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Role of nuclear binding in the European-Muon-Collaboration effect

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We present a new derivation of the convolution formula for the contributions of nuclear binding to the structure functions measured in the deep inelastic scattering of leptons from nuclei. The derivation, which is manifestly covariant, gives a new binding correction. This new correction, which depends on the mass of the recoiling nucleon fragments, gives corrections that are numerically significant, and that improve the agreement between theory and experiment at large x. We conclude that nuclear binding effects may be sufficient to explain the European-Muon-Collaboration effect at large x.

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I. INTRODUCTION

The importance of nuclear binding to the classical European-Muon-Collaboration (EMC) effect is still very much in question. On one hand, even from the beginning there were many explanations for this effect based on "new" physics associated with the quark structure of the nucleon [1], and, on the other hand, early calculations which produced large contributions from nuclear binding [2] are in doubt because of the incorrect treatment of wave-function normalization (sometimes referred to as omission of the "flux factor") [3]. Connected with these questions is the role of pions, which must be included somehow whenever nuclear binding is discussed [4–6]. Recently it has been found that the high momentum components arising from correlations can enhance the effect at large x [7, 8], but these results still fall somewhat short of the experimental data [9, 10]. Finally, there is uncertainty surrounding the derivation of the "smearing," or convolution formula, from which the nuclear binding effects are calculated. Attempts to derive it from the "instant form" of quantum mechanics have difficulties treating Lorentz covariance and current conservation in a consistent manner [11, 12], and usually employ a rather *ad hoc* prescription introduced by de Forest [13]. The popular "front form" derivations [11, 14] employ spectral functions which may be difficult to relate to the nonrelativistic densities normally available from nuclear structure models.

Regardless of the role that it will ultimately be found to play, it is essential to have a good treatment of nuclear binding. Fermi motion and nuclear binding are minimal effects which will always be present in the EMC data, and it is essential to know the contributions from these effects before new physics can be extracted. To be believable, nuclear binding must be calculated in a covariant, gauge-invariant manner, using a formalism in which there is a clear connection between the relativistic spectral function which necessarily enters the calculation and the nonrelativistic spectral function available from present nuclear theory.

In this paper, we present a new derivation of this important formula. Our method is based on relativistic Feynman diagrams, in which covariance is manifest and exact. This method is fully developed and has already been applied to a treatment of the two-body problem [15, 16], and extensions to the three- [17] and many-body problem [18] are being developed. The formalism has a smooth nonrelativistic limit, and can be used to treat electromagnetic interactions in a fully gauge-invariant manner [19], so it is ideal for the calculation of binding contributions to the EMC effect. Using this method, we obtain a new convolution formula which includes significant new effects. Evaluation of the new formula for realistic cases with realistic parameters shows that nuclear binding can account for the EMC effect for $x \geq 0.5$.

In this approach, deep inelastic scattering (DIS) is described by relativistic Feynman diagrams in which both the structure of the nucleon and the nuclear target are described by relativistic vertex functions in which one constituent is off-shell, as illustrated in Figs. 1(a)–1(c). The nucleon will be described by two such functions, $\Gamma_N^V(p, p_2)$ and $\Gamma_N^S(p, p_4)$, describing valence and sea contributions, respectively. As Fig. 1(a) suggests, the valence vertex function has two "spectators" while the sea function [shown in Fig. 1(b)] has a minimum of four, and we assume that the internal degrees of freedom of the spectators (either two or four) can be ignored, so that each Γ_N can be expanded as a sum of tensor operators multiplied by invariant functions which depend on p^2 and p_X^2 only, where $p_X = p_2$ or p_4. The structure of the nuclear target is similarly described by the vertex function $\Gamma_A(P, P_{A-1})$ shown in Fig. 1(c).

The use of vertex functions with one particle off-shell to describe nuclear structure has been developed extensively for the two-body problem, and provides a clear way to

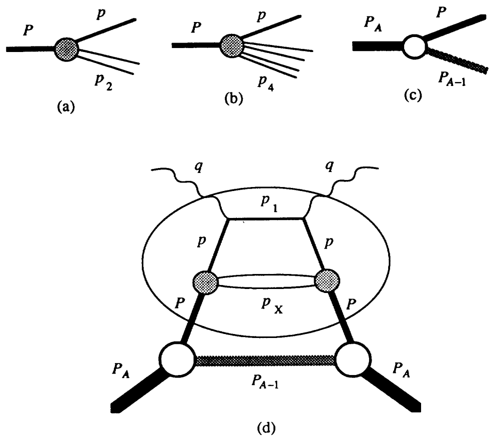

FIG. 1. The vertex functions and Feynman diagram used in the derivation. (a) The valence and (b) sea vertex functions have one quark with four-momentum p off-shell (represented by the dark line) and the remaining spectators on-shell. (c) The vertex function for the nucleus has one nucleon off-shell (the dark line) and the residual $A-1$ system on-shell. (d) The Feynman diagram with the momenta labeled, includes the off-shell nucleon structure function (inclosed by the oval).

