned} \tag{2.23}$$

We note that the recoil angle θ_r and consequently θ_p rather strongly depend on the energy transfer ω.

If one further assumes that the mass of X is infinitely heavy and neglects the energy transfer ($\omega\approx 0$), one gets $\theta=\theta_{\rm lab}$ and $\theta_p=\theta/2$ and

$$\begin{aligned} D_q &= \tfrac{1}{4}[1-D_{NN}+D_{SS'}-D_{LL'}]\ , \\ D_p &= \tfrac{1}{4}[1-D_{NN}-D_{SS'}+D_{LL'}]\ , \end{aligned} \tag{2.24}$$

which coincide with Eq. (2.14) of Bleszynski *et al.* [9].

For the nucleon-nucleon elastic scattering,

$$\theta_p=\frac{\theta}{2},\quad \Omega=\theta-2\theta_{\rm lab},\quad D_{pq}+D_{qp}=0\ , \tag{2.25}$$

then one gets from Eq. (2.19)

$$\begin{aligned} D_q &= \tfrac{1}{4}[1-D_{NN}+(D_{SS'}-D_{LL'})\sec\theta_{\rm lab}]\ , \\ D_p &= \tfrac{1}{4}[1-D_{NN}-(D_{SS'}-D_{LL'})\sec\theta_{\rm lab}]\ . \end{aligned} \tag{2.26}$$

These equations are nothing new but the formulas (1.1) by identifying D_q and D_p to S_L and S_T of Eq. (1.1).

From the above consideration, one can say that Carey's formulas (1.1) work only in the approximation where a nucleus is treated simply as an ensemble of free nucleons at rest and only one free NN scattering occurs in the quasifree scattering. They do not hold in general. An important difference between Eq. (1.1) and our exact expression (2.20) is that even for a given $\theta_{\rm lab}$, the angles θ_p and Ω still depend on the energy transfer ω and affect the energy spectra of D_q and D_p in Eq. (2.20) but their energy dependence disappears in Eq. (1.1).

In Table I the relevant angles are shown at several energy transfers ω for ^{40}Ca(p,p') with the incident energy 500 MeV and the fixed angle $\theta_{\rm lab}=18.5^\circ$, which corresponds to the LAMPF experiment [1,2]. For this example, the spin rotation angle Ω is small enough to be neglected but θ_p strongly depends on ω and this effect

TABLE I. Rotation angles associated with the transformation from the NA c.m. to the laboratory frame for ^{40}Ca(N,N') with the incident energy 500 MeV and the scattering angle $\theta_{\rm lab}=18.5°$.

ω (MeV)	θp (deg)	Ω (deg)	$2\theta p$-$\theta_{\rm lab}$-Ω (deg)
30	15.72	0.24	12.70
60	22.39	0.24	26.03
90	29.09	0.23	39.44
120	35.65	0.23	52.58
150	41.94	0.22	65.16

must be taken into account. Unfortunately observed values of D_{ij} have not fully been reported. Therefore we cannot see the effect at the moment.

III. OPTIMAL FACTORIZATION IN THE IMPULSE APPROXIMATION

In this section we discuss the nucleon-nucleus inelastic and charge-exchange reactions in PWIA. Therefore X in the preceding section represents a nucleus with the mass number A. As is depicted in Fig. 2, the T matrix in the NA c.m. frame is written in terms of the NN t matrix as

$$T_{n0}(\mathbf{k}',\mathbf{k})=A\int\prod_i^A d^3\mathbf{p}_i d^3\mathbf{p}'_i \Psi^*_{n,-\mathbf{k}'}(\mathbf{p}'_1,\mathbf{p}'_2,\ldots,\mathbf{p}'_A)\langle\mathbf{k}',\mathbf{p}'_1|t|\mathbf{k},\mathbf{p}_1\rangle\Psi_{0,-\mathbf{k}}(\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_A)\prod_{i\neq 1}^A\delta(\mathbf{p}'_i-\mathbf{p}_i)\ , \tag{3.1}$$

in the momentum representation where $\mathbf{p}_i$ ($\mathbf{p}'_i$) is the momentum of the ith nucleon in the initial (final) state of the nucleus with respect to the NA c.m. frame. The wave function $\Psi_{n,\mathbf{P}}$ represents the nuclear state with the center-of-mass momentum $\mathbf{P}$ and the internal state n and is written as

$$\Psi_{n,\mathbf{P}}(\mathbf{p}_1,\ldots,\mathbf{p}_A)=\delta\left[\sum\mathbf{p}_i-\mathbf{P}\right]\Phi_n(\mathbf{p}^r_1,\ldots,\mathbf{p}^r_A)\ , \tag{3.2}$$

