Comparison of nucleon-nucleon potential models in proton-proton bremsstrahlung

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(Received 4 April 1991)

The sensitivity of the $pp\gamma$ process to the different off-shell behaviors of existing nucleon-nucleon interactions based on modern potentials is investigated within a relativistic approach. We find that the analyzing power is sufficiently sensitive to distinguish between the off-shell behaviors of those interactions considered. The $pp\gamma$ spin-correlation coefficients are also found to be sensitive to off-shell differences of nucleon-nucleon interactions so that, if data for these observables were available, they could be used to disentangle different off-shell nonequivalent interactions. The cross sections are unable to discriminate between those interactions considered based on their off-shell differences.

PACS number(s): 25.20.$-$x, 25.20.Lj

I. INTRODUCTION

Recent investigations of the proton-proton (pp) bremsstrahlung reaction [1–7] have demonstrated that off-shell effects of the nucleon-nucleon (NN) interaction can be clearly seen in this type of reaction under certain kinematical conditions. In coplanar-geometry experiments, where the emitted photon and scattered protons are detected in a plane, the pp bremsstrahlung reaction becomes sensitive to the off-shell behavior of the NN interaction for very small proton-scattering angles. Also, in inclusive experiment, where only the emitted photon is detected in the final state, the $pp\gamma$ cross section is found to be very sensitive to off-shell effects for photon energies near the maximum value allowed by the kinematics. In particular, the inclusive cross section becomes more sensitive to off-shell effects than the coplanar-geometry exclusive cross section [6]. Theoretical predictions, however, may be less reliable for these kinematical conditions than for those conditions investigated in the past [8–10]. In particular, some additional corrections (two-body currents, etc.) not included in the theory may become more important when nucleons are scattered at very small angles and/or photons are produced with energies near the maximum value allowed by the kinematics.

Once the sensitivity of the $pp\gamma$ process to the off-shell behavior of the NN interaction is established for different observables, the next interesting and natural question to ask is, to what extent can the NN bremsstrahlung reaction distinguish different off-shell behaviors of modern realistic NN interactions? This question has been addressed by a number of authors in the past and, more recently, by the TRIUMF group [3,4] and is also the issue of the present work. We calculate the pp bremsstrahlung process within a relativistic approach so that the important relativistic corrections are taken into account; also, the one-body rescattering contribution is included.

In the present work, we consider three different interactions, two of which are based on the one-boson-exchange (OBE) model for the NN force developed by the Bonn group [11,12]; the third one is based on the Paris potential [13]. The two Bonn potentials considered are OBEPQ [14], which is a variation of the potential developed in Refs. [11,12], and OBEPT [11], which is an energy-dependent potential. In Sec. II we outline briefly the formalism. In Sec. III the differences between those NN interactions considered in this work are discussed. The results are presented in Sec. IV, and the conclusions are drawn in Sec. V.

II. FORMALISM

In the present work, the pp bremsstrahlung transition amplitude is calculated in momentum space within the framework of potential models. The calculation includes the single-scattering contribution [second and third terms in Eq. (1b)] as well as the double-scattering, or rescattering, contribution [last term in Eq. (1b)] within a relativistic approach. In order to facilitate a close comparison with the bremsstrahlung amplitude obtained within the nonrelativistic approach, we write the invariant relativistic transition amplitude $\tilde{M}$ for producing a photon of momentum $\mathbf{k}$ and polarization $\boldsymbol{\epsilon}$ in an NN collision as

$$\tilde{M}=\sqrt{\varepsilon_1'\varepsilon_2'\omega}M\sqrt{\varepsilon_1\varepsilon_2}\ , \tag{1a}$$

where ω denotes the energy of the emitted photon and $\varepsilon_1', \varepsilon_2'$ ($\varepsilon_1, \varepsilon_2$) are the energies of the two interacting nucleons, 1 and 2, in the final (initial) state; they are defined as $\varepsilon_i=(\mathbf{p}_i^2+m^2)^{1/2}$, with $\mathbf{p}_i$ being the momentum of ith nucleon and m the nucleon mass. In the above equation, M denotes the relativistic bremsstrahlung transition amplitude, which can be expressed as

$$M=\langle \epsilon,\mathbf{k};\phi_f|V_{\rm em}|0;\phi_i\rangle+\langle \epsilon,\mathbf{k};\phi_f|(T^-)^\dagger \mathcal{G}_f V_{\rm em}|0;\phi_i\rangle$$
$$+\langle \epsilon,\mathbf{k};\phi_f|V_{\rm em}\mathcal{G}_i T^+|0;\phi_i\rangle$$
$$+\langle \epsilon,\mathbf{k};\phi_f|(T^-)^\dagger \mathcal{G}_f V_{\rm em}\mathcal{G}_i T^+|0;\phi_i\rangle\ , \tag{1b}$$

where ϕ denotes the two-nucleon nonrelativistic unperturbed wave function; $\mathcal{G}$ is the energy denominator (or two-nucleon Green's function) consistent with the approximation used for solving the relativistic T-matrix integral equation. T^+ and T^- stand for the NN T matrices associated with the outgoing (+) and incoming (−) waves, respectively. The subscript i (f) refers to the initial (final) two-nucleon state. $V_{\rm em}$ in Eq. (1b) denotes the effective relativistic electromagnetic transition potential to be used with nonrelativistic two-component wave functions. It is obtained from the relativistic nuclear Hamiltonian via the minimal substitution $\mathbf{p}\rightarrow\mathbf{p}-e\,\mathbf{A}$ and by considering the electromagnetic coupling to the nucleon only to first order. We write $V_{\rm em}$ as the sum of four terms,

