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Bosonic kinetics and the pion transverse momentum in heavy ion collisions

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Using a new technique to solve the kinetic equation including Bose statistics, we show that the shape of the pion transverse momentum spectrum is sensitive to the hadronization time in ultrarelativistic heavy ion collisions. For a pure pion gas, the magnitude of the soft pion component observed in central 200 GeV/nucleon O+Au collisions is reproduced by an effective pionization time of about 7 fm/c. This explanation implies that the hadrons are produced out of chemical equilibrium.

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Negative particle and neutral pion spectra from ultrarelativistic heavy ion collisions at CERN show a markedly enhanced low transverse momentum component when compared to corresponding minimum bias proton-proton spectra [1–3]. This "soft pion" puzzle has been much discussed in the literature. One suggestion has been that the hadronic gas would cool by collective expansion [4], but more detailed considerations of the hydrodynamics found too small an effect [5]. The decay of excited baryons gives low momentum pions [6], and this mechanism may well account for the soft pions in situations where the baryon density is high, such as near target rapidity, or at the lower energies of Alternating Gradient Synchrotron experiments [7]. For Super Proton Synchrotron energies, on the other hand, there are probably not enough baryons in the central region to account for the data [8]. We note that Sollfrank *et al.* [9] reach the opposite conclusion. We also mention the possibility that soft pion interactions within hot, dense matter affect the dispersion relation and the momentum distribution in the final state [10]. Finally, several authors [11,12] have pointed out that the transverse momentum spectra are compatible with a thermal Bose-Einstein distribution if there is a strong excess of pions with respect to chemical equilibrium at freezeout. However, collective flow would wash out the enhancement thus obtained [11] while the corresponding mean free paths ($\lambda \sim 1$ fm) are too short to be compatible with estimated freezeout radii ($R \sim 7$ fm), [13,14], and the rapidity distributions are too narrow by a factor of 2.

In this work, we shall seek a more detailed picture of the hadronic evolution of the collision to see whether the assumptions of Kataja and Ruuskanen can plausibly be supported in models of kinetics and to what extent the observed spectra might reflect those at hadronization. One such kinetic equation is the bosonic Boltzmann equation [15], which describes the time evolution of the phase-space distribution function $f(\mathbf{r},\mathbf{p},t)$ for bosons:

$$\frac{Df_1}{Dt}=\frac{1}{4E_1}\int d\omega_2 d\omega_3 d\omega_4 (2\pi)^4 \delta^4(k_1+k_2-k_3-k_4)|T|^2 \times\{f_3 f_4 \tilde{f}_1 \tilde{f}_2 - f_1 f_2 \tilde{f}_3 \tilde{f}_4\}\ , \qquad (1)$$

where $d\omega_i = d^3k/[(2\pi)^3 2E_i]$, $E_i^2=|\mathbf{p}|^2+m^2$, $|T|^2$ is the square of the invariant scattering amplitude, and the factors $\tilde{f}_i \equiv 1+f_i$ account for Bose enhancement. These factors will drive particles to occupy preferentially regions of higher phase-space density, i.e., toward small transverse momenta and rapidities. Gavin and Ruuskanen [16] have solved Eq. (1) using a relaxation time approximation for the right-hand side. Here we shall use a more general test-particle method to simulate the evolution of the system. To our knowledge, the only previous treatment of the kinetics for bosons was by Zeldovich and Levich in the theory of plasmas [17].

We shall consider specifically the central 200 GeV/nucleon ^{16}O+Au data of the NA35 Collaboration [1]. This experiment measures negative particles, mostly pions, with a $\sim 10\%$ admixture of negative kaons, electrons, and antiprotons [18]. For simplicity, we solve Eq. (1) for an isospin-symmetric pion gas only, and assume that all particles in the experimental data are negative pions. An integration of the rapidity distribution therefore shows that the system should be initialized with ~ 440 pions. Inclusion of other negatives would reduce this number, and hence the phase-space density of pions, by 10%. The results obtained here are thus subject to a systematic error of this order of magnitude. Further, we have ignored the excited-state mesons and baryons which will be present in the initial state [19,20] and influence the evolution of the system. Exclusion of the baryons is probably justified for the midrapidity range considered here. The heavy mesons omitted are mainly ρ's, which will decay rapidly. We therefore consider the evolution of the pions only with an effective average hadronization, or "pionization," time τ_0^{eff} for the system which is $\gtrsim 1-3$ fm/c larger than the actual hadronization time.