describe the relativistic structure of a bound state. The relativistic wave function of the nucleon can be related to the quantities $\Psi_N \cong S_m(p)\,\Gamma_N^X(p,p_X)$, where $S_m(p)$ is the propagator of the virtual quark, and $X = V$ or S. For the nucleus, the relation is $\Psi_A \cong S_M(P)\,\Gamma_A(P,P_{A-1})$. These bound-state wave functions are manifestly covariant, and satisfy (known) relativistic wave equations and normalization conditions. Our assumption that the spectator quarks or spectator nucleons can be treated as a single system (with a variable mass) means that we may carry over all of this formalism to this problem.

The additional approximations made in the derivation of our convolution formula are common to almost all other treatments of nuclear binding. We assume that (i) final-state interactions and meson exchange currents can be ignored, so that the one-body current operator gives the leading contribution; (ii) interference terms can be ignored, so that the cross section is the incoherent sum of squared terms, as illustrated in Fig. 1(d); and (iii) any explicit dependence of the vertex functions on the mass of the bound state can be ignored, so that the vertex functions describing the nucleon structure will be assumed to have no direct dependence on the mass of the nucleon, P^2. It is known how to use final-state interactions and meson exchange currents to ensure gauge invariance of inelastic processes [19], and the first of these assumptions means that any effects arising from the gauge dependence of the plane wave processes are assumed to vanish in the Bjorken limit. This assumption is supported by specific estimates, when they exist [20], but is a subject for further study. The second assumption has not been widely discussed and might also benefit from further study. The third is the essence of the nuclear binding approximation, where it is assumed that the EMC effect can be explained by the binding and Fermi motion of nucleons without assuming any change in their intrinsic structure. This is the assumption we hope to be testing.

With these assumptions, the structure functions of the nucleus can be calculated from the diagram shown in Fig. 1(d). This calculation will be carried out in Sec. II. We obtain a new convolution formula which includes a dependence on the mass, m_X, of the fragments of the nucleon left behind by the struck quark. Numerical results obtained from our new formula will be given and discussed in Sec. III. Section IV includes further discussion and conclusions.

II. DERIVATION OF THE CONVOLUTION FORMULA

The inelastic cross section [21] from which the EMC data is obtained is

$$\frac{d^2\sigma}{d\Omega' dE'} = \sigma_M(W_2 + 2W_1 \tan^2\tfrac{1}{2}\theta\,), \tag{2.1}$$

where σ_M is the Mott cross section, and the structure function W_2 is a sum of the three amplitudes $W_{\lambda\lambda} = \epsilon_\lambda^{\mu *}\, W_{\mu\nu}\, \epsilon_\lambda^\nu$, where $\lambda = 0, \pm$ are the three polarization states of the virtual photon:

$$4\pi M\, W_2 = \frac{Q^2}{q_L^2}[W_{00} + \tfrac{1}{2}(W_{++} + W_{--})]\,. \tag{2.2}$$

With the normalization implied by Eq. (2.2), the $W_{\mu\nu}$ tensor for a spin-0 nucleus and a spin-$\frac{1}{2}$ residual system of fixed mass M_{A-1}, denoted by $W_{\mu\nu}^A$, can be obtained directly from diagram Fig. 1(d)

$$W_{\mu\nu}^A = \left(\frac{A}{2M_A}\right)\int \frac{d^3P_{A-1}}{(2\pi)^3}\left(\frac{M_{A-1}}{E_{A-1}}\right) \mathrm{tr}[\Lambda_{M_{A-1}}^T(P_{A-1})\overline{\Gamma}_A(P,P_{A-1})S_M(P)\, W_{\mu\nu}^N\, S_M(P)\Gamma_A(P,P_{A-1})]\,, \tag{2.3}$$

where we assume that there are A contributions, one for each nucleon, and $\Lambda_{M_{A-1}}(P_{A-1}) = (M_{A-1}+ \not{P}_{A-1})/2M_{A-1}$ is the projection operator for a spin-$\frac{1}{2}$ particle of mass M_{A-1} (the residual nuclear system in this case) and $S_M(P) = (M+\not{P})/(M^2-P^2)$ is the propagator of a spin-$\frac{1}{2}$ particle of mass M (the nucleon in this case). The tensor for a single nucleon, $W_{\mu\nu}^N$, is

$$W_{\mu\nu}^N = \overline{e}_q^{\,2}\int \frac{d^3p_X}{(2\pi)^3 2E_X}\frac{m_1}{E_1}\, 2\pi\delta(E_1+E_X-P_0-\nu)\,\overline{\Gamma}_N(p,p_X)S_{m_1}(p)\gamma_\mu\Lambda_{m_1}(p+q)\gamma_\nu S_{m_1}(p)\Gamma_N(p,p_X)\,, \tag{2.4}$$