$$V_{\rm em}=V_{\rm conv}+V_{\rm mag}+V_{\rm rsc}+V_{\rm rem}\ , \tag{2a}$$

where, in the NN center-of-mass (c.m.) frame,

$$V_{\rm conv}=-\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\epsilon\cdot(\mathbf{p}+\mathbf{p}')\left[\tilde{e}_1^-\delta\left[\mathbf{p}-\mathbf{p}'-\frac{\mathbf{k}}{2}\right]-\tilde{e}_2^+\delta\left[\mathbf{p}-\mathbf{p}'+\frac{\mathbf{k}}{2}\right]\right]\ , \tag{2b}$$

$$V_{\rm mag}=-i\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\left[\tilde{\mu}_1^-\epsilon\cdot(\mathbf{k}\wedge\boldsymbol{\sigma}_1)\delta\left[\mathbf{p}-\mathbf{p}'-\frac{\mathbf{k}}{2}\right]+\tilde{\mu}_2^+\epsilon\cdot(\mathbf{k}\wedge\boldsymbol{\sigma}_2)\delta\left[\mathbf{p}-\mathbf{p}'+\frac{\mathbf{k}}{2}\right]\right]\ , \tag{2c}$$

and

$$V_{\rm rsc}=i\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\left[\tilde{v}_1^-\epsilon\cdot(\mathbf{p}'\wedge\boldsymbol{\sigma}_1)\delta\left[\mathbf{p}-\mathbf{p}'-\frac{\mathbf{k}}{2}\right]-\tilde{v}_2^+\epsilon\cdot(\mathbf{p}'\wedge\boldsymbol{\sigma}_2)\delta\left[\mathbf{p}-\mathbf{p}'+\frac{\mathbf{k}}{2}\right]\right]\ . \tag{2d}$$

In the above equations, $\mathbf{p}$ and $\mathbf{p}'$ denote the relative momenta of the two interacting nucleons before and after the emission of a photon. We have used the transversality condition $\epsilon\cdot\mathbf{k}=0$. The nucleon electric charge is denoted by e, and σ_i stands for the Pauli spin matrix for nucleon i (=1,2). The factors $\tilde{e}_i^\pm$, $\tilde{\mu}_i^\pm$, and $\tilde{v}_i^\pm$ are functions of nucleon and photon momenta. The leading terms in $\tilde{e}_i^\pm$ and $\tilde{\mu}_i^\pm$ are given by $\tilde{e}_i^\pm=1$ and $\tilde{\mu}_i^\pm=\mu_i$, which give rise to the convection and magnetization current operators of the conventional nonrelativistic approach. Here μ_i denotes the anomalous nuclear magnetic moment in units of nuclear magnetons (for protons $\mu_1=\mu_2=\mu_p=2.793$); in $\tilde{v}_i^\pm$ the leading term is given by $\tilde{v}_i^\pm=(\mu_p-\frac{1}{2})k/m$. If we consider terms through p/m in the factors $\tilde{e}_i^\pm$, $\tilde{\mu}_i^\pm$, and $\tilde{v}_i^\pm$, one recovers the usual expression for the interaction Hamiltonian obtained via the Foldy-Wouthuysen reduction when terms through p/m are kept.

In Eq. (2), the dominant relativistic spin correction is denoted by $V_{\rm rsc}$. It results from cross terms between the upper and lower components of the Dirac spinors when taking the matrix element of the relativistic four-component electromagnetic transition operator. Inclusion of $V_{\rm rsc}$ introduces a "composite" current (convection ⊗ spin) term into $V_{\rm em}$. The composite currents also appear in other reactions such as (p,p') when the spin-dependent part of the NN coupling is momentum dependent [15] as arises from an analogous two-component reduction [16] of the Dirac spinors, for example. The remaining relativistic correction $V_{\rm rem}$ in Eq. (2) has a different operator structure than the first three terms, and its contribution is negligible compared to those from the other terms. Further details of the formalism will be given in Ref. [17].

The relativistic electromagnetic interaction Hamiltonian for a two-nucleon system consists not only of the sum of the interaction Hamiltonians for each nucleon as given by Eq. (2), but also of a correction term which accounts for Lorentz boosting effects [18–22]. We neglect this correction since it is known to be very small [9]. Moreover, there is a problem of the nonuniqueness of this correction term, as has been discussed in Refs. [21,23]. One should keep in mind, however, that in the present work this correction may be much larger than that found in Ref. [9] because kinematical conditions relevant for the present investigation are different from those studied in Ref. [9]. The Coulomb correction, which is known to reduce the $pp\gamma$ cross section [2,10] is also neglected in the present work; at a proton incident energy of $T_{\rm lab}=280$ MeV and proton-scattering angles of $\theta_1\sim\theta_2\sim 12°$ (which is in the kinematical region of interest of the present work), this correction reduces the cross section by $\sim$5–8 % [24].

