Nuclear structure properties of the double-charge-exchange transition amplitudes

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Nuclear structure aspects of the double-charge-exchange (DCX) reaction on nuclei are studied. Using a variety of DCX-type two-body transition operators, we explore the influence of two-body correlations among valence nucleons on the DCX transition amplitudes to the isobaric analog state and to other nonanalog $J^\pi=0^+$ states. In particular, the question of the spin dependence and of the range of the DCX transition operators is explored and the behavior of the transition amplitudes as a function of the valence nucleon number is studied. It is shown that the two-amplitude DCX formula derived by Auerbach, Gibbs, and Piasetzky for a single j^n configuration holds also in some cases when configuration mixing is strong. DCX-type transitions from the Ca and Ni isotopes to the Ti and Zn isotopes and from ^{56}Fe to ^{56}Ni are the subject of this study.

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I. INTRODUCTION

Several years ago, theoretical progress was made in the understanding of the pion double charge exchange (DCX) on nuclei [1–3]. In particular, for transitions to the double isobaric analog state (DIAS), it was shown that the DCX transition amplitude separates basically into two parts, one representing the contributions of the nuclear mean field and the other incorporating the effects of two-body correlations [1,3]. The first part is large when the two-body DCX transition operator is of long range, while the correlation term is important when the DCX transition operator has a short range. In the case of a pure j^n (n even) configuration in the seniority scheme, a formula was derived in Ref. [1] (we will refer to it as the AGP) for the DCX amplitude to the DIAS:

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[A+\frac{2j+3-2n}{(n-1)(2j-1)}B\right], \quad (1)$$

where A and B are complex amplitudes dependent on the pion energy and angle, but independent of the number of valence nucleons n. This formula explicitly exhibits the division of the DCX amplitude into a mean-field term (A) and a correlation term (B). The expression in Eq. (1) can also be written in the form [1]

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[\alpha+\frac{\beta}{n-1}\right], \quad (2)$$

where again α and β are two complex amplitudes independent of n. The connection between A,B and α,β is

$$A=\alpha+\frac{2}{2j+1}\beta,\qquad B=\frac{2j-1}{2j+1}\beta. \quad (3)$$

In terms of the matrix elements of a two-body DCX transition operator $\hat{O}_{12}$ one can write [4]

$$A=\frac{1}{2j+1}(2j\bar{O}+O_0),\qquad B=\frac{2j}{2j+1}(O_0-\bar{O}), \quad (4)$$

where the following definitions were used:

$$O_0=\langle j^2J=0|\hat{O}_{12}|j^2J=0\rangle,\qquad \bar{O}=\frac{\sum_{J\text{even}}(2J+1)\langle j^2J|\hat{O}_{12}|j^2J\rangle}{\sum_{J\text{even}}(2J+1)}. \quad (5)$$

One sees from these expressions that the relative size of A and B depends on the range of $\hat{O}_{12}$. Consider a real DCX transition operator which possesses the "pairing" property [4], i.e., an operator that for even n satisfies

$$\langle j^nJ=0|\hat{O}_{12}|j^nJ=0\rangle=\frac{n}{2}O_0, \quad (6)$$

where $n/2$ is the number of neutron pairs. Operators such as $\delta(\mathbf{r}_1-\mathbf{r}_2)$ and $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ fall into this category. For such an operator, $\bar{O}=O_0/(2j)$, $\alpha=0$, $\beta=O_0$, and

$$B/A=j-\tfrac{1}{2}. \quad (7)$$

The DCX amplitude M then takes the following simple form:

$$M=\left[\frac{n}{2(n-1)}\right]^{1/2}\beta. \quad (8)$$

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Thus the n dependence is distinctly different for the mean-field part and the correlation part of the nuclear wave function. The n behavior of the cross section provides therefore a clear indication as for the range of the pion-nucleus interaction. For a long-range interaction, the DCX cross section is sensitive to the mean-field part of the wave function, and thus one should expect that the cross section in this case will have an $n(n-1)/2$ dependence, but when the pion-nucleus interaction is of short range, the correlative part of the wave function will contribute substantially, and the DCX cross section will have an $n/(n-1)$ dependence [1,3].

This characteristic behavior of the DCX cross sections to the DIAS as a function of the number of valence nucleons is exact in the case of a pure j^n configuration. It was shown [5] that in the case of weak configuration mixing of a special type, Eqs. (1) and (2) still hold, but the amplitudes A and B (or α and β) are replaced by effective amplitudes $\tilde{A}$ and $\tilde{B}$ (or $\tilde{\alpha}$ and $\tilde{\beta}$). Consider a mixed-configuration state of the type

$$|\psi\rangle = |j^n 0\rangle + \sum_{j',J'} x_{j',J'} |[j^{n-2}J';j'^2J']0\rangle \ , \tag{9}$$

with $|x_{j',J'}|^2 \ll \frac{1}{2}$. In first-order perturbation theory,

$$x_{j',J'} = \frac{\langle j^n 0 | \sum_{i>k} V_{ik} | [j^{n-2}J';j'^2J']0\rangle}{\Delta E} \ , \tag{10}$$

where ΔE is the energy difference between the unperturbed configuration $|j^n 0\rangle$ and $|[j^{n-2}J';j'^2J']0\rangle$. When the single-$j$-shell model provides a good description for the ground state, as is the case for the Ca isotopes, ΔE only depends on n and J' weakly and it can be approximated by $\Delta E \simeq 2\epsilon_j - 2\epsilon_{j'}$, where ϵ_j and $\epsilon_{j'}$ are the single-particle energies for the orbitals (n,l,j) and (n',l',j'), respectively. One can express $x_{j',J'}$ in terms of the two-nucleon coefficient of fractional parentage (2CFP) and the interaction two-body matrix element as

$$x_{j',J'} = \left[\frac{n(n-1)}{2}\right]^{1/2} \langle [j^{n-2}J';j^2J']0 | \} j^n 0\rangle \frac{\langle j'^2J'|V_{12}|j^2J'\rangle}{\Delta E} \ . \tag{11}$$