where $\overline{e}_q^{\,2}$ is the average of the square of the quark charge, and we assume that the spectator system with fixed mass m_X has spin zero. An important feature of this method is that *both* energy and momentum are conserved at every vertex, and hence the four-vector q transferred to the quark is identical to the q transferred to the nucleus, removing ambiguities or effects of the kind encountered using instant form treatments employing the de Forest [13] prescription. Also, we are assured that the P_0 which enters into the δ function in (2.4) is equal to $M_A - E_{A-1}$, so that the convolution formula (2.3) is an exact statement of the fact that the only difference between scattering from a bound nucleon and a free nucleon is that $P^2 \neq M^2$.

One of the difficulties with the covariant formalism, in which the struck quark is initially off-shell, is that the elementary current "operator" γ_μ does not conserve current. This could be corrected by introducing the current conserving operator $\gamma_\mu - \not{q} q_\mu/q^2$, but such a procedure is *ad hoc*, and does not do justice to this approach, where it has recently been learned [19] how to assure current conservation naturally by including interaction currents and final-state interactions. We postpone this discussion for a later time in the expectation that these terms can be shown to vanish in the DIS limit, justifying some effective treatment such as the one we are using. In any case, expanding the virtual photon in terms of its four polarization vectors, as discussed in Ref. [21], leads to the observation that all terms proportional to q_μ can be dropped, making the use of the effective current operator $\gamma_\mu - \not{q} q_\mu/q^2$ completely equivalent to using γ_μ.

From studies of the relativistic equation satisfied by $\Gamma_A(P, P_{A-1})$, we can derive the following relativistic normalization condition [22]:

$$A = \left(\frac{A}{2M_A}\right) \int \frac{d^3P_{A-1}}{(2\pi)^3} \left(\frac{M_{A-1}}{E_{A-1}}\right) \mathrm{tr}[\Lambda^T_{M_{A-1}}(P_{A-1})\overline{\Gamma}_A(P, P_{A-1}) S_M(P)\gamma^0 S_M(P)\Gamma_A(P, P_{A-1})] \,. \tag{2.5}$$

Note that this is a different condition from that obtained in the light front formalism. It involves the charge (hence the operator γ^0) instead of the $(-)$ component of the current $(\gamma^0 - \gamma^z)$. However, our final result will be almost equivalent to the normalization obtained in light front theory (see below).

While the relativistic wave functions are known for the deuteron [23], this is not so for complex nuclei, so we approximate Eq. (2.3) and (2.5) by introducing a covariant nuclear spectral function, ρ_A,

$$S_M(P)\Gamma_A(P, P_{A-1})\Lambda^T_{M_{A-1}}(P_{A-1})\overline{\Gamma}_A(P, P_{A-1})S_M(P) = \rho_A(P^2, M^2_{A-1})\, \tfrac{1}{2}\Lambda_M(P), \tag{2.6}$$

where $P^2 \neq M^2$ in the projection operator on the right-hand side (RHS) of the equation. The matrix product on the left-hand side (LHS) of the equation is the relativistic density matrix of the bound nucleon, and the equation says that this can be approximated by the density matrix of a pure spin-$\frac{1}{2}$ system with four-momentum P. The remaining, spin independent, scalar function ρ_A can depend only on P^2 (provided M_{A-1} is fixed). In fact, the relativistic structure of the nuclear target will result, in general, in a more complicated spin dependence for the density matrix, but if the spin of the target is zero, the approximation (2.6) should be very good. [For the study of the spin-dependent EMC effect, Eq. (2.6) would not be sufficient.] With this definition, the convolution formula (2.3) reduces to

$$W^A_{\mu\nu} = \left(\frac{A}{2M_A}\right) \times \int \frac{d^3P_{A-1}}{(2\pi)^3} \left(\frac{M_{A-1}}{E_{A-1}}\right) \rho_A(P^2, M^2_{A-1})\, \overline{W}^N_{\mu\nu}, \tag{2.7}$$

where $\overline{W}^N_{\mu\nu}$ is the spin-averaged nucleon structure function for a bound nucleon

$$\overline{W}^N_{\mu\nu} = \tfrac{1}{2}\,\mathrm{tr}\left[W^N_{\mu\nu}\,\Lambda_M(P)\right] . \tag{2.8}$$

The normalization condition (2.5) similarly reduces to

$$A = \left(\frac{A}{2M_A}\right) \times \int \frac{d^3P_{A-1}}{(2\pi)^3} \left(\frac{M_{A-1}}{E_{A-1}}\right) \left(\frac{P_0}{M}\right) \rho_A(P^2, M^2_{A-1}). \tag{2.9}$$

The next step is to introduce a quark spectral function, ρ_N, similar to Eq. (2.6), but with a different normalization

$$S_{m_1}(p)\Gamma_N(p, p_X)\Lambda_M(P)\overline{\Gamma}^T_N(p, p_X)S^T_{m_1}(p) = \rho_N(p^2, p_X^2)\,(m_1 + \not{p}) \,. \tag{2.10}$$

This equation relates the relativistic density matrix for a quark in a spin-averaged nucleon (written on the LHS) to the product of a scalar spectral function of the two variables p^2 and p_X^2, multiplied by the relativistic density matrix for a pure spin-$\frac{1}{2}$ particle of four-momentum p. This is the simplest way of treating the spin, and is consistent with both the parton and nuclear binding models.

