

Ground state proton emission from heavy nuclei

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We examine the phenomenon of proton emission from the ground states of heavy nuclei, using a model of charged particle emission which has previously given a good description of unhindered *s*-wave alpha and exotic decays in heavy nuclei. We have extended the model formalism by including an angular momentum term, because the emitted proton seldom comes from an *s* orbital. Using a realistic proton-nucleus potential, obtained from proton-nucleus scattering, we calculate partial half-lives and find generally good agreement with the currently available experimental measurements.

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The phenomenon of proton emission from the nuclear ground state limits the possibilities of creation of ever more exotic nuclei on the proton-rich side of the valley of beta stability. By its very nature it is an intrinsically difficult decay mode to observe, and yet its simplicity, together with the clean-cut information it can bring concerning nuclear masses and single-particle orbitals, makes its detection and interpretation well worth pursuing. A few ground state proton emitters were discovered [1] in the early 1980s with masses around 110 and 150, but all subsequent searches [2] had proved fruitless until the recent discovery [3] of two more examples at the rather larger mass numbers of 156 and 160. This experimental development has stimulated us to examine the situation anew from the theoretical standpoint.

In addition to this experimental prompting, we were also motivated to examine proton emission by our recent results using a simple cluster model to calculate the half-lives for unhindered *s*-wave alpha decay of both even-even and odd-mass nuclei [4–7]. We were generally able to achieve agreement between our calculated values and the corresponding measured half-lives to within a factor of ~ 2 using a fixed set of parameters. We have also shown [8] that the model can be extended to reproduce the half-lives of all the known *s*-wave exotic decays of even-even nuclei (involving emission of heavy clusters such as ^{14}C , ^{24}Ne , etc.), to within a factor of 3. It is therefore a logical step to investigate the possibility of applying an extended version of this same model to the presumably more straightforward case of proton emission from nuclear ground states, where both the relevant proton-nucleus potentials and spectroscopic factors are known with a greater degree of confidence.

In order to carry out our proposed program, the model of Refs. [4–7] must be extended to include a centrifugal

barrier (since the majority of proton emissions are likely to come from orbitals with $L \neq 0$). However, in many other respects the calculations are less ambiguous than those for alpha and exotic decay. The appropriate proton-nucleus potential is independently very well known from scattering data, and the quantum numbers (node number n and orbital angular momentum L) defining the orbital motion of the proton can be limited to a small number of possible values by consideration of the level ordering in the simplest version of the spherical shell model. We also expect the spectroscopic factor S_p for the occupation of the appropriate orbital to be large and, therefore, take the limiting value $S_p = 1$ in all cases.

We describe the interaction between the odd proton and remaining core nucleons by the potential

$$V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2} (L + \frac{1}{2})^2, \quad (1)$$

where $V_N(r)$ is the nuclear (strong interaction) potential, for which we employ a Woods-Saxon real part and a related Thomas spin-orbit term

$$V_N(r) = -V_0 f_0(r) + V_s \left[\frac{\hbar}{m_\pi c} \right]^2 \frac{1}{r} \left[\frac{d}{dr} f_s(r) \right] (\boldsymbol{\sigma} \cdot \mathbf{L}), \quad (2)$$

with

$$f_i(r) = \frac{1}{1 + \exp[(r - R_i)/a]}, \quad (3)$$

in standard notation, and $V_C(r)$ is a Coulomb potential appropriate to a point proton interacting with a uniformly charged spherical core,

$$V_C(r) = \begin{cases} \frac{Ze^2}{r} & \text{for } r \geq R_c, \\ \frac{Ze^2}{2R_c} \left[3 - \left(\frac{r}{R_c} \right)^2 \right] & \text{for } r \leq R_c, \end{cases} \quad (4)$$

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Z being the charge of the core. The final term in Eq. (1) is a Langer modified centrifugal barrier, in which $L(L+1)$ is replaced by $(L+\frac{1}{2})^2$, and μ is the reduced mass of the proton-core system. The radius parameters appropriate to the Woods-Saxon and Coulomb potentials are initially unknown and have to be determined within our model. However, so as to keep the number of adjustable parameters to a minimum, we constrain them to be equal and hereafter write them both as R .