Although, both the one-body rescattering and two-body current terms should be included to the same order in the photon momentum in order to preserve the gauge invariance of the theory [25,26], we neglect the two-body current contribution entirely in the present work. Unlike the neutron-proton bremsstrahlung reaction where a large (two-body) exchange current contribution is present, the two-body current is expected to contribute very weakly to the pp bremsstrahlung reaction. Accordingly, the violation of gauge invariance in the theory introduced by omitting the two-body current term in the pp bremsstrahlung process is not expected to be serious. Moreover, a fully consistent treatment of the two-body current within a realistic potential model calculation is not yet available; see, however, Ref. [27], where consistent two-body currents have been obtained for separable interactions.

III. OFF-SHELL NONEQUIVALENT INTERACTIONS

The different off-shell behavior exhibited by different *NN* interactions arises either from the differences in the structure of the corresponding *NN* potentials or from the different approximations made in obtaining a *T*-matrix interaction from a particular *NN* potential. The *T*-matrix interaction based on the Paris potential is obtained by solving the nonrelativistic two-body Schödinger equation or, equivalently, the Lippmann-Schwinger equation which in the *NN* c.m. system reads

$$T_{\rm NR}(\mathbf{p}',\mathbf{p}) = V_{\rm NR}(\mathbf{p}',\mathbf{p}) + \int d^3p'' V_{\rm NR}(\mathbf{p}',\mathbf{p}'') \frac{1}{p^2/m - p''^2/m + i\eta} \times T_{\rm NR}(\mathbf{p}'',\mathbf{p}) , \tag{3}$$

where $\mathbf{p}$ ($\mathbf{p}'$) denotes the relative momentum of the two interacting nucleons in the initial (final) state. The *T*-matrix interaction $T_{\rm NR}$ is related to the *NN* cross section in the c.m. system via [28]

$$\frac{d\sigma}{d\Omega} = \left[\frac{m}{4\pi}\right]^2 |T_{\rm NR}(\mathbf{p}',\mathbf{p})|^2 , \tag{4}$$

with $|\mathbf{p}'| = |\mathbf{p}|$.

However, as has been discussed in Refs. [29,17], in order to comply with the requirement of the theory of special relativity and still reproduce the *NN* data using Eq. (4), we convert the nonrelativistic Lippmann-Schwinger equation into a relativistic Blankenbecler-Sugar-type equation

$$\tilde{T}_{\rm NR}(\mathbf{p}',\mathbf{p}) = \tilde{V}_{\rm NR}(\mathbf{p}',\mathbf{p}) + \int \frac{d^3p''}{\varepsilon_{p''}} \tilde{V}_{\rm NR}(\mathbf{p}',\mathbf{p}'') \frac{m}{p^2/m - p''^2/m + i\eta} \times \tilde{T}_{\rm NR}(\mathbf{p}'',\mathbf{p}) , \tag{5}$$

where we define

$$\begin{aligned} \tilde{V}_{\rm NR}(\mathbf{p}',\mathbf{p}) &= \left[\frac{\varepsilon_{p'}}{m}\right]^{1/2} V_{\rm NR}(\mathbf{p}',\mathbf{p}) \left[\frac{\varepsilon_p}{m}\right]^{1/2} , \\ \tilde{T}_{\rm NR}(\mathbf{p}',\mathbf{p}) &= \left[\frac{\varepsilon_{p'}}{m}\right]^{1/2} T_{\rm NR}(\mathbf{p}',\mathbf{p}) \left[\frac{\varepsilon_p}{m}\right]^{1/2} , \end{aligned} \tag{6}$$

with $\varepsilon_p = (\mathbf{p}^2 + m^2)^{1/2}$. We note that the conversion of the Lippmann-Schwinger equation into a relativistic integral equation, although not unique, is needed for constructing a Lorentz-invariant transition amplitude from a nonrelativistically constructed *T* matrix and yet be consistent with Eq. (4). This is particularly important in the description of the *NN* bremsstrahlung process at high incident energies, where the calculations are based on *NN* potential models and one makes use of the invariant nature of the transition amplitude [29,17].

The *T*-matrix interaction associated with the OBEPQ version of the Bonn potential is obtained [11] from the Blanckenbecler-Sugar equation [as Eq. (5)]

$$\tilde{T}_{\rm OB}(\mathbf{p}',\mathbf{p}) = \tilde{V}_{\rm OB}(\mathbf{p}',\mathbf{p}) + \int \frac{d^3p''}{\varepsilon_{p''}} \tilde{V}_{\rm OB}(\mathbf{p}',\mathbf{p}'') \frac{m}{p^2/m - p''^2/m + i\eta} \times \tilde{T}_{\rm OB}(\mathbf{p}'',\mathbf{p}) , \tag{7}$$

which is obtained from a three-dimensional reduction of the relativistic Bethe-Salpeter equation. In this approximation the exchanged mesons in intermediate states are also put on the mass shell, so that they transfer only three-momenta in the NN c.m. frame.

Therefore we see that, if the modified potential $\tilde{V}_{\rm NR}$ and the OBEPQ potential $\tilde{V}_{\rm OB}$ had the same structure, the associated interactions $\tilde{T}_{\rm NR}$ and $\tilde{T}_{\rm OB}$ would have the same off-shell behavior since both interactions obey the same integral equation. In other words, the off-shell differences between these two interactions arise only from the differences in the structure of the corresponding *NN* potentials. We mention that the Paris potential we consider here [see Eq. (6)] is based on the Yukawa-parametrized version of the original Paris potential [13]. Hereafter, we simply refer to the *T*-matrix interaction $\tilde{T}_{\rm NR}$ calculated with the parametrized Paris potential as the Paris interaction.

