# Generalized potential-model description of mutual scattering of the lightest $p+d$, $d+{}^3$He nuclei and the corresponding photonuclear reactions

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For the problem of interaction between the lightest $p+d$, $d+{}^3$He nuclei supermultiplet expansion of the reaction amplitude $T_{LS}$ is proposed. We have disclosed the nonpotential character of channels with $S=\frac{1}{2}$, where the scattering amplitude is written as a superposition of the potential amplitudes $T_L^{[f]}$ with different symmetries $[f]$. We have constructed the interaction potentials $V_L^{[f]}$, $[f]=[21]$, [3] and $[f]=[32]$, [41] for $p+d$ and $d+{}^3$He, respectively. The differential cross sections for $p+d$ inelastic scattering with the deuteron spin-isospin flip, which correspond to the available experimental data in the low-energy region, are predicted at higher energies and also for the $d+{}^3$He system in a broad energy interval. The potential-model approach well describes quantitatively the photonuclear ${}^3\mathrm{He}\gamma\rightarrow p+d$ and $d+{}^3\mathrm{He}\rightarrow{}^5\mathrm{Li}\,\gamma$ processes in a broad photon energy range $E_\gamma \leq 30$ MeV.

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## I. INTRODUCTION

The objective of the present paper is to elucidate new possibilities of the potential-model description of the interaction between lightest nuclei $p+d$, $d+{}^3\mathrm{H}({}^3\mathrm{He})$ and others. In this case, by the potential-model description, we imply a theoretical treatment, based on local or quasilocal (i.e., dependent on orbital angular momentum parity) potentials, which enables one to obtain the energy dependence of scattering phase shifts in a fairly wide energy range. This treatment generalizes the traditional optical-model description.

The main expedient of describing the interaction between light nuclei such as $d+t$, $d+{}^4$He, $N+{}^4$He, ${}^3\mathrm{H}+{}^3$He, and others has long been so far the resonating-group method (RGM) [1,2], which leads to the nonlocal and energy-dependent interaction. This was quite consistent with the familiar Feshbach formalism [3] for treating the optical potential of nucleon-nucleus scattering, which also predicts a nonlocal and energy-dependent nucleon-nucleus interaction potential.

In contrast with this common viewpoint, the optical potentials of the interaction between nucleons (deuterons, helions, and alpha particles) and medium and heavy nuclei, which have been found and well grounded in the last two decades, turn out to be practically local with a rather weak energy dependence [4]. In quite the same way, the interaction potentials between light clusters such as ${}^4\mathrm{He}+{}^4$He, ${}^3\mathrm{H}+{}^4$He, $d+{}^4$He, and others also turned out to be local (or quasilocal with some even-odd splitting [5,6] and also with a weak energy dependence. The Pauli exclusion principle in the above potentials manifests itself through the occurrence in them of deeply lying nodeless bound states which simulate the states forbidden by the Pauli exclusion principle [5]. Owing to the hermiticity of the Hamiltonian, the scattering wave functions turn out to be orthogonal to the wave functions of the forbidden states, which agrees with the Pauli exclusion principle. This description, based on the concept of deep forbidden-state attractive potentials, generalizes the previous important step—the orthogonality condition model of Saito [7].

The question arises as to the interrelation of these two approaches, i.e., microscopic and phenomenological. The common viewpoint adopted in the literature is that both the local optical-model interaction potentials and cluster interaction potentials are phase-shift equivalent potentials for complicated nonlocal and energy-dependent microscopic potentials (see, in particular, Refs. [8,9]). Indeed, the solution of the inverse scattering problem, as given by Newton [9] and Chadan and Sabatier [9], makes it possible to construct a purely local potential at fixed energy for any set of partial phase shifts $\{\delta_l\}$. And, thereby, the only problem that remains is that the local potentials, constructed at different energies (from some not too broad interval), turn out to be fairly close to each other, which is most likely due to weak dependence of nuclear dynamics on energy outside the resonance region.

In this context, the idea is tempting of extending this potential-model approach to include the interaction between lightest nuclei such as $p+d$ and $d+{}^3\mathrm{H}({}^3\mathrm{He})$. At first glance it seems also that the potential-model approach is hardly possible and fruitful because exchange effects are strong and the deuteron is loosely bound. It will be shown, however, that if one separates out channels with different permutation symmetries $[f_i]$ of the spatial part of the wave function, then one can construct a local potential in each channel $[f_i]$ so that the scattering amplitude in the channel with a given orbital angular

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momentum $L$ and spin-isospin $ST$ is a superposition of the potential scattering amplitudes corresponding to different values of $[f_i]$. We base our analysis here on the well-known fact that supermultiplet symmetry for the lightest nuclei is a good quantum number.

Attempts have been made in the literature to construct a local potential for the $p+d$ and $n+d$ interactions [10,11], but with neglect of the supermultiplet structure. We introduce a supermultiplet description of scattering for different cluster systems, construct its own potential for each channel $[f_i]$, and verify these potentials in inelastic scattering accompanied by the transition of the deuteron to the singlet state ($TS=10$) and in photonuclear processes.

Indeed, because of the substantial supermultiplet splitting of the above-mentioned potential, the high inelasticity of scattering in the doublet channels (at low $L$ values) follows from the theory which is opposite to the quartet channels whose elasticity is still preserved. This explicit property of scattering is quite in correspondence with the present-day experimental evidence for the $d+p$ system and predicted in the case of the $^3\mathrm{He}(t)+d$ system. As regards photonuclear reactions, our theory predicts that, say, in the case of $^3\mathrm{He}(\gamma,p)^2\mathrm{H}$ electric dipole photoabsorption, the final-state interaction is defined by the shallow $p$-$d$ potential exhibiting the lowest spatial symmetry $[f]=[21]$ reconstructable just from the quartet phase shifts.

So, thereby, calculating the $^3\mathrm{He}(\gamma,p)^2\mathrm{H}$ and $^3\mathrm{He}(d,\gamma)^6\mathrm{Li}$ photonuclear reactions fits the experimental data quantitatively and enables one to predict the cross sections for any energy $E_\gamma \leq 30$ MeV.

The present paper is organized as follows. In Sec. II we give a general description of supermultiplet symmetry manifestations in the interaction between the lightest clusters. Section III describes a formalism for summing up the amplitudes in terms of the super-multiplet model and for calculating the inelastic cross sections. Section IV examines the $p+d$ and $d+^3\mathrm{He}$ interactions. The final Sec. V contains a detailed analysis of such photonuclear processes as $^3\mathrm{He}(\gamma,p)^2\mathrm{H}$ and $^3\mathrm{He}(d,\gamma)^5\mathrm{Li}$. In conclusion, some perspectives are outlined.

## II. GENERAL CONSIDERATION OF THE SUPERMULTIPLET SCHEME

At present, use is widely made of the potential-model description of mutual scattering of light clusters such as $^4\mathrm{He}+^4\mathrm{He}$, $^4\mathrm{He}+^3\mathrm{H}(^3\mathrm{He})$, and $d+^4\mathrm{He}$ (see Refs. [5,6,12,13]). The microscopic basis of the potential-model approach are RGM investigations [8]. Actually, the RGM is a certain model approach where, for example, the effective nucleon-nucleon interaction involved is far from being the free $NN$ interaction, thereby implying the importance of the virtual excitations of interacting clusters [14]. At the same time, the RGM treatment proves to be of assistance because it permits a careful examination of the total antisymmetrization. The relevant studies have shown that the interaction between heavy clusters, e.g., $^{16}\mathrm{O}+^{16}\mathrm{O}$, are substantially nonlocal [15]. This is exhibited by strong damping of the oscillations of the wave function for the relative motion of clusters in their overlap region. At the same time, in the case of light clusters, there is practically no damping of oscillations, and the description of interaction using the local potential is adequate. Such a description made it possible to invoke the powerful results obtained in potential scattering theory and to show that the $E$ dependence of phase shifts $\delta_L(E)$ obeys the generalized Levinson theorem [16]: All the significant phase shifts at not too small energies are positive, starting at $E=0$ from different values of $(n+m)\pi$ (where $n$ is the number of bound states observed and $m$ is the number of bound states present in the potential, but forbidden by the Pauli exclusion principle) and, as the energy $E$ increases, reaching eventually the common region of Born values $\delta_L(E)<1$.

For all that, the potential-model description, practically very efficient, was not used for a long time to describe scattering of lightest nuclei such as $p(n)+d$, $d+d$, $d+^3\mathrm{H}(^3\mathrm{He})$, etc., in a wide energy range. Recently, it has been remarked [17,18] that here it is necessary to undertake a new important step, namely, to introduce into the channels with the minimum possible total spin $S$ the interference of several potential amplitudes which correspond to possible permutation symmetries of the system with various Young schemes $[f]$, the potentials for which considerably differ from each other. This comes from the important circumstance that the nucleon-nucleon interaction is of Serber character, i.e., strong in even waves and rather weak in odd ones. As a result, a pronounced supermultiplet structure [19,20] is peculiar to the lightest nuclei which is characterized, for example,, by the fact that for the breakup of the $^4\mathrm{He}$ nucleus, possessing space symmetry $[f]=[4]$, a high energy of $\Delta E \approx 20$ MeV is required, while the $^5\mathrm{He}$ or $^8\mathrm{Be}$ nuclei are unbound. The significant role of this circumstance in the theory of nuclear reactions with clusters was repeatedly analyzed elsewhere [21–23].

The use of symmetry quantum number $[f]$ in scattering theory is supported by the fact that the $J$ splitting of phase shifts is minor in the case of mutual scattering of the examined lightest clusters. The observation [23] that some compound states of $^6\mathrm{Li}$-$^9\mathrm{Be}$ nuclei exhibit a strong mixing of symmetries $[f]$ cannot be an argument against considering relatively fast processes. Indeed, e.g., the isospin $T$ is a rather bad quantum number for some $^8\mathrm{Be}$ long-living compound states [24], but is quite perfect in the case of direct nuclear reactions on light nuclei.

The key point is that if the scattering wave function of two clusters can have two possible permutation symmetries (i.e., the Young schemes $[f]$), then, owing to the strong supermultiplet effects mentioned above and to the different numbers of even and odd $NN$ pairs for different $[f]$, the corresponding interaction potentials $V_L^{[f]}$ should also be essentially different. A particular great difference between the potentials corresponds to the nucleon "quarteting" break $[41]\leftrightarrow[32]$ for $td$ scattering, $[4]\leftrightarrow[22]$ for even-wave $dd$ scattering, $[4]\leftrightarrow[31]$ for $tp$ scattering, etc.

Based on the results of previous investigations [17,18], we introduce a scattering amplitude with fixed total Young scheme $[f]$, assume that this scattering amplitude

is a potential one, and make a natural step forward, namely, represent the total scattering amplitude as a superposition of the potential amplitudes with different Young schemes $[f]$. We shall see that the elastic scattering becomes nonunitary.