We determine the classical turning points (r_1 , r_2 , and r_3 in order of increasing distance from the origin) by solution of the equation $V(r)=Q$, where Q is the energy available for proton emission. We deduce Q from the measured proton kinetic energy E_p (MeV) by applying a standard recoil correction and an electron shielding correction, i.e.,

$$Q = \left[\frac{M_A}{M_A - m_p} \right] E_p + E_{SC}, \quad (5)$$

where M_A and m_p are the masses of the decaying nucleus and proton, respectively. We follow Hoffman [1] in estimating the screening correction (SC) from the tables of Huang *et al.* [9]. For the nuclei of interest to us below, we use values of $E_{SC} = 8.8, 9.2, 13.1, 13.7, 14.3,$ and 15.0 keV for emission from I, Cs, Tm, Lu, Ta, and Re, respectively.

Although we intend to take most of the parameters appearing in Eqs. (1)–(4) from global optical-model fits to proton-scattering data, the radius parameter R appearing in both $V_N(r)$ and $V_C(r)$ should be evaluated separately for each decay, so as to obtain a quasibound state at the observed energy. By applying the Bohr-Sommerfeld quantization condition to generate a state of relative motion with n nodes and orbital angular momentum L at the energy Q , we obtain

$$\int_{r_1}^{r_2} dr \left[\frac{2\mu}{\hbar^2} [Q - V(r)] \right]^{1/2} = (G - L + 1) \frac{\pi}{2}, \quad (6)$$

where $G = 2n + L$. This equation for R may be solved to the required level of accuracy by a few iterations of the Newton-Raphson method. In practice, we find that $R = 1.2A^{1/3}$ provides a good starting value. We can then check the consistency of our calculation by comparing the value of R required to fit the quasibound state at the specified Q value with that prescribed by the optical potential.

Having determined the value of R , we may now calculate the width of the quasibound state in semiclassical approximation following the procedure of Gurvitz and Kälbermann [10]. The partial width for proton decay Γ_p is given by

$$\Gamma_p = S_p F \frac{\hbar^2}{4\mu} \exp \left[-2 \int_{r_2}^{r_3} dr k(r) \right], \quad (7)$$

where S_p is the spectroscopic factor for finding the proton in the orbital specified by n and L in the parent nucleus ground state. The normalization factor F is given by

$$F \int_{r_1}^{r_2} dr \frac{1}{k(r)} \cos^2 \left[\int_{r_1}^r dr' k(r') - \frac{\pi}{4} \right] = 1, \quad (8)$$

where the squared cosine term may be replaced by its average value of $\frac{1}{2}$ without significant loss of accuracy, giving

$$F \int_{r_1}^{r_2} \frac{dr}{2k(r)} = 1, \quad (9)$$

and the wave number $k(r)$ is given by

$$k(r) = \left[\frac{2\mu}{\hbar^2} |Q - V(r)| \right]^{1/2}. \quad (10)$$

The partial decay half-life for proton emission is then related to the partial width by the relation $T_{1/2}^p = \hbar \ln 2 / \Gamma_p$.

An evaluation of our extended model can now be made by comparing its predicted half-lives for ground state proton emission with those measured. Until mid-1991, ^{109}I , ^{113}Cs , ^{147}Tm , and ^{151}Lu were the only ground state proton emitters with known lifetimes. In addition, proton emission had been observed from the ground state of ^{150}Lu and a lower limit placed on its half-life. These five examples are all included in the review by Hofmann [1], which summarizes the experimental searches up until 1987. Subsequent searches for ground state proton emitters had all proved negative (see [2] for a recent search in the range $31 \leq Z \leq 38$) until very recently, when two further emitters were identified (^{156}Ta and ^{160}Re) [3].

Before proceeding with the calculational scheme outlined in the last section, we present, as benchmarks, the half-lives for proton emission calculated without adjustment of the radius of the nuclear potential, but simply using the Becchetti-Greenlees potential as it stands. Although this means that a quasibound state will generally be produced at an energy which is a little different from that found experimentally, in practice we find that the discrepancy is not so large as to change the resulting half-life enormously. We assign values of G , or more precisely n and L , to the emitted proton in accordance with the spherical shell model. Thus the protons emitted from ^{109}I and ^{113}Cs are expected to be in $1d_{5/2}$ ($n=1, L=2, G=4$) or $0g_{7/2}$ ($n=0, L=4, G=4$) orbitals, those emitted from ^{147}Tm , ^{150}Lu , and ^{151}Lu in $0h_{11/2}$ orbitals ($n=0, L=5, G=5$), while those coming from ^{156}Ta and ^{160}Re might also be expected to be in $0h_{11/2}$ orbitals, but actually seem to be expelled from $1d_{3/2}$ orbitals ($n=1, L=2, G=4$). We are also implicitly assuming that the wave functions describing the core of the parent nucleus and ground state of the daughter nucleus are essentially identical.