The *T*-matrix interaction based on the OBEPT version of the Bonn potential differs off shell from that based on the OBEPQ potential. The deviation of the OBEPT model from the OBEPQ version arises from the different underlying dynamics of the two models. In particular, the OBEPT potential is based on a field-theoretical Hamiltonian within noncovariant time-ordered perturbation theory with the inclusion of only OBE processes [30], whereas the OBEPQ potential is based on the Blankenbecler-Sugar reduction of the Bethe-Salpeter equation, as mentioned above, with only OBE processes. Since in the case of the OBEPT potential the full Hilbert space contains mesonic states, this leads to an energy-dependent *NN* potential when the problem is reduced to the *NN* subspace. In this way meson retardation effects are effectively included in a way quite differently than in the Blankenbecler-Sugar approximation to the Bethe-Salpeter equation. Although the exchanged meson transfers only three-momenta in the *NN* c.m. frame in the Blankenbecler-Sugar approximation, this does not imply that retardation effects are neglected. In fact, it has been shown in Ref. [31] that in an arbitrary frame the energy transferred by the exchanged meson does not vanish in this approximation. The basic difference between the energy-dependent OBEPT and OBEPQ potentials is that, while in the OBEPQ version the propagation of exchanged mesons is described by Feynman propagators, in the OBEPT version this propagation is described by propagators of the form [11,30] $P_\alpha \propto 1/[\omega_\alpha(z - \varepsilon_{p'} - \varepsilon_p - \omega_\alpha)]$, with $\omega_\alpha = [(\mathbf{p}'-\mathbf{p})^2 + \mu_\alpha^2]^{1/2}$ and μ_α is the mass of the meson exchanged; α stands for the type of the meson (α = scalar, vector, etc.).

Explicitly, the integral equation for the OBEPT-based interaction reads (in the NN c.m. frame) [11,30]

$$\tilde{T}_{\rm OT}(\mathbf{p}',\mathbf{p};z)=\tilde{V}_{\rm OT}(\mathbf{p}',\mathbf{p};z)+\int d^3p''\tilde{V}_{\rm OT}(\mathbf{p}',\mathbf{p}'';z)\frac{1}{z-2\varepsilon_{p''}+i\eta}\times\tilde{T}_{\rm OT}(\mathbf{p}'',\mathbf{p};z)\ , \qquad (8)$$

where $\tilde{V}_{\rm OT}$ denotes the energy-dependent OBEPT potential and $z=2\varepsilon_p$. Therefore the off-shell differences between the T matrix based on the energy-dependent OBEPT potential and that based on the OBEPQ potential arise not only from the different structure of the associated NN potentials, but also from the different integral equations they obey.

When the electromagnetic coupling to a nucleon is taken to first order the NN bremsstrahlung process involves extra intermediate states [described by the propagators $\mathcal{G}_{i,f}$ in Eq. (1b)] compared with the NN elastic-scattering process. Most of the existing NN bremsstrahlung calculations, either within the relativistic or nonrelativistic approaches, assume an expression of the form [2,8,32]

$$\mathcal{G}_{i,f}=\frac{1}{E_{i,f}-\varepsilon_1''-\varepsilon_2''\pm i\eta}\ , \qquad (9)$$

for the energy denominators, irrespective of the type of propagator entering the T-matrix integral equation. In the above equation, $E_i=\varepsilon_1+\varepsilon_2$ and $E_f=\varepsilon_1'+\varepsilon_2'$. The double prime denotes intermediate states. In a consistent approach, however, the above propagator $\mathcal{G}$ should be of the same type as that used for solving the T-matrix integral equation. This is particularly clear if one solves, for example, the Bethe-Salpeter equation or some approximation to it for the bremsstrahlung process. Indeed, for the bremsstrahlung amplitude based on the OBEPQ T matrix discussed before, $\mathcal{G}$ should be the Blankenbecler-Sugar propagator; an analogous observation holds for the Paris-interaction-based amplitude. For the amplitude based on the OBEPT interaction, $\mathcal{G}$ is of the form given in Eq. (9). Therefore the NN bremsstrahlung reaction probes not the differences in the T-matrix interactions used, but the composite differences in the T-matrix interactions and corresponding propagators $\mathcal{G}$ in Eq. (1b). Of course, differences in the propagators which enter in the corresponding T-matrix integral equations are responsible for part of the differences in the T matrices as discussed before.

At this point it is appropriate to mention that when we refer to the on- and off-shell behavior of a given T matrix, we refer to the on- and off-shell behavior of the transition amplitude T, which is related to the different T matrices discussed above ($\tilde{T}$) through

$$T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)=\left[\frac{m}{\varepsilon_1'}\right]^{1/2}\left[\frac{m}{\varepsilon_2'}\right]^{1/2}\tilde{T}(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)\left[\frac{m}{\varepsilon_1}\right]^{1/2}\left[\frac{m}{\varepsilon_2}\right]^{1/2}. \qquad (10)$$

$\tilde{T}$ denotes $\tilde{T}_{\rm NR}$, $\tilde{T}_{\rm OB}$, or $\tilde{T}_{\rm OT}$.