Taking the Bjorken limit, defined by Q^2 and $\nu \to \infty$ with $x = Q^2/2M\nu$ fixed, and choosing a coordinate system so that $q = (\nu, 0_\perp, q_L)$, leads to the following approximations:

$$\frac{1}{2}\sum_{\lambda} \not\epsilon^*_\lambda\,[m_q+\not p+\not q]\,\not\epsilon_\lambda \to \nu\,(\gamma_0-\gamma_z),$$
$$\delta(E_1+E_X-P_0-\nu)\to\delta(Mx-p_-), \tag{2.11}$$
$$E_1\to\nu,$$
$$\frac{Q^2}{q_L^2}\to\frac{2Mx}{\nu},$$

where the sum over λ is the same weighted sum that occurs in the definition of W_2, Eq. (2.2). Note that the light cone variable $p_- = p_0 - p_z$ appears naturally in the energy conservation relation. This suggests defining the momentum fractions y and z by

$$(p_X)_-\equiv P_-(1-y),\quad (P_{A-1})_-\equiv M_A\left(1-\frac{z}{A}\right), \tag{2.12}$$
$$p_-=P_-y,\quad P_-=\frac{M_A}{A}z.$$

We emphasize that our covariant formalism is *not* related directly to light front dynamics, and hence these momentum fractions should be viewed only as convenient substitutions for the p_- and P_- variables, motivated by the appearance of these variables in the energy-conserving delta function. Furthermore, the momentum fractions should be regarded as *defined* by the on-shell four-momenta p_X and P_{A-1} so that all four components of these momenta are known, and later we will be able to express $(p_X)_+$ and $(P_{A-1})_+$ in terms of y and z. Finally, note that even though y and z are defined by $(p_X)_-$ and $(P_{A-1})_-$, their relations to p_- and P_- are exact, because *energy and momentum are conserved at every vertex.*

With these definitions it is straightforward to obtain the following expression for $F_2^A(x)=\nu W_2^A$:

$$\begin{aligned}F_2^A(x)&=\int\frac{d^2k_\perp\,dz}{2(2\pi)^3}\left(\frac{z\,M_{A-1}}{(A-z)\,M}\right)\rho_A(P^2,M_{A-1}^2)\\&\quad\times\int dy\,\overline{F}_2^N(y,k)\,\delta\left(y-\frac{AMx}{zM_A}\right)\\&\equiv\int dz\,f(z,x),\end{aligned}\tag{2.13}$$

where $k_\perp\equiv P_\perp$, $k=(k_\perp,z)$, $f(z,x)$ is a shorthand notation for the entire integrand, which will prove useful later, and $\overline{F}_2^N(y,k)$ is the structure function of a bound nucleon, which depends explicitly on the nuclear motion through p^2, which is a function of k:

$$\overline{F}_2^N(y,k)=\overline{e}_q^{\,2}\,\frac{y^2}{(1-y)}\int\frac{d^2p_\perp}{4(2\pi)^3}\rho_N(p^2(k),p_X^2)\,. \tag{2.14}$$

The normalization condition (2.9) similarly reduces to

$$A=\int\frac{d^2k_\perp\,dz}{2(2\pi)^3}\left(\frac{z\,M_{A-1}}{(A-z)\,M}\right)\rho_A(P^2,M_{A-1}^2)\,. \tag{2.15}$$

To obtain these equations, it is necessary to express the integration over $(p_X)_z$ and $(P_{A-1})_z$ in terms of y and z, which can be done using (2.12) and the energy relations $E_X=\sqrt{m_X^2+\mathbf{p}_X^2}$ and $E_{A-1}=\sqrt{M_{A-1}^2+\mathbf{P}_{A-1}^2}$. Also, to obtain (2.15) from (2.5) observe that the factor P_0 in the numerator of (2.5) can be replaced by $P_0-P_z=P_-$ because the additional factor P_z integrates to zero.

Our results (2.13) and (2.15) are similar to those usually obtained from the parton model, with a few important differences. The most significant of these is the explicit dependence of the bound nucleon structure function, $\overline{F}_2^N$, on the momentum of the bound nucleon, k. As we will show now, this dependence gives additional corrections to the EMC effect, as anticipated in Ref. [24]. If these new effects are ignored our formula reduces to a form identical to the standard convolution formula. Another difference, which is not numerically important, is that the range of integration over the variable z is not the same.

