Modified random phase approximation for multipole excitations at finite temperature

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The modified finite-temperature random phase approximation (FT-RPA) has been constructed by taking the influence of thermostat on the structure of quasiparticles into account. The modified FT-RPA linear response for electric quadrupole ($\lambda^{\pi}=2^{+}$) and octupole ($\lambda^{\pi}=3^{-}$) excitations in ^{58}Ni has been calculated as a function of the nuclear temperature. As compared to the conventional FT-RPA, the modified FT-RPA has given a stronger spreading for the strength distribution of quadrupole excitations at finite temperature $T \leq 3$ MeV.

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I. INTRODUCTION

The finite-temperature random phase approximation (FT-RPA) has been the background of many microscopic theories studying the temperature-dependent effects upon collective vibrations in hot finite nuclear systems [1–6]. Based on it, interesting features related to the nonvanishing thermal superfluid pairing gap [7], to the thermal splitting and collapse of low-lying vibrational states [1,3,5], and to the shift and broadening of strength distributions for isoscalar (IS) and isovector (IV) giant resonances (GR) have been investigated in detail [8–10]. The difficulty associated with discrete FT-RPA calculations of collective roots and strength distributions in a pair configuration space with large dimension has been avoided by applying alternatives of linear response techniques [4,11,12].

However, in the conventional FT-RPA [1–7] and in the approaches constructed on it [8–10] the thermal single-particle or quasiparticle excitations have been generated by operators of zero-temperature particles or quasiparticles immersed in the thermal Hartree-Fock or Hartree-Fock-Bogolubov mean field. The temperature-dependent effects, therefore, have been accounted for only by averaging over the statistical ensemble appearing in the single-particle or quasiparticle occupation numbers. The influence of the thermodynamical ensemble, playing the role of thermostat, on the structure of quasiparticle operators has been ignored so far.

Recently, based on the thermal Bogolubov canonical transformation in the formalism of thermofield dynamics (TFD) [13], we have constructed the microscopic FT-RPA operators from the zero-temperature two-quasiparticle ones and derived for them a Dyson boson realization [14]. The adoption of the thermal Bogolubov transformations allowed us to take into account the effects of the thermostat on the structure of thermal quasiparticles.

In this paper we are going to construct a modified version of the FT-RPA making use of the FT-RPA microscopic operators constructed in [14] under the influence of thermostat. We shall apply this modified FT-RPA to study the quadrupole ($\lambda^{\pi}=2^{+}$) and octupole ($\lambda^{\pi}=3^{-}$) vibrations in the case of separable multipole interactions for a superfluid spherical nucleus. Applying the linear response techniques in conjunction with Bethe-Salpeter's equation method [12], we shall compare the results obtained in the modified FT-RPA with those of the conventional one.

The paper is organized as follow. In Sec. II we discuss the theoretical details of the modified FT-RPA in the linear response treatment. Results of calculations of strength distributions for multipole vibrations $\lambda^{\pi}=2^{+}$ and 3^{-} for ^{58}Ni are shown in Sec. III. Conclusions are drawn in Sec. IV.

II. FORMALISM

In this section for sake of convenience we describe briefly the main features of the model Hamiltonian we use as well as the conventional FT-RPA equation [5] based on this Hamiltonian in Sec. II A. This summary will make easier the comparison with the modified version of FT-RPA derived in Sec. II B.

A. Model Hamiltonian and conventional FT-RPA

For the investigations of electric ($E\lambda-$) transitions with multipolarity λ we adopt the Hamiltonian [15] which consists of terms describing, respectively, the nucleon motion in the mean field $H_{\rm av}$, the monopole superfluid pairing interaction $H_{\rm pair}$, and the residual interactions in the form of separable multipole particle-hole (p-h) forces $H_{\rm ph}$,

$$H=H_{\rm av}+H_{\rm pair}+H_{\rm ph}\ , \tag{1}$$

where

$$H_{\rm av}=\sum_{jmt_z}[E_j(t_z)-\lambda_{t_z}]a^{\dagger}_{jm}a_{jm}\ ,$$
$$H_{\rm pair}=-\tfrac{1}{4}\sum_{t_z}G(t_z)\sum_{jmj'm'}a^{\dagger}_{jm}a^{\dagger}_{j\bar{m}}a_{j'\bar{m}'}a_{j'm'}\ , \tag{2}$$
$$H_{\rm ph}=-\tfrac{1}{2}\sum_{\lambda\mu}\sum_{t_z\rho=\pm1}(\kappa_0^{(\lambda)}+\rho\kappa_1^{(\lambda)})M^{\dagger}_{\lambda\mu}(t_z)M_{\lambda\mu}(\rho t_z)\ .$$

In Eqs. (1) and (2) we have used the standard notations widely employed in [15]. Namely, $a^{\dagger}_{jm}$ and a_{jm} are the single-particle operators; t_z is the z projection of the nucleon isospin operator; $E_j(t_z)$ are single-particle energies for neutron (n) and proton (p); $G(t_z)$ are the superfluid pairing constants; $\kappa_0{}^{(\lambda)}$ and $\kappa_1{}^{(\lambda)}$ denote the IS and IV constants of separable multipole forces; $M^{\dagger}_{\lambda\mu}(t_z)$ and $M_{\lambda\mu}(\rho t_z)$ are the creation and annihilation multipole operators. The tilde stands for the time-reversing operation: $a_{j\tilde{m}}=(-)^{j-m}a_{j-m}$. Using the standard canonical Bogolubov transformation

$$a^{\dagger}_{jm}=u_j\alpha^{\dagger}_{jm}+v_j\alpha_{j\tilde{m}} \tag{3}$$

the Hamiltonian (1) is expressed in terms of the quasiparticle creation $\alpha^{\dagger}_{jm}$ and annihilation α_{jm} operators. At finite temperature the quasiparticle energies ε_j are found by solving the temperature-dependent BCS equations (FT-BCS) [16]