In terms of the original nonrelativistic T matrix $T_{\rm NR}$, which obeys the Lippmann-Schwinger equation, Eq. (10) becomes

$$T(\mathbf{p}',\mathbf{p})=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}T_{\rm NR}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2}, \qquad (11)$$

in the NN c.m. frame.

In terms of the transition amplitude T, as defined by Eq. (10), the NN differential cross section reads (in the NN c.m. frame)

$$\frac{d\sigma}{d\Omega}=\left[\frac{\varepsilon_p}{4\pi}\right]^2|T(\mathbf{p}',\mathbf{p})|^2\ , \qquad (12)$$

with $|\mathbf{p}'|=|\mathbf{p}|$.

The NN bremsstrahlung reaction requires NN T-matrix elements (in the NN c.m. frame) of the form [32]

$$T^{JST}_{L'L}(|\mathbf{p}'\pm\mathbf{k}/2|,p;E_p)\ ,\quad T^{JST}_{L'L}(|\mathbf{p}\pm\mathbf{k}/2|,p';E_{p'})\ , \qquad (13)$$

corresponding to the process in which the photon is emitted after and before the strong interaction takes place. In Eq. (13), $\mathbf{k}$ denotes the momentum of the emitted photon. The starting energy E_q, with $\mathbf{q}$ denoting either $\mathbf{p}$ or $\mathbf{p}'$, is related to $\mathbf{q}$ via $E_q=\mathbf{q}^2/m$; E_q is also related to z in Eq. (7) via $z=2\sqrt{m(E_q+m)}$. The quantum numbers $S,L(L')$, J, and T stand for the spin, orbital angular momentum, total angular momentum, and total isospin of two interacting nucleons. "Half off the energy shell" or simply "off shell" means that $|\mathbf{q}'\pm\mathbf{k}/2|\neq|\mathbf{q}|$. In NN elastic scattering, where no photon is present, $|\mathbf{q}'|=|\mathbf{q}|$ (on shell) by energy-momentum conservation.

Since the on-shell limit of the NN T-matrix interaction from Eq. (13) is not unique, we have to specify which on-shell limit we are considering when discussing off-shell effects. The on-shell limit we are interested in here is

$$T^{JST}_{L'L}(|\mathbf{p}'\pm\mathbf{k}/2|,p;E_p)\rightarrow T^{JST}_{L'L}(p,p;E_p)\ ,\quad T^{JST}_{L'L}(|\mathbf{p}\pm\mathbf{k}/2|,p';E_{p'})\rightarrow T^{JST}_{L'L}(p',p';E_{p'})\ . \qquad (14)$$

As in Ref. [6], we refer to these on-shell limits as the OESA. In this OESA limit, the T-matrix elements, and only the T-matrix elements, are forced to be on shell according to Eq. (14).

IV. RESULTS

Keeping in mind the uncertainty in the theory, especially in the present calculation as has been discussed before, we compare in this section the results obtained for $pp\gamma$ observables with different off-shell nonequivalent realistic NN interactions. Since in the present work we consider the pp bremsstrahlung reaction at a proton incident energy of $T_{\rm lab}=280$ MeV, all two-body partial-wave states through total angular momentum $J=11$ are included in the calculation in order to assure convergence.

Although realistic *NN* interactions are said to be equivalent on the energy shell (phase-shift equivalent), they are not completely equivalent because of the uncertainties arising in fitting the phase shifts. Since we want to test the sensitivity of the *pp* bremsstrahlung reaction to off-shell differences of these interactions, we have to check whether they yield the same on-shell results. Before we check the on-shell results, however, we note that, if the *NN* interactions have the same on-shell values, the corresponding bremsstrahlung results will still differ from each other unless the *NN* propagators $\mathcal{G}$ associated with these interactions are the same [see Eq. (1b)]. Figure 1(a) shows a comparison of the OESA [Eq. (14)] on-shell results for the coplanar-geometry $pp\gamma$ cross section for *NN* interactions based on the OBEPQ (solid curve), OBEPT (dashed curve), and Paris (dot-dashed curve) potentials. The incident energy is $T_{\rm lab}=280$ MeV, and the proton-scattering angles are $\theta_1=12.4°$ and $\theta_2=12.0°$. The data are from Ref. [5]. We see that as far as the *pp* bremsstrahlung process in the coplanar geometry is concerned, the OBEPQ- and Paris-based *T* matrices yield practically the same on-shell results. We observe that the OBEPQ and Paris interactions have the same *NN* propagators. The differences observed between the on-shell results based on the OBEPT interaction and those based on the other two interactions is due only to the difference in the corresponding *NN* propagators $\mathcal{G}$ in the extra intermediate states which are present in the *NN* bremsstrahlung process compared with the *NN* elastic-scattering process because of the emission of the photon [see Eq. (1b)]. Indeed, if we force $\mathcal{G}$ in those intermediate states (and only in those intermediate states) to be the same for all these three interactions, the resulting $pp\gamma$ cross sections in the OESA limit are practically the same for all these interactions. Therefore, as far as the *pp* bremsstrahlung process is concerned, all the three interactions have the same on-shell values.