with

$$\mathbf{p}^r_i\equiv\mathbf{p}_i-\mathbf{P}/A\ , \tag{3.3}$$

where $\mathbf{p}^r_i$ is the momentum of the ith nucleon with respect to the center of mass of the nucleus. Here nucleons in the nucleus are treated nonrelativistically. The T matrix is rewritten as

$$T_{n0}(\mathbf{k}',\mathbf{k})=A\int\prod_i^A d^3\mathbf{p}^r_i\Phi^*_n(\mathbf{p}^r_1-\mathbf{q}+\mathbf{q}/A,\mathbf{p}^r_2+\mathbf{q}/A,\ldots,\mathbf{p}^r_A+\mathbf{q}/A)$$
$$\times\langle\mathbf{k}',\mathbf{p}^r_1-\mathbf{q}-\mathbf{k}/A|t|\mathbf{k},\mathbf{p}^r_1-\mathbf{k}/A\rangle\Phi_0(\mathbf{p}^r_1,\mathbf{p}^r_2,\ldots,\mathbf{p}^r_A)\delta\left[\sum\mathbf{p}^r_i\right]\ . \tag{3.4}$$

The integration over $\mathbf{p}^r_1$ is cumbersome since it appears in t as well as in the wave functions. Furthermore, the t matrix is off the energy shell in general. A way to deal with these difficulties is an optimum factorization approximation, in which $\mathbf{p}^r_1$ in the t matrix is replaced by a certain averaged value $\tilde{\mathbf{p}}^r$ so that it becomes on-shell. Then the t matrix is factored out from the integral as

$$T_{n0}(\mathbf{k}',\mathbf{k})\approx\sum_{s_1,s'_1}\langle\mathbf{k}',s',\tilde{\mathbf{p}}',s'_1|t|\mathbf{k},s,\tilde{\mathbf{p}},s_1\rangle F^{s'_1 s_1}_{n0}(\mathbf{q})\ , \tag{3.5}$$

with

$$\tilde{\mathbf{p}}=\tilde{\mathbf{p}}^r-\frac{\mathbf{k}}{A},\quad \tilde{\mathbf{p}}'=\tilde{\mathbf{p}}-q\ , \tag{3.6}$$

and the nuclear transition form factor

$$F^{s'_1 s_1}_{n0}(\mathbf{q})=A\int\prod_i^A d^3\mathbf{p}^r_i\Phi^*_n(\mathbf{p}^r_1-\mathbf{q}+\mathbf{q}/A,s'_1,\mathbf{p}^r_2+\mathbf{q}/A,\ldots,\mathbf{p}^r_A-\mathbf{q}/A)\Phi_0(\mathbf{p}^r_1,s_1,\mathbf{p}^r_2,\ldots,\mathbf{p}^r_A)\delta\left[\sum\mathbf{p}^r_i\right]\ . \tag{3.7}$$

Here only the spin projections, s,s',s_1,s'_1, of interacting nucleons are explicitly written. Hereafter when these s's are suppressed, t and F are understood as matrices with respect to the spin projection.

For NA elastic scattering, the optimum momentum $\tilde{\mathbf{p}}$ is usually chosen [6] so that the scattering, $(\mathbf{k},\tilde{\mathbf{p}}\rightarrow\mathbf{k}',\tilde{\mathbf{p}}')$, becomes the one in the Breit frame, namely,

$$\tilde{\mathbf{p}}=\frac{1}{2}\left[\mathbf{q}-\frac{\mathbf{q}}{A}\right]-\frac{\mathbf{k}}{A}=\frac{1}{2}\mathbf{q}-\frac{\mathbf{k}_a}{A}\ , \tag{3.8}$$

with

$$\mathbf{k}_a=\frac{\mathbf{k}+\mathbf{k}'}{2}\ . \tag{3.9}$$

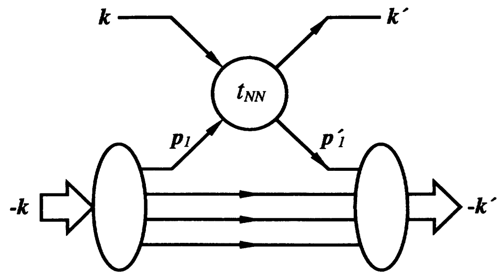

FIG. 2. PWIA in the NA c.m. frame.

This choice is often used even for inelastic scatterings, but it is questionable for large energy transfer processes.