$$\varepsilon_j(T)=\sqrt{(E_j-\lambda)^2+\Delta_T^2}\ , \tag{4}$$

where the superfluid pairing gap Δ_T is defined as

$$\Delta_T=G\sum_j(j+\tfrac{1}{2})u_jv_j(1-2n_j) \tag{5}$$

with n_j being the quasiparticle occupation number at temperature T:

$$n_j=[\exp(\varepsilon_j/T)+1]^{-1} \tag{6}$$

for both neutron and proton components. The single-particle energies E_j and the chemical potential λ are found by the average nucleon number conserving condition

$$N=\sum_j(j+\tfrac{1}{2})\left[1-\frac{E_j-\lambda}{\varepsilon_j}(1-2n_j)\right] \tag{7}$$

and depend, in general, also on T. Due to the finiteness of realistic nuclei the thermal fluctuations must be included [17,18]. Consequently, the thermal average pairing gap $\langle\Delta_T\rangle$ which does not collapse at the critical temperature $T_c\sim\frac{1}{2}\Delta_{T=0}$ should be adopted instead of Δ_T from Eq. (5). It is defined as [17,18]

$$\langle\Delta_T\rangle=\int_0^{\infty}\Delta_TP(\Delta_T,T)d\Delta_T\Big/\int_0^{\infty}P(\Delta_T,T)d\Delta_T\ , \tag{8}$$

where $P(\Delta_T,T)$ is the probability that the pairing gap in the system with the free energy $F(\Delta_T)$ takes the value Δ_T at T:

$$P(\Delta_T,T)\sim\exp[-F(\Delta_T)/T]\ . \tag{9}$$

The gap $\langle\Delta_T\rangle$ will be used in our calculations in Sec. III.

The multipole operator $M^{\dagger}(t_z)$ from Eq. (2) in the quasiparticle representation has the form [15]

$$M^{\dagger}_{\lambda\mu}(t_z)=\frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda+1}}\sum_{jj'}{}^{t_z}f^{(\lambda)}_{jj'}\{\tfrac{1}{2}u^{(+)}_{jj'}[A^{\dagger}_{\lambda\mu}(jj')+A_{\lambda\tilde{\mu}}(jj')]+v^{(-)}_{jj'}B_{\lambda\mu}(jj')\} \tag{10}$$

with $f^{(\lambda)}_{jj'}=\langle j'\|iR_\lambda(r)Y_\lambda\|j\rangle$ being the single-particle matrix elements corresponding to the separable multipole interactions; $u^{(+)}_{jj'}=u_jv_{j'}+u_{j'}v_j$, $v^{(-)}_{jj'}=u_ju_{j'}-v_jv_{j'}$ and the well-known two-quasiparticle operators $A^{\dagger}$, A, B, $B^{\dagger}$:

$$\begin{aligned}A^{\dagger}_{\lambda\mu}(jj')&=\sum_{jj'}\langle jmj'm'|\lambda\mu\rangle\alpha^{\dagger}_{jm}\alpha^{\dagger}_{j'm'}\ ,\\ B_{\lambda\mu}(jj')&=-\sum_{jj'}\langle jmj'm'|\lambda\mu\rangle\alpha^{\dagger}_{jm}\alpha_{j'\tilde{m}'}\ .\end{aligned} \tag{11}$$

In Ref. [5] we have introduced the microscopic collective operators called thermal phonon ones of type

$$\begin{aligned}Q^{\dagger}_{\lambda\mu i}(T)&=\tfrac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}A^{\dagger}_{\lambda\mu}(jj')-\varphi^{\lambda i}_{jj'}A_{\lambda\tilde{\mu}}(jj')+\xi^{\lambda i}_{jj'}B_{\lambda\mu}(jj')-\zeta^{\lambda i}_{jj'}B^{\dagger}_{\lambda\tilde{\mu}}(jj')\}\ ,\\ Q_{\lambda\mu i}(T)&=[Q^{\dagger}_{\lambda\mu i}(T)]\end{aligned} \tag{12}$$

and linearized the equations of motion for operators (12) to obtain the FT-RPA equation

$$\mathcal{F}[X_T(\omega)]\equiv1+(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})[X_T^{0(n)}(\omega)+X_T^{0(p)}(\omega)]+4\kappa_0^{(\lambda)}\kappa_1^{(\lambda)}X_T^{0(n)}(\omega)X_T^{0(p)}(\omega)=0 \tag{13}$$

with

$$X_T^0(\omega)=-\frac{1}{2\lambda+1}\sum_{jj'}{}^{t_z}[f^{(\lambda)}_{jj'}]^2\left[\frac{u^{(+)2}_{jj'}(1-n_j-n_{j'})(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2}-\frac{v^{(-)2}_{jj'}(n_j-n_{j'})(\varepsilon_j-\varepsilon_{j'})}{(\varepsilon_j-\varepsilon_{j'})^2-\omega^2}\right]\ . \tag{14}$$

The quasiparticle occupation number n_j in Eq. (14) appeared as a result of averaging over statistical ensemble and by using the commutation relations valid to $O(n_j)$ [1,5]

$$\begin{aligned}&\langle[A_{\lambda\mu}(j_aj_b),A^{\dagger}_{\lambda'\mu'}(j_cj_d)]\rangle\\&\quad=\delta_{\lambda\lambda'}\delta_{\mu\mu'}[\delta_{j_aj_c}\delta_{j_bj_d}-(-)^{j_a+j_b-\lambda}\delta_{j_aj_d}\delta_{j_bj_c}]\\&\qquad\times(1-n_{j_a}-n_{j_b})\ ,\\&\langle[B^{\dagger}_{\lambda\mu}(j_aj_b),B_{\lambda'\mu'}(j_cj_d)]\rangle=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{j_aj_c}\delta_{j_bj_d}(n_{j_b}-n_{j_a})\ ,\end{aligned} \tag{15}$$

where

$$\langle\cdots\rangle=\mathrm{Tr}[\cdots\exp(-H/T)]/\mathrm{Tr}[\exp(-H/T)]\ . \tag{16}$$

We note that the same equations as (13) and (14) have been also obtained in [1,2] by using the techniques of temperature-dependent Green's-function method. From Eqs. (12)–(16) one can see that the only temperature effects included in the FT-RPA equation (13) are accounted for by using the thermal average (16) leading to the quasiparticle occupation numbers of type (6). At the same time the two-quasiparticle operators forming the thermal phonons (12) have the same form as those at zero temperature with the temperature-dependent quasiparticle energies $\varepsilon_j(T)$ calculated from Eq. (4).