In fact, for example, only one permutation symmetry $[f]=[44]$ is characteristic of the system $^4$He+$^4$He and all the orbital angular momenta $L$ are even [5]. For the $^3$H+$^4$He system, there is, also, only one symmetry $[f]=[43]$ permissible, but possible now are both even and odd orbital angular momenta $L$, and in this case the potential is splitted with respect to parity $L$ (similarly, for the $d+^4$He system) [6]. This even-odd effect, as a rule, causes an increase in the cross section in the backward hemisphere [25]. Even more interesting is the $^3$H+$^3$He system [6], where even values of $L$ are assigned to the Young scheme $[f]=[42]$ and odd ones to $[f]=[33]$, but here to the channel with the given values of $L$ and $S$ corresponding to only one Young scheme $[f]$ too.

We now turn to the $p+d$ system. In this system we are faced with the most complicated and interesting situation: In each of the channels with $S=\frac{1}{2}$, two Young schemes $[f]=[21]$ and $[3]$ are now possible. Indeed,

$$[2]\times[1]=[21]+[3]\ .$$

Both even and odd values of $L$ correspond to two possible Young schemes $[f]=[21]$ and $[3]$. It is important that the scattering amplitude in the channels with $S=\frac{3}{2}$ is unitary because only the Young scheme $[f]=[21]$ is permissible, and consequently, the given scattering amplitude $T_{L,S=3/2}$ is determined by the interaction potentials $V_L^{[21]}$, with $L$ even and odd. We take phase shifts in these channels, required to construct the above potentials from the experiment [26], shifting them up by $n\pi$ in accordance with the number of forbidden states. At the same time, in the channels with $S=\frac{1}{2}$, we have a superposition of the potential amplitudes $T_L^{[f]}$ with symmetries $[f]=[3]$ and $[21]$. This creates a very fruitful situation which has not been discussed previously. Specifically, there appears coupling with the inelastic channels of the spin-isospin flip $p+d\to p+d_s$, the $d_s$-singlet deuteron with $S=0$ and $T=1$, and of the charge exchange $p+d\to pp+n$. The amplitudes of these inelasticities, as will be seen, are proportional to $T_L^{[3]}-T_L^{[21]}$. For not too small energies $E\geq 5$ MeV (hereafter $E$ is the energy in the c.m. system), the elastic nonunitary amplitude $T_{L,1/2}$ for the lowest partial waves $L$ may be small because of destructive interference of the potential amplitudes $T_L^{[3]}$ and $T_L^{[21]}$. As a result, the use of the law of the composition of phase shifts for elastic doublet scattering, which could make it possible to determine the phase shifts $\delta_L^{[3]}$ from the experimental phase shifts $\delta_{L,1/2}$ and $\delta_{L,3/2}\equiv\delta_L^{[21]}$, needs some caution in this region. Nevertheless, the potential $V_L^{[3]}$, with even $L$, is determined sufficiently reliably from the photonuclear data (disintegration of $^3$He and radiative capture $p+d\to{}^3\mathrm{He}+\gamma$), as well as from the known binding energy of the $^3$He nucleus and from the momentum distribution of nucleons in it. The potential $V_{L\,\mathrm{odd}}^{[31]}$ is determined more or less reasonably from the differential cross sections of the spin-isospin flip reaction $p+d\to p+d_s$. From this standpoint the $d+d$ system has been considered preliminary [17]; however, it is characterized by a fairly high inelasticity threshold $d+d\to d_s+d_s$ equal to 4.46 MeV. At the same time, our treatment implies that the inelasticity is "soft"; that is, the scattering energy $E$ significantly exceeds the above threshold. By virtue of this, the singlet channel of the $d+d$ system may be considered only at energies of $E\geq 10$ MeV. Unfortunately, the only attempt to use the phase-shift analysis [27] covers a narrow energy interval $E=4$–$6$ MeV, and there are no data whatever on inelastic scattering and charge exchange. In this respect the $p+d$ system has a considerable advantage: The phase-shift analysis for it has been made in a broad energy region $E=0$–$30$ MeV (see Ref. [26]), the inelasticity threshold $p+d\to p+d_s$ is equal to 2.23 MeV, and the differential deuteron breakup cross sections $p+d\to np+p$ have been measured [28].

It would be of interest to discuss in general terms the possibility of the potential-model description of the interaction between light composite particles in systems such as $p+d$, $d+d$, $d+^3$He, and others. From the general principles of the many-body problem (the Faddeev-Yakubovsky formalism for the $p+d$ or $d+d$ system [29]), it follows that, generally speaking, the interaction should be nonlocal and dependent on energy and orbital angular momentum $L$. However, there are no theoretical estimates as yet which would allow one to understand just how important this nonlocality is. At the same time the reconstruction of the potentials shows that the $L$ and $E$ dependences of potentials are weak, and consequently, the potential is quasilocal. Recently, Tomio *et al.* [10] have undertaken an attempt to give a potential-model description of $p+d$ scattering in the low-energy region $E\leq 2$ MeV according to the conventional viewpoint, implying asymptotic behavior of the potential in the region of large distance, explanation of the Phillips plot, etc. A similar attempt for $n+d$ scattering has recently been undertaken by Petrov [11] who used the results of theoretical calculations by Faddeev equations with separable potentials. These above-mentioned authors have demonstrated that for both the $p+d$ and $n+d$ systems it becomes possible only in a very limited energy region $E\leq 3$ MeV and for the lowest partial waves to obtain effective local potentials which not only reproduce well theoretical phase shifts at low energies, but also describe rather adequately the other properties of interaction, in particular, the vertex constant of the virtual decay $^3$H$\to n+d$ of the $^3$H ground state. In our treatment of the $p+d$ system, quartet scattering is considered at all energies, while the description of doublet scattering is valid starting from energies a few times above the threshold. However, elastic scattering is considered jointly with inelastic scattering (and with the photodisintegration process $^3\mathrm{He}\,\gamma\to p+d$) in a wide energy range which is bounded above only by the general conditions for applicability of the potential-model description.

Note that the $p+d$ system is an ideal subject for using the Faddeev formalism [29], which enables one to describe both elastic and inelastic scattering, including the three-particle breakup [30]. And this circumstance

makes it possible to investigate in detail the correspondence between the accurate many-particle and effective potential-model descriptions. In this way, in particular, the notion of forbidden states is well resolved [5], which is important for our approach. At the same time the solution of Faddeev equations is extremely difficult to find for systems with the Coulomb interaction. Our generalized potential-model description enables one to elucidate in a simple manner the physics of the processes under consideration and to give a quantitatively correct description of both elastic and inelastic scattering (excluding the truly three-particle breakup) and of photonuclear processes.

The outlined scheme is suitable for describing the mutual scattering of other pairs of lightest nuclei such as $p+{}^6\mathrm{Li}$, $d+{}^6\mathrm{Li}$, $d+{}^7\mathrm{Li}$, etc.

## III. SUPERMULTIPLET DESCRIPTION OF SCATTERING

Consider the mutual scattering of two nuclei $A$ and $B$, which are described by the wave functions $|\alpha[\tilde{f}_A]S_A\sigma_A t_A\tau_A\rangle$ and $|\beta[\tilde{f}_B]S_B\sigma_B t_B\tau_B\rangle$, respectively, where $S_i$, $t_i$, and $[\tilde{f}_i]$ are, respectively, spin, isospin, and spin-isospin Young scheme of the nucleus $i$, and $\alpha$ and $\beta$ characterize spatial parts of the wave functions. In view of the small splitting of phase shifts with respect to the total angular momentum $J$ for the $p+d$ and $d+{}^3\mathrm{He}$ systems, we use the supermultiplet scheme of calculation [22,23,31]. In the supermultiplet notation [20,32], the partial scattering amplitude in the $A+B$ system is written as

$$T_L=\sum_{s,\sigma,t,\tau,[f]}(t_A\tau_A,t_B\tau_B|t\tau)(S_A\sigma_A,S_B\sigma_B|S\sigma)\langle[\tilde{f}_A]S_At_A,[\tilde{f}_B]S_Bt_B|[\tilde{f}]St\rangle T_L^{[f]}(\alpha,\beta)$$
$$\times\langle[\tilde{f}]St|[\tilde{f}_A]S'_At'_A,[\tilde{f}_B]S'_Bt'_B\rangle(S\sigma|S'_A\sigma'_A,S'_B\sigma'_B)(t\tau|t'_A\tau'_A,t'_Bt'_B)\ . \tag{1}$$

where $\langle[\tilde{f}_A]S_At_A,[\tilde{f}_B]S_Bt_B|[\tilde{f}]St\rangle$ are the isoscalar factors of the SU(4) group [29], and $S$, $t$, and $[\tilde{f}]$ are spin, isospin, and spin-isospin Young scheme of the compound $A+B$ system. The partial amplitudes $T_L$ occur in the expansion of the total scattering $f(\theta)$ in terms of orbital angular momenta $L$:

$$f(\theta)=-\frac{i}{2p_0}\sum_L(2L+1)T_LP_L(\cos\theta)\ ,$$

where $p_0$ is the momentum of the relative motion of $A$ and $B$ particles in their c.m. system.

The cross section, averaged over the initial orientations of spins and summed over the final ones, is of the form

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{1}{(2S_A+1)(2S_B+1)}\sum_{\sigma_A,\sigma_B,\sigma'_A,\sigma'_B}|f(\theta)|^2\ , \tag{2a}$$

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{1}{4p_0^2}\sum_{l=0}^{\infty}B_lP_l(\cos\theta)\ , \tag{2b}$$

$$B_l=\frac{1}{(2S_A+1)(2S_B+1)}\sum_{L=0}^{\infty}\sum_{L'=|L-l|}^{L+l}(2L+1)(2L'+1)(L0,L'0|l0)^2\sum_{\substack{[f],t,\tau\\[f'],t',\tau'}}(t_A\tau_A,t_B\tau_B|t\tau)(t'_A\tau'_A,t'_Bt'_B|t\tau)$$
$$\times(t_A\tau_A,t_B\tau_B|t'\tau')(t'_A\tau'_A,t'_Bt'_B|t'\tau')\sum_S(2S+1)T^{[f]}_{L,t,s}(\alpha,\beta)[T^{[f']}_{L',t',s}(\alpha,\beta)]^*$$
$$\times\langle[\tilde{f}_A]S_At_A,[\tilde{f}_B]S_Bt_B|[\tilde{f}]St\rangle^2\langle[f_A]S'_At'_A,[\tilde{f}_B]S'_Bt'_B|[\tilde{f}']St'\rangle^2\ . \tag{2c}$$

Considering the values of the isoscalar factors of the SU(4) group [33], outlined in Appendix A, we find that in the doublet ($S=\frac{1}{2}$) channels of the $d+p$ and $d+{}^3\mathrm{He}$ systems the partial scattering amplitudes are determined by the interference of potential amplitudes,

$$T_L^{[f]}\equiv T_L^{[f]}(\alpha,\beta)=\exp(2i\delta_L^{[f]})-1\ , \tag{3}$$

with two different permutation symmetries $[f_1]$ and $[f_2]$,

$$T_{L,1/2}=\langle[\tilde{f}_A]S_At_A,[\tilde{f}_B]S_Bt_B|[\tilde{f}_1]\tfrac{1}{2}\tfrac{1}{2}\rangle^2T_L^{[f_1]}$$
$$+\langle[\tilde{f}_A]S_At_A,[\tilde{f}_B]S_Bt_B|[\tilde{f}_2]\tfrac{1}{2}\tfrac{1}{2}\rangle^2T_L^{[f_2]}\ , \tag{4}$$

whereas the quartet channels ($S=\frac{3}{2}$) are unitary and are described directly by the potential amplitudes $T_L^{[f_1]}$. Here $[f_1]=[21]$, $[f_2]=[3]$ for $p+d$ and $[f_1]=[32]$, $[f_2]=[41]$ for $d+{}^3\mathrm{He}$. Note that Eq. (4) is valid even in the presence of absorption in the channels with fixed $[f]$.