The choice of the optimum momentum $\tilde{\mathbf{p}}$ for inelastic scatterings was carefully discussed by Gurvitz [8] nonrelativistically in the NA laboratory frame, and by Zhu, Mobed, and Wong [7] relativistically for infinitely heavy target. Here we follow Gurvitz's argument but in the NA c.m. frame and treat the interacting NN system relativistically as Zhu *et al.* did. In their formalism the optimum momentum $\tilde{\mathbf{p}}$ is chosen to satisfy the on-shell condition

$$E(\mathbf{k})+E(\tilde{\mathbf{p}})=E(\mathbf{k}')+E(\tilde{\mathbf{p}}') \tag{3.10}$$

with a special choice

$$\tilde{\mathbf{p}}=(\tfrac{1}{2}-\eta)\mathbf{q}-\frac{\mathbf{k}_a}{A} , \tag{3.11}$$

and thus

$$\tilde{\mathbf{p}}'=-(\tfrac{1}{2}+\eta)\mathbf{q}-\frac{\mathbf{k}_a}{A} . \tag{3.12}$$

Then η is given by

$$\eta=\frac{|\Delta|}{q}\left\{\frac{E_a}{Aq}+\left[\frac{1}{4}-\frac{1}{t_{\text{eff}}}\left[m^2+\frac{k_a^2}{A^2}-\frac{E_a^2\Delta^2}{A^2q^2}\right]\right]^{1/2}\right\} , \tag{3.13}$$

with

$$t_{\text{eff}}\equiv\Delta^2-q^2,\quad E_a\equiv\frac{E(\mathbf{k})+E(\mathbf{k}')}{2} , \tag{3.14}$$

where the relation $E_a\Delta-\mathbf{k}_a\cdot\mathbf{q}=0$ is used. For the elastic scattering $\Delta=0$ and thus $\eta=0$, hence the given optimum frame becomes the Breit frame as is usually used.

Next we should express the t matrix at the optimum frame (to be called η frame), $\langle\mathbf{k}',\tilde{\mathbf{p}}'|t|\mathbf{k},\tilde{\mathbf{p}}\rangle$, in Eq. (3.5) by the known one at the NN c.m. frame, $\langle\boldsymbol{\kappa}',-\boldsymbol{\kappa}'|t|\boldsymbol{\kappa},-\boldsymbol{\kappa}\rangle$. The former has the general form

$$\begin{aligned}\langle\mathbf{k}',\tilde{\mathbf{p}}'|t|\mathbf{k},\tilde{\mathbf{p}}\rangle=&A^{\eta}+B^{\eta}\sigma_{1n}\sigma_{0n}+C_1^{\eta}\sigma_{0n}+C_2^{\eta}\sigma_{1n}\\&+D_1^{\eta}\sigma_{1p}\sigma_{0q}+D_2^{\eta}\sigma_{1q}\sigma_{0p}\\&+E^{\eta}\sigma_{1q}\sigma_{0q}+F^{\eta}\sigma_{1p}\sigma_{0p} ,\end{aligned} \tag{3.15}$$

while the latter has [see Eq. (2.10)]

$$\begin{aligned}\langle\boldsymbol{\kappa}',-\boldsymbol{\kappa}'|t|\boldsymbol{\kappa},-\boldsymbol{\kappa}\rangle=&A'+B'\sigma_{1n_c}\sigma_{0n_c}+C'(\sigma_{1n_c}+\sigma_{0n_c})\\&+E'\sigma_{1q_c}\sigma_{0q_c}+F'\sigma_{1p_c}\sigma_{0p_c} .\end{aligned} \tag{3.16}$$

Here we represent the momenta of the incident and outgoing nucleon in the NN c.m. frame by $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$, respectively, and introduce the three orthnormal vectors

$$\begin{aligned}&\hat{\mathbf{q}}_c=\frac{\mathbf{q}_c}{|\mathbf{q}_c|},\quad \hat{\mathbf{n}}_c=\frac{\boldsymbol{\kappa}\times\boldsymbol{\kappa}'}{|\boldsymbol{\kappa}\times\boldsymbol{\kappa}'|}=\hat{\mathbf{n}},\\&\hat{\mathbf{p}}_c=\hat{\mathbf{q}}_c\times\hat{\mathbf{n}}_c=\frac{\mathbf{k}_c^a}{|\mathbf{k}_c^a|} ,\end{aligned} \tag{3.17}$$

with

$$\mathbf{q}_c=\boldsymbol{\kappa}'-\boldsymbol{\kappa},\quad \mathbf{k}_c^a=\boldsymbol{\kappa}+\boldsymbol{\kappa}' . \tag{3.18}$$