A word of caution is in order at this point because the first proton emitter ever discovered, a $\frac{19}{2}^-$ isomeric state of ^{53}Co , provides a salutary counterexample. The leading term in the shell-model configuration for $^{53}\text{Co}^m$ is $[vf_{7/2}^{-2}]_6 \otimes [\pi f_{7/2}^{-1}]$; i.e., the spin of $\frac{19}{2}^-$ is achieved by coupling two neutron holes (themselves coupled to a spin of 6) with one $\frac{7}{2}^-$ proton hole. The ^{52}Fe ground state, on the other hand, has a spin of 0, so that the two neutron holes in the leading configuration are here very differently

aligned. The core-daughter overlap is thus expected to be very small, leading to a strong suppression of the emission rate. This effect shows us that S_p may be much less than unity and can lead to our calculations underestimating the true half-life for proton emission. The possibility of a low value of S_p must be borne in mind when comparing our results with the data. In fact, it leads to the general principle that if our calculated half-life for proton emission from a given orbital is grossly in excess of the experimental value, then that orbital was almost certainly not occupied by the proton prior to emission.

Table I compares the half-lives, calculated using the Becchetti-Greenlees potential, with the measured values for the seven presently known ground state proton emitters. The results for ^{113}Cs , ^{147}Tm , ^{150}Lu , ^{151}Lu , ^{156}Ta , and ^{160}Re are all satisfactory and suggest that we have indeed correctly identified the odd proton orbital in these cases. Furthermore, there is an isomeric state in ^{147}Tm with spin either $\frac{1}{2}^+$ or $\frac{3}{2}^+$, which also decays by proton emission, with a half-life of $(3.6 \pm 0.8) \times 10^{-4}$ s. Assuming that the odd proton here is in a $1d_{3/2}$ orbital, we calculate a half-life of 1.5×10^{-4} s, which is also in good agreement with the measured value. The result for ^{109}I , however, is only within a factor of 12 of the experimental value and leads us to reexamine the assignment of quantum numbers to the odd proton in this case.

We now investigate the sensitivity of the calculated proton emission half-lives to the orbital from which the emitted proton is assumed to have been expelled. From here on we operate within the scheme described above. To this end we retain the values of Becchetti and Greenlees [11] for the depth of the real potential,

$$V_0 = 54.0 - 0.32E + 0.4 \frac{Z}{A^{1/3}} + 24.0 \left[\frac{N-Z}{A} \right] \text{ MeV}, \quad (11)$$

together with a depth of the spin-orbit potential, $V_{s.o.} = 6.2$ MeV, and diffusenesses $a = a_{s.o.} = 0.75$ fm. We tie the spin-orbit radius parameter $R_{s.o.}$ to that of the real potential by the relation $R_{s.o.} = 1.01R / 1.17$ so as to maintain the same ratio between the two as in Ref. [11]. The only difference is that now we adjust the radius R to produce a quasibound state with the specified quantum numbers at exactly the same energy as that measured experimentally.

Inspection of a shell-model single-particle level diagram suggests that the most likely orbitals for the odd proton in the known ground state proton emitters should be $0g_{9/2}$, $1d_{5/2}$, $0g_{7/2}$, $0h_{11/2}$, $2s_{1/2}$, $1d_{3/2}$, or $0h_{9/2}$ [12]. To check out these possibilities, while examining the sensitivity of our calculations to the precise values of the quantum numbers selected, we present in Table II calculations for all ground state proton emitters, using the above assignments.