To obtain a practical formula from Eq. (2.14), we exploit the fact that it displays the nucleon structure function as a product of two terms, the "kinematic factor" $y^2/(1-y)$ and the integral over the unknown function ρ_N of p^2. From the diagram shown in Fig. 1(d), and the definitions (2.12), it is a simple matter to show that

$$\begin{aligned}m_1^2-p^2&=m_1^2+p_\perp^2+\frac{y}{1-y}[m_X^2+(k_\perp-p_\perp)^2]-y(M^2+k_\perp^2)+yM^2\Delta\\&=\xi_0(y)+yM^2\Delta+\frac{1}{1-y}(p_\perp^2-2y\mathbf{p}_\perp\cdot\mathbf{k}_\perp+y^2k_\perp^2),\end{aligned}\tag{2.16}$$

where $\xi_0(y)=m_1^2+[y/(1-y)]m_X^2-yM^2$ and Δ is

$$\Delta=\frac{1}{M^2}\left(z\frac{M_{A-1}^2+k_\perp^2}{A-z}+M^2+k_\perp^2-z\frac{M_A^2}{A}\right). \tag{2.17}$$

Note that $\Delta=0$ if the nucleon is at rest, and there is no nuclear binding. The integral over $d^2p_\perp$ can be reexpressed by shifting $\mathbf{p}_\perp\to\mathbf{p}_\perp+y\mathbf{k}_\perp$, giving the following simple form:

$$\overline{F}_2^N(y,k)=y^2\int_{\xi_0(y)+yM^2\Delta}d\xi\;\frac{\overline{e}_q^{\,2}}{8(2\pi)^2}\,\rho_N(m_1^2-\xi,\,p_X^2)\,. \tag{2.18}$$

A similar formula (with $\Delta=0$) holds for the free structure function $F_2^N(y)$.

Using Eq. (2.18) it is therefore possible to express

$\overline{F}_2^N(y,k)$ in terms of the free structure function $F_2^N(y)$, evaluated at a shifted value of y. The shifted value, denoted by y', satisfies the equation

$$\xi_0(y) + yM^2\Delta = \xi_0(y'). \tag{2.19}$$

This transformation depends on the parameter m_X^2. The condition that the quark be bound is that $m_1+m_X \geq M$, and it seems most appropriate, in the DIS application, to regard the scattering as taking place from a current quark, in which case it is reasonable to take $m_1 = 0$ and $m_X \geq M$. The physical picture emerging from this choice is that the spectators and glue remaining behind constitute "most" of the nucleon, and hence most of its mass.

An additional reason for requiring $m_X \geq M$ is that this condition is necessary and sufficient for the mapping between $\xi_0(y)$ and y to be one to one. If this mapping is not one to one, the nucleon structure function, which is a function of y through $\xi_0(y)$ (except for the kinematic factor y^2), would be constrained by kinematics. The removal of such constraints, which occurs naturally when $m_X \geq M$, further supports the physical picture outlined above.

If $m_X \geq M$, the transformation determined by (2.19) becomes

$$y' = 1 - a(y) + \sqrt{a^2(y) - \frac{m_X^2}{M^2}},$$
$$a(y) = \frac{1}{2}\left((1-y) + \frac{m_X^2}{M^2}\frac{1}{1-y} + y\Delta\right) . \tag{2.20}$$

Since the y^2 factor in front of Eq. (2.18) does not get transformed, the transformation law is

$$\overline{F}_2^N(y,k) = \frac{y^2}{y'^2}F_2^N(y') . \tag{2.21}$$

This result, when combined with the basic convolution formula (2.13), will be referred to as the "fixed mass" formula, and the fact that $y' \neq y$ in (2.21) is an additional consequence of nuclear binding which seems to have been overlooked in previous treatments using this formalism.

The fixed mass formula (2.21) assumes a fixed mass m_X^2, which is a parameter unconstrained by the derivation except for the requirement that it be larger than M^2. If this mass is infinite, then the transformation (2.20) reduces to $y' = y$, and the convolution formula reduces to the familiar form obtained by many previous investigators. However, the physical picture of the process given in Fig. 1(d) suggests that the minimal value $m_X^2 = M^2$ is a better choice, and if m_X is finite, $y' > y$, as shown in Fig. 2 for an illustrative case. Since the free nucleon structure function decreases as y increases (at least at large y), this new result will predict a larger EMC effect. This is shown in Fig. 3, which gives the integrand $f(z,x)$, Eq. (2.13), for iron, evaluated at $x = 0.7$. The dotted line is the old result ($y' = y$), and the new result ($y' > y$) is smaller, particularly at large z. The resulting EMC effect will be larger.