In Fig. 1(b) the corresponding off-shell results as prescribed by the *NN* bremsstrahlung reaction are shown. Here we do not see any significant differences among these results, indicating that the $pp\gamma$ cross section in this geometry is not sensitive enough to distinguish among the differences in the off-shell behavior of these three interactions (as has been discussed in Ref. [5], the cross-section data may suffer from an ambiguity in the overall normalization). Figure 1(c) and 1(d) show the on- and off-shell results, respectively, for the analyzing power in the same kinematical condition of Figs. 1(a) and 1(b) except for the value of $\theta_2=14°$. A comparison of these two

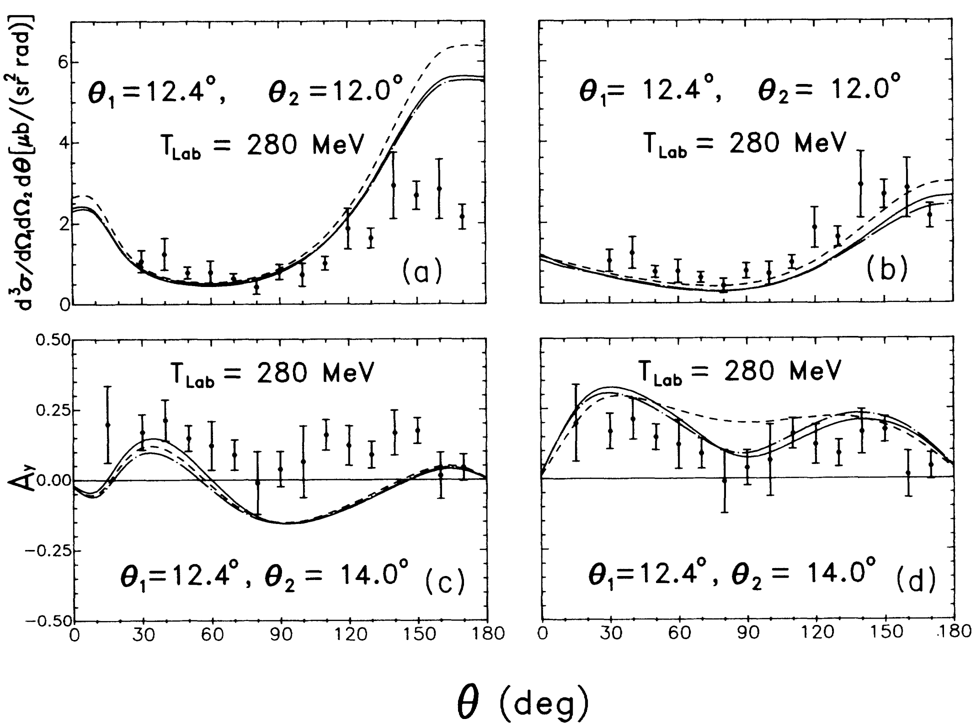

FIG. 1. Coplanar-geometry $pp\gamma$ cross section and analyzing power in the laboratory frame as a function of photon-emission angle θ at an incident energy of $T_{\rm lab}=280$ MeV. The solid, dashed, and dot-dashed curves are the results corresponding to the OBEPQ [14], OBEPT [11], and Paris [13] potentials, respectively. Parts (a) and (b) correspond to the OESA [Eq. (14)] and exact (off-shell) cross sections for $\theta_1=12.4°$ and $\theta_2=12.0°$, respectively. The cross-section data have not been renormalized as was done in [5]. Parts (c) and (d) correspond to the OESA and exact (off-shell) results, respectively, for the analyzing power for $\theta_1=12.4°$ and $\theta_2=14.0°$. The data [5] are multiplied by a factor of -1 to agree with our convention.

figures shows clearly that this quantity is sufficiently sensitive to reveal the off-shell difference of the OBEPT-based interaction from the other two interactions manifested in the results around $\theta=90°$. Here we stress that, although the OBEPT version predicts a result which is in disagreement with the data, a caution must be taken to conclude from this that its off-shell behavior is wrong. This is because the theory of the *NN* bremsstrahlung reaction and, in particular, the present potential model calculation, is not yet complete, as has been discussed in Refs. [7,17].

In order to see how much these three interactions differ from each other off shell (at least in the region of energy sampled by the bremsstrahlung reaction), we show in Fig. 2 the magnitude of the relevant off-shell *T*-matrix elements in the $^1S_0$ [Fig. 2(a)] and $^3FP_2$ [Fig. 2(b)] states for these three interactions as they enter the calculation of the $pp\gamma$ process in the case of Fig. 1(b) for the $^1S_0$ state and in the case of Fig. 1(d) for the $^3FP_2$ state. We observe that the cross section becomes sensitive to the $^1S_0$ state for small proton-scattering angles and for forward and backward photon angles, while the analyzing power becomes very sensitive to the coupled tensor states [6]. We see in Fig. 2(a) that the OBEPQ-based interaction (solid curve) differs most from the other two interactions in the $^1S_0$ state. Moreover, the difference here is basically a shift of the OBEPQ *T* matrix with respect to those of the OBEPT (dashed curve) and Paris (dot-dashed curve) results by an amount of ~ 10 fm^2. This fact, together with the results of Fig. 1, shows that the $pp\gamma$ cross section in the coplanar geometry is not sufficiently sensitive to distinguish between the off-shell differences of this amount. We also note that, given the error bars involved in the data (see Fig. 1), it is unlikely that the $pp\gamma$ cross section in this geometry can be used to differentiate between different realistic *NN* interactions unless these interactions differ off shell by a much larger amount than that exhibited by the interactions considered here. In the $^3FP_2$ partial-wave state, the OBEPQ- and the Paris-potential-based interactions yield practically the same off-shell values, while the OBEPT interaction is weaker by ~ 2.5 fm^2. However, the analyzing power is sensitive enough to disentangle this difference, as we have seen in Fig. 1(d). We mention that the *T*-matrix elements in the $^3PF_2$ state (not shown here) are similar to those in the $^3FP_2$ state; i.e., the OBEPT-based interaction differs from those based on the OBEPQ and Paris potentials. The OBEPQ- and Paris-based interactions are very close to each other. The same observation also holds in the $^3P_0$ state for most photon-emission angles.