Some further model assumptions are needed to specify the initial conditions for Eq. (1). We shall suppose a Bjorken-like picture is applicable [21], and take the initial distribution of pions to be given by

$$\frac{d^6N}{dy_b dr_\perp^2 dy\, dp_\perp^2}=\mathcal{N}\{\exp[\beta m_\perp \cosh(y-y_b)]-1\}^{-1} \times\theta(\Delta y_b/2+y_b)\theta(\Delta y_b/2-y_b) \times\int_{-\infty}^{\infty}dz'\rho_0(r_\perp^2+z'^2)\ , \qquad (2)$$

as a function of rapidity y, transverse momentum $p_\perp^2 = m_\perp^2 - m_\pi^2$, longitudinal position $z = \tau_0^{\rm eff} \sinh y_b$, and transverse position $r_\perp$. Here, $\tau_0^{\rm eff}$ is the effective pionization time, while $\mathcal{N}$ is a normalization constant that follows from the total particle number, and ρ_0 is the empirical projectile baryon density [22]. The initial state of the calculation is defined on the hyperbola $t_i^2 - z_i^2 = (\tau_0^{\rm eff})^2$, i.e., a given particle i only begins to participate in the evolution of the system at the laboratory frame time $t_i = \tau_0^{\rm eff} \cosh y_b^i$.

In pp collisions the effect of Bose statistics is not expected to be significant because the small size of the source means that there will be too few collisions to even partially thermalize the system. We choose to fix the initial momentum distribution (i.e., β and Δy_b) from the minimum bias pp $p_\perp$ spectrum [1,23] below ~ 1 GeV/c [24], and the pp rapidity distribution [25]. Resulting values of $\beta^{-1} = 135$ MeV and $\Delta y_b = 3.6$ in Eq. (2) describe these experimental pp data well. We note that even locally the initial distribution is not necessarily in equilibrium because the particle density is arbitrary; in particular, the form of the local momentum distribution merely provides a good fit to the pp data and, strictly speaking, β^{-1} may not be identified with a physical temperature. In Fig. 1(a), the dashed line shows the initial $(t = \tau_0^{\rm eff})$ rapidity distribution, including a factor of $\frac{1}{3}$ to account for the pion degeneracy. The dashed line in Fig. 1(b) similarly shows the corresponding initial transverse momentum distribution in a rapidity interval of $-0.3 < y < 0.7$. These values follow from the shift to c.m. frame and rapidity interval for the experimental AB rapidity data.

In Eq. (2), the initial pion density at transverse distances $r_\perp$ is assumed to be proportional to the product $N_P^z(r_\perp) N_T^z$ of projectile and target nucleon numbers that overlap along the beam direction and ignores the target geometry for central collisions. On the other hand, the particle distribution in the z direction follows from the fixed boost distribution and the effective pionization time $\tau_0^{\rm eff}$. This free parameter therefore controls the pion collision rate via the pion density. The magnitude of the low-$p_\perp$ enhancement may thus be expected to depend sensitively on $\tau_0^{\rm eff}$, and one should be able to extract it from the AB data given the initial conditions discussed above. Of course, the collision rate, and hence the size of the enhancement, also depend on the value of the cross section that appears in Eq. (1). We use $\sigma = 23$ mb, which is a thermal average [26] at $T = 135$ MeV of the π-π cross section determined from scattering phase-shift data.

Equation (1) is solved using the "full-ensemble" test-particle method [27,28]. To do this, assume that the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ is bounded by a constant $F_{\rm max}$ at all times, and replace the transition matrix in (1) by $|T'|^2 \equiv (1 + F_{\rm max})^2 |T|^2$. Collisions between test particles in phase-space elements $d\omega_1, d\omega_2$ are then accepted with a probability of $(1+f_3)(1+f_4)/(1+F_{\rm max})^2$, where f_3, f_4 are the final phase-space occupations. In order to sample phase space adequately, and to avoid surface effects [28], the number of test particles per pion must be chosen to be rather large, $\gtrsim 100$ for $F_{\rm max} \sim 10$. This method has been shown to yield the correct collision rates in thermal equilibrium, and the correct Bose-Einstein equilibrium states [29], but ceases to be practical as $\mu_\pi \to m_\pi$, when f becomes unbounded.

We first show the results of the calculation for $\tau_0^{\rm eff} = 7$ fm/c, which give our best fit. The solid line in Fig. 1(a) shows the rapidity distribution at final time $t_f = 29$ fm/c from an evolution of a system with the initial conditions given by the dashed curve. The circles are the experimental AB data of NA35 [1] and the solid line has been subjected to the NA35 software cuts ($|\mathbf{p}_{\rm lab}| > 0.1$ GeV/c and $0.5 < y_{\rm lab} < 4.5$). We see that the rapidity distribution has narrowed slightly relative to the input distribution to a full width at half maximum of $\Delta y_{\rm FWHM} = 3.4$. An effect in this direction is also seen in the heavy ion data relative to the pp data [1]. The solid line in Fig. 1(b) shows the corresponding transverse momentum distribution together with the experimental AB data [1]. The effect of Bose enhanced scattering to low transverse momenta has been to double the cross section in the lowest-