In such a case the phase shifts $\delta_L^{[f]}$ in (3) are complex. As will be seen below, the account of such inelasticities is important for the quantitatively correct description of mutual $p+d$ scattering. At this point we suggest, for the sake of simplicity, that the phase shifts $\delta_L^{[f]}$ are real.

Turning to the partial $S$ matrix for elastic scattering, we have

$$S_{LS}^{\text{elastic}}=\eta_{LS}\exp(2i\delta_{LS})\equiv T_{LS}+1 , \tag{5}$$

$$\delta_{L,1/2}=\tfrac{1}{2}\delta_L^{[f_1]}+\tfrac{1}{2}\delta_L^{[f_2]} , \tag{6}$$

$$\delta_{L,3/2}=\delta_L^{[f_1]} , \tag{7}$$

$$\eta_{L,1/2}=|\cos(\delta_L^{[f_1]}-\delta_L^{[f_2]})| , \tag{8}$$

$$\eta_{L,3/2}=1 . \tag{9}$$

The nonunitarity of the scattering amplitudes in the doublet channels (8) is due to the inclusion of spin-isospin flip $p+d\to p+d_s$, $d+{}^3\text{He}\to d_s+{}^3\text{He}$ and charge exchange $p+d\to pp+n$, $d+{}^3\text{He}\to pp+{}^3\text{H}$:

$$\begin{aligned} S_{L,1/2}^{\text{flip}}&=(\tfrac{1}{2}\tfrac{1}{2},10|\tfrac{1}{2}\tfrac{1}{2})(\tfrac{1}{2}T_L^{[f_1]}-\tfrac{1}{2}T_L^{[f_2]}) ,\\ S_{L,1/2}^{\text{ch exc}}&=(\tfrac{1}{2}\tfrac{1}{2},1-1|\tfrac{1}{2}\tfrac{1}{2})(\tfrac{1}{2}T_L^{[f_1]}-\tfrac{1}{2}T_L^{[f_2]}) \end{aligned} \tag{10}$$

[the coefficients are the Clebsch-Gordan coefficients of the SU(2) group]. The full $S$ matrix is unitary:

$$\begin{aligned} &|S_{L,1/2}^{\text{elastic}}|^2+|S_{L,1/2}^{\text{flip}}|^2+|S_{L,1/2}^{\text{ch exc}}|^2=1 ,\\ &|S_{L,3/2}^{\text{elastic}}|^2=1 . \end{aligned} \tag{11}$$

Thus we obtain the natural generalization of the known approach of Lane [34] to optical scattering where the isospin term $(\mathbf{T}\times\boldsymbol{\tau})$ connected the charge-exchange cross section and corresponding contribution to the elastic-scattering amplitude. On this basis it is possible to predict the cross sections for charge exchange or spin-isospin flip processes:

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\theta)=&\frac{1}{4p_0^2}\frac{(2S'_A+1)(2S'_B+1)}{(2S_A+1)(2S_B+1)}(t'_A\tau'_A,t'_B\tau'_B|t\tau)^2\\ &\times\Big|\sum_L(2L+1)P_L(\cos\theta)(\langle[\tilde f_A]S_At_A,[\tilde f_B]S_Bt_B|[\tilde f_1]St\rangle\langle[\tilde f\,'_A]S'_At'_A,[\tilde f\,'_B]S'_Bt'_B|[\tilde f_1]St\rangle T_L^{[f_1]}\\ &+\langle[\tilde f_A]S_At_A,[\tilde f_B]S_Bt_B|[\tilde f_2]St\rangle\langle[\tilde f\,'_A]S'_At'_A,[\tilde f\,'_B]S'_Bt'_B|[\tilde f_2]St\rangle T_L^{[f_2]})|^2 , \end{aligned} \tag{12}$$

where $S_B=1$, $S'_B=0$, $t'_B=1$, $[\tilde f_B]=[\tilde 2]$, $t_B=0$, $[\tilde f\,'_B]=[\tilde 2]$; $S_A=\frac{1}{2}$, $S'_A=\frac{1}{2}$, $t_A=\frac{1}{2}$, $t'_A=\frac{1}{2}$, $[\tilde f_A]=[\tilde f\,'_A]=[\tilde 1]$, $t=\frac{1}{2}$, $\tau=\frac{1}{2}$, $S=\frac{1}{2}$; $\tau'_A=\frac{1}{2}$, $\tau'_B=0$ for the deuteron spin-isospin flip without charge exchange $p+d\to p+d_s$ and $d+{}^3\text{He}\to d_s+{}^3\text{He}$, and $\tau'_A=\frac{1}{2}$, $\tau'_B=1$ in the case of charge exchange $p+d\to pp+n$ and $d+{}^3\text{He}\to pp+{}^3\text{H}$.

## IV. RECONSTRUCTION OF THE POTENTIALS

Here we discuss the problem of reconstruction of the potentials which correspond to various permissible space permutational symmetries in the *pd* and *dh* systems.

### A. Analysis of *pd* scattering

For this system there exist extensive experimental data on the scattering phase shifts $\delta_{LS}$ and the reflection coefficients (inelasticity parameters) $\eta_{LS}$ in a broad energy range (up to 30 MeV) and for a large set of partial waves $L=0$–8 (see Ref. [26] and references therein). On the other hand, the properties of the *pd* system are well described in calculations based on the use of Fadeev equations [35]. The binding energy of the ${}^3$He nucleus [35], the phase shifts [35], the *pd* elastic scattering cross sections [36], and the differential $pd\to{}^3\text{He}\,\gamma$ cross sections [37] are well reproduced with the aid of the nucleon-nucleon separable potentials. We do not intend, of course, to compete with the three-body theory, but demonstrate the efficiency of a much more simple potential-model description which interconnects the elastic and inelastic scattering with the deuteron spin-isospin flip into the singlet state (which has not so far been considered in the Faddeev approach), as well as the above-mentioned photonuclear reaction. In this connection the problem of obtaining the effective potentials of $p+d$ and $d+{}^3$He interaction is possibly one of the most significant applications of three-body theory [38].

In analyzing the experimental data, we revealed substantial inelasticity of the doublet channels ($\eta_{L,1/2}\le 0.5$ at $E\ge 7$ MeV) and unitarity of the quartet channels ($\eta_{L,3/2}\cong 1$ throughout the energy range), which corresponds well to our approach (8) and (9). Furthermore, of interest is the behavior of $S$- and $D$-phase shifts in the quartet channels: The $D$-phase shift is negative at small energies ($\approx -10°$) and next it changes sign, and the $S$-phase shift is characterized by an abrupt decrease as the energy increases. This indicates the presence of the peripheral repulsion which is connected with nucleon exchange (the influences of this exchange is well studied in the reactions with heavy ions [25]). The corresponding potentials are constructed as

$$V_L^{[f]}(R)=V_1\exp(-\alpha_1R^2)+V_2\exp(-\alpha_2R) , \tag{13}$$

where the exponential addition models the repulsion in the periphery. The conclusion that the peripheral repulsion need be allowed for in certain $n+{}^4$He scattering channels has been made in Ref. [39] too. In our view the

peripheral repulsion in the $n+{}^4$He system is far less than those in some channels of the $p+d$ system. In constructing the potentials we allow for their splitting with respect to parity of the orbital momentum $L$ (see Ref. [5]).

When we reconstruct the potential $V_L^{[21]}$ with $L$ even, we proceed from the fact that it should have, by the Pauli exclusion principle, a forbidden $S$ state since the symmetry $[f]=[21]$ is realized starting with the configuration $0s(1p)^2$. In the fractional parentage expansion of this three-nucleon configuration into the product of the deuteron $(0s)^2$ state by the $\chi(R)$ function of the $p$-$d$ relative motion, the latter contains, owing to conservation of the number of oscillator quanta, the dominating nodal $2S$ component so that the nodeless $0S$ component turns out to be forbidden by the Pauli principle. In the potential-model description of the $N$-$d$ interaction, this nodeless $0S$ state manifests itself as a deeply lying eigenstate of the $N$-$d$ Hamiltonian so that the continuum wave function $\chi(k,R)$ in this channel will be orthogonal to the wave function of this nodeless state (see also Ref. [38]). In other words, the forbidden state shows itself as a node of the $S$-wave function which was previously perceived as the repulsive core [5]. A very interesting circumstance is that in the rigorous three-particle theory of scattering it becomes possible [6,40] to give an interpretation of these forbidden states in the form of so-called ghost states.

Table I lists the parameters of the potential $V_L^{[21]}$ (with $L$ even) which was reconstructed from the experimental quartet phase shifts taken from [26] [the parameters are given in the form of (13)]. Figure 1 shows the calculated scattering phase shifts for the potential $V_L^{[21]}$, $L$ even, with allowance for the Coulomb interaction in the $p+d$ system.

The potential $V_L^{[21]}$ with $L$ odd evidently has no forbidden states. The phase-shift behavior [26] permits the potential to be chosen as attractive in the Gaussian form. Figure 1 shows the phase shifts for this potential with allowance for the Coulomb interaction. All the potentials $V_L^{[21]}$ found here resemble qualitatively the quartet potentials of Ref. [38] produced within the schematical three-body approach.