B. Modified FT-RPA

We are going now to consider how the effects of the thermostat on the quasiparticle excitations modify the FT-RPA equations. According to the conception of the TFD [13] the influence of temperature appears in the structure of quasiparticle operators $\alpha_{jm}^\dagger$ and α_{jm} through the thermal Bogolubov transformation [13]

$$\begin{aligned} \alpha_{jm}^\dagger(T) &= \sqrt{1-n_j}\,\alpha_{jm}^\dagger - \sqrt{n_j}\,\alpha_{j\tilde{m}} , \\ \alpha_{j\tilde{m}}(T) &= \sqrt{1-n_j}\,\alpha_{j\tilde{m}} + \sqrt{n_j}\,\alpha_{jm}^\dagger . \end{aligned} \tag{17}$$

The thermal quasiparticles $\alpha_{jm}^\dagger(T)$ and $\alpha_{j\tilde{m}}(T)$ in the left-hand side of Eq. (17) describe the states with quasiparticle excitation $\alpha_{jm}^\dagger$ and quasihole annihilation $\alpha_{j\tilde{m}}$ existed due to the thermostat. Using Eqs. (3) and (17) we find that the transformation from particle creation and annihilation operators $a_{jm}^\dagger$ and a_{jm} to the thermal quasiparticle creation and annihilation operators $\alpha_{jm}^\dagger(T)$ and $\alpha_{jm}(T)$ has the same form as Eq. (3)

$$a_{jm}^\dagger = \tilde{u}_j \alpha_{jm}^\dagger(T) + \tilde{v}_j \alpha_{j\tilde{m}}(T) , \tag{18}$$

where, however,

$$\begin{aligned} \tilde{u}_j &= \sqrt{1-n_j}\,u_j + \sqrt{n_j}\,v_j , \\ \tilde{v}_j &= \sqrt{1-n_j}\,v_j - \sqrt{n_j}\,u_j . \end{aligned} \tag{19}$$

The thermal two-quasiparticle operators (with subscript T) $A_{\lambda\mu}^\dagger(jj')_T$, $A_{\lambda\mu}(jj')_T$, $B_{\lambda\mu}(jj')_T$, and $B_{\lambda\mu}^\dagger(jj')_T$ have been expressed in terms of operators $A_{\lambda\mu}^\dagger(jj')$, $A_{\lambda\mu}(jj')$, $B_{\lambda\mu}(jj')$, and $B_{\lambda\mu}^\dagger(jj')$ from Eq. (11) based on Eq. (17) in Ref. [14] as

$$\begin{aligned} A_{\lambda\mu}^\dagger(jj')_T &= \sqrt{(1-n_j)(1-n_{j'})}\,A^\dagger_{\lambda\mu}(jj') - \sqrt{n_j n_{j'}}\,A_{\lambda\tilde{\mu}}(jj') \\ &\quad + \sqrt{(1-n_j)n_{j'}}\,B_{\lambda\mu}(jj') + \sqrt{(1-n_{j'})n_j}\,B_{\lambda\tilde{\mu}}(jj') \\ &\quad - \sqrt{2j+1}\sqrt{(1-n_j)n_j}\,\delta_{jj'}\delta_{\lambda 0} , \\ B_{\lambda\mu}(jj')_T &= \sqrt{(1-n_j)(1-n_{j'})}\,B_{\lambda\mu}(jj') - \sqrt{n_j n_{j'}}\,B_{\lambda\tilde{\mu}}^\dagger(jj') \\ &\quad - \sqrt{(1-n_j)n_{j'}}\,A^\dagger_{\lambda\mu}(jj') - \sqrt{(1-n_{j'})n_j}\,A_{\lambda\tilde{\mu}}(jj') \\ &\quad + \sqrt{2j+1}\,n_j\delta_{jj'}\delta_{\lambda 0} , \\ A_{\lambda\mu}(jj')_T &= [A_{\lambda\mu}^\dagger(jj')_T]^\dagger, \quad B_{\lambda\mu}^\dagger(jj')_T = [B_{\lambda\mu}(jj')_T]^\dagger . \end{aligned} \tag{20}$$

Due to the canonical transformation (17) the thermal two-quasiparticles $A_T^\dagger$, A_T, B_T, and $B_T^\dagger$ from Eq. (20) satisfy the same commutation relations (15) of the zero-temperature operators $A^\dagger$, A, B, and $B^\dagger$, immersed in the thermostat. In fact, by using the Dyson boson realizations obtained in Ref. [14] for thermal two-quasiparticle operators $A_T^\dagger$, A_T, B_T, and $B_T^\dagger$ one can easily verify the validity [to $O(n_j)$] of Eqs. (15) for operators $A_T^\dagger$, A_T, B_T and $B_T^\dagger$.