To relate the above t matrices we need the rotation angle ψ from the $\hat{\mathbf{q}}_c,\hat{\mathbf{n}}_c,\hat{\mathbf{p}}_c$ to $\hat{\mathbf{q}},\hat{\mathbf{n}},\hat{\mathbf{p}}$ triad and the relativistic spin rotation angles χ,χ' for the initial and final states of the incident nucleon and ρ,ρ' for those of the target nucleon. To obtain them it is convenient to introduce four-vectors $(E,\mathbf{K})$ and $(\tilde{E},\tilde{\mathbf{K}})$ in the η frame as

$$\begin{aligned}&E\equiv E(\mathbf{k})+E(\tilde{\mathbf{p}})=E(\mathbf{k}')+E(\tilde{\mathbf{p}}') ,\\&\mathbf{K}\equiv\mathbf{k}+\tilde{\mathbf{p}}=\mathbf{k}'+\tilde{\mathbf{p}}' ,\\&\tilde{E}\equiv E(\mathbf{k})-E(\tilde{\mathbf{p}}')=E(\mathbf{k}')-E(\tilde{\mathbf{p}}) ,\\&\tilde{\mathbf{K}}\equiv\mathbf{k}-\tilde{\mathbf{p}}'=\mathbf{k}'-\tilde{\mathbf{p}} ,\end{aligned} \tag{3.19}$$

and Lorentz invariant variables

$$s_{\text{eff}}\equiv E^2-\mathbf{K}^2,\quad u_{\text{eff}}\equiv\tilde{E}^2-\tilde{\mathbf{K}}^2 . \tag{3.20}$$

The four-vectors $(\Delta,\mathbf{q})$, $(\tilde{E},\tilde{\mathbf{K}})$, and $(\tilde{E},\tilde{\mathbf{K}})$ correspond to the four-vectors $(0,\mathbf{q}_c)$, $(\sqrt{s_{\text{eff}}},0)$, and $(0,\mathbf{k}_c^a)$ in the NN c.m. frame, respectively. Therefore the relations

$$\begin{aligned}&E\Delta-\mathbf{K}\cdot\mathbf{q}=0,\quad \tilde{E}\Delta-\tilde{\mathbf{K}}\cdot\mathbf{q}=0,\quad E\tilde{E}-\mathbf{K}\cdot\tilde{\mathbf{K}}=0 ,\\&t_{\text{eff}}=-(\mathbf{q}_c)^2,\quad u_{\text{eff}}=-(\mathbf{k}_c^a)^2\end{aligned} \tag{3.21}$$

hold. Noting the identity

$$\mathbf{K}=\frac{A-1}{A+1}\tilde{\mathbf{K}}-\frac{2A\eta}{A+1}\mathbf{q} ,$$

it is helpful to introduce another four-vector in the η frame

$$(\xi,0)\equiv(E,\mathbf{K})-\frac{A-1}{A+1}(\tilde{E},\tilde{\mathbf{K}})+\frac{2A\eta}{A+1}(\Delta,\mathbf{q}) , \tag{3.22}$$

which corresponds to

$$\left[\sqrt{s_{\text{eff}}},-\frac{A-1}{A+1}\mathbf{k}_c^a+\frac{2A\eta}{A+1}\mathbf{q}_c\right]$$

in the NN c.m. frame. So one gets $s_{\text{eff}}=E\xi$. Using this, one can write

$$(E,\mathbf{K})-(\xi,0)=(E\beta_{\text{c.m.}}^2,\mathbf{K}) , \tag{3.23}$$

where $\beta_{\text{c.m.}}=\mathbf{K}/E$ is the velocity of the c.m. frame with respect to the η frame. This corresponds to

$$\left[0, \frac{A-1}{A+1}\mathbf{k}_c^a - \frac{2A\eta}{A+1}\mathbf{q}_c\right]$$

in the c.m. frame. The Lorentz transformation of the four-vectors $(\Delta, \mathbf{q})$ and $(E\beta^2_{\text{c.m.}}, \mathbf{K})$ in the η frame to those in the c.m. frame gives

$$\mathbf{q}_c = \mathbf{q} - \frac{\Delta}{\sqrt{s_{\text{eff}}}+E}\mathbf{K},$$
$$\frac{A-1}{A+1}\mathbf{k}_c^a - \frac{2A\eta}{A+1}\mathbf{q}_c = \frac{\sqrt{s_{\text{eff}}}}{E}\mathbf{K}. \tag{3.24}$$

As $\mathbf{q}_c \cdot \mathbf{k}_c^a = 0$, the angle between $\mathbf{q}$ and $\mathbf{q}_c$, which is the rotation angle ψ, is given by

$$\tan\psi = \frac{-(A-1)E\Delta}{(A+1)\sqrt{s_{\text{eff}}}(\sqrt{s_{\text{eff}}}+E) - 2A\eta E\Delta}\sqrt{u_{\text{eff}}/t_{\text{eff}}}. \tag{3.25}$$

Relativistic transformation of the t matrix in one frame to that in the other is explained in detail by McNeil, Ray, and Wallace [15] and by Zhu *et al.* [7]. We follow the latter below.