Figure 3 shows the angular distribution of the inclusive $pp\gamma$ cross section in the initial *pp* c.m. frame at an incident energy of $T_{\rm lab}=280$ MeV and at a photon energy of $\omega=125$ MeV. The solid, dashed, and dot-dashed curves are the results based on the OBEPQ, OBEPT, and Paris potentials, respectively. The upper three curves correspond to the OESA on-shell results according to Eq. (14). The lower three curves are the results when the half off-shell *T* matrices are used. We see similar features to those observed in Fig. 1 for coplanar-geometry exclusive cross sections. The inclusive cross sections also cannot disentangle the off-shell differences in these interactions even at photon energies very close to the end-point energy.

Very recently, we investigated the sensitivity of spin-correlation coefficients in *pp* bremsstrahlung to off-shell effects of the *NN* interaction [7] and found that these

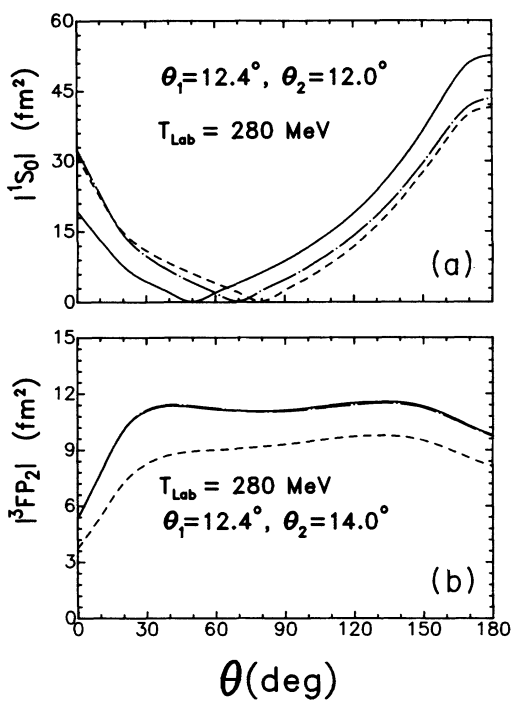

FIG. 2. Absolute magnitude of the nucleon-nucleon *T*-matrix elements corresponding to the OBEPQ (solid curve), OBEPT (dashed curve), and Paris (dot-dashed curve) potentials as they enter the calculation of the $pp\gamma$ process in Fig. 1. (a) $T(|\mathbf{p}+\mathbf{k}/2|,p';E_{p'})$ in the $^1S_0$ state for proton-scattering angles of $\theta_1=12.4°$ and $\theta_2=12.0°$. (b) $T(|\mathbf{p}+\mathbf{k}/2|,p';E_{p'})$ in the $^3FP_2$ state for proton-scattering angles of $\theta_1=12.4°$ and $\theta_2=14.0°$.

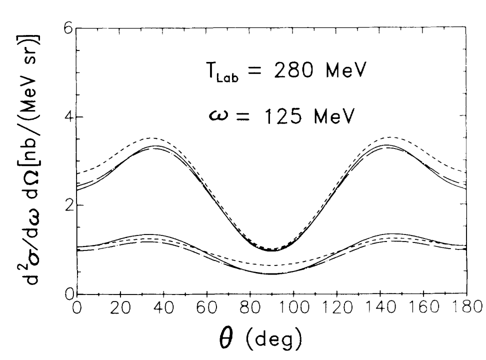

FIG. 3. Angular distribution of the inclusive $pp\gamma$ cross section in the initial proton-proton center-of-mass frame at an incident energy of $T_{\rm lab}=280$ MeV for a photon energy of $\omega=125$ MeV. The solid, dashed, and dot-dashed curves correspond to the OBEPQ, OBEPT, and Paris potentials, respectively. The upper curves denote the OESA results [Eq. (14)], while the lower ones denote the exact (off-shell) calculations.