Employing Eqs. (20) we have shown in Ref. [14] that the thermal phonon operators (12) have the same form as the conventional microscopic RPA phonon ones:

$$\begin{aligned} Q_{\lambda\mu i}^\dagger(T) &= \tfrac{1}{2}\sum_{jj'} [x_{jj'}^{\lambda i} A_{\lambda\mu}^\dagger(jj')_T - y_{jj'}^{\lambda i} A_{\lambda\tilde{\mu}}(jj')_T] , \\ Q_{\lambda\tilde{\mu} i}(T) &= \tfrac{1}{2}\sum_{jj'} [x_{jj'}^{\lambda i} A_{\lambda\tilde{\mu}}(jj')_T - y_{jj'}^{\lambda i} A_{\lambda\mu}^\dagger(jj')_T] , \end{aligned} \tag{21}$$

where the relations between the amplitudes $x_{jj'}^{\lambda i}$, $y_{jj'}^{\lambda i}$ and the amplitudes ψ, ϕ, ξ, and ζ in the former expression (12) of thermal phonon operators $Q_{\lambda\mu i}^\dagger(T)$ and $Q_{\lambda\mu i}(T)$ are [14]

$$\begin{aligned} \psi_{jj'}^{\lambda i} &= x_{jj'}^{\lambda i}\sqrt{(1-n_j)(1-n_{j'})} + y_{jj'}^{\lambda i}\sqrt{n_j n_{j'}} , \\ \varphi_{jj'}^{\lambda i} &= y_{jj'}^{\lambda i}\sqrt{(1-n_j)(1-n_{j'})} + x_{jj'}^{\lambda i}\sqrt{n_j n_{j'}} , \\ \xi_{jj'}^{\lambda i} &= x_{jj'}^{\lambda i}\sqrt{(1-n_j)n_{j'}} - y_{jj'}^{\lambda i}\sqrt{(1-n_{j'})n_j} , \\ \zeta_{jj'}^{\lambda i} &= y_{jj'}^{\lambda i}\sqrt{(1-n_j)n_{j'}} - x_{jj'}^{\lambda i}\sqrt{(1-n_{j'})n_j} . \end{aligned} \tag{22}$$

It is noteworthy that thermal phonon operators (21) consist only of the thermal two quasiparticle $A_{\lambda\mu}^\dagger(jj')_T$ and $A_{\lambda\tilde{\mu}}(jj')_T$ from Eqs. (21).

Applying now the usual procedure of linearizing the equations of motion for operators (21) with Hamiltonian (1) or the standard variational procedure (see, e.g., Ref. [15]) we obtain after some transformations the modified FT-RPA equation, which has the same form as Eq. (13) where, however, instead of matrix $X_T^0(\omega)$ (14) the matrix

$$\tilde{X}_T^0(\omega) = \frac{1}{2\lambda+1}\sum_{jj'}{}^{t_z} [f_{jj'}^{(\lambda)}]^2 \frac{\tilde{u}_{jj'}^{(+)2}(1-n_j-n_{j'})(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2} \tag{23}$$

is understood. In Eq. (23) functions $\tilde{u}_{jj'}^{(+)}$ are combinations of coefficients $\tilde{u}_j$ and $\tilde{v}_j$ from Eq. (19): $\tilde{u}_{jj'}^{(+)} = \tilde{u}_j\tilde{v}_{j'} + \tilde{u}_{j'}\tilde{v}_j$. We note that in Eq. (23) there are only the poles of type $(\varepsilon_j+\varepsilon_{j'})$, while the poles $(\varepsilon_j-\varepsilon_{j'})$ as in the conventional FT-RPA equation (14) are absent. The appearance of the poles $(\varepsilon_j-\varepsilon_{j'})$ in (14) are caused by the presence of operators B and $B^\dagger$ in the definition (12) [1,2,5] while in Eqs. (21) these operators are involved in operators $A_T^\dagger$ and A_T by the transformation (20). Working with operators $A_T^\dagger$ and A_T from the definition (21), we obtain only the poles $(\varepsilon_j+\varepsilon_{j'})$. This interesting feature of the present modified FT-RPA allows us to avoid the problem with (p-p) and (h-h) states which appear between the poles $\varepsilon_j-\varepsilon_{j'}$ and can be localized near zero when $\varepsilon_j \sim \varepsilon_{j'}$. The presence of a great number of (p-p) and (h-h) states, which are generally less energetic than the (p-h) ones, has increased too much the dimension of pair configuration space in the conventional FT-RPA. Therefore they have complicated the computation procedure and put under question the traditional choice of parameters for multipole interactions. In the modified FT-RPA the contribution of operators B and $B^\dagger$ is accounted effectively by using the definition (21). The dimension of two-quasiparticle space is the same as in zero-temperature RPA.

Writing the modified FT-RPA equation in terms of the unperturbed linear response matrix $\tilde{X}_T^0$ from (23) on the complex energy plane $\Omega=\omega+i\eta$

$$\tilde{X}_T^0(\Omega)=\frac{1}{2(2\lambda+1)}\sum_{jj'}{}^{t_z}[f_{jj'}^{(\lambda)}]^2\tilde{u}_{jj'}^{(+)2}(1-n_j-n_{j'})\left[\frac{1}{\omega-(\varepsilon_j+\varepsilon_{j'})+i\eta}-\frac{1}{\omega+(\varepsilon_j+\varepsilon_{j'})+i\eta}\right], \quad (24)$$

we can use the linearized Bethe-Salpeter equation to obtain the coupled system of linear response matrices. Since the application of this method has been elaborated in detail in Ref. [12], we give here only the final formulas of the linear response matrices $\tilde{X}_T(\Omega)_{t_z,t_{z'}}$ in neutron-neutron, proton-proton, and neutron-proton channels. They read (cf. Ref. [12])