Let M^{RI} be the relativistic invariant scattering matrix, the t matrix component in any frame is given by

$$\langle \mathbf{k}_0', s_0', \mathbf{k}_1', s_1' | t | \mathbf{k}_0, s_0, \mathbf{k}_1, s_1 \rangle = \chi^\dagger_{s_0'}\chi^\dagger_{s_1'}\langle \mathbf{k}_0', \mathbf{k}_1' | t | \mathbf{k}_0, \mathbf{k}_1 \rangle \chi_{s_0}\chi_{s_1}$$
$$= \frac{m^2}{\sqrt{E(\mathbf{k}_0)E(\mathbf{k}_1)E(\mathbf{k}_0')E(\mathbf{k}_1')}}\bar{u}^0_{s_0'}(\mathbf{k}_0')\bar{u}^1_{s_1'}(k_1')M^{\text{RI}}u^0_{s_0}(\mathbf{k}_0)u^1_{s_1}(\mathbf{k}_1), \tag{3.26}$$

where χ_s is a Pauli spinor and $u_s(\mathbf{k})$ is a Dirac spinor with positive energy which is normalized as $\bar{u}u = 1$. The Dirac spinor is expressed as

$$u_s(\mathbf{k}) = L(\mathbf{k})\begin{bmatrix}\chi_s \\ 0\end{bmatrix} \tag{3.27}$$

with

$$L(\mathbf{k}) = \frac{1}{\sqrt{2m[E(\mathbf{k})+m]}}\begin{bmatrix}E(\mathbf{k})+m & \sigma\cdot\mathbf{k} \\ \sigma\cdot\mathbf{k} & E(\mathbf{k})+m\end{bmatrix}. \tag{3.28}$$

Following the procedure of Ref. [7], the t matrix in the η frame is expressed by that in the c.m. frame as

$$\langle \mathbf{k}', \tilde{\mathbf{p}}' | t | \mathbf{k}, \tilde{\mathbf{p}} \rangle = J(\mathbf{k}, \mathbf{q}, \eta)\exp\left[\frac{i}{2}\chi'\sigma_{0n}\right]\exp\left[-\frac{i}{2}\rho'\sigma_{1n}\right]\langle \boldsymbol{\kappa}', -\boldsymbol{\kappa}' | t | \boldsymbol{\kappa}, -\boldsymbol{\kappa} \rangle \exp\left[\frac{i}{2}\chi\sigma_{0n}\right]\exp\left[-\frac{i}{2}\rho\sigma_{1n}\right] \tag{3.29}$$

with Möller factor

$$J(\mathbf{k}, \mathbf{q}, \eta) = \frac{E(\boldsymbol{\kappa})^2}{\sqrt{E(\mathbf{k})E(\mathbf{k}')E(\tilde{\mathbf{p}})E(\tilde{\mathbf{p}}')}}. \tag{3.30}$$

The spin rotation angles, χ, χ', ρ, ρ', around the normal $\hat{\mathbf{n}}$ are given by

$$\exp\left[\frac{i}{2}\chi\sigma_{0n}\right]\mathbf{1} = L_0(-\boldsymbol{\kappa})L_0(-\mathbf{P})L_0(\mathbf{k}),$$
$$\exp\left[\frac{i}{2}\chi'\sigma_{0n}\right]\mathbf{1} = L_0(-\mathbf{k}')L_0(\mathbf{P})L_0(\boldsymbol{\kappa}),$$
$$\exp\left[-\frac{i}{2}\rho\sigma_{1n}\right]\mathbf{1} = L_1(\boldsymbol{\kappa})L_1(-\mathbf{P})L_1(\tilde{\mathbf{p}}),$$
$$\exp\left[-\frac{i}{2}\rho'\sigma_{1n}\right]\mathbf{1} = L_1(-\tilde{\mathbf{p}}')L_1(\mathbf{P})L_1(-\boldsymbol{\kappa}), \tag{3.31}$$