The expectation value of the DCX operator $\hat{O} = \sum_{i>k} \hat{O}_{ik}$ in ψ can be written to first order in $x_{j',J'}$ as

$$\begin{aligned}\langle \psi|\hat{O}|\psi\rangle &= \langle j^n 0|\hat{O}|j^n 0\rangle + 2\sum_{j',J'} x_{j'J'} \langle [j^{n-2}J';j'^2J']0|\hat{O}|j^n 0\rangle \\ &= \frac{n(n-1)}{2} \sum_{J'} \langle [j^{n-2}J';j^2J']0|\} j^n 0\rangle^2 \langle j^2J'|\hat{O}_{12}|j^2J'\rangle \\ &\quad + 2\left[\frac{n(n-1)}{2}\right]^{1/2} \sum_{j',J'} x_{j',J'} \langle [j^{n-2}J';j^2J']0|\} j^n 0\rangle \langle j'^2J'|\hat{O}_{12}|j^2J'\rangle \ .\end{aligned} \tag{12}$$

Using Eq. (11) in Eq. (12), one finds [5]

$$\langle \psi|\hat{O}|\psi\rangle = \frac{n(n-1)}{2} \sum_{J'} \langle [j^{n-2}J';j^2J']0|\} j^n 0\rangle^2 \left[\langle j^2J'|\hat{O}_{12}|j^2J'\rangle + \sum_{j'} 2\langle j^2J'|\frac{V_{12}}{\Delta E}|j'^2J'\rangle\langle j'^2J'|\hat{O}_{12}|j^2J'\rangle\right] . \tag{13}$$

This equation can be expressed in the form

$$\langle \psi|\hat{O}|\psi\rangle = \frac{n(n-1)}{2} \sum_{J'} \langle [j^{n-2}J';j^2J']0|\} j^n 0\rangle^2 \langle j^2J'|\tilde{O}_{12}|j'^2J'\rangle \ , \tag{14}$$

where $\tilde{O}_{12}$ is an *effective* operator given by

$$\tilde{O}_{12} = \left[1 + V_{12} \sum_{j'} \frac{2}{\Delta E} |j'^2J'\rangle\langle j'^2J'|\right] \hat{O}_{12} \ . \tag{15}$$

We see that the expectation value of the operator $\hat{O}_{12}$ in a weakly mixed configuration can be written as the expectation value of an effective operator $\tilde{O}_{12}$ in the pure single-j configuration, provided that the energy denominator ΔE is not dependent strongly on n.

The DCX transition amplitude to the DIAS is given in the case of a pure j^n configuration by the ground-state expectation value of $\hat{O}_{12}$ divided by the factor $\sqrt{n(n-1)/2}$. This leads to the expressions in Eqs. (1) and (2) [1]. We see that also in the case of weak configuration mixing, Eqs. (1) and (2) will hold for the effective two-body operator $\tilde{O}_{12}$. In this work we will demonstrate through numerical calculations that the form of Eq. (1) or (2) is approximately preserved also in some cases when configuration mixing is strong. The effective DCX operator $\tilde{O}_{12}$ that will emerge in such cases will have *shorter* ranges than the operators employed in the pure j^n case.

Our purpose in this work is to study the influence of purely nuclear structure effects on the DCX transition amplitudes. In particular, we wish to learn about the influence that configuration mixing has on the DCX cross sections and on the ranges of the effective transition operators.

To keep things simple and in order to concentrate on the nuclear structure effects, we will use *real* two-body transition operators of isotensor type. Only transitions

between $0^+T_i \rightarrow 0^+T_f$ will be considered, and therefore only scalar operators will enter the calculations. Of course, the actual DCX transition operator is complex, and therefore our calculations should not be compared directly with the experimental results. Nevertheless, many of the results we obtain can be related to certain characteristic trends in the existing experimental data and can be used to predict such trends in the data to emerge from future experiments.

II. TRANSITION OPERATORS

In this work we calculate the amplitude

$$M=\langle 0_f^+,T_f,T_{zf}|\sum_{ik}\hat{O}_{ik}|0_i^+,T_i,T_{zi}\rangle\ , \tag{16}$$

using the following two-body double-charge-exchange operators:

$$\begin{aligned}
\hat{O}_{12}^{(1)}&=\mathcal{N}_1\delta(\mathbf{r})t_+(1)t_+(2)\equiv\delta tt\ ,\\
\hat{O}_{12}^{(2)}&=\mathcal{N}_2\delta(\mathbf{r})(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)t_+(1)t_+(2)\equiv\delta\sigma\sigma tt\ ,\\
\hat{O}_{12}^{(3)}&=\mathcal{N}_3(e^{-m_\rho r}/r)t_+(1)t_+(2)\equiv Y_\rho tt\ ,\\
\hat{O}_{12}^{(4)}&=\mathcal{N}_4(e^{-m_\rho r}/r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)t_+(1)t_+(2)\equiv Y_\rho\sigma\sigma tt\ ,\\
\hat{O}_{12}^{(5)}&=\mathcal{N}_5(e^{-m_\pi r}/r)t_+(1)t_+(2)\equiv Y_\pi tt\ ,\\
\hat{O}_{12}^{(6)}&=\mathcal{N}_6(e^{-m_\pi r}/r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)t_+(1)t_+(2)\equiv Y_\pi\sigma\sigma tt\ ,\\
\hat{O}_{12}^{(7)}&=\mathcal{N}_7(1/r)t_+(1)t_+(2)\equiv Ctt\ ,\\
\hat{O}_{12}^{(8)}&=\mathcal{N}_8(1/r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)t_+(1)t_+(2)\equiv C\sigma\sigma tt\ ,\\
\hat{O}_{12}^{(9)}&=t_+(1)t_+(2)\equiv tt\ ,\\
\hat{O}_{12}^{(10)}&=(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)t_+(1)t_+(2)\equiv\sigma\sigma tt\ .
\end{aligned} \tag{17}$$

In the above, $t_+|n\rangle=|p\rangle$, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, m_ρ and m_π refer to the masses of the rho and pion mesons, and $\mathcal{N}_1,\ldots,\mathcal{N}_8$ are certain normalization constants. We have also given an abbreviated notation for each operator. Y denotes a Yukawa-type potential, and C stands for a Coulomb-type potential.