We see that the number of nodes has a far more profound effect on the half-lives than does the angular momentum. The different quantum numbers generally produce half-lives which differ quite markedly, and even though this does not allow us to pinpoint unique values of n and L , it may often limit the choice to two or three alternatives. In addition to this criterion, we should also compare the radius required to generate the quasibound state at the known Q value with that of the Becchetti-Greenlees potential (i.e., $1.17 A^{1/3}$). Insistence on having a value of R close to the optical potential value can significantly restrict the possible values of the orbital quantum numbers in question still further. For example, the $0h_{9/2}$ orbitals can be ruled out for all the nuclei under discussion on this basis alone, since it requires radii which are far too large, as one would indeed have expected from simple shell-model considerations. Similarly, the $0g_{9/2}$ orbital can be seen to be inappropriate for emission from all the heavier nuclei, since it requires radii which are far too small, again in line with simple shell-model expectations. Together with the requirement that the predicted half-life not be greatly in excess of the measured value, we are then able, on occasion, to select a unique set of quantum numbers for the odd proton orbital. The radius consistency check is a distinct advantage of our model. Its violation indicates that some very abnormal nuclear structure effects, requiring a more microscopic treatment, are present.

The Tm and Lu decays certainly indicate a strong preference for $0g_{7/2}$ or $0h_{11/2}$ orbitals, in terms both of predicted half-life and of potential radius. The results with the $0h_{11/2}$ orbital are rather better and lead us to favor it, as expected from the shell-model occupancies in this mass region. Similarly, the Ta and Re decays point to proton emission from d orbitals, suggesting that the $1d_{3/2}$ levels in these nuclei are becoming occupied, before the $0h_{11/2}$ level is full. Likewise, the ^{113}Cs half-life and

TABLE I. Comparison of calculated and measured half-lives for all currently known ground state proton emitters using the prescription of Gurvitz and Kälbermann [10] and the unmodified Becchetti-Greenlees potential [11]. Experimental half-lives and Q values are from Refs. [1,3].

System	Q (MeV)	nL_j	$T_{1/2}^{\text{calc}}$ (s)	$T_{1/2}^{\text{expt}}$ (s)
$^{109}\text{I} \rightarrow ^{108}\text{Te} + p$	0.829	$1d_{5/2}$	9.0×10^{-6}	$(1.09 \pm 0.17) \times 10^{-4}$
$^{113}\text{Cs} \rightarrow ^{112}\text{Xe} + p$	0.977	$0g_{7/2}$	1.3×10^{-4}	$(3.3 \pm 0.7) \times 10^{-5}$
$^{147}\text{Tm} \rightarrow ^{146}\text{Er} + p$	1.071	$0h_{11/2}$	1.7×10^0	$(2.7^{+2.4}_{-0.9}) \times 10^0$
$^{150}\text{Lu} \rightarrow ^{149}\text{Yb} + p$	1.285	$0h_{11/2}$	1.8×10^{-2}	$\geq 1.0 \times 10^{-2}$
$^{151}\text{Lu} \rightarrow ^{150}\text{Yb} + p$	1.255	$0h_{11/2}$	3.6×10^{-2}	$(1.2^{+1.2}_{-0.4}) \times 10^{-1}$
$^{156}\text{Ta} \rightarrow ^{155}\text{Hf} + p$	1.015	$1d_{3/2}$	1.3×10^{-1}	$(1.65^{+1.65}_{-0.55}) \times 10^{-1}$
$^{160}\text{Re} \rightarrow ^{159}\text{W} + p$	1.250	$1d_{3/2}$	4.8×10^{-4}	$(8.7^{+7.3}_{-1.7}) \times 10^{-4}$

TABLE II. Calculated proton emission half-lives for a variety of possible n and L assignments. Experimental half-lives and Q values are from Refs. [1,3]. The real potential depth used in the calculations is given by Eq. (3.1), with corresponding radii chosen to produce a quasibound state at the Q value, as shown.