In going over to nonunitary doublet channels, we see that the potentials $V_L^{[3]}$ cannot be reconstructed directly from the experimental phase shifts since in the $N$-$d$ scattering experiment the states with pure symmetry, $[f]=[3]$, are not realized. Using, however, the supermultiplet approximation (6) and identifying the phase shifts $\delta_L^{[21]}$ with the quartet ones, we can, at energies $E$ a few times above the threshold, separate out "pure" phase shifts $\delta_L^{[3]}$, i.e., the ones which should be used for the reconstruction of the potentials $V_L^{[3]}$. Besides, we should

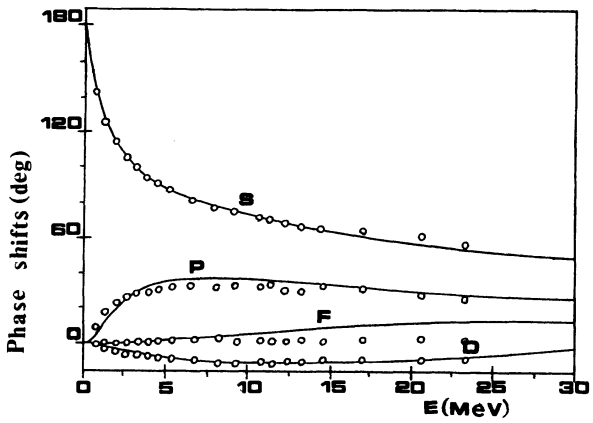


FIG. 1. Phase shifts for the potentials $V_L^{[21]}$ (with allowance for the Coulomb interaction). The points show the experimental phase-shift values, taken from Ref. [26].

make allowance for the inelasticity of the doublet channels for the lowest partial waves $L \leq 2$, caused by the remarkable influence of the three-body breakup $p+d \rightarrow p+p+n$, comparable in magnitude with the inelasticities due to deuteron spin-isospin flip in the singlet spin state, and the charge exchange. The influence of this three-body breakup can be allowed for by introduction of an imaginary part $W_I \exp(-\alpha_I R^2)$ into the interaction potentials $V_L^{[3]}$ for $L$ even and odd. Because of the increased number of parameters in the reconstructed potentials, it is necessary to use, in addition, other independent data, in order to determine unambiguously the potentials $V_L^{[3]}$. For example, in the reconstruction of the potential $V_L^{[3]}$ for $L$ even, we used the momentum distribution of nucleons in the $^3$He nucleus [41] and also the differential cross sections for photodisintegration of the $^3$He nucleus (see Sec. V). The potential $V_L^{[3]}$ for $L$ odd does not prove to be sensitive to photonuclear data, and therefore, the reconstruction of this potential should be directly based on the inelastic cross sections for the deuteron spin-isospin flip into the singlet state [25]. To reconstruct the potential $V_L^{[3]}$ with $L$ even, one should take into account that the $S$ state in this channel is the ground state of the $^3$He nucleus as the bound $p+d$ system with binding energy $E \simeq 5.5$ MeV and the well-known momentum distribution. The parameters of the potentials $V_L^{[3]}$ are listed in Table I and the corresponding phase shifts are shown in Fig. 2 (with allowance for the Coulomb interaction). The calculated phase shifts show reasonable stability. For instance, the 5–10% variation of the potential parameters produces only 2°–5° change of $S$- and $P$-phase shifts. Figure 3 presents the reflection

TABLE I. $p+d$ interaction potentials.

| No. | $[f]$ | $L$ | $V_1$ (MeV) | $\alpha_1$ (fm$^{-2}$) | $V_2$ (MeV) | $\alpha_2$ (fm$^{-1}$) | $W_I$ (MeV) | $\alpha_I$ (fm$^{-2}$) | Power |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 21 | Even | −57.0 | 0.37 | 7.2 | 0.36 | | | −53 |
| 2 | 21 | Odd | −8.8 | 0.06 | | | | | −73 |
| 3 | 3 | Even | −55.8 | 0.31 | | | −17.0 | 0.43 | −90 |
| 4 | 3 | Odd | −13.8 | 0.16 | 1.6 | 0.09 | −5.24 | 0.33 | −30 |

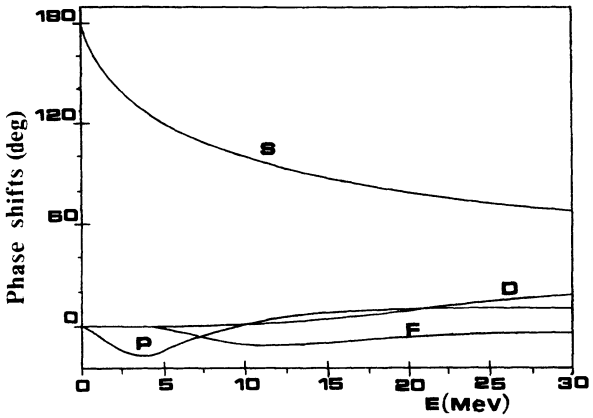


FIG. 2. Phase shifts for the potentials $V_L^{[3]}$ (with allowance for the Coulomb interaction).

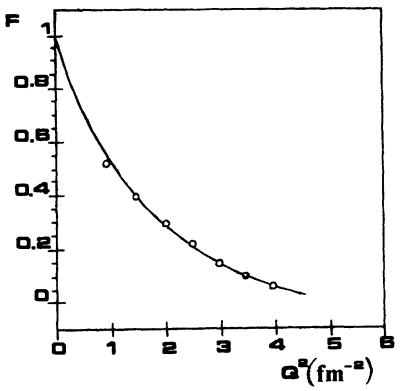


FIG. 4. Form factor of the $^3$He nucleus as a $p+d$ system. The experimental values, marked by the points, are taken from Ref. [41].

parameters for the lowest partial waves $L=0,1,2$ that were calculated by expression (8). As is seen from Fig. 3, nonunitarity of the elastic channel $S=\frac{1}{2}$ is the most substantial in the lowest partial waves $L=0,1$. Figure 4 shows the form factor of the $^3$He nucleus as the $p+d$ system. The experimental values are taken from Ref. [41].

Note that the phase shifts of the potential $V_L^{[3]}$ ($L$ even), which describes well, simultaneously, the binding energy of $^3$He, the $p+d$ momentum distribution in this nucleus, the inelastic-scattering cross section for the deuteron spin-isospin flip into the singlet state, and also the photonuclear processes $^3\mathrm{He}\,\gamma \rightarrow pd \rightarrow {}^3\mathrm{He}\,\gamma$, represent qualitatively the behavior of the phase shifts that were obtained by expression (6) from the experimental doublet and quartet phases [26]. The phase shifts of the potential $V_L^{[3]}$ for odd $L$ behave analogously.

The differential cross sections for elastic scattering of a proton on deuteron [Eq. (2)] for four scattering energies $E=10.83$, 11.4, 12.35, and 13.28 MeV are shown in Fig. 5 as compared with the experimental data [36]. Figure 6 illustrates the results of our calculations of the differential cross sections for the deuteron spin-isospin flip into the singlet spin state at energies $E=10.83$, 11.4, and 13.28 MeV. The normalization of the density function of the final states of the deuteron is here the same as in Ref. [28]. Note that the experimental data are available in the angular range which does not cover the characteristic dip in the cross section in the range of $\theta \approx 120°$. It would be highly desirable to carry out more complete measurements in this region.

Thus we have obtained the interaction potentials and corresponding $p+d$ scattering amplitudes in all allowed channels with $S=\frac{1}{2}$ and $\frac{3}{2}$ and $[f]=[3]$ and [21]. The reliability and effectiveness of the obtained $dp$ interaction potentials will be studied in Sec. V as applied to the $^3$He photodisintegration.

### B. $^3$He+$d$ interaction

Here we proceed to analyze the $d+{}^3$He system. In view of the insufficient accuracy of the experimental data on the elastic-scattering cross section and polarizations, the phase-shift analysis for this system was made only in the low-energy region of $E \leq 3.4$ MeV and the results turned out to be ambiguous [42]. When we reconstruct the interaction potentials in unitary quartet channels

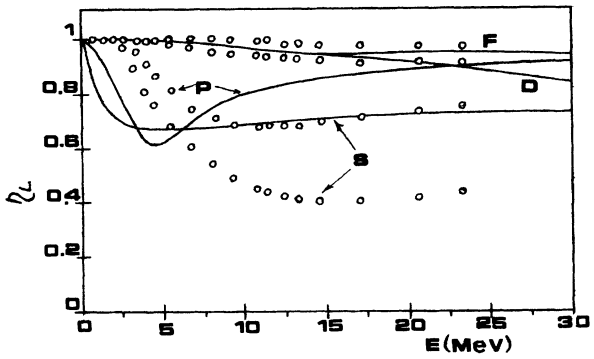


FIG. 3. Inelasticity parameters $\eta_L$ in the doublet channels of the $p+d$ system, calculated by (8).

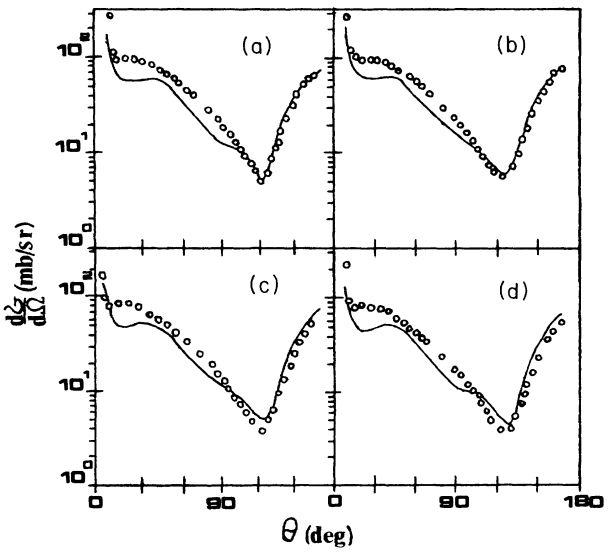


FIG. 5. Differential cross sections for elastic $p+d$ scattering at (a) $E=10.83$, (b) 11.41 MeV, (c) 12.35 MeV, and (d) 13.28 MeV. The experimental points are taken from Ref. [36].

($S=\frac{3}{2}$), we rely, to a significant extent, on the phase shifts obtained in Ref. [42]. In nonunitary doublet channels ($S=\frac{1}{2}$), however, our description is applicable in the region of energies a few times above the $d+{}^3\mathrm{He}\to d_s+{}^3\mathrm{He}$ inelasticity threshold, and in virtue of this, we have to base our analysis directly on the experimental data for the differential elastic-scattering cross sections [43] and also for the phase shifts calculated with the resonating-group method in Refs. [44,45].

In constructing the potential $V_L^{[32]}$ with $L$ even, we proceed from the fact that it should have by the Pauli principle, a forbidden $S$ state since five nucleon states with the Young scheme $[f]=[32]$ are realized starting with the configuration $s^3p^2$. The lowest observable positive-parity state, which has the symmetry [32], is the known resonance $J^\pi=\frac{3}{2}^+$ ($E=0.26$ MeV) in the $d+{}^3\mathrm{He}$ system [44]. This resonance has recently attracted general interest in connection with the problem of thermonuclear synthesis [46]. In our formalism we describe correctly the position and reduced width of the $\frac{3}{2}^+$ resonance and, moreover, obtain its wave function. The parameters of this wave function, which is represented as an expansion in 15 Gaussians,

$$\phi(R)=\sum_i c_i\exp(-a_iR^2),\quad 0\le R\le 10\text{ fm},$$

are listed in Table II.

Using as an initial approximation the quartet phase shifts calculated in the RGM [44,45], we next determine more accurately the potentials $V_L^{[32]}$ directly from the differential cross sections for elastic scattering $d+{}^3\mathrm{He}$ (see Ref. [43]). The resulting scattering phase shifts for the channels $S=\frac{3}{2}$ with $L$ even and $[f]=[32]$ are shown in Fig. 7, and the potential itself is given in Table I. The energy of the forbidden states of this potential is equal to $-4.61$ MeV. For the potential $V_L^{[32]}$ with $L$ odd, we allow for the presence of the forbidden $P$ state (lowest allowed configuration, $s^2p^3$). The parameters of the potential $V_L^{[32]}$ for $L$ odd are given in Table I, and the corresponding phase shifts $\delta_L^{[32]}$, $L$ odd, are shown in Fig. 7.