The transition amplitude M in (16) for an isotensor operator $\hat{O}$ in Eq. (17) can be written

$$\begin{aligned}
M&=\langle 0_f^+,T_f,T_{zf}|\hat{O}_{+2}^{(2)}|0_i^+,T_i,T_{zi}\rangle\\
&=(-1)^{T_f-T_{zf}}\begin{pmatrix}T_f & 2 & T_i\\ -T_{zf} & +2 & T_{zi}\end{pmatrix}\langle 0_f^+,T_f|||\hat{O}^{(2)}|||0_i^+,T_i\rangle\ ,
\end{aligned} \tag{18}$$

where the triply reduced matrix element (in the convention of Edmonds) can be expressed as a product of normalized antisymmetric two-body matrix elements of the operator under consideration and the isotensor two-body transition density matrix [6]:

$$\langle 0_f^+,T_f|||\hat{O}^{(2)}|||0_i^+,T_i\rangle=\sum_{j_1\le j_2,j_3\le j_4,J}\rho^{(2)}(j_1,j_2,j_3,j_4,J)\langle j_1j_2|||\hat{O}^{(2)}|||j_3j_4\rangle_{J,T=1}\ . \tag{19}$$

The isotensor two-body transition density matrix $\rho^{(2)}$ is defined as

$$\rho^{(2)}(j_1,j_2,j_3,j_4,J)=-\frac{\langle 0_f^+,T_f|||[(a_{j_1}^+\otimes a_{j_2}^+)^{J,T=1}\otimes(\tilde{a}_{j_3}\otimes\tilde{a}_{j_4})^{J,T=1}]^{\mathcal{J}=0,\mathcal{T}=2}|||0_i^+,T_i\rangle}{\sqrt{5(1+\delta_{j_1,j_2})(1+\delta_{j_3,j_4})}}\ , \tag{20}$$

where $\tilde{a}_j$ is a tensor operator which creates a hole state $|j_{-1}\rangle$. The matrix $\rho^{(2)}$ can be obtained by running a shell-model program (OXBASH) [6].

III. TRANSITIONS TO THE DOUBLE ISOBARIC ANALOG STATE

We first present the various calculations of transitions from initial 0^+ states to the corresponding DIAS being the final states.

A. Ca isotopes

We start with the Ca isotopes. Many of the properties of the low-lying states are described in terms of the pure $(f_{7/2})^n$ configuration. The description of the low-energy DCX experiments [7] performed on ^{42}Ca, ^{44}Ca, and ^{48}Ca was therefore less complicated as far as nuclear structure goes, and Eq. (1) was applied successfully to the above nuclei [1,3]. We will therefore start the presentation of our results with the $(f_{7/2})^n$ configuration and apply it to the Ca isotopes.

In Table I we show the transition amplitudes from the even-mass Ca isotopes to the corresponding DIAS in the Ti isotopes for the various two-body operators listed in Sec. II. The column labeled "SJ" denotes the single-j^n-configuration description of the even-mass Ca isotope ground states. The results have been normalized so that, for each spin-independent transition operator Xtt,

$$\begin{aligned}
\langle\mathrm{DIAS}|Xtt|f_{7/2}^2 0^+\rangle&\equiv\langle\mathrm{DIAS}|tt|f_{7/2}^2 0^+\rangle\\
&=\sqrt{2n(n-1)}=2\ ,
\end{aligned} \tag{21}$$

and for each spin-dependent operator $X\sigma\sigma tt$,

$$\begin{aligned}
\langle\mathrm{DIAS}|X\sigma\sigma tt|f_{7/2}^2 0^+\rangle&\equiv\langle\mathrm{DIAS}|\sigma\sigma tt|f_{7/2}^2 0^+\rangle\\
&=-\frac{j+1}{j}\left[\frac{2n}{n-1}\right]^{1/2}=-\frac{18}{7}\\
&\quad\text{for } j=\tfrac{7}{2},\ n=2\ .
\end{aligned} \tag{22}$$

In Eqs. (21) and (22), X represents the spatial part of the two-body operator, i.e., $X=\delta, Y_\rho, Y_\pi, C$. Of course, the numbers shown in the SJ column obey exactly the formula in Eq. (1) for each of the transition operators. We should note, however, the following: All the numbers obtained with the transition operator containing a $\sigma\sigma$ term are the same for the same number of particles. This is because in the case of equivalent particles occupying a single orbit, any operator of the $\sigma\sigma$ type has the "pairing" property and thus it has the same diagonal matrix elements as a $\delta(r)$ function. Indeed, as can be seen from Table I, the matrix elements evaluated for the δtt operator are proportional to the matrix elements for all the $X\sigma\sigma tt$-type operators. It is very useful to evaluate the ratios of B/A and β/α for each of the operators:

$$\frac{B}{A}=\frac{2j(O_0-\bar{O})}{2j\bar{O}+O_0}, \quad \frac{\beta}{\alpha}=\frac{(2j+1)(B/A)}{(2j-1)-2(B/A)} . \tag{23}$$

For a transition operator which possesses the pairing property (6), $\bar{O}=O_0/(2j)$, and so $B/A=j-\frac{1}{2}$ and β/α is infinite. For an operator Xtt with $X=$const, $\bar{O}=O_0$; therefore $B/A=0$ and also $\beta/\alpha=0$. For operators which have intermediate ranges, we therefore expect

$$0<B/A<j-\tfrac{1}{2} . \tag{24}$$

The shorter the range of the interaction, the larger is the ratio of B/A. In Table I we show the B/A ratios for the various operators.