$$\begin{aligned}\tilde{X}_T(\Omega)_{nn}&=[1+(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})\tilde{X}_T^{0(p)}(\Omega)]\tilde{X}_T^{0(n)}(\Omega)/\mathcal{F}[\tilde{X}_T^0(\Omega)]\ ,\\ \tilde{X}_T(\Omega)_{pp}&=[1+(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})\tilde{X}_T^{0(n)}(\Omega)]\tilde{X}_T^{0(p)}(\Omega)/\mathcal{F}[\tilde{X}_T^0(\Omega)]\ ,\\ \tilde{X}_T(\Omega)_{np}&=-(\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})\tilde{X}_T^{0(n)}(\Omega)\tilde{X}_T^{0(p)}(\Omega)/\mathcal{F}[\tilde{X}_T^0(\Omega)]\ ,\end{aligned} \quad (25)$$

where $\mathcal{F}[\tilde{X}_T^0(\Omega)]$ is defined from the left-hand side of Eq. (13) with replacing $X_T^0(\Omega)$ by $\tilde{X}_T^0(\Omega)$ (24) for neutron and proton components, respectively.

The corresponding IS and IV linear response matrices have the form [12]

$$\begin{aligned}\tilde{X}_T(\Omega,\tau=0)&=\tilde{X}_T(\Omega)_{nn}+\tilde{X}_T(\Omega)_{pp}+2\tilde{X}_T(\Omega)_{np}\ ,\\ \tilde{X}_T(\Omega,\tau=1)&=\tilde{X}_T(\Omega)_{nn}+\tilde{X}_T(\Omega)_{pp}-2\tilde{X}_T(\Omega)_{np}\ .\end{aligned} \quad (26)$$

From Eqs. (24)–(26) we can calculate the unperturbed and FT-RPA strength functions $S^0(E\lambda,\omega,\tau)$ and $S(E\lambda,\omega,\tau)$ as the imaginary part of linear response matrices $\tilde{X}_T^0(\Omega)$ and $\tilde{X}_T(\Omega)$, respectively,

$$\begin{aligned}S^0(E\lambda,\omega,\tau)&=-\frac{1}{\pi}\mathrm{Im}\tilde{X}_T^0(E\lambda,\Omega,\tau)\ ,\\ S(E\lambda,\omega,\tau)&=-\frac{1}{\pi}\mathrm{Im}\tilde{X}_T(E\lambda,\Omega,\tau)\ .\end{aligned} \quad (27)$$

The FT-RPA moments $m_k(T)_\tau$ are defined in the energy interval $E_1\le\omega\le E_2$ as

$$m_k(T)_\tau=\int_{E_1}^{E_2}S(E\lambda,\omega,\tau)\omega^k d\omega\ . \quad (28)$$

The centroid energy $\bar{E}_T$ and the spreading width Γ of the multipole strength distributions are calculated based on Eq. (28) in a standard way

$$\begin{aligned}\bar{E}_T&=m_1(T)_\tau/m_0(T)_\tau\ ,\\ \Gamma&=\left[m_0^{-1}(T)_\tau\int_{E_1}^{E_2}S(E\lambda,\omega,\tau)(\omega-\bar{E}_T)^2d\omega\right]^{1/2}\\ &=\{m_2(T)_\tau/m_0(T)_\tau-\bar{E}_T^2\}^{1/2}\ .\end{aligned} \quad (29)$$

III. NUMERICAL RESULTS

We are now going to present and discuss the numerical results of our calculations of the linear response functions in the modified FT-RPA for $E2$ and $E3$ excitations in hot spherical ^{58}Ni nucleus. These results will be shown in comparison with those of the calculations based on the conventional FT-RPA equations (13) and (14).

The single-particle energies have been calculated in the Woods-Saxon potential W, whose parameters have been

TABLE I. Energy-weighted sum rules (EWSR) for electric quadrupole ($\lambda^\pi=2^+$) and octupole ($\lambda^\pi=3^-$) excitations in ^{58}Ni. Neutron (n), proton (p), isoscalar ($\tau=0$), and isovector ($\tau=1$) EWSR are displayed at $T=0$ and 3 MeV. Columns (a) show the values obtained in the conventional FT-RPA. Columns (b) present the results corresponding to the modified FT-RPA. All values are given in single-particle units.

		Unperturbed values				FT-RPA values							
		(n)		(p)		(n)		(p)		($\tau=0$)		($\tau=1$)	
λ^π	T (MeV)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
2^+	0	475	475	321	321	474	474	321	321	799	799	791	791
	3	402	551	262	471	402	549	261	470	666	1023	661	1099
3^-	0	969	969	387	387	963	963	385	385	1363	1363	1332	1332
	3	867	964	386	408	862	958	384	406	1259	1379	1232	1347

defined following Ref. [19]. This basis includes discrete and quasidiscrete states arising from the centrifugal and Coulomb barriers. The radial part of the single-particle matrix elements $f_{jj'}^{(\lambda)}$ is described by $R_\lambda(r) \sim r^\lambda$. For choosing the IS and IV constants $\kappa_{0,1}^{(\lambda)}$ of multipole interactions we have followed the method presented previously in Ref. [15]. Namely, we have first defined the dipole IS constant $\kappa_0^{(1)}$ by putting the first discrete RPA root for dipole modes at zero temperature equal to zero to exclude the "spurious" 1^- state. The IV constant $\kappa_1^{(1)}$ has been defined to reproduce the location of the IV dipole resonance. After that by fixing the ratio $\kappa_1^{(1)}/\kappa_0^{(1)}$ for higher multipolarities we have found the quadrupole and octupole IS and IV constants so as to reproduce in the discrete RPA calculations the lowest experimental 2_1^+ and 3_1^- energies [20]. At finite temperature several numerical estimations [21,22] have shown that the temperature dependence of the single-particle energies E_j as well as of the multipole constants is rather smooth and weak up to $T \sim 6$ MeV. We have therefore used in our calculations at finite temperature the same single-particle basis and values of multipole constants $\kappa_{0,1}^{(\lambda)}$ defined at zero temperature.