TABLE II. Wave function of the $J^\pi=\frac{3}{2}^+$ resonance in $d+{}^3\mathrm{He}$ system which is represented as an expansion in 15 Gaussians $\chi(r)=\sum_{i=1}^{15}c_i\exp(-a_ir^2)$, $0\le r\le 10$ fm.

| No. | $a_i$ (fm$^{-2}$) | $c_i$ (fm$^{-3/2}$) |
|---|---|---|
| 1 | 0.003 417 | −2.301 717 |
| 2 | 0.017 955 | 4.177 554 |
| 3 | 0.039 508 | −8.331 693 |
| 4 | 0.067 745 | 15.640 023 |
| 5 | 0.103 600 | −26.106 954 |
| 6 | 0.149 069 | 36.436 111 |
| 7 | 0.207 570 | −38.517 836 |
| 8 | 0.284 846 | 36.156 042 |
| 9 | 0.390 891 | −24.335 301 |
| 10 | 0.544 294 | 11.987 791 |
| 11 | 0.783 179 | −5.376 892 |
| 12 | 1.197 688 | 1.268 938 |
| 13 | 2.053 673 | −0.443 513 |
| 14 | 4.518 988 | −0.091 352 |
| 15 | 23.741 958 | −0.121 243 |

We now turn to nonunitary doublet channels. Using expression (6), we reconstruct the phase shifts $\delta_L^{[41]}$ corresponding to the Young scheme $[f]=[41]$. In reconstructing the potentials $V_L^{[41]}$, we shall proceed from the fact that the potential $V_L^{[41]}$ with $L$ even possesses a forbidden $S$ state and the potential $V_L^{[41]}$ with $L$ odd—the observable bound $P$ state corresponding to the $^5$Li ground state $J^\pi=\frac{3}{2}^-$, which is bound in the $d+{}^3\mathrm{He}$ channel with the binding energy $E\approx 16$ MeV (see Ref. [47]). The $^5$Li state $J^\pi=\frac{3}{2}^-$ decays through the $p+{}^4\mathrm{He}$ channel with a

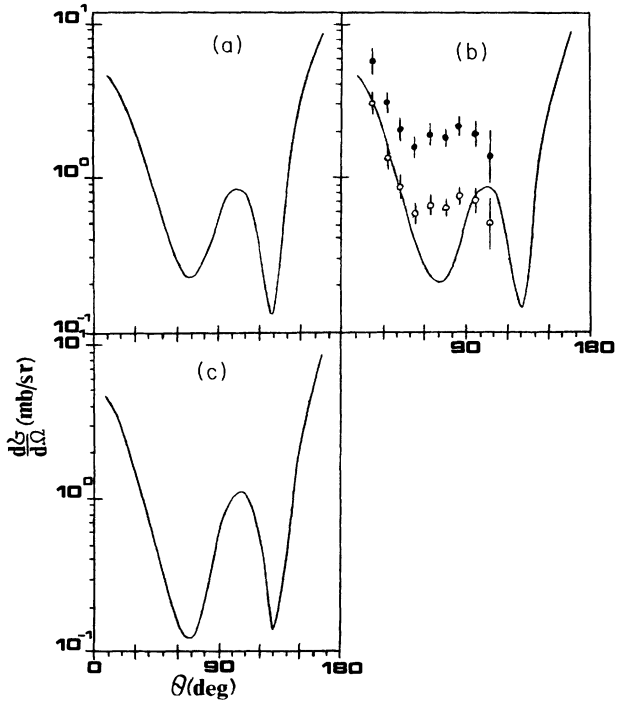


FIG. 6. Differential cross sections for the reaction $p+d\to p+d_s$ at (a) $E=10.83$, (b) 11.4 MeV and (c) 13.28 MeV. The points show the experimental values of the cross section for $E=11.4$ MeV (see Ref. [28]).

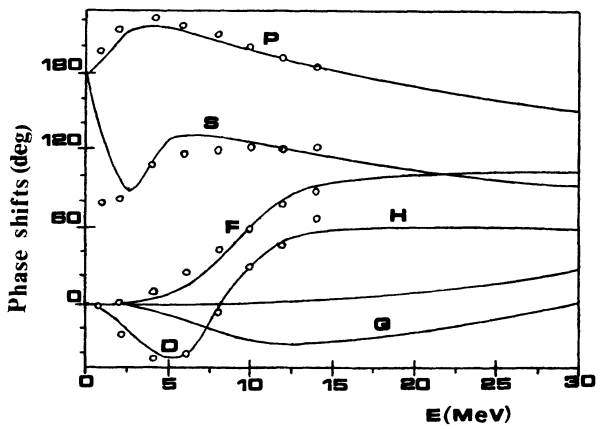


FIG. 7. Phase shifts for the potentials $V_L^{[32]}$ (with allowance for the Coulomb interaction). The points show the RGM phase shifts from Ref. [45].

large spectroscopic factor $S_p \simeq 1$ (see Ref. [24]). Note that we do not discuss the possible resonance $J^\pi = \frac{1}{2}^+$ since it is below the inelasticity threshold $d + {}^3\mathrm{He} \to d_s + {}^3\mathrm{He}$ in the doublet $S$ channel where our treatment is not valid. The energy of the forbidden $S$ state of the potential $V_L^{[41]}$ for $L$ even is equal to $E = -6$ MeV.

The parameters of the present potentials $V_L^{[41]}$ are listed in Table I, and the corresponding phase shifts are shown in Fig. 8 (with allowance for the Coulomb interaction). Here again the peripheral repulsion for $L$ odd plays the significant role. This accounts for the intricate behavior of the $S$-phase shifts and negative values of the $D$-phase shift at low energies, which changes sign as $E$ increases (see Fig. 8). By comparing the theoretical result with the phase shifts of $V_L^{[41]}$, inferred from the RGM calculations, we note that we have got $\eta_{L,1/2} \ll 1$ (Fig. 9) and that our microscopical approach seems in this case to be in advance of the RGM where the given property is reflected as a purely phenomenological strong absorption.

As one can see from Fig. 10, the phase shifts obtained in our approach lead to a good description of the differential elastic-scattering cross sections. This figure shows the differential cross sections for elastic scattering of deuterons on $^3$He for energies $E = 8.64$, 10, and 13.8 MeV and, also, the experimental data of Ref. [43]. Figure 11 presents our predictions for the inelastic cross sections for spin-isospin flip reaction $d + {}^3\mathrm{He} \to d_s + {}^3\mathrm{He}$, measured by means of the triton (or $pp$-pair) registration. Setting up this experiment would be very desirable. Above, we everywhere neglected the coupling of channels $d + {}^3\mathrm{He}$ and $p + {}^4\mathrm{He}$. For the quartet channels this assumption is justified and confirmed by the RGM calculations [44,45]. In doublet channels this coupling can be allowed for by the introduction of a small imaginary part into the corresponding potentials $V_L^{[f]}$, but in fact, it appears to be practically unnecessary; i.e., the deuteron spin-isospin flip exhausts the inelasticity here.

In going over to the potentials $V_L^{[f]}$ (Tables I and III), we note that the potentials, which correspond to different

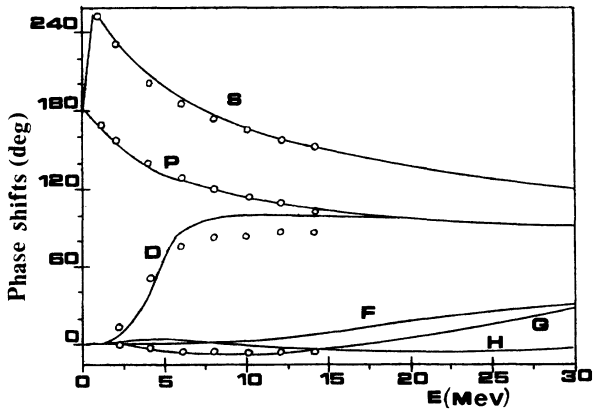


FIG. 8. Phase shifts for the potentials $V_L^{[41]}$ (with allowance for the Coulomb interaction). The points show the values of the "pure" phase shifts that were obtained according to (6) from the RGM phase shifts of Ref. [45].

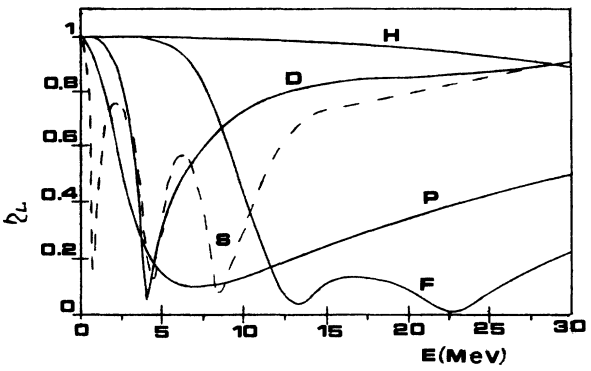


FIG. 9. Inelasticity parameters in the doublet channels of the $d + {}^3\mathrm{He}$ system.

space symmetries $[f]$, differ in power very remarkably. It is convenient to define this power using the following integral:

$$W_{L,t,s}^{[f]} = \int V_{L,t,s}^{[f]}(r) r \, dr \ .$$

This power decreases substantially in going from the states with a more symmetric Young scheme $[f]$ to the states with a less symmetric scheme, and as a result, the scattering amplitudes for different $[f]$ are very different. This effect is particularly strong in the case of the $^3$He$+d$ system where the symmetry $[f]=[32]$ vs $[41]$ is characterized by a broken "quarteting" ($\alpha$ clustering). That is

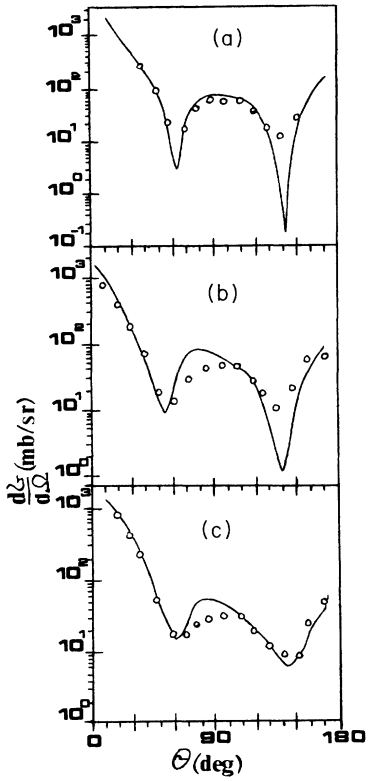


FIG. 10. Differential cross sections for elastic $d + {}^3\mathrm{He}$ scattering at (a) $E = 8.64$ MeV, (b) 10 MeV and (c) 13.84 MeV. The experimental values of the cross sections, marked by points, are taken from Ref. [43].