Obviously, for all the operators having the pairing property (6), this ratio is given by Eq. (7); thus, for $j=f_{7/2}$, $B/A=3$. However, for operators with a finite range, as shown in Table I, this ratio decreases from 3 to 2.75 for a Yukawa-type operator with the range of $1/m_\rho$, to 1.18 for a Yukawa-type operator with the range of $1/m_\pi$, and to 0.22 for a Coulomb-type operator $1/r$.

B. Configuration mixing in the Ca isotopes

In Table I are also shown the results of calculations for the Ca→Ti transitions performed in a larger shell-model space. For $A=42$ and 44, the full fp configuration space $(0f_{7/2}, 1p_{3/2}, 0f_{5/2}, 1p_{1/2})$ is used, while for $A=46$ and 48, because of limited computer memory, only up to two $f_{7/2}$ nucleons are allowed to leave the $f_{7/2}$ orbit and occupy other orbits in the fp shell. A recent fp-shell effective interaction of Richter *et al.*, "FPD6" [8], is employed in the calculation. The ground-state wave func-

TABLE I. $^{40+n}\mathrm{Ca}(0^+_{g.s.})\rightarrow{}^{40+n}\mathrm{Ti}$(DIAS) transition amplitudes M for various types of operators in the single-$f_{7/2}$ shell-model (SJ) and in the extended fp space shell-model (CM) calculations using the "FPD6" interaction. In the column labeled "AGP" is shown the fit obtained using Eq. (1) with A,B parameters (denoted by $\tilde{A},\tilde{B}$) determined by the $n=2$ and 8 CM values. The ratios B/A and $\tilde{B}/\tilde{A}$ are also shown for each operator.

n	$\hat{O}_{12}$	SJ	CM	AGP	$\hat{O}_{12}$	SJ	CM	AGP
2	δtt	2.000	3.090	3.090	$\delta\sigma\sigma tt$	−2.571	−3.973	−3.973
4		1.633	2.340	2.33		−2.099	−3.008	−3.00
6		1.549	1.974	2.03		−1.992	−2.538	−2.61
8		1.512	1.800	1.800		−1.944	−2.314	−2.314
$B/A(\tilde{B}/\tilde{A})$		3.00		3.52		3.00		3.52
2	$Y_\rho tt$	2.000	3.008	3.008	$Y_\rho\sigma\sigma tt$	−2.571	−3.992	−3.992
4		1.707	2.382	2.35		−2.099	−3.023	−3.01
6		1.690	2.099	2.14		−1.992	−2.549	−2.62
8		1.718	1.990	1.990		−1.944	−2.324	−2.324
$B/A(\tilde{B}/\tilde{A})$		2.75		3.27		3.00		3.52
2	$Y_\pi tt$	2.000	2.581	2.581	$Y_\pi\sigma\sigma tt$	−2.571	−4.183	−4.183
4		2.539	3.026	2.92		−2.099	−3.166	−3.15
6		3.268	3.577	3.54		−1.992	−2.648	−2.73
8		4.028	4.206	4.206		−1.944	−2.413	−2.413
$B/A(\tilde{B}/\tilde{A})$		1.18		1.54		3.00		3.54
2	Ctt	2.000	2.185	2.185	$C\sigma\sigma tt$	−2.571	−4.147	−4.147
4		4.101	4.276	4.22		−2.099	−3.172	−3.15
6		6.232	6.345	6.32		−1.992	−2.680	−2.76
8		8.367	8.428	8.428		−1.944	−2.471	−2.471
$B/A(\tilde{B}/\tilde{A})$		0.22		0.31		3.00		3.47
2	tt	2.000	2.000	2.000	$\sigma\sigma tt$	−2.571	−4.021	−4.021
4		4.899	4.899	4.90		−2.099	−3.109	−3.08
6		4.746	7.746	7.75		−1.992	−2.662	−2.73
8		10.583	10.583	10.583		−1.944	−2.481	−2.481
$B/A(\tilde{B}/\tilde{A})$		0.00		0.00		3.00		3.41

tions for the even isotopes that result from these computations have about 10–20% of admixture of the configurations different from $(f_{7/2})^n$. Using the resulting wave functions, the transition amplitudes to the DIAS for the various operators are calculated and shown in Table I in the column labeled "CM." Note that because of configuration mixing, the various amplitudes have increased in magnitude by about 50% for the Xtt-type operators of the short range and the $X\sigma\sigma tt$-type operators. However, the resulting amplitudes, also in this case when configuration mixing is presented, obey approximately Eq. (1) or (2). In the column labeled "AGP" are shown the results predicted by Eq. (1) when A and B parameters are determined by fitting the CM amplitudes for $n=2$ and 8. We see that the agreement for $n=4$ and 6 between the results obtained in the CM calculation and obtained in the fit using the AGP formula (1) is excellent for all the transitions considered. The largest deviation of the AGP predictions from the CM amplitudes, given by the $Y_\pi tt$, $n=4$ case is only 3.5%.