The results for the strength functions $S^0(E\lambda,\omega,\tau)$ and $S(E\lambda,\omega,\tau)$ calculated in the conventional and modified FT-RPA are depicted in Figs. 1–4 at $T=0$ and 3 MeV. In Figs. 1 and 2 the unperturbed strength functions $S^0(E\lambda,\omega,\tau)$ of 2^+ and 3^- excitations are shown for neutron and proton channels separately. The strength functions for IS and IV energy-weighted strength distributions are presented in Figs. 3 and 4, respectively. In cal-

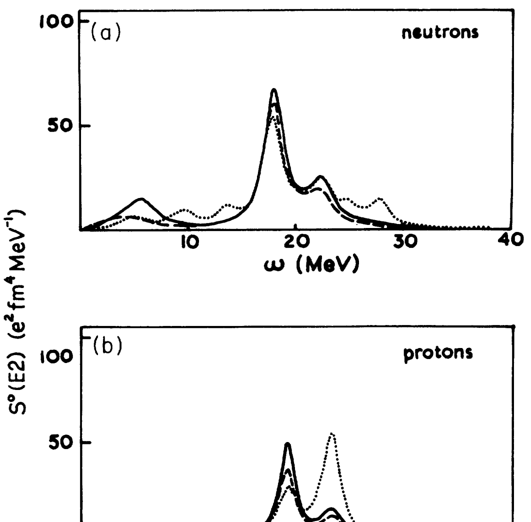

FIG. 1. Unperturbed strength distributions corresponding to (a) neutron-neutron and (b) proton-proton configurations for electric quadrupole ($\lambda^\pi = 2^+$) excitations in ^{58}Ni. Solid lines show the results calculated at zero temperature. Dashed lines are the conventional FT-RPA results at $T=3$ MeV. Dotted lines present the results obtained within the modified FT-RPA at $T=3$ MeV.

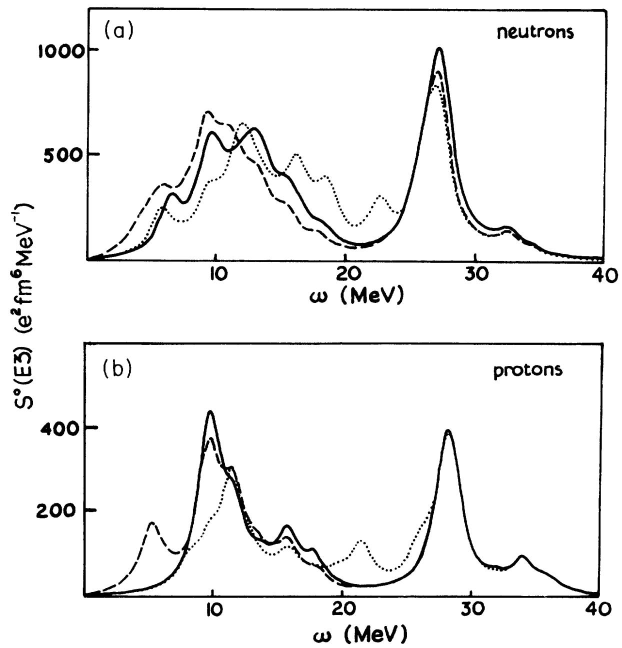

FIG. 2. Unperturbed strength distribution for electric octupole ($\lambda^\pi = 3^-$) excitations in ^{58}Ni. The results are shown with the same notation used in Fig. 1.

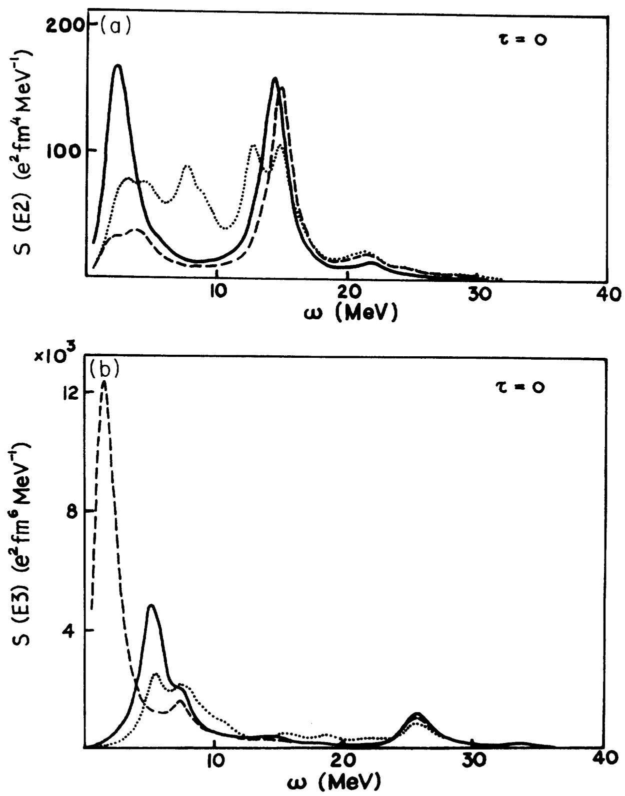

FIG. 3. Strength distributions for isoscalar ($\tau=0$) transitions in ^{58}Ni. Solid lines show the (a) quadrupole $\lambda^\pi = 2^+$ and (b) octupole $\lambda^\pi = 3^-$ linear response functions at $T=0$. Dashed lines are the conventional FT-RPA results at $T=3$ MeV. Dotted lines present the results obtained within the modified FT-RPA at $T=3$ MeV.