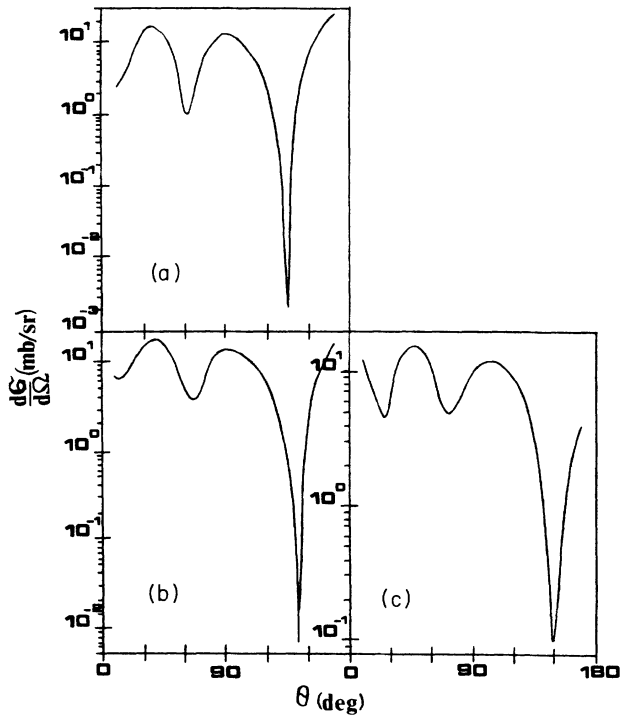


FIG. 11. Differential cross sections for the reaction $d+{}^3\mathrm{He}\rightarrow d_s+{}^3\mathrm{He}$ at (a) $E=8.644$ MeV, (b) 10 MeV, and (c) 13.84 MeV.

why the deuteron spin-isospin flip amplitude is so large here that it exhausts the inelasticity (see the above discussion of $d+p$ scattering). As a result, the ${}^3\mathrm{He}(d,\gamma){}^5\mathrm{Li}$ photonuclear cross sections can be calculated without any free parameters in the entire energy range mentioned in the Introduction; similar features should be also characteristic of the $d+d$ and $t+p$ systems (see Refs. [17,48] for some tentative estimates). All these circumstances justify the employment of the supermultiplet scheme despite its approximate character.

Our results clarify why the attempts to describe doublet channels in the $p+d$ system with a unique potential $V_{L,S=1/2}$ (see Refs. [10,11]) encountered severe difficulties.

## V. PHOTONUCLEAR PROCESSES IN THE $p+d$ AND $d+{}^3\mathrm{He}$ SYSTEMS

The photonuclear reactions in systems of lightest nuclei, starting from the paper by Gorbunov [49], has been the subject of interest to both experimentalists [50] and theorists [51,52]. The theoretical description is, as a rule, based on the use of composite many-particle wave functions for the ground states of nuclei [51] or is made in the framework of microscopic approaches of the RGM or $K$-harmonic type [53]. These approaches enable one to understand some characteristic features of the phenomenon, but at the same time possess strong limitations. For example, the approach based on the use of many-particle wave functions [51] fails to give a practical recipe for how to allow for the final-state interaction and, hence, cannot pretend to quantitative description. The microscopic calculations [53] do not include the coupling of the elastic-scattering channels and the inelastic ones [Eq. (10)] in the final state. This coupling, as was shown in Sec. IV, plays a significant role, defining the form of the corresponding potentials and wave functions. That is why we use the above-mentioned potential-model approach and consider the photonuclear processes in two aspects: (1) as a good test of the constructed interaction potentials, especially off shell, and (2) *per se*, i.e., trying to construct a quantitatively exact and at the same time simple formalism for describing photonuclear processes of given type.

Note that our approach is to some extent close to the approach used in Ref. [54] for studying ${}^6\mathrm{Li}$ electromagnetic form factors in the framework of the antisymmetrized multicluster dynamical model with Pauli projection. The main point of our approach, as opposed to the RGM, is "the antisymmetrization after variation" when the dynamical problem is first solved which does not include explicitly the antisymmetrization (but allowing for it through the effective potentials and orthogonality conditions to the forbidden states). Next, in calculating the electromagnetic transitions, the complete antisymmetrization of the initial- and/or final-state wave functions is carried out. This approach is much more simple and physically lucid.

The supermultiplet quantum number $[f]$ has long been known to be a rather efficient tool for describing the cluster decay properties of giant dipole resonance on the lightest $p$-shell nuclei [55,56].

### A. Two-particle photodisintegration of ${}^3\mathrm{He}$

We shall use here a standard formalism [57]. The differential cross section for two-particle photodisintegration of ${}^3\mathrm{He}$, which is averaged over the projections $M_J$ of the total nuclear momentum $J$, the photon polarizations $\lambda=\pm1$ and summed over the spin orientations of final particles $\sigma'_A$ and $\sigma'_B$ is of the form

TABLE III. The $d+{}^3\mathrm{He}$ interaction potentials.

| No. | $[f]$ | $L$ | $V_1$ (MeV) | $\alpha_1$ (fm$^{-2}$) | $V_2$ (MeV) | $\alpha_2$ (fm$^{-1}$) | Power |
|---|---|---|---|---|---|---|---|
| 1 | 32 | Even | −50.0 | 0.15 | | | −167 |
| 2 | 32 | Odd | −73.1 | 0.23 | 18.1 | 0.56 | −123 |
| 3 | 41 | Even | −57.0 | 0.16 | 8.4 | 0.21 | −138 |
| 4 | 41 | Odd | −69.0 | 0.14 | | | −246 |

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{\mu p_0}{4\pi k_\gamma}\frac{1}{2J+1}\sum_{\sigma'_A,\sigma'_B,\lambda,M_J}|\langle f|\hat{O}|i\rangle|^2 , \quad (14)$$

where $\mu$ is the reduced mass and $p_0$ is momentum of final particles in their c.m. system.

Among the possible multipole transitions which produce the photodisintegration of $^3$He, the electric dipole $E1$, electric quadrupole $E2$, and magnetic dipole $M1$ play the main role in the energy region $E_\gamma \le 30$ MeV (see, for example, Ref. [49]). The matrix elements of the corresponding one-particle operators

$$\hat{T}^{E1}_{1\lambda}=-\sqrt{(2/3)}k_\gamma\sum_{j=1}^{3}\hat{e}_j r_j Y_{1\lambda}(\Omega_{r_j}) , \quad (15)$$

$$\hat{T}^{E2}_{2\lambda}=-\frac{1}{5\sqrt{6}}k_\gamma^2\sum_{j=1}^{3}\hat{e}_j r_j^2 Y_{2\lambda}(\Omega_{r_j}) , \quad (16)$$

$$\hat{T}^{M1}_{1\lambda}=-\frac{1}{2\sqrt{6\pi}\mu}k_\gamma i\sum_{j=1}^{3}\hat{\mu}_j\sigma_{j\lambda} \quad (17)$$

are calculated in the long-wave approximation. Equations (15) and (16) are written in the form prescribed by the Siegert's theorem [58].

As the wave function of the ground state of $^3$He we use the cluster wave function of $p+d$ the system

$$|i\rangle=\{\varphi(23)\chi_{00}(R)Y_{00}(\Omega_R)\}^{[3]}|[\tilde{3}]\tfrac{1}{2}\sigma\tfrac{1}{2}\tfrac{1}{2}\rangle , \quad (18)$$

where the curly brackets mean the symmetrization of the spatial part of the wave function; $|[\tilde{f}]S\sigma t\tau\rangle$ is the corresponding spin-isospin function of the $^3$He nucleus. It appears to be sufficient to take the deuteron wave function $\varphi(23)$ in the simplest form [17]:

$$\varphi(23)=\left[\frac{\alpha}{\pi}\right]^{3/4}\exp(-\tfrac{1}{2}\alpha r_{23}^2), \quad \alpha=0.154\ \text{fm}^{-2} . \quad (19)$$

It is assumed that the deuteron is made up of nucleons with indices 2 and 3.

The relative motion function $\chi_{00}(R)$ is determined as a result of the variational numerical solution of the Schrödinger equation with the $V^{[3]}_{L\,\text{even}}$ potential. Furthermore, it is expanded in terms of the sum of Gaussians:

$$\chi_{00}(R)=\sum_{k=1}^{15}c_k\exp(-a_k R^2) . \quad (20)$$

Symmetrization in (18) is realized by means of Young projection operators in the nonstandard basis [32] corresponding to the reduction chain $U(3)\to U(1)\times U(2)$:

$$P^{[3]}_{[1][2];[1][2]}=\tfrac{1}{3}(E+P_{12}+P_{13}) , \quad (21)$$

where $P_{ij}$ is the permutation of nucleons with indices $i$ and $j$.

Note that as a result of antisymmetrization the normalization parameter of the wave function (18) is changed and this should be taken into account when calculating the cross section (14). A normalization parameter is easily calculated:

$$(N^{[3]})^2=\frac{1}{3}N_{\pi 0}^2+\frac{2^9}{9\pi}(\alpha\pi)^{3/2}\sum_{k,k'=1}^{15}c_k c_{k'}([10\alpha+9a_{k'}](2\alpha+a_{k'}+4a_k)-(3a_{k'}-2\alpha)^2]^{-3/2}$$
$$+\tfrac{1}{2}\{(4\alpha+a_{k'}+a_k)[4\alpha+9(a_k+a_{k'})]-9(a_{k'}-a_k)^2\}^{-3/2}) . \quad (22)$$

The final-state wave function has the asymptotic behavior which corresponds to an ingoing spherical wave [59]; in the partial-wave expansion

$$|f\rangle\equiv\psi^{(-)}_{\mathbf{p}_0}=4\pi\sum_{lm}i^l\exp(-i\delta_l)Y^*_{lm}(\Omega_{p_0})Y_{lm}(\Omega_R)\chi_{nl}(p_0R)\varphi(23)|[\tilde{1}]\tfrac{1}{2}\sigma'_A\tfrac{1}{2}\tfrac{1}{2}\rangle|[\tilde{2}]1\sigma'_B00\rangle . \quad (23)$$

This function is not antisymmetric with respect to all possible permutations of nucleons. We can antisymmetrize it in two stages. At first, we symmetrize the spatial part of the wave function (23) by virtue of Young projection operators (21) and

$$P^{[21]}_{[1][2];[1][2]}=\tfrac{1}{3}(2E-P_{12}-P_{13}) . \quad (24)$$

Second, we construct the spin-isospin part of the wave function (23) corresponding to the representation of the SU(4) group with total Young scheme $[\tilde{f}]$ ($[\tilde{f}]$) conjugated to $[f]$):

$$|[\tilde{1}]\tfrac{1}{2}\sigma'_A\tfrac{1}{2}\tfrac{1}{2}\rangle|[\tilde{2}]1\sigma'_B00\rangle=\sum_{s,\sigma,t,\tau,[f]}\langle[\tilde{1}]\tfrac{1}{2}\tfrac{1}{2},[\tilde{2}]10|[\tilde{f}]St\rangle$$
$$\times(\tfrac{1}{2}\sigma'_A,1\sigma'_B|S\sigma)(\tfrac{1}{2}\tfrac{1}{2},00|t\tau)|[\tilde{1}]\tfrac{1}{2}\sigma'_A\tfrac{1}{2}\tfrac{1}{2};[\tilde{2}]1\sigma'_B00:[\tilde{f}]S\sigma t\tau\rangle , \quad (25)$$

where the total isospin $t=\frac{1}{2}$, its projection $\tau=\frac{1}{2}$, and total spatial Young scheme $[f]=[21]$ or $[3]$.