The interesting feature that emerged from the fit is the ratio of $\tilde{B}/\tilde{A}$, where $\tilde{A}$ and $\tilde{B}$ are the A,B parameters obtained in the fit of the CM results to Eq. (1). Also, in Table I we compare the B/A for the pure $(f_{7/2})^n$ configuration to $\tilde{B}/\tilde{A}$ We see that in all cases the effective $\tilde{B}/\tilde{A}$ increases with respect to the B/A obtained when configuration mixing is not present. Thus the effective transition operators $\tilde{O}_{12}$ have shorter ranges than the corresponding $\hat{O}_{12}$ operators. The additional correlations among nucleons resulting from configuration mixing lead to the shortening of the range of the effective two-body transition operators. We learn that for the pion DCX, the effective range of the transition operator is determined not only by the range of the basic pion-nucleon interaction, but also by the two-body nucleon-nucleon correlations.

C. Ni isotopes

In the case of the Ni isotopes, configuration mixing is very strong [9]. Assuming ^{56}Ni to be an inert core, one is successful in describing the low-energy properties of the spectrum by placing the valence neutrons in the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbits [9]. In the past it was noticed that the Ni-isotope spectra are well described by using a generalized seniority scheme [9,10]. It is of interest therefore to probe the Ni-isotope wave functions with DCX operators.

The calculation of the wave function for the even-mass $^{56+n}$Ni isotopes (^{58}Ni–^{68}Ni) are performed in the full $(1p_{3/2},0f_{5/2},1p_{1/2})$ space using the "ASDI" interaction [11].

The results for the transition matrix elements to the DIAS for the various operators are shown in Table II. The results have been normalized in such a way that

$$\langle \text{DIAS}|Xtt|p^2_{3/2}0^+\rangle \equiv \langle \text{DIAS}|tt|p^2_{3/2}0^+\rangle = \sqrt{2n(n-1)}=2 , \quad (25)$$

for operators of the Xtt type, and

$$\langle \text{DIAS}|X\sigma\sigma tt|p^2_{3/2}0^+\rangle \equiv \langle \text{DIAS}|\sigma\sigma tt|p^2_{3/2}0^+\rangle = -\frac{j+1}{j}\left[\frac{2n}{n-1}\right]^{1/2} = -\frac{10}{3} \quad \text{for } j=\tfrac{3}{2},\ n=2 , \quad (26)$$

for operators of the $X\sigma\sigma tt$ type. In the above equations, X represents δ, Y_ρ, Y_π, and C. The same features are apparent in the Ni isotopes as in the Ca isotopes. For the interactions having the pairing property (6), the amplitudes behave approximately as $\sqrt{n}/(n-1)$, but for the longer-range operators, the amplitudes increase rapidly with n. Taking the approach of the generalized seniority scheme, we try to fit the results with the AGP-type formulas [1]:

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[\tilde{A}+\frac{2\tilde{j}+3-2n}{(n-1)(2\tilde{j}-1)}\tilde{B}\right] \quad (27)$$

or, equivalently,

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[\tilde{\alpha}+\frac{\tilde{\beta}}{n-1}\right] , \quad (28)$$

where $\tilde{j}$ is an effective j quantum number in the generalized seniority scheme given by

$$\tilde{j}=\Omega-\tfrac{1}{2} , \quad (29)$$

and where Ω is the size of the chosen shell-model space:

$$\Omega=\sum_i (j_i+\tfrac{1}{2}) . \quad (30)$$

In our case, $j_i=1p_{3/2},0f_{5/2},1p_{1/2}$, and so $\tilde{j}=\frac{11}{2}$. The quantities $\tilde{A},\tilde{B}$ and $\tilde{\alpha},\tilde{\beta}$ are defined as effective A,B and α,β amplitudes in the generalized seniority scheme. They are determined by fitting the results for ^{58}Ni and ^{64}Ni ($n=2$ and 8) of the full calculation with formula (27) or (28). With those parameters determined, we calculate using Eq. (27) or (28) the transition amplitudes for the rest of the isotopes ($n=4,6,10,12$). The results are also shown in Table II in the column labeled "AGP." The agreement between the "exact" calculations and those resulting from Eq. (27), for all six isotopes and for all types of operators, is remarkably good (within 2.6%). The ratios $\tilde{B}/\tilde{A}$ and $\tilde{\beta}/\tilde{\alpha}$ are also shown in Table II. We see that for those operators that possess the pairing property in the single-j configuration, the ratio $\tilde{B}/\tilde{A}$ is close to 5, which would be the case for a generalized seniority scheme with $\tilde{j}=\frac{11}{2}$ used in Eq. (27). We note that for spin-independent operators, as the range of the operator becomes longer, the ratio $\tilde{B}/\tilde{A}$ or $\tilde{\beta}/\tilde{\alpha}$ decreases.

IV. NONANALOG TRANSITIONS

In this section we will discuss transitions induced by the same types of operators as before to 0^+ states which are not the DIAS. These will include also the ground states in nuclei with $T>1$ ($n>2$) where the ground state is not the DIAS.

A. Ca→Ti transitions

In Table III we present the DCX-type transitions from the 0^+ ground state in $^{40+n}$Ca ($n \geq 2$, even) to the 0^+ nonanalog states in $^{40+n}$Ti for the pure j^n configurations. The calculation includes transitions to all the 0^+ states that have isospin $T_f = |T_i - 2|$, where $T_i = n/2$ is the isospin of each $^{40+n}$Ca parent state. From Table III we see that the transition amplitudes for the long-range Xtt-type operators to the nonanalog 0^+ states are small. This is because in the extreme case of X=const, the operator tt can only induce the transition between double isobaric analog states. However, we observe that for the short-range Xtt-type operators and for the $X\sigma\sigma tt$-type operators, the nonanalog transitions are comparable to the analog ones. We also note that while in the transitions to the DIAS, it was impossible to distinguish between a δ-type operator and any double spin-flip operator (i.e., the operator containing a $\sigma\sigma$ term); now the relative transition strength to various excited 0^+ states depends on more detailed properties of a given operator. We mention this to emphasize the fact that as opposed to the