and explicitly written as

$$\tan\left[\frac{\chi}{2}\right]=\left[\frac{A-1}{A+1}-\frac{2A}{A+1}\eta\right]\frac{\sqrt{t_{\text{eff}}u_{\text{eff}}}}{s_{\text{eff}}}\frac{E}{E_P+\sqrt{s_{\text{eff}}}/2+m+E(\mathbf{k})}\ ,$$
$$\tan\left[\frac{\chi'}{2}\right]=\left[\frac{A-1}{A+1}+\frac{2A}{A+1}\eta\right]\frac{\sqrt{t_{\text{eff}}u_{\text{eff}}}}{s_{\text{eff}}}\frac{E}{E_P+\sqrt{s_{\text{eff}}}/2+m+E(\mathbf{k}')}\ ,$$
$$\tan\left[\frac{\rho}{2}\right]=\left[\frac{A-1}{A+1}-\frac{2A}{A+1}\eta\right]\frac{\sqrt{t_{\text{eff}}u_{\text{eff}}}}{s_{\text{eff}}}\frac{E}{E_P+\sqrt{s_{\text{eff}}}/2+m+E(\tilde{\mathbf{p}})}\ ,$$
$$\tan\left[\frac{\rho'}{2}\right]=\left[\frac{A-1}{A+1}+\frac{2A}{A+1}\eta\right]\frac{\sqrt{t_{\text{eff}}u_{\text{eff}}}}{s_{\text{eff}}}\frac{E}{E_P+\sqrt{s_{\text{eff}}}/2+m+E(\tilde{\mathbf{p}}')}\ , \tag{3.32}$$

where

$$\mathbf{P}\equiv\frac{m}{\sqrt{s_{\text{eff}}}}\mathbf{K},\quad E_P\equiv\frac{m}{\sqrt{s_{\text{eff}}}}E\ . \tag{3.33}$$

Now each amplitude of the η frame t matrix is expressed by those of the c.m. frame t matrix as

$$\begin{bmatrix}A^\eta\\B^\eta\\C_1^\eta\\C_2^\eta\end{bmatrix}=J(\mathbf{k},\mathbf{q},\eta)\begin{bmatrix}\cos\varphi_+^0\cos\varphi_+^1 & \sin\varphi_+^0\sin\varphi_+^1 & i\sin(\varphi_+^0-\varphi_+^1)\\ \sin\varphi_+^0\sin\varphi_+^1 & \cos\varphi_+^0\cos\varphi_+^1 & i\sin(\varphi_+^0-\varphi_+^1)\\ i\sin\varphi_+^0\cos\varphi_+^1 & -i\cos\varphi_+^0\sin\varphi_+^1 & \cos(\varphi_+^0-\varphi_+^1)\\ -i\cos\varphi_+^0\sin\varphi_+^1 & i\sin\varphi_+^0\cos\varphi_+^1 & \cos(\varphi_+^0-\varphi_+^1)\end{bmatrix}\begin{bmatrix}A'\\B'\\C'\end{bmatrix},$$
$$\begin{bmatrix}D_1^\eta\\D_2^\eta\\E^\eta\\F^\eta\end{bmatrix}=J(\mathbf{k},\mathbf{q},\eta)\begin{bmatrix}-\cos\varphi_-^0\sin\varphi_-^1 & \sin\varphi_-^0\cos\varphi_-^1\\ -\sin\varphi_-^0\cos\varphi_-^1 & \cos\varphi_-^0\sin\varphi_-^1\\ \cos\varphi_-^0\cos\varphi_-^1 & \sin\varphi_-^0\sin\varphi_-^1\\ \sin\varphi_-^0\sin\varphi_-^1 & \cos\varphi_-^0\cos\varphi_-^1\end{bmatrix}\begin{bmatrix}E'\\F'\end{bmatrix}, \tag{3.34}$$

where

$$\varphi_+^0=(\chi+\chi')/2,\quad \varphi_-^0=\psi+(\chi-\chi')/2\ ,$$
$$\varphi_+^1=(\rho+\rho')/2,\quad \varphi_-^1=\psi-(\rho-\rho')/2\ . \tag{3.35}$$

In Ref. [7], an infinitely heavy target ($A\rightarrow\infty$) is assumed and the Möller factor is omitted and the directions of $\hat{\mathbf{n}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{q}}_c$ are opposite to ours. Some misprints are also corrected.

Then one gets

$$\hat{T}_0=\sum_i^A(A^\eta 1_i+C_2^\eta\sigma_{in})e^{-i\mathbf{q}\cdot\mathbf{r}_i},\quad \hat{T}_n=\sum_i^A(B^\eta\sigma_{in}+C_1^\eta 1_i)e^{-i\mathbf{q}\cdot\mathbf{r}_i}\ ,$$
$$\hat{T}_q=\sum_i^A(E^\eta\sigma_{iq}+D_1^\eta\sigma_{ip})e^{-i\mathbf{q}\cdot\mathbf{r}_i},\quad \hat{T}_p=\sum_i^A(D_2^\eta\sigma_{iq}+F^\eta\sigma_{ip})e^{-i\mathbf{q}\cdot\mathbf{r}_i}\ . \tag{3.36}$$

These equations say that $\hat{T}_q$ ($\hat{T}_p$) does not excite the pure spin-longitudinal (-transverse) mode and consequently ID_q and ID_p of Eq. (3.5) do not represent exclusively the spin-longitudinal nor spin-transverse nuclear responses but their mixture.