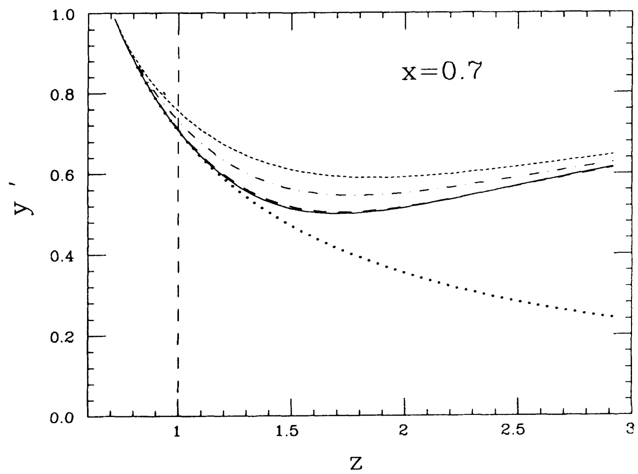

FIG. 2. The variable y' ploted as a function of the nucleon momentum fraction z for $x = 0.7$, $m_X = M$ and different values of $k_\perp = 0$ (solid), 1 (dashed), 3 (dash-dotted), and 4.5 (short dashed) fm^{-1}. The dotted line is $y' = y$, shown for comparison.

The model also suggests that a different convolution formula should hold for the sea and valence contributions. Since the sea contribution has at least four spectators, we might expect the effective value of $m_4 = 2m_2$, and this greatly reduces the EMC contribution for sea quarks (as will be discussed in the next section).

Finally, the fixed mass formula is only an approximation to Fig. 1(d); the full calculation requires that we integrate over m_X^2, weighting the integral by the appropriate phase-space factor (two body for the valence contribution, and four body for the sea contribution). If the dependence of the vertex function on m_X^2 can be ignored, this procedure gives an answer independent of the detailed structure of the nucleon. Making the approximation that the two-body relativistic phase space $\sqrt{(m_X^2 - M^2)/m_X^2}$ is a constant (which is good because of the rapid rise of the square root), the structure of (2.18) shows that the m_X^2 integral scales like $y/(1-y)$,

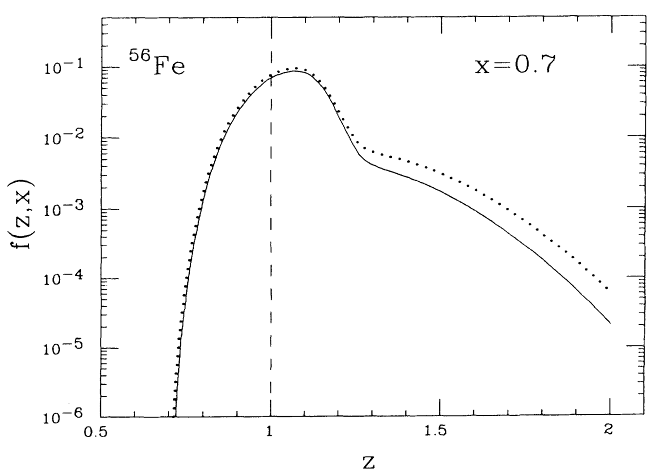

FIG. 3. The integrand $f(z,x)$ for iron, Eq. (2.13), evaluated at $x = 0.7$. The dotted and solid lines are the same cases discussed in Figs. 4 and 5.

giving an improved transformation law for the valence part

$$\overline{F}_{2V}^{N}(y,k) = \frac{y(1-y)}{y'(1-y')} F_{2V}^{N}(y') . \tag{2.22}$$

This result, when used in the basic convolution formula (2.13), will be referred to as the "phase-space" formula. The sea contribution involves a four-body phase-space integral, which is a convolution of three two-body phase-space factors, and does not scale unless all masses are ignored. If we make this naive approximation, just to get a feeling for the effect of the four-body phase space, it scales like $[y/(1-y)]^3$, leading to

$$\overline{F}_{2S}^{N}(y,k) = \frac{y'(1-y)^3}{y(1-y')^3} F_{2S}^{N}(y') . \tag{2.23}$$

However, this transformation law gives an infinite contribution from the sea quarks as $x \to 0$ (which implies $y \to 0$) because in this limit the ratio y'/y is very large. Physically, this arises because the formula (2.18) is independent of m_X at $y = 0$, and the phase-space integral diverges. (Even the valence contribution is unreliably evaluated at this point, but this is less serious because the valence part of F_2^N is also zero at $y = 0$.) Since the sea quarks make important contributions at small x (or y), we conclude that we do not yet have a reliable calculation of their contribution, and this part requires further study.

We now turn to a discussion of the numerical results obtained with the new convolution formula.