TABLE II. $^{56+n}$Ni($0^+_{g.s.}$)→$^{56+n}$Zn(DIAS) transition amplitudes M for various types of operators in the full ($1p_{3/2}, 0f_{5/2}, 1p_{1/2}$) space shell-model (SM) calculation using the "ASDI" interaction. In the column labeled "AGP" is shown the fit obtained using Eq. (27) or (28) with $\tilde{A}, \tilde{B}$ and $\tilde{\alpha}, \tilde{\beta}$ amplitudes determined by the n=2 and 8 SM values. The ratios $\tilde{B}/\tilde{A}$ and $\tilde{\beta}/\tilde{\alpha}$ are also shown for each operator.

n	$\hat{O}_{12}$	SM	AGP	$\hat{O}_{12}$	SM	AGP
2	δtt	3.611	3.611	$\delta\sigma\sigma tt$	−6.018	−6.018
4		2.969	2.97		−4.948	−4.94
6		2.835	2.83		−4.724	−4.71
8		2.776	2.776		−4.626	−4.626
10		2.734	2.75		−4.557	−4.59
12		2.698	2.74		−4.497	−4.57
$\tilde{B}/\tilde{A}$			4.92			4.92
$\tilde{\beta}/\tilde{\alpha}$			352			354
2	$Y_\rho tt$	3.597	3.597	$Y_\rho\sigma\sigma tt$	−6.050	−6.050
4		3.099	3.08		−4.973	−4.97
6		3.075	3.06		−4.747	−4.74
8		3.118	3.118		−4.648	−4.648
10		3.173	3.21		−4.578	−4.61
12		3.226	3.31		−4.517	−4.59
$\tilde{B}/\tilde{A}$			4.35			4.92
$\tilde{\beta}/\tilde{\alpha}$			39.9			367
2	$Y_\pi tt$	3.077	3.077	$Y_\pi\sigma\sigma tt$	−5.536	−5.536
4		3.847	3.80		−4.488	−4.50
6		4.858	4.82		−4.251	−4.26
8		5.894	5.894		−4.140	−4.140
10		6.923	6.98		−4.054	−4.07
12		7.932	8.08		−3.968	−4.02
$\tilde{B}/\tilde{A}$			1.63			5.05
$\tilde{\beta}/\tilde{\alpha}$			2.91			−561
2	Ctt	2.290	2.290	$C\sigma\sigma tt$	−4.650	−4.650
4		4.589	4.57		−3.688	−3.73
6		6.912	6.90		−3.455	−3.48
8		9.234	9.234		−3.338	−3.338
10		11.550	11.57		−3.236	−3.23
12		13.858	13.91		−3.122	−3.15
$\tilde{B}/\tilde{A}$			0.30			5.26
$\tilde{\beta}/\tilde{\alpha}$			0.38			−120
2	tt	2.000	2.000	$\sigma\sigma tt$	−4.239	−4.239
4		4.899	4.90		−3.322	−3.38
6		7.746	7.75		−3.094	−3.13
8		10.583	10.583		−2.978	−2.978
10		13.416	13.42		−2.869	−2.86
12		16.248	16.25		−2.743	−2.76
$\tilde{B}/\tilde{A}$			0.00			5.38
$\tilde{\beta}/\tilde{\alpha}$			0.00			−85.9

DIAS transitions, the DCX transitions to other states may distinguish between very-short-range operators, but spin-independent and spin-dependent ones. It is also interesting to note that among the various $J=0^+$ states, there are usually a few that have strong transitions induced by the double Gamow-Teller $[\sigma(1)\cdot\sigma(2)t_+(1)t_+(2)]$ operator [12,13]. For $^{48}Ca\rightarrow{}^{48}Ti$, for example, the highest $J=0^+$, $T=2$ state (in the $f_{7/2}$ model) in ^{48}Ti carries a double Gamow-Teller (DGT) transition strength that is much larger than the strength found in any other final 0^+ state, including the DIAS.

In Table IV we present the results of the extended shell-model space calculations for the Ca→Ti transitions.

TABLE III. $^{40+n}Ca(0^+_{g.s.})\rightarrow{}^{40+n}Ti(0^+_f, T_f=|T_i-2|)$ transition amplitudes M for various types of operators in the single-$f_{7/2}$ shell-model calculation using the "FPD6" interaction. The calculated excitation energies for the final 0^+ states are also given (in MeV).

n	Final state	0^+_1	0^+_2	0^+_3	0^+_4	0^+_5
2	E_x	0.000				
	δtt	2.000				
	$Y_\rho tt$	2.000				
	$Y_\pi tt$	2.000				
	Ctt	2.000				
	tt	2.000				
	$\delta\sigma\sigma tt$	−2.571				
	$Y_\rho\sigma\sigma tt$	−2.571				
	$Y_\pi\sigma\sigma tt$	−2.571				
	$C\sigma\sigma tt$	−2.571				
	$\sigma\sigma tt$	−2.571				
4	E_x	0.000	4.758	8.628		
	δtt	1.677	0.821	0.386		
	$Y_\rho tt$	1.628	0.827	0.390		
	$Y_\pi tt$	1.153	0.748	0.319		
	Ctt	0.382	0.277	0.108		
	tt	0.000	0.000	0.000		
	$\delta\sigma\sigma tt$	2.156	1.056	0.496		
	$Y_\rho\sigma\sigma tt$	2.160	1.045	0.490		
	$Y_\pi\sigma\sigma tt$	2.093	1.191	0.619		
	$C\sigma\sigma tt$	1.784	1.743	1.527		
	$\sigma\sigma tt$	1.558	2.121	2.261		
6	E_x	0.000	3.798	4.982	6.930	8.976
	δtt	1.771	0.754	0.610	0.060	0.648
	$Y_\rho tt$	1.713	0.764	0.621	0.058	0.666
	$Y_\pi tt$	1.178	0.728	0.585	0.003	0.608
	Ctt	0.384	0.277	0.219	0.014	0.219
	tt	0.000	0.000	0.000	0.000	0.000
	$\delta\sigma\sigma tt$	2.277	0.969	0.785	0.077	0.833
	$Y_\rho\sigma\sigma tt$	2.285	0.957	0.774	0.078	0.819
	$Y_\pi\sigma\sigma tt$	2.171	1.100	0.929	0.116	1.081
	$C\sigma\sigma tt$	1.658	1.492	1.596	0.648	2.670
	$\sigma\sigma tt$	1.283	1.723	2.077	1.115	3.918
8	E_x	0.000	3.795	8.063		
	δtt	1.509	0.712	0.855		
	$Y_\rho tt$	1.456	0.740	0.892		
	$Y_\pi tt$	0.979	0.828	0.868		
	Ctt	0.315	0.335	0.319		
	tt	0.000	0.000	0.000		
	$\delta\sigma\sigma tt$	1.941	0.916	1.099		
	$Y_\rho\sigma\sigma tt$	1.949	0.898	1.075		
	$Y_\pi\sigma\sigma tt$	1.827	1.106	1.518		
	$C\sigma\sigma tt$	1.294	1.483	4.147		
	$\sigma\sigma tt$	0.908	1.634	6.204		