Hence the total antisymmetric wave function of the $p+d$ system is written as

$$|f^{[f]}\rangle=\sum_{s,\sigma}\left[4\pi\sum_{lm}i^l\exp(-i\delta_l)Y^*_{lm}(\Omega_{p_0})Y_{lm}(\Omega_R)\chi_{nl}(p_0R)\varphi(23)\right]^{[f]}$$
$$\times\langle[\tilde{1}]\tfrac{1}{2}\tfrac{1}{2},[\tilde{2}]10|[\tilde{f}]S\tfrac{1}{2}\rangle(\tfrac{1}{2}\sigma'_A,1\sigma'_B|S\sigma)|[\tilde{1}]\tfrac{1}{2}\sigma'_A\tfrac{1}{2}\tfrac{1}{2};[\tilde{2}]1\sigma'_B00:[\tilde{f}]S\sigma\tfrac{1}{2}\tfrac{1}{2}\rangle . \quad (26)$$

The functions $\chi_{nl}(p_0R)$ are determined as a result of the accurate numerical solution of the Schrödinger equation with the potentials $V_L^{[f]}$ of Sec. IV. To carry out the antisymmetrization of the complete wave functions (26), the relative motion functions $\chi_{nl}(p_0R)$ are expanded in an analytic basis of the form

$$\chi_{nl}(R)=R^l\sum_{k=1}^{K}\xi_k(p_0)\exp[-\kappa_k(p_0)R^2]\ , \tag{27}$$

using a convenient net of scale coefficients $\xi_k$ and $\kappa_k$. In the series of our previous papers (see, for example, Ref. [60]), we used the Tchebycheff discretization net

$$\kappa_k(p_0)=\kappa_1(p_0)\left[\tan\left[\pi\frac{2k-1}{4K}\right]\right]^Q\ ,$$
$$k=1,2,\ldots,K\ , \tag{28}$$

which proved to be rather economical in the sense that a moderate number of basis Gaussoids is required to approximate exactly a very wide class of wave functions. If we approximate the functions on the finite interval (even large), then it does not matter which asymptotic character of the wave functions is considered. Here it is to be noted that in the calculation of the transitions with high multipolarity (staring from $E2$) the requirements to accuracy of the expansion (27) in the asymptotic region become more stringent. Therefore, to provide the acceptable accuracy of calculation of the matrix elements, it is necessary to take for the exact approximation a rather wide interval of $R$: $0\le R\le 20$ fm. Appendix B gives the expressions for the corresponding matrix elements of the operators (15)–(17).

Figure 12 shows the differential $^3\mathrm{He}\,\gamma\to p+d$ cross section at the ejection angle of proton $\theta=90°$ in the laboratory frame for proton energies $E_\gamma$ from the reaction threshold up to 30 MeV. The choice of the angle is conditioned by the available experimental data [37,49,61,62]. It is to be noted that the experimental data for the reaction under consideration consist of two very different groups (see Fig. 12). Our results lie between the experimental points of these groups.

Here we analyze the angular distributions. Using the detailed balance ratio [63]

$$\frac{d\sigma^{\text{capture}}}{d\Omega}(\theta)=\frac{2}{3}\left[\frac{k_\gamma}{k_p}\right]^2\frac{d\sigma^{\text{photodisint}}}{d\Omega}(\theta)\ , \tag{29}$$

we get the angular distributions for the $p+d\to{}^3\mathrm{He}\,\gamma$ reaction. Figure 13 shows the calculated results and experimental data [64] for proton energies $E_\gamma=12.1$ and 15.3 MeV. As one can see, the calculated results are in reasonable agreement with experiment.

The $E1$ transition is dominant in the energy region $E_\gamma\le 30$ MeV. The contribution of the $E2$ transition to the total cross section makes up, according to our estimates, 5–8 % and the transition itself shows up mainly through the interference with the $E1$ transition which leads to a certain asymmetry of the angular distributions. The $M1$ transition in our model is absent.

### B. Radiative capture $d\,{}^3\mathrm{He}\to{}^5\mathrm{Li}\,\gamma$

The calculation of the differential radiative capture cross section is in fact analogous to the calculation of the differential two-particle photodisintegration cross section.

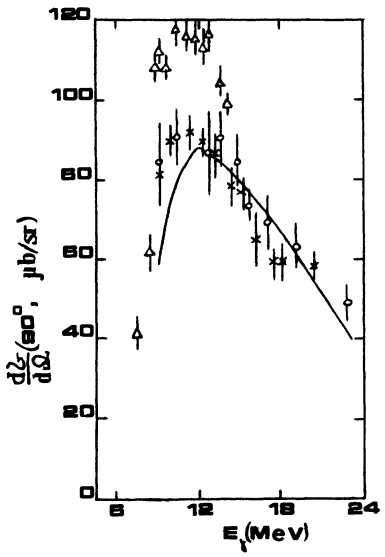


FIG. 12. Differential cross sections for the reaction $^3\mathrm{He}\,\gamma\to p+d$ at $\theta^{\text{lab}}=90°$. The experimental data △, ×, and ○ are taken from Refs. [37], [61], and [62], respectively.

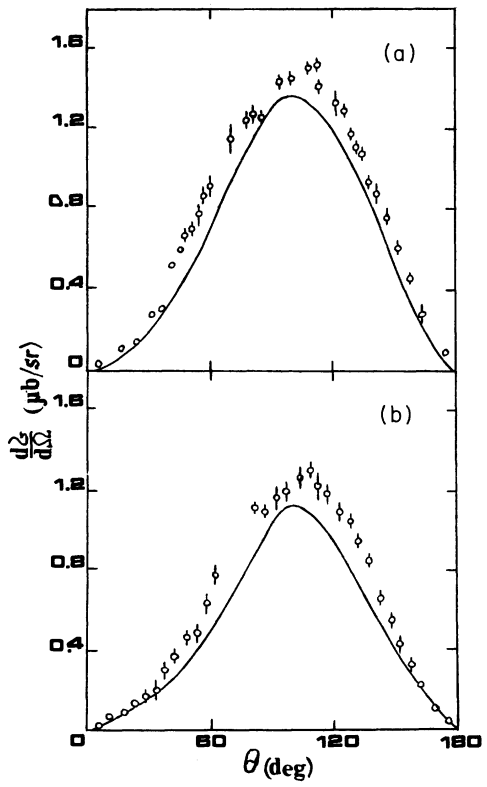


FIG. 13. Angular distributions of the $p+d\to{}^3\mathrm{He}\,\gamma$ reaction products at (a) $E_\gamma=12.1$ MeV and (b) 15.3 MeV. The points show the experimental data from Ref. [64].

Therefore, we shall make only brief comments on the initial- and final-state wave functions used in the calculation.

In order to prepare wave functions of the needed symmetry, we shall use, just as in the calculation of the two-particle photodisintegration of $^3$He, the projection Young operators in the nonstandard basis which corresponds to the reduction chain U(5)→U(3)×U(2):

$$P^{[32]}_{[3][2];[3][2]}=\tfrac{1}{12}(6E-2P_{14}-2P_{24}-2P_{34}-2P_{15}-2P_{25}-2P_{35}+P_{14}P_{25}+P_{24}P_{15}+P_{14}P_{35}+P_{34}P_{15}+P_{24}P_{35}+P_{34}P_{25}) , \tag{30}$$

$$P^{[41]}_{[3][2];[3][2]}=\tfrac{1}{15}(6E+P_{14}+P_{24}+P_{34}+P_{15}+P_{25}+P_{35}-2P_{14}P_{25}-2P_{24}P_{15}-2P_{14}P_{35}-2P_{34}P_{15}-2P_{24}P_{35}-2P_{34}P_{25}) . \tag{31}$$

Using these operators, we prepare the functions with the needed symmetry from the nonsymmetrized initial- and final-state wave functions:

$$|i\rangle\equiv\psi^{(+)}_{\mathbf{p}_0}=4\pi\sum_{lm} i^l\exp(i\delta_l)Y^*_{lm}(\Omega_{p_0})Y_{lm}(\Omega_R)\chi_{nl}(p_0R)\varphi_A(123)|[\tilde{3}]\tfrac{1}{2}\sigma_A\tfrac{1}{2}\tfrac{1}{2}\rangle\varphi_B(45)|[\tilde{2}]1\sigma_B00\rangle , \tag{32}$$

$$|f^{[41]}\rangle=P^{[41]}_{[3][2];[3][2]}\varphi_A(123)\varphi_B(45)\chi_{11}(R)Y_{1M}(\Omega_R)|[\tilde{4}1]\tfrac{1}{2}\sigma\tfrac{1}{2}\tfrac{1}{2}\rangle . \tag{33}$$

Here it appears again sufficient to use the simplest internal wave function of the $^3$He nucleus [65], but not the functions (18)–(20):

$$\varphi_A(123)=\left[\frac{2\beta^4}{\pi^2}\right]^{3/2}\exp[-\beta^2(\tfrac{3}{4}r_{12}^2+\rho^2)] ,$$
$$\beta=0.385\ \text{fm}^{-1},\quad \mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2 , \tag{34}$$
$$\boldsymbol{\rho}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)-\mathbf{r}_3 .$$

Figure 14 shows the angular distributions of the reaction $d+{}^3\text{He}\rightarrow{}^5\text{Li}\,\gamma$ for three excitation energies of $^5$Li: $E_x=E_\gamma+Q=19.9$, 20.7, and 23.2 MeV. The experimental data are taken from Ref. [47]. Note that the electric transitions involving changes of the space symmetry $[f]$ provide the main contribution to the cross section. The contribution of the transitions, which give an increase in the total momentum of the system $J$, is somewhat smaller in the low-energy region ($E_\gamma\leq 3.9$ MeV) as compared with the transitions which produce a decrease in $J$ [this situation is depicted in Fig. 14(a)]. The opposite is the case in the region of high energies [Figs. 14(b) and 14(c)]. Figures 14(d) and 14(e) present our predictions for $p+d\rightarrow{}^3\text{He}\,\gamma$ angular distributions at $E_x=24$ and 26 MeV, respectively. The electric quadrupole transition $E2$ is weak, and the resultant asymmetry of the angular distribution is small.