III. NUMERICAL RESULTS

Figure 4 shows the results for several different calculations of the EMC effect for an illustrative nucleus (^{12}C). The data are from Ref. [25]. All of these calculations were carried out using the same nuclear model [7, 8] in which the (nonrelativistic) spectral function is assumed to be composed of two parts: (i) contributions from the sum over the discrete bound states of the $A-1$ system, and (ii) more complex configurations including $A-1$ breakup channels. When correlations are included in the ground-state wave function, both of these configurations will play an important role, and guided by recent experiments and theory, the nuclear model assumes that 80% of the spectral function comes from the contributions of discrete final states, with an average removal energy of $< E_0 >= 23$ MeV (for ^{12}C), and the remaining 20% comes from the breakup channels, with a mean removal energy of $< E_1 >= 153$ MeV, giving an overall average removal energy of $< E >= 49$ MeV (see Refs. [7, 8] for details). The calculation of Refs. [7, 8], which uses the usual convolution formula (identical to ours if $y' = y$) and is our standard of comparison, is the dotted line in Fig. 4. This calculation already gives a larger contribution from nuclear binding (because of the effect of correlations) than most preceding ones, yet falls short of explaining the full effect at large x, where it should work best.

Two of the remaining four curves (the short and long dashed lines) in Fig. 4 use the "fixed mass" formula de-

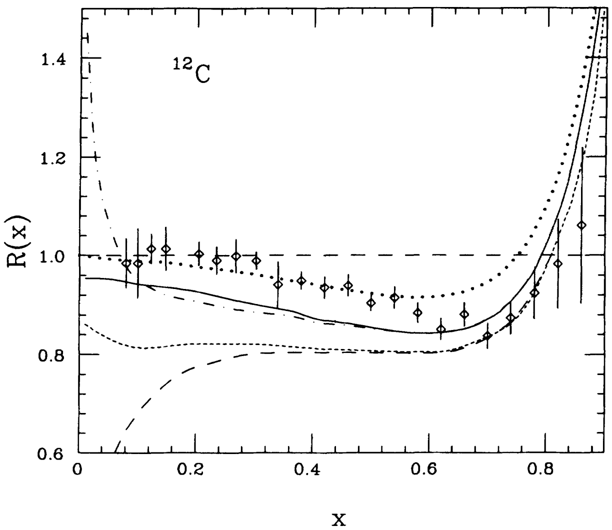

FIG. 4. The ratio R of F_2^A (for ^{12}C) divided by $6F_2^D$ (for the deuteron) plotted as a function of the scaling variable x. The five curves are discussed in the text. The data are from Ref. [25].

scribed in the preceding section, and two use the "phase-space" formula (the solid and dot-dashed lines). As we discussed above, the phase-space formula is preferred, and the two fixed mass cases are presented for comparison only. The long dashed curve is the fixed mass result when *both* the valence and sea quark contributions are evaluated with the same fixed mass $m_2 = m_4 = M$. This gives the maximum effect possible, and deviates strongly from the experimental data at small x. This deviation is due almost entirely to the sea contribution, as illustrated by the short dashed curve, which shows the fixed mass result when $m_2 = M$ and $m_4 = 2M$. The choice of a larger value for m_4 is strongly suggested by the model, and shows how sensitive the fixed mass formula is to the choice of m_4 (note that the two fixed mass curves are almost equal for $x > 0.5$, a reflection of the fact that the sea quark contribution vanishes there).

However, even the valence contribution is over emphasized by the fixed mass formula when we choose $m_2 = M$. A more realistic result is obtained from the phase-space formula, and the solid line shows the result when this formula (2.22) is used for the valence part, and *the sea part is not smeared (no EMC effect)*. (This latter idea is suggested by the absence of any observed EMC effect in Drell-Yan processes [26], which are dominated by sea contributions, but if we smear the sea part with a fixed mass $m_4 = 2M$, the result is quite similar to the curve shown.) This is the treatment most faithful to the physics contained in our approach, and hence is our theoretically preferred result. Finally, if we smear the sea part using (2.23), and use (2.22) for the valence part, we obtain the dot-dashed curve. This gives an unrealistic result at $x = 0$, as explained in Sec. III, and illustrates once again the sensitivity of the sea part to the treatment of the m_4 dependence.

Figure 5 shows how the results of Refs. [7, 8] and the preferred results of this paper compare with the data [25,