The transition amplitudes to the lowest six nonanalog $J=0^+$ states with isospin $T_f=|T_i-2|$ are shown. In the case of ^{42}Ca, the DIAS is the ground state and is included for comparison in the table. For $^{44}\text{Ca}\rightarrow{}^{44}\text{Ti}$, we see that the fourth and fifth 0^+ states in ^{44}Ti carry substantial DGT strength. For $^{46}\text{Ca}\rightarrow{}^{46}\text{Ti}$ and $^{48}\text{Ca}\rightarrow{}^{48}\text{Ti}$, the main part of the DGT $0^+\rightarrow 0^+$ strength is found in higher 0^+ states not shown in the table.

It is interesting to compare the ground state to ground state (g.s. to g.s.) transition amplitudes in the single-$f_{7/2}$ shell-model calculation (Table III) and in the extended fp-shell-model calculation (Table IV). For all the opera-

TABLE IV. Same as Table III, but for the extended fp space shell-model calculation. The results in this table should be compared to those in Table III for the pure $(f_{7/2})^n$ configuration. Only the lowest six final 0^+ states are included.

n	Final state	0_1^+	0_2^+	0_3^+	0_4^+	0_5^+	0_6^+
2	E_x	0.000	5.961	10.931	14.906		
	δtt	3.090	0.899	0.680	1.503		
	$Y_\rho tt$	3.008	0.834	0.618	1.375		
	$Y_\pi tt$	2.581	0.559	0.333	0.660		
	Ctt	2.185	0.205	0.098	0.178		
	tt	2.000	0.000	0.000	0.000		
	$\delta\sigma\sigma tt$	−3.973	1.156	0.875	1.932		
	$Y_\rho\sigma\sigma tt$	−3.992	1.149	0.887	1.992		
	$Y_\pi\sigma\sigma tt$	−4.183	1.267	1.041	2.434		
	$C\sigma\sigma tt$	−4.147	1.007	1.012	2.962		
	$\sigma\sigma tt$	−4.021	0.664	0.884	3.270		
4	E_x	0.000	5.273	8.460	9.749	10.596	10.891
	δtt	2.070	1.028	0.575	0.747	0.898	1.175
	$Y_\rho tt$	1.961	0.996	0.579	0.718	0.859	1.149
	$Y_\pi tt$	1.237	0.697	0.530	0.538	0.592	0.863
	Ctt	0.393	0.231	0.199	0.194	0.198	0.298
	tt	0.000	0.000	0.000	0.000	0.000	0.000
	$\delta\sigma\sigma tt$	2.662	1.322	0.739	0.960	1.155	1.510
	$Y_\rho\sigma\sigma tt$	2.638	1.348	0.721	0.966	1.204	1.527
	$Y_\pi\sigma\sigma tt$	2.354	1.425	0.745	0.861	1.726	1.909
	$C\sigma\sigma tt$	1.469	1.212	0.897	0.026	2.352	3.192
	$\sigma\sigma tt$	0.808	1.011	0.975	0.755	2.661	4.145
6	E_x	0.000	4.891	5.712	6.967	7.340	7.992
	δtt	2.074	1.371	0.478	0.150	0.361	0.535
	$Y_\rho tt$	1.971	1.334	0.472	0.136	0.372	0.500
	$Y_\pi tt$	1.250	0.990	0.408	0.036	0.368	0.291
	Ctt	0.396	0.340	0.154	0.004	0.141	0.090
	tt	0.000	0.000	0.000	0.000	0.000	0.000
	$\delta\sigma\sigma tt$	2.666	1.763	0.614	0.192	0.464	0.688
	$Y_\rho\sigma\sigma tt$	2.648	1.753	0.590	0.210	0.494	0.686
	$Y_\pi\sigma\sigma tt$	2.342	1.688	0.582	0.222	0.587	0.618
	$C\sigma\sigma tt$	1.363	1.478	0.740	0.270	0.116	0.305
	$\sigma\sigma tt$	0.642	1.309	0.828	0.340	0.323	0.058
8	E_x	0.000	4.575	6.818	7.464	7.620	7.979
	δtt	1.801	1.019	0.156	0.241	1.083	0.209
	$Y_\rho tt$	1.709	1.018	0.128	0.219	1.067	0.197
	$Y_\pi tt$	1.064	0.906	0.008	0.044	0.839	0.147
	Ctt	0.333	0.334	0.006	0.010	0.294	0.056
	tt	0.000	0.000	0.000	0.000	0.000	0.000
	$\delta\sigma\sigma tt$	2.315	1.310	0.201	0.310	1.393	0.269
	$Y_\rho\sigma\sigma tt$	2.298	1.297	0.193	0.339	1.388	0.230
	$Y_\pi\sigma\sigma tt$	1.987	1.313	0.117	0.344	1.377	0.152
	$C\sigma\sigma tt$	1.082	1.025	0.009	0.182	1.406	0.587
	$\sigma\sigma tt$	0.430	0.721	0.057	0.104	1.427	0.947