The energy dependence of the differential cross section for the reaction $d+{}^3\text{He}\rightarrow{}^5\text{Li}\,\gamma$ at $\theta=90°$ is shown in Fig. 15. The experimental data are taken from Ref. [47]. Just as in the case with the angular distributions, the agreement with experiment is reasonable too.

It is very easy now to calculate the differential cross section at any energy $E_\gamma$ below 30 MeV. We repeat that we have no free parameters here—potentials are defined entirely from the scattering data.

As a general conclusion, let us note two points.

First, the definite spatial symmetry $[f]$ of the total system has sense only if there is a multiplicate nucleon exchange between clusters. It means that the energy $E_{\text{c.m.}}$

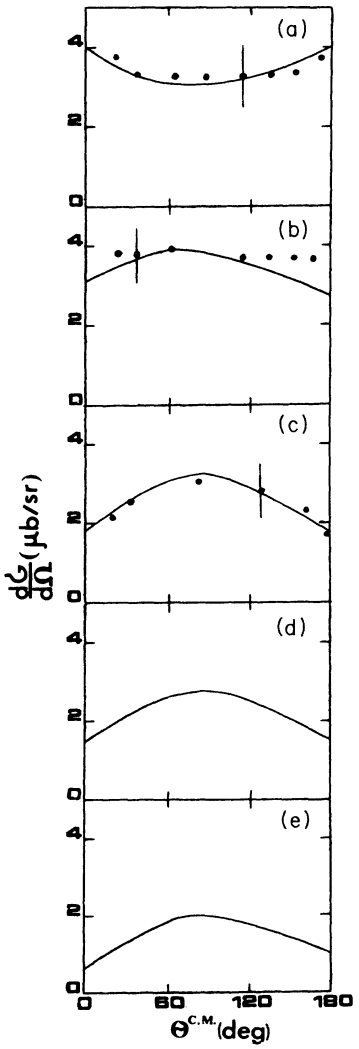


FIG. 14. Angular distributions of the $d+{}^3\text{He}\rightarrow{}^5\text{Li}\,\gamma$ reaction products at (a) $E_x=19.9$ MeV, (b) 20.7 MeV, (c) 23.2 MeV, (d) 24 MeV and (e) 26 MeV. The experimental data are taken from Ref. [47].

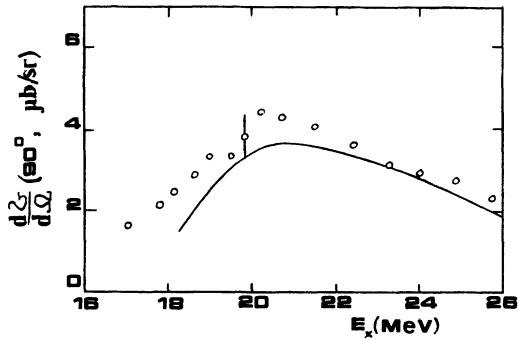


FIG. 15. Differential cross sections for the reaction $d+{}^3\mathrm{He}\rightarrow{}^5\mathrm{Li}\,\gamma$ at $\theta=90°$. The experimental data are taken from Ref. [47].

must be comparable to the value of $\hbar\omega_{\mathrm{osc}}$ (practically, probably, $E_{\mathrm{c.m.}}\leq 30$ MeV).

As a second remark, we note that our supermultiplet approach can be probably of interest also in the theory of baryon-baryon collisions at the intermediate energies, where the quark permutational symmetry quantum numbers $[f]_X$, $[f]_{\mathrm{CS}}$, etc., are widely used [66]. In particular, the "soft" spin-isospin flip process looks like $N+N\rightarrow\Delta+\Delta$ with the further possible generalization of isospin SU(2) symmetry to the flavor SU(3) [67] and so on.

## APPENDIX A

In the calculations of the elastic $p+d$ and $d+{}^3\mathrm{He}$ scattering, we used the following values of the spin-isospin fractional parentage coefficients [32,33]:

$$\langle[\tilde{2}]10,[\tilde{1}]_{\frac12\,\frac12}|[2\tilde{1}]_{\frac32\,\frac12}\rangle=1\ ,$$
$$\langle[\tilde{2}]10,[\tilde{1}]_{\frac12\,\frac12}|[\tilde{3}]_{\frac32\,\frac12}\rangle=0\ ,$$
$$\langle[\tilde{2}]10,[\tilde{1}]_{\frac12\,\frac12}|[2\tilde{1}]_{\frac12\,\frac12}\rangle=\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]10,[\tilde{1}]_{\frac12\,\frac12}[\tilde{3}]_{\frac12\,\frac12}\rangle=\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]10,[\tilde{3}]_{\frac12\,\frac12}|[3\tilde{2}]_{\frac32\,\frac12}\rangle=-1\ ,$$
$$\langle[\tilde{2}]10,[\tilde{3}]_{\frac12\,\frac12}|[4\tilde{1}]_{\frac32\,\frac12}\rangle=0\ ,$$
$$\langle[\tilde{2}]10,[\tilde{3}]_{\frac12\,\frac12}|[\tilde{3}2]_{\frac12\,\frac12}\rangle=\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]10,[\tilde{3}]_{\frac12\,\frac12}|[4\tilde{1}]_{\frac12\,\frac12}\rangle=-\sqrt{1/2}\ .$$

To calculate the differential cross sections for the deuteron spin-isospin flip and charge exchange, the following nonzero spin-isospin fractional parentage coefficients are needed [32,33]:

$$\langle[\tilde{2}]01,[\tilde{1}]_{\frac12\,\frac12}|[\tilde{3}]_{\frac12\,\frac12}\rangle=-\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]01,[\tilde{1}]_{\frac12\,\frac12}|[\tilde{2}1]_{\frac12\,\frac12}\rangle=\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]01,[\tilde{3}]_{\frac12\,\frac12}|[\tilde{3}2]_{\frac12\,\frac12}\rangle=-\sqrt{1/2}\ ,$$
$$\langle[\tilde{2}]01,[\tilde{3}]_{\frac12\,\frac12}|[4\tilde{1}]_{\frac12\,\frac12}\rangle=-\sqrt{1/2}\ .$$

## APPENDIX B

The matrix elements for the $E1$, $E2$, and $M1$ transitions in the photodisintegration of ${}^3\mathrm{He}$ are the following:

$$\langle f|\hat{T}^{E1}_{1\lambda}|i^{[3]}\rangle=\frac{2\pi}{9}\left[\frac{2}{3}\right]^{1/2}k_\gamma e\exp(i\delta_1^{[21]})Y_{1\lambda}(\Omega_{p_0})(\tfrac12\sigma'_A,1\sigma'_B|\tfrac12\sigma)$$
$$\times\sum_{k,k'}c_k^{[3]}\xi_{k'}^{[21]}\left[\mathfrak{X}_4^{[21]}+\frac{2\alpha^{3/2}}{\pi^{1/2}}\left[3\frac{2\alpha-3\kappa_{k'}^{[21]}}{10\alpha+9\kappa_{k'}^{[21]}}+1\right]\mathfrak{B}_4^{[21]}\mathfrak{Z}_2^{[21]}+\frac{3\alpha^{3/2}}{2\pi^{1/2}}\left[3+\frac{2\alpha-3\kappa_{k'}^{[21]}}{10\alpha+9\kappa_{k'}^{[21]}}\right]\mathfrak{P}_4^{[21]}\mathfrak{D}_2^{[21]}\right]\ , \tag{B1}$$

$$\langle f|\hat{T}^{E2}_{2\lambda}|i^{[3]}\rangle=\frac{\sqrt{2\pi}}{27\sqrt{15}}k_\gamma^2 eY_{2\lambda}(\Omega_{p_0})(\tfrac12\sigma'_A,1\sigma'_B|\tfrac12\sigma)$$
$$\times\sum_{k,k'}c_k^{[3]}\left[\xi_{k'}^{[21]}\exp(i\delta_2^{[21]})\left[\mathfrak{X}_6^{[21]}+\frac{\alpha^{2/3}}{\pi^{1/2}}\mathfrak{M}^{[21]}\right]+2\xi_{k'}^{[3]}\exp(i\delta_2^{[3]})\left[2\mathfrak{X}_6^{[3]}-\frac{3\alpha^{3/2}}{\pi^{1/2}}\mathfrak{M}^{[3]}\right]\right]\ , \tag{B2}$$

$$\langle f|\hat{T}^{M1}_{1\lambda}|i^{[3]}\rangle=0\ , \tag{B3}$$

where

$$\mathfrak{M}^{[f]}=\frac{2}{2\alpha-3\kappa_{k'}^{[f]}}\left[\frac{8}{10\alpha+9\kappa_{k'}^{[f]}}\mathfrak{Z}_0^{[f]}-\mathfrak{Z}_2^{[f]}\right]\left[\frac{18}{2\alpha-3\kappa_{k'}^{[f]}}\mathfrak{B}_2^{[f]}-\frac{38\alpha-9\kappa_{k'}^{[f]}}{10\alpha+9\kappa_{k'}^{[f]}}\mathfrak{B}_4^{[f]}\right]$$
$$+8\frac{2\alpha-3\kappa_{k'}^{[f]}}{(10\alpha+9\kappa_{k'}^{[f]})^2}\left[3\frac{2\alpha-3\kappa_{k'}^{[f]}}{10\alpha+9\kappa_{k'}^{[f]}}+2\right]\mathfrak{B}_6^{[f]}\mathfrak{Z}_0^{[f]}+\tfrac13\mathfrak{B}_6^{[f]}\mathfrak{Z}_2^{[f]}\ , \tag{B4}$$

and

$$\mathfrak{B}_q^{[f]}=\int_0^{R_{\max}} y^q \exp\left[-\frac{1}{4}\left[2\alpha+4a_k^{[3]}+\kappa_{k'}^{[f]}-\frac{(2\alpha-3\kappa_{k'}^{[f]})^2}{10\alpha+9\kappa_{k'}^{[f]}}\right]y^2\right]dy\ , \tag{B5}$$

$$\mathfrak{P}_q^{[f]}=\int_0^{R_{\max}} y^q \exp\left[-\frac{1}{16}\left[10\alpha+9\kappa_{k'}^{[f]}-\frac{(2\alpha-3\kappa_{k'}^{[f]})^2}{2\alpha+4a_k^{[3]}+\kappa_{k'}^{[f]}}\right]y^2\right]dy\ , \tag{B6}$$

$$\mathfrak{Z}_q^{[f]}=\int_0^{R_{\max}} y^q \exp[-\tfrac{1}{16}(10\alpha+9\kappa_{k'}^{[f]})y^2]dy\ , \tag{B7}$$

$$\mathfrak{D}_q^{[f]}=\int_0^{R_{\max}} y^q \exp[-\tfrac{1}{4}(2\alpha+4a_k^{[3]}+\kappa_{k'}^{[f]})y^2]dy\ , \tag{B8}$$

$$\mathfrak{X}_q^{[f]}=\int_0^{R_{\max}} y^q \exp[-(a_k^{[3]}+\kappa_{k'}^{[f]})y^2]dy\ . \tag{B9}$$


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