Now let us set the spin of the target nucleus J_x to be 0. The spin-longitudinal and -transverse response functions, R_L and R_T, are defined as

$$
\begin{aligned}
R_L(q,\omega) &= \frac{1}{2}\sum_n \left| \langle \Psi_{n,-\mathbf{k}'} | \sum_i \sigma_{iq} e^{-i\mathbf{q}\cdot\mathbf{r}_i} | \Psi_{0,-\mathbf{k}} \rangle \right|^2 \delta(\omega-(E_A^n-E_A^0)) \\
&= \frac{1}{2}\sum_n \left| \sum_{s,s'} (\sigma_q)_{s's} F_{n0}^{s's}(\mathbf{q}) \right|^2 \delta(\omega-(E_A^n-E_A^0)) , \\
R_T(q,\omega) &= \frac{1}{2}\sum_n \left| \langle \Psi_{n,-\mathbf{k}'} | \sum_i \sigma_{ip} e^{-i\mathbf{q}\cdot\mathbf{r}_i} | \Psi_{0,-\mathbf{k}} \rangle \right|^2 \delta(\omega-(E_A^n-E_A^0)) \\
&= \frac{1}{2}\sum_n \left| \sum_{s,s'} (\sigma_p)_{s's} F_{n0}^{s's}(\mathbf{q}) \right|^2 \delta(\omega-(E_A^n-E_A^0)) .
\end{aligned} \tag{3.37}
$$

One can show that the interference part of the response function

$$
R_{qp}(\mathbf{q},\omega) = \sum_n \langle \Psi_{0,-\mathbf{k}} | \sigma_q e^{-i\mathbf{q}\cdot\mathbf{r}_i} | \Psi_{n,-\mathbf{k}'} \rangle \langle \Psi_{n,-\mathbf{k}'} | \sigma_p e^{i\mathbf{q}\cdot\mathbf{r}_i} | \Psi_{0,-\mathbf{k}} \rangle \delta(\omega-(E_A^n-E_A^0)) = 0 , \tag{3.38}
$$

by using the formula for the momentum diagonal part of the density-current polarization propagator derived by Alberico *et al.* [16] [see Eqs. (2.20) and (2.24) of Ref. [16]]. We remark that R's defined in Ref. [16] are the above one times q^2 except for the isospin factor. We also note that in the distorted-wave impulse approximation (DWIA) the response functions nondiagonal with respect to $\mathbf{q}$ are needed and the interference term R_{qp} remains in principle, though it could be small.

From Eqs. (2.29), (3.36), (3.37), and (3.38), ID_q and ID_p are expressed as

$$
\begin{aligned}
ID_q &= 4K(|D_1^\eta|^2 R_T + |E^\eta|^2 R_L) , \\
ID_p &= 4K(|D_2^\eta|^2 R_L + |F^\eta|^2 R_T) .
\end{aligned} \tag{3.39}
$$

Though they are not proportional to R_L and R_T, respectively, one can deduce R_L and R_T from them by knowing theoretical values of $|D_1^\eta|^2$, $|D_2^\eta|^2$, $|E^\eta|^2$, and $|F^\eta|^2$.

The undesirable amplitudes D_1^η and D_2^η are evaluated by the rotating angles φ_-^0 and φ_-^1 as well as the amplitudes E' and F'. If these angles are so small that the approximation

$$
|E^\eta|^2 \approx |JE'|^2, \quad |F^\eta|^2 \approx |JF'|^2, \quad |D_1^\eta|^2 \approx |D_2^\eta|^2 \approx 0 \tag{3.40}
$$

holds, then ID_q and ID_p exclusively extract the spin-longitudinal and -transverse strength as $ID_q \approx 4K|E^\eta|^2 R_L$ and $ID_p \approx 4K|F^\eta|^2 R_T$. If the effect of distortion is simply represented by the common reduction factor N_e due to absorption, they are approximated as

$$
ID_q \approx 4KN_e|E^\eta|^2 R_L, \quad ID_p \approx 4KN_e|F^\eta|^2 R_T . \tag{3.41}
$$