coefficients are very sensitive to off-shell effects, even at proton-scattering angles where the analyzing power is weakly sensitive to these effects. In Fig. 4 we show the on- and off-shell results for the spin-correlation coefficients C_{xx}, C_{yy}, and C_{zz}, as defined in Ref. [7], at an incident energy of $T_{\rm lab}=280$ MeV and for symmetric proton-scattering angles of $\theta_1=\theta_2=10°$. In the upper part of the figure, we also display the corresponding analyzing power for comparison. We see that all three interactions considered yield practically the same on-shell results [Fig. 4(a)]. In Fig. 4(b), where the off-shell results are shown, we see a clear difference between results based on the OBEPT interaction and those based on the OBEPQ and Paris interactions. The latter two interactions exhibit basically the same off-shell results. The off-shell difference we observe in the spin-correlation coefficients, especially in C_{yy}, is larger than that in the corresponding analyzing power. Moreover, this difference is seen over a much wider region of the photon-emission angle than is seen in the analyzing power. Therefore, if data were available, spin-correlation coefficients could be used to disentangle different off-shell nonequivalent interactions. Of course, measurements of spin-correlation coefficients are more difficult than measurements of analyzing powers. However, we mention that even at larger proton-scattering angles of $\theta_1=\theta_2=20°$, where the analyzing power shows practically no off-shell difference between the interactions considered and where the experiments are easier to perform than at smaller proton-scattering angles, the spin-correlation coefficients still show an off-shell difference similar to those exhibited in Fig. 4(b).

V. CONCLUSION

We have tested the sensitivity of the $pp\gamma$ reaction to the different off-shell behaviors of three *NN* interactions based on modern potentials. We have considered two OBE versions of the *NN* potential developed by the Bonn group [11,12] and another developed by the Paris group [13]. We have found that cross-section measurements are unable to distinguish between these off-shell differences. In order to discriminate between different interactions, one would need much larger off-shell differences than those generated by modern interactions. The analyzing power, however, is sufficiently sensitive to distinguish between the off-shell differences in the tensor component of different *NN* interactions (at least for those interactions

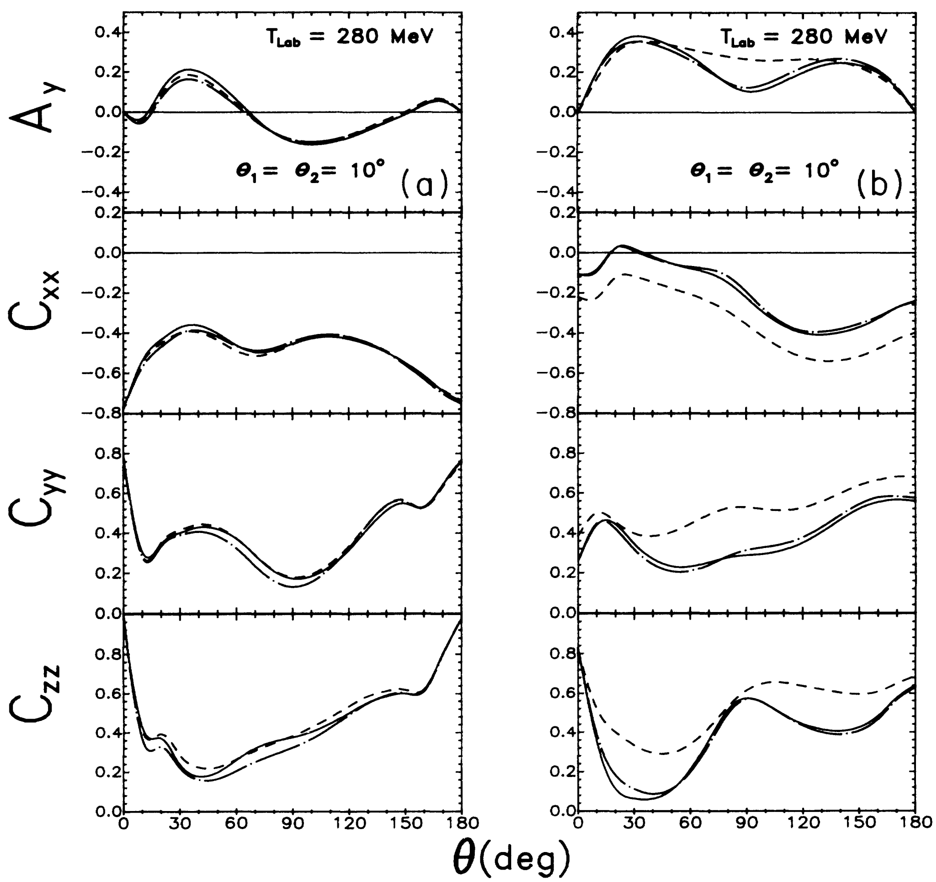

FIG. 4. Coplanar-geometry $pp\gamma$ spin-correlation coefficients and analyzing power in the laboratory frame as a function of photon-emission angle θ at an incident energy of $T_{\rm lab}=280$ MeV and at symmetric proton-scattering angles of $\theta_1=\theta_2=10°$. From the top to bottom are the analyzing power A_y and the spin-correlation coefficients C_{xx}, C_{yy}, and C_{zz}. Part (a) corresponds to the on-shell results, while part (b) corresponds to the exact (off-shell) results. For details, see caption of Fig. 1.

we have considered). It is shown that measurements of spin-correlation coefficients offer a potential means of distinguishing between different interactions based on their off-shell differences.

ACKNOWLEDGMENTS

We are indebted to T. Cheon for providing us with the Paris *NN* potential routine. We would like to thank W. G. Love for many discussions and a careful reading of the manuscript. One of us (V.H.) acknowledges the kind hospitality of the members of the Department of Physics and Astronomy during his stay at the University of Georgia. This work was supported in part by NATO Grant No. RG85/0093 and a grant from the Faculty Research Foundation (UGA). We also appreciate a grant for computing time provided by the University of Georgia.

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