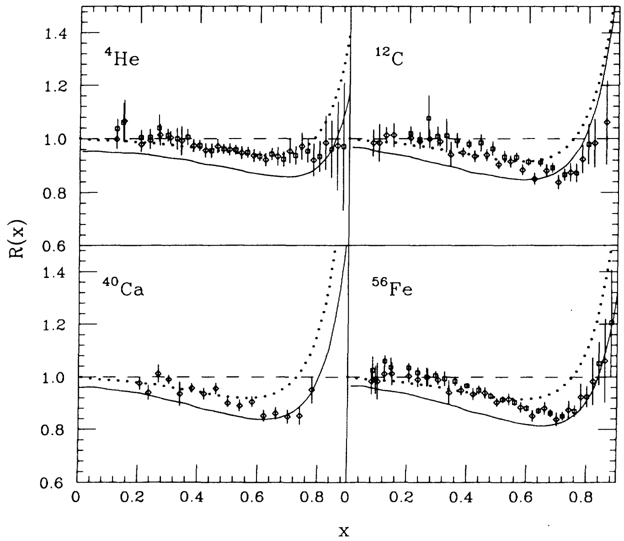

FIG. 5. The ratio R of F_2^A divided by $(A/2)F_2^D$ for ^{4}He, ^{12}C, ^{40}Ca, and ^{56}Fe. The dotted lines are the calculations of [7, 8], given for comparison, and the solid lines are the best results of this calculation (with the phase-space formula for the valence quarks and no EMC effect for the sea quarks). The two curves for ^{12}C are identical to the corresponding cases shown in Fig. 4. The data are from Ref. [25] (diamonds) and Ref. [27] (boxes).

27] for four illustrative nuclei: ^{4}He, ^{12}C, ^{40}Ca, and ^{56}Fe. In each case the mean removal energies and spectral functions of Refs. [7,8] were used. Note that the large x data are systematically well explained by the new convolution formula, but that the predicted effect is too big at low x.

Note that a feature of the results shown in Figs. 4 and 5 is that $R(0) \neq 1$. This is a consequence of the structure of the basic convolution formula (2.13) and occurs because of the behavior of the integrand as $y, y' \to 0$. It does *not* reflect any violation of baryon conservation, which is ensured by the normalization condition (2.15).

IV. CONCLUSIONS AND DISCUSSION

The derivation of the convolution formula given in Sec II, and the results shown in Figs. 4 and 5, suggest that nuclear binding can indeed explain the EMC effect at large x. In this region only the valence quarks can contribute, and the additional binding effects which we find are sufficient to give the extra contributions needed to reach agreement with data. Note that the results shown in Fig. 4 are rather insensitive to how the valence part is treated, and all methods give a substantial new contribution. The results given in Fig. 5 show that our success with ^{12}C is replicated for an illustrative set of nuclei. The success with ^{40}Ca and ^{56}Fe is particularly striking.

The same cannot be said for the description of the low x region. Here the additional effects spoil the rather good description obtained in most previous calculations. This may be due to the fact that many other processes contribute to this region, including the sea contributions and possible contributions from the mesons which partly account for the nuclear force, and are in any case present in the nuclear medium. We take the sensitivity of our model to the way in which the sea contributions are handled as a signature of the fact that a better description is needed before it is possible to carry out a reliable calculation of the EMC effect in this region. The model itself suggests additional work which needs to be done, including (i) the development of a more microscopic description of the sea contribution, (ii) a detailed treatment of the additional interactions which are known to contribute because of gauge invariance (final state interactions and interaction currents), with a possible demonstration that they do not contribute in the DIS limit, and (iii) inclusion of meson (pion) contributions and the restoration of the momentum sum rule.

There are some indications that the pion contributions may have the behavior needed to correct the low x results. Using the estimate worked out by Llewellyn Smith [4] (which may, however, not apply here) it looks like a small pion enhancement of 4% per nucleon would bring the ratios R up to unity at $x = 0$. It is known that a pion enhancement will only contribute at small x, but the toy model worked out in Ref. [6] suggests that this contribution might well extend out to $x \cong 0.6$. Such a contribution would also correct the violation of the momentum sum rule which is a feature of the present result. Furthermore, a small pion enhancement may not be contradicted by the recent Drell-Yan measurements [26], which show that the EMC effect for sea quarks must be very small. Before any definite conclusions can be drawn, a completely new calculation, in which these effects are treated in a manner consistent with the covariant formalism, is needed.

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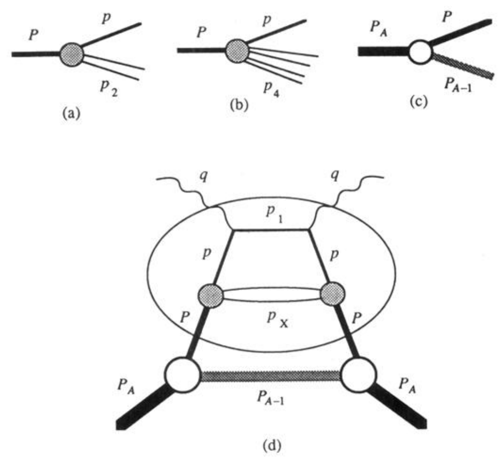

FIG. 1. The vertex functions and Feynman diagram used in the derivation. (a) The valence and (b) sea vertex functions have one quark with four-momentum p off-shell (represented by the dark line) and the remaining spectators on-shell. (c) The vertex function for the nucleus has one nucleon off-shell (the dark line) and the residual $A-1$ system on-shell. (d) The Feynman diagram with the momenta labeled, includes the off-shell nucleon structure function (inclosed by the oval).