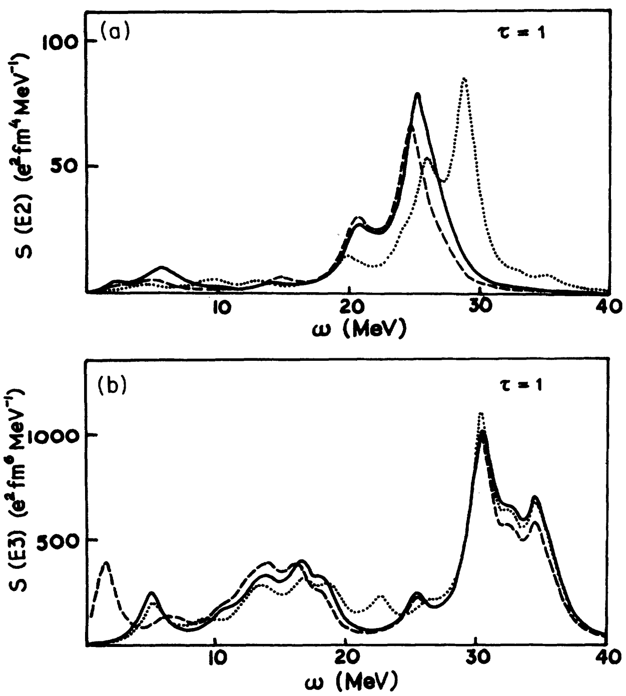

FIG. 4. Strength distributions for electric quadrupole and octupole isovector ($\tau=1$) transitions in ^{58}Ni. The results are presented with the same notation used in Fig. 3.

culating these strength functions we used a value $\eta=1$ MeV for the averaging parameter in Eq. (24). The choice of the averaging parameter η in the linear response method has been discussed thoroughly in Refs. [12,15] and we do not repeat it here. From Figs. 1 and 2 we see that the unperturbed neutron and proton strength functions in the conventional FT-RPA are rather stable with varying temperature except for the low-lying region under 10 MeV (cf. Ref. [12]). At the same time the n- and p-strength functions calculated within the modified FT-RPA are more sensitive to temperature. In Fig. 1(b), for example, there is a clear transference of the $E2$-proton strength from the lower-lying region localized at $E_x \approx 19$ MeV at $T=0$ to the higher-lying one in the domain $E_x \approx 23$ MeV at $T=3$ MeV within the modified FT-RPA. In general, the temperature dependence of $E3$-strength distributions are weaker as compared to the $E2$ ones in the high-lying domain.

The same feature is observed for the IS and IV strength functions (Figs. 3 and 4). Figure 3(a) shows a collapse of the first quadrupole state in the conventional RPA at $T=3$ MeV; an effect which has been already discussed in several works [3,5]. At the same time the $E2$ IS strength function calculated in the modified FT-RPA is changed significantly at $T=3$ MeV as compared to the one at zero temperature. We see in this case a large splitting of 2_1^+ state and a rather strong damping of the IS giant quadrupole resonance (IS-GQR) in such a way that the fragmented states are nearly fused together in a large bump up to $E_x \sim 20$ MeV. For the $E3$ IS states due to a great number of (p-p) and (h-h) states the conventional FT-RPA calculations give a too strong low-lying region at $E_x \sim 2$ MeV at $T=3$ MeV as a result of shifting down the 3_1^- state at nearly 5 MeV at $T=0$. This peculiarity is absent in the modified FT-RPA $E3$ IS strength function at $T=3$ MeV although the distribution is slightly broadened.

The IV-GQR calculated within the modified FT-RPA at $T=3$ MeV is shifted up while the result obtained in the conventional FT-RPA is shifted down as compared to the one calculated at $T=0$. The spreading width Γ of the former is also greater as will be discussed quantitatively later by analyzing the tables. The behavior of $E3$ IV strength distributions is nearly independent of temperature in both versions of FT-RPA except for some little changes at the tail of the resonance in the lower-energy domain.

In the calculations of the centroid energies and the widths of IS and IV $E2$ and $E3$ states we have chosen the energy interval $(E_1;E_2)$ in Eqs. (28) and (29) such that the corresponding energy-weighted sum rule (EWSR) exhausted 88–90 % of the model-independent values. Thus, this interval is ($E_1=10$ MeV; $E_2=20$ MeV) for the quadrupole IS states and (18 MeV; 30 MeV) for the IV ones in the conventional FT-RPA calculations. For the calculations in the modified FT-RPA version these values are (10 MeV; 20 MeV) and (18 MeV; 33 MeV), respectively. For the octupole IV states the energy interval in both versions has been chosen to be ($E_1=18$ MeV; $E_2=38$

TABLE II. Centroid energies ($\bar{E}_T$) and widths (Γ) of the strength distributions corresponding to isoscalar ($\tau=0$) and isovector ($\tau=1$) 2^+ and 3^- excitations in ^{58}Ni. Columns (a) show the values obtained in the conventional FT-RPA while the results of the modified FT-RPA are collected in columns (b). All values are given in MeV.

λ^π	τ	T	$\bar{E}_T$		Γ	
			(a)	(b)	(a)	(b)
2^+	0	0	14.48	14.48	1.84	1.84
		3	14.91	12.83	1.79	4.50
	1	0	24.61	24.61	2.65	2.65
		3	23.94	26.81	2.64	3.28
3^-	0	0	11.09	11.09	8.80	8.80
		3	6.32	12.51	8.26	8.30
	1	0	31.74	31.74	3.13	3.13
		3	31.46	31.64	3.14	3.11

MeV). In fact, these are regions where the corresponding giant multipole resonances are localized.

The linear response EWSR for quadrupole and octupole excitations are shown in Table I. We have used Eq. (28) for $m_k(T)$ which is the integrated energy-weighted strength of the multipole field λ^π. Here also the temperature dependence of the EWSR for the quadrupole excitations calculated in the modified FT-RPA [columns (b)] is more pronounced as compared to the results of the conventional FT-RPA [columns (a)]. The $E2$ EWSR obtained in the modified version increases remarkably at finite temperature.