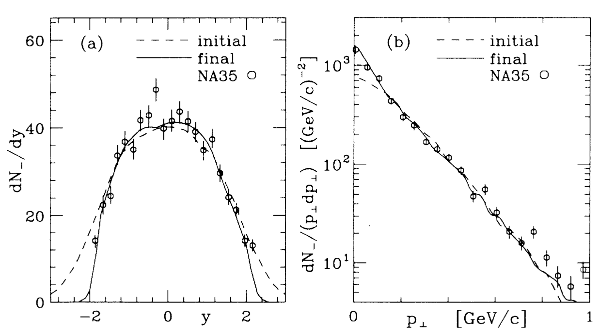

FIG. 1. (a) The input (dashed line) and calculated final (solid curve) rapidity distributions for negative pions with $\tau_0^{\rm eff} = 7$ fm/c. The circles are the NA35 negatives data for central 200 GeV/nucleon ^{16}O+Au collisions [1] shifted by $\Delta y = -2.3$ to the cm frame. (b) The initial (dashes) and calculated final (solid line) transverse momentum distributions for $\tau_0^{\rm eff} = 7$ fm/c in the rapidity interval $-0.3 < y < 0.7$. The circles are the NA35 AB data for $2 < y_{\rm lab} < 3$ [1].

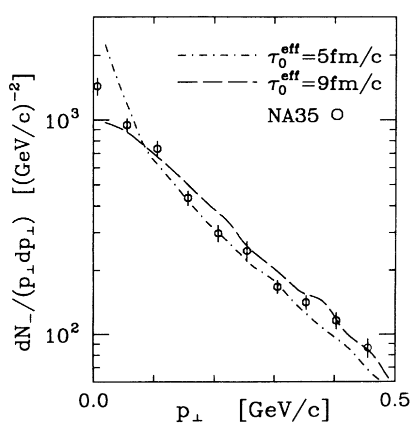

FIG. 2. Final transverse momentum spectra. Long-dashed curve: $\tau_0^{\rm eff} = 9$ fm/c and $\Delta y_b = 3.4$; dot-dashed line: $\tau_0^{\rm eff} = 5$ fm/c and $\Delta y_b = 3.8$. The normalization and rapidity cuts are as in Fig. 1(b). The circles are the NA35 O+Au data for $2 < y_{\rm lab} < 3$ [1].

$p_\perp$ bin and qualitatively change the low-$p_\perp$ shape of the spectrum. The simplified calculations here therefore suggest a pionization time of ~ 7 fm/c.

To see the sensitivity to τ_0^{eff} we show in Fig. 2 the $p_\perp$ spectra obtained from a longer pionization time of 9 fm/c (long-dashed curve). Since the accompanying narrowing of the rapidity distribution is less pronounced than before, we begin with a smaller initial Δy_b of 3.4. Note, however, that variations on the order of 5% in Δy_b do not qualitatively affect the $p_\perp$ spectrum. The Bose enhancement effect of the $p_\perp$ curve is reduced in comparison to the calculation of Fig. 1, and no longer consistent with the data [30]. Finally, for a smaller τ_0^{eff} of 5 fm/c, the soft pion enhancement is too pronounced, as is seen from the dot-dashed line in Fig. 2. Here we have taken $\Delta y_b = 3.8$ since the rapidity distribution narrows considerably during the evolution of the system. The curves in Fig. 2 clearly indicate that the soft pion enhancement is rather sensitive to τ_0^{eff}. These calculations therefore suggest limits of $6 \lesssim \tau_0^{\mathrm{eff}} \lesssim 8$ fm/c for the effective pionization time. This range is consistent with source size and duration parameters extracted from π-π correlations [13]. Of course, one has to bear in mind that mesonic resonances will be present in the hadron gas [19,20]. Introduction of these will decrease the pion phase-space density and hence the collision rate, at least for initial times. One may further expect intermediate-state Bose enhancement effects to reduce the in-medium π-π cross section [31], which will similarly reduce the enhancement. The above range for τ_0^{eff} may therefore well be an upper bound for the actual hadronization time [32].

Our findings have a number of implications for the main objective of ultrarelativistic collisions to discover a dense phase of matter such as the quark-gluon plasma and measure its properties. The result that the hadronization time scale is rather long and the QCD time scale is encouraging because the system then has adequate time to come to a local equilibrium. However, the finding that the pions are apparently produced out of chemical equilibrium suggests that any phase transition is a weak one. Conversion of matter via a strong first-order transition would occur across a well-defined spatial boundary via a slow evaporative process, with detailed balance. The produced particles would then be expected to be near chemical equilibrium. A pionization time of ~ 7 fm/c is somewhat short if a first-order transition with a large latent heat is involved. Of course, this time represents some sort of average and small fractions of the quark-gluon plasma could persist to much greater times.

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