System	nL_j	$R_0 = R / A^{1/3}$ (fm)	$T_{1/2}^{\text{calc}}$ (s)
$^{109}\text{I} \rightarrow ^{108}\text{Te} + p$ $Q = 0.829 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (1.09 \pm 0.17) \times 10^{-4} \text{ s}$	$2s_{1/2}$	1.25	5.3×10^{-7}
	$1d_{3/2}$	1.25	5.7×10^{-6}
	$1d_{5/2}$	1.21	6.4×10^{-6}
	$0g_{7/2}$	1.23	1.5×10^{-3}
	$0g_{9/2}$	1.15	2.1×10^{-3}
	$0h_{9/2}$	1.39	7.5×10^{-3}
	$0h_{11/2}$	1.29	1.2×10^{-2}
$^{113}\text{Cs} \rightarrow ^{112}\text{Xe} + p$ $Q = 0.977 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (3.3 \pm 0.7) \times 10^{-5} \text{ s}$	$2s_{1/2}$	1.24	3.2×10^{-8}
	$1d_{3/2}$	1.24	3.3×10^{-7}
	$1d_{5/2}$	1.21	3.6×10^{-7}
	$0g_{7/2}$	1.22	7.2×10^{-5}
	$0g_{9/2}$	1.14	1.0×10^{-4}
	$0h_{9/2}$	1.37	3.3×10^{-4}
	$0h_{11/2}$	1.28	5.5×10^{-4}
$^{147}\text{Tm} \rightarrow ^{146}\text{Er} + p$ $Q = 1.071 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (2.7_{-0.9}^{+2.4}) \times 10^0 \text{ s}$	$2s_{1/2}$	1.19	1.2×10^{-4}
	$1d_{3/2}$	1.18	9.9×10^{-4}
	$1d_{5/2}$	1.15	1.1×10^{-3}
	$0g_{7/2}$	1.16	2.4×10^{-1}
	$0g_{9/2}$	1.08	2.4×10^{-1}
	$0g_{9/2}$	1.29	6.1×10^{-1}
	$0h_{11/2}$	1.21	1.0×10^0
$^{150}\text{Lu} \rightarrow ^{149}\text{Yb} + p$ $Q = 1.285 \text{ MeV}$ $T_{1/2}^{\text{expt}} \geq 1.0 \times 10^{-2} \text{ s}$	$2s_{1/2}$	1.18	1.7×10^{-6}
	$1d_{3/2}$	1.18	1.4×10^{-5}
	$1d_{5/2}$	1.15	1.5×10^{-5}
	$0g_{7/2}$	1.15	2.0×10^{-3}
	$0g_{9/2}$	1.08	3.0×10^{-3}
	$0h_{9/2}$	1.29	6.8×10^{-3}
	$0h_{11/2}$	1.21	1.1×10^{-2}
$^{151}\text{Lu} \rightarrow ^{150}\text{Yb} + p$ $Q = 1.255 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (1.2_{-0.4}^{+1.2}) \times 10^{-1} \text{ s}$	$2s_{1/2}$	1.18	3.5×10^{-6}
	$1d_{3/2}$	1.18	2.9×10^{-5}
	$1d_{5/2}$	1.15	3.2×10^{-5}
	$0g_{7/2}$	1.15	4.3×10^{-3}
	$0g_{9/2}$	1.08	6.2×10^{-3}
	$0h_{9/2}$	1.28	1.5×10^{-2}
	$0h_{11/2}$	1.20	2.4×10^{-2}
$^{156}\text{Ta} \rightarrow ^{155}\text{Hf} + p$ $Q = 1.015 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (1.65_{-0.55}^{+1.65}) \times 10^{-1} \text{ s}$	$2s_{1/2}$	1.18	1.1×10^{-2}
	$1d_{3/2}$	1.17	9.2×10^{-2}
	$1d_{5/2}$	1.15	1.0×10^{-1}
	$0g_{7/2}$	1.15	$1.5 \times 10^{+1}$
	$0g_{9/2}$	1.08	$2.2 \times 10^{+1}$
	$0h_{9/2}$	1.28	$5.2 \times 10^{+1}$
	$0h_{11/2}$	1.19	$8.7 \times 10^{+1}$
$^{160}\text{Re} \rightarrow ^{159}\text{W} + p$ $Q = 1.250 \text{ MeV}$ $T_{1/2}^{\text{expt}} = (8.7_{-1.7}^{+2.3}) \times 10^{-4} \text{ s}$	$2s_{1/2}$	1.17	3.9×10^{-5}
	$1d_{3/2}$	1.17	3.1×10^{-4}
	$1d_{5/2}$	1.14	3.5×10^{-4}
	$0g_{7/2}$	1.14	4.3×10^{-2}
	$0g_{9/2}$	1.07	6.3×10^{-2}
	$0h_{9/2}$	1.27	1.4×10^{-1}
	$0h_{11/2}$	1.19	2.3×10^{-1}

TABLE III. Comparison of calculated and measured half-lives for all currently known ground state proton emitters and $^{147}\text{Tm}^*$. Experimental half-lives and Q values are from Refs. [1,3]. The real potential has a constant depth of 70 MeV, with corresponding radii chosen so as to produce a quasibound state at the Q value, as shown.