Now the ratio R_L/R_T is given by

$$
\frac{R_L(q,\omega)}{R_T(q,\omega)} = \frac{D_q/|E^\eta|^2}{D_p/|F^\eta|^2} = \frac{D_q/|E|^2}{D_p/|F|^2} . \tag{3.42}
$$

Note that for NN scattering,

$$
\begin{aligned}
I^{NN}D_q^{NN} &= I^{NN}S_L^{NN} = 4K|E'|^2 = |E|^2 , \\
I^{NN}D_p^{NN} &= I^{NN}S_T^{NN} = 4K|F'|^2 = |F|^2 ,
\end{aligned} \tag{3.43}
$$

which coincide with Eq. (2.13), then Eq. (3.41) gives

$$
ID_q \approx I^{NN}D_q^{NN}N_e|J|^2 R_L, \quad ID_p \approx I^{NN}D_p^{NN}N_e|J|^2 R_T , \tag{3.44}
$$

which corresponds to the relation (1.2).

The rotation angles for the case of 500 MeV nucleon colliding on ^{40}Ca are shown in Table II for some transferred energies ω and the fixed scattering angle $\theta_{lab} = 18.5°$. For the charge-exchange channel, the η frame amplitudes near the incident energy $E_{inc} = 500$ MeV, momentum transfer $q \approx 1.75$ fm^{-1}, and $\omega = 70$ MeV are estimated from the c.m. frame amplitudes [17] at 515 MeV as

$$
\frac{|E^\eta|^2}{|F^\eta|^2} \approx 2.4, \quad \frac{|D_1^\eta|^2}{|F^\eta|^2} \approx 0.017, \quad \frac{|D_2^\eta|^2}{|F^\eta|^2} \approx 0.007 . \tag{3.45}
$$

Therefore the contribution from the terms with D^η's may safely be neglected and the approximation (3.40) works for this case. It was utilized in our previous DWIA calculation [5].

For higher incident energies, one may expect larger effect of relativistic spin rotations. For a given θ_{lab}, χ's and ρ's naturally increase as $E(\mathbf{k})$ does. However, the ratio $\omega/E(\mathbf{k})$ decrease and consequently η becomes smaller. This means that the η frame comes close to the Breit frame. In this limit, one sees $\chi-\chi'\to 0$, $\rho-\rho'\to 0$, and $\psi\to 0$ and thus the frame transformation does not affect the E and F terms. McNeil *et al.* [15] have given the explicit formula for the transformation from the c.m. to

TABLE II. Rotation angles associated with the transformation from the NN c.m. to the η frame with the incident energy 500 MeV and the scattering angle $\theta_{lab} = 18.5°$.

ω (MeV)	ψ (deg)	χ (deg)	χ' (deg)	ρ (deg)	ρ' (deg)	φ_-^0 (deg)	φ_-^1 (deg)
30	1.00	1.11	2.99	1.24	3.29	0.06	2.03
60	2.01	0.11	3.90	0.12	4.24	0.12	4.07
90	2.90	−0.80	4.72	−0.89	5.07	0.14	5.88
120	3.62	−1.55	5.40	−1.73	5.71	0.14	7.33
150	4.11	−2.12	5.89	−2.35	6.12	0.11	8.35

the Breit frame in Table III of their paper, and have shown no change of the E and F terms. From this consideration, the approximation (3.40) may hold for wide range of energies. On the contrary one must note that A,B,C terms are affected by this transformation.

IV. SUMMARY

We investigated how to extract information about the spin-longitudinal and spin-transverse nuclear responses, R_L and R_T, from the polarization transfers D_{ij} of nucleon-nucleus quasifree scatterings. The quantities S_L and S_T defined by the formulas (1.1) of Carey *et al.* have been used for this purpose. We derived the more reliable formulas (2.20) for the corresponding quantities D_q and D_p, and considered some limiting cases for which they reduce to those derived by Moss and by Bleszynsky *et al.* We pointed out that change of the direction of transferred momentum as the change of the energy transfer for a fixed scattering angle must be taken into account. This effect is missing in Carey's formulas.

We then questioned if D_q and D_p truly represent the spin responses in the framework of the optimum factorization approximation in PWIA. Complete formulas for the transformation of the NN t matrix from the c.m. frame to the optimal frame were derived with full consideration of the relativistic spin rotations. We found that for a wide range of incident energies D_q and D_p reasonably well represent the spin responses R_L and R_T, respectively.

Finally, we comment on Eq (1.4) for the ratio R_L/R_T. We recommend use of formula (3.42) with explicit calculation of $|E|^2$ and $|F|^2$ from the phase shift rather than using Eq. (1.4) because assumption (1.3) has not yet been confirmed. The D-state component of the deuteron may affect the assumption. This must be investigated.

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