In Table II we present the values of the centroid energy $\bar{E}_T$ and the spreading width Γ for quadrupole and octupole excitations. The most significant difference between the results obtained in two versions of the FT-RPA is found in the calculations of quadrupole excitations. The centroid energy for the $E2$ excitations ($\tau=0$) at $T=3$ MeV calculated in the conventional FT-RPA is increased to 14.91 MeV while the one for the IV states is decreased to 23.94 MeV as compared to the zero-temperature values 14.48 and 24.61 MeV, respectively. At the same time the modified FT-RPA gives an inverse result, namely, $\bar{E}_{T=3\ \mathrm{MeV}}=12.83$ MeV for IS excitations is less than its value at $T=0$, while for IV excitations $\bar{E}_{T=3\ \mathrm{MeV}}=26.81$ MeV is greater than $\bar{E}_{T=0}$. The spreading widths for IS and IV strength functions obtained in the modified FT-RPA increase significantly at finite temperature, while their values in the conventional FT-RPA are nearly stable with varying T. As a result of calculations discussed above based on Figs. 1–4, the calculated values of $\bar{E}_T$ and for octupole excitations are nearly independent of temperature in both versions of FT-RPA. Moreover, the preference of the modified FT-RPA can be approved here due to the absence of the strong low-lying IS state, which appeared in the conventional FT-RPA calculations, as has been discussed above.

IV. CONCLUSIONS

In the present paper, based on the thermal canonical Bogolubov transformation in the TFD, we have derived explicitly the modified FT-RPA equation and applied it to calculate the linear response strength functions for electric quadrupole ($\lambda^\pi=2^+$) and octupole ($\lambda^\pi=3^-$) excitations in hot ^{58}Ni nucleus. The advantage of the constructed modified FT-RPA is twofold. On one hand, the new equation does not contain obviously the (p-p) and (h-h) poles; an unintended effect, which appeared in the conventional FT-RPA so far. On the other hand, due to the influence of the thermostat on the structure of thermal quasiparticles participating in the construction of modified FT-RPA microscopic phonon operators, the new version has given a stronger spreading for the distribution of multipole excitations, particularly for the quadrupole field, at finite temperature. Of course, to describe the broadening of GR in hot nuclei one has to involve at least the coupling to more complicated states and the shape fluctuations as has been done in several papers [8–11]. However, as a base, the modified version of the FT-RPA constructed in this paper by the reasons discussed above seems to be more adequate as compared to the conventional one, employed so far in the literature.

The application of this modified FT-RPA in the construction of states containing the coupling to (2p-2h) configurations is the goal for our future study.

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[1] A. V. Ignatyuk, Proc. USSR Acad. Sci. **38**, 2613 (1974).
[2] M. Sommermann, Ann. Phys. (N.Y.) **151**, 163 (1983).
[3] D. Vautherin and N. Vinh Mau, Phys. Lett. **120B**, 261 (1983); Nucl. Phys. **A422**, 140 (1984).
[4] O. Civitarese, R. A. Broglia, and C. H. Dasso, Ann. Phys. (N.Y.) **156**, 142 (1984).
[5] Nguyen Dinh Dang, J. Phys. G **11**, L125 (1985).
[6] F. Alasia, O. Civitarese, and M. Reboiro, Phys. Rev. C **36**, 2555 (1987).
[7] Nguyen Dinh Dang and Nguyen Zuy Thang, J. Phys. G **14**, 1471 (1988).
[8] P. F. Bortignon, R. A. Broglia, G. F. Bertsch, and J. Pacheco, Nucl. Phys. **A460**, 149 (1986).
[9] M. Gallardo, F. J. Luis, and R. A. Broglia, Phys. Lett. B **191**, 222 (1987).
[10] Nguyen Dinh Dang, Nucl. Phys. **A504**, 143 (1989).
[11] G. F. Bertsch, P. F. Bortignon, and R. A. Broglia, Rev. Mod. Phys. **55**, 287 (1983).
[12] F. Alasia, O. Civitarese, and M. Reboiro, Phys. Rev. C **39**, 1012 (1989).
[13] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermofield Dynamics and Condensed States* (North-Holland, Amsterdam, 1982).
[14] Nguyen Dinh Dang, Phys. Lett. B **229**, 181 (1989).
[15] V. G. Soloviev, *Theory of Complex Nuclei* (Pergamon, Oxford, 1976); V. G. Soloviev, Prog. Part. Nucl. Phys. **19**, 107 (1987); V. G. Soloviev, *Theory of Atomic Nuclei-Quasiparticles and Phonons* (Energoatomizdat, Moscow, 1989).
[16] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics,* Vol. 5: Statistical Physics (Nauka, Moscow, 1964).
[17] L. G. Moretto, Nucl. Phys. **A185**, 145 (1972); **A226**, 9 (1974); Phys. Lett. **40B**, 1 (1972); **44B**, 494 (1973).
[18] A. L. Goodman, Phys. Rev. C **29**, 1887 (1984).
[19] V. A. Chepurnov, Yad. Fiz. **6**, 955 (1966) [Sov. J. Nucl. Phys. **6**, 696 (1967)]; K. Takeuchi and P. A. Moldauer, Phys. Lett. **28B**, 384 (1969).
[20] D. C. Kocher and R. L. Auble, Nuclear Data Sheets for $A=58$, **4**, 445 (1976).
[21] M. Brack and P. Quentin, Phys. Lett. **42B**, 159 (1974); Nucl. Phys. **A361**, 35 (1981); P. Bonche, S. Levit, and D. Vautherin, *ibid.* **A427**, 178 (1984).
[22] A. I. Blokhin and A. V. Ignatyuk, *Problems of Nuclear Physics and Cosmic Rays* (Vyshaya Shkola, Kharkov, 1977), Vol. 7, p. 111.