System	Q (MeV)	nL_j	$R_0 = R/A^{1/3}$ (fm)	$T_{1/2}^{\text{calc}}$ (s)	$T_{1/2}^{\text{expt}}$ (s)
$^{109}\text{I} \rightarrow ^{108}\text{Te} + p$	0.829	$1d_{5/2}$	1.10	1.2×10^{-5}	$(1.09 \pm 0.17) \times 10^{-4}$
$^{113}\text{Cs} \rightarrow ^{112}\text{Xe} + p$	0.977	$0g_{7/2}$	1.11	1.6×10^{-4}	$(3.3 \pm 0.7) \times 10^{-5}$
$^{147}\text{Tm} \rightarrow ^{146}\text{Er} + p$	1.071	$0h_{11/2}$	1.12	2.4×10^0	$(2.7^{+2.4}_{-0.9}) \times 10^0$
$^{147}\text{Tm}^* \rightarrow ^{146}\text{Er} + p$	1.139	$1d_{3/2}$	1.08	2.7×10^{-4}	$(3.6 \pm 0.8) \times 10^{-4}$
$^{150}\text{Lu} \rightarrow ^{149}\text{Yb} + p$	1.285	$0h_{11/2}$	1.11	2.8×10^{-2}	$\geq 1.0 \times 10^{-2}$
$^{151}\text{Lu} \rightarrow ^{150}\text{Yb} + p$	1.255	$0h_{11/2}$	1.11	5.8×10^{-2}	$(1.2^{+1.2}_{-0.4}) \times 10^{-1}$
$^{156}\text{Ta} \rightarrow ^{155}\text{Hf} + p$	1.015	$1d_{3/2}$	1.08	1.8×10^{-1}	$(1.65^{+1.65}_{-0.55}) \times 10^{-1}$
$^{160}\text{Re} \rightarrow ^{159}\text{W} + p$	1.250	$1d_{3/2}$	1.07	6.1×10^{-4}	$(8.7^{+2.3}_{-1.7}) \times 10^{-4}$

radius seem to favor a $0g_{7/2}$ orbital, when one would have expected the extra five valence protons beyond $Z = 50$ in ^{113}Cs to go into the $1d_{5/2}$ orbital. The ^{109}I decay is not well reproduced in our model, but the half-life seems to favor a $1d$ orbital, and the resulting radius is also consistent for this orbital. It would certainly be interesting to see if a more microscopic model can suggest any reason for the small spectroscopic factor required to ameliorate the discrepancies between our calculation and the data.

Apart from the difficulty with ^{109}I , even if the proton orbitals have been correctly identified, there is a systematic tendency for our calculated half-lives to be a little smaller than the measured ones. This could be a manifestation of a spectroscopic factor which is large but less than 1 (about 0.5, say), or it could be an indication that the potential is not completely adequate. In Table III we show the results of repeating our calculations with a Woods-Saxon potential of fixed depth $V_0 = 70$ MeV instead of the prescription of Eq. (11), which means we are proposing an increase of around 10 MeV in most cases. This enables us to give a better overall description of the various proton emission half-lives, with a correspondingly smaller potential radius of about $R = 1.1A^{1/3}$ fm. It would be interesting to see whether the available proton

scattering data in the mass region $100 \leq A \leq 160$ could also be adequately described with such a modified potential.

In summary, we have extended the model of alpha and exotic decay [4–7] to deal with proton emission by including an angular momentum term. Using physically motivated parameter values that are consistent with the real part of the Becchetti-Greenlees global proton-nucleus optical potential, we are able to give a reasonable account of the half-lives of the seven ground state proton emitters (and the $^{147}\text{Tm}^*$ isomer) known to date. Taking into account the radius consistency check afforded by our model together with the requirement that we do not grossly overestimate the half-life, the model predictions can be used to restrict considerably the values of the quantum numbers of the orbital from which the proton is emitted. Spin-parity assignments can thus be made, and moderate discrepancies between half-life calculations and measurements may be indicative of a smaller than expected spectroscopic factor.

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