tors of the Xtt type, configuration mixing more or less increases the transition strength. This is especially the case for the short-range operators and for nuclei with a small number of valence nucleons (e.g., $A=42,44$). For instance, for the operator δtt, the g.s.-to-g.s. transition *strength* (M^2) is increased by a factor of 2.39 for $^{42}\mathrm{Ca}\rightarrow{}^{42}\mathrm{Ti}$, but only by a factor of 1.42 for $^{48}\mathrm{Ca}\rightarrow{}^{48}\mathrm{Ti}$. For the operator Ctt, the g.s.-to-g.s. transition strength is increased only by a factor of 1.19 for $^{42}\mathrm{Ca}\rightarrow{}^{42}\mathrm{Ti}$ and by a factor of 1.12 for $^{48}\mathrm{Ca}\rightarrow{}^{48}\mathrm{Ti}$.

The results of the g.s.-to-g.s. transition amplitudes for the $X\sigma\sigma tt$-type operators are more complex. For $^{42}\mathrm{Ca}\rightarrow{}^{42}\mathrm{Ti}$, where the g.s. is also the DIAS, the transition strength is increased by about a factor of 2.4 when we go from Table III (SJ) to Table IV (CM), nearly independent of the range of X. But for other nuclei, configuration mixing increases the g.s.-to-g.s. transition strength for the short-range $X\sigma\sigma tt$ operators, but it decreases the strength for the long-range $X\sigma\sigma tt$ operators.

Of particular interest is the DGT transition amplitude from the ground state in ^{48}Ca to the ground state in ^{48}Ti since it is the nuclear transition amplitude of the double beta decay in the closure approximation. As we see from Tables III and IV, this transition carries very small DGT strength and this strength decreases by about a factor of 4.46 when we go from the single-$f_{7/2}$ calculation (0.908^2 in Table III) to the extended fp space shell-model calculation (0.430^2 in Table IV). It should be pointed out, however, that our result depends on the effective interaction that is used. In previous work [13], we used a modified renormalized Kuo-Brown (MKB) interaction [16] to study the nuclear double-beta-decay amplitude in ^{48}Ca. We found that configuration mixing slightly *increases* the g.s.-to-g.s. DGT amplitude from the single-j value of 0.361 to 0.393 in contrast to the case when the FPD6 interaction of Richter *et al.* [8] is used.

B. $^{56}\mathrm{Fe}\rightarrow{}^{56}\mathrm{Ni}$ transition

This transition is of special interest because of the recent low-energy pion DCX measurements [14,15]. Using pion energies of 50 MeV, forward-angle $^{56}\mathrm{Fe}(\pi^+,\pi^-)^{56}\mathrm{Ni}$ measurements were performed. In addition to the DIAS transition, the three lowest $J=0^+$, $T=0$ states including the ground state were observed in a very recent experiment [15]. The cross sections for the above nonanalog transitions were each about 0.7 μb/sr, while to the DIAS about 2 μb/sr. We explore the $^{56}\mathrm{Fe}\rightarrow{}^{56}\mathrm{Ni}$ transition using, again, the class of operators discussed previously. Two different types of interactions are used in computing the nuclear wave functions [8,16]. The configuration space includes the $0f_{7/2}$ and $1p_{3/2}$ orbits, and up to four nucleons are allowed to leave the $f_{7/2}$ orbit and occupy the $p_{3/2}$ orbit.

In Table V are shown the results of the calculation for

TABLE V. $^{56}\mathrm{Fe}(0^+_{\mathrm{g.s.}})\rightarrow{}^{56}\mathrm{Ni}(0^+_f, T_f=0,2)$ transition amplitudes M for various types of two-body operators using the "FPD6" interaction and the modified Kuo-Brown (MKB) interaction [16]. Only up to four nucleons are allowed to leave the $f_{7/2}$ orbit and occupy the $p_{3/2}$ orbit. The notation $J^+_n(T)$ is used for the nth J^+, isospin T state [so $0^+_1(2)$ is the DIAS]. The excitation energies of the final 0^+ states are also shown (in MeV).

Interaction	Operator	Final states				
		$0^+_1(0)$	$0^+_2(0)$	$0^+_3(0)$	$0^+_4(0)$	$0^+_1(2)$
FPD6	E_x	0.000	8.229	9.604	12.896	13.333
	δtt	3.012	2.789	4.402	1.192	5.013
	$Y_\rho tt$	2.777	2.553	4.127	1.107	4.998
	$Y_\pi tt$	1.177	1.004	1.855	0.500	4.938
	Ctt	0.233	0.188	0.363	0.103	4.910
	tt	0.000	0.000	0.000	0.000	4.899
	$\delta\sigma\sigma tt$	1.396	1.293	2.040	0.552	−2.323
	$Y_\rho\sigma\sigma tt$	1.389	1.270	2.033	0.581	−2.317
	$Y_\pi\sigma\sigma tt$	1.225	1.109	2.050	0.582	−2.256
	$C\sigma\sigma tt$	0.743	1.008	2.349	0.212	−2.206
	$\sigma\sigma tt$	0.405	0.968	2.595	0.879	−2.189
MKB	E_x	0.000	6.579	7.441	11.279	11.633
	δtt	2.619	3.417	3.959	0.184	4.595
	$Y_\rho tt$	2.401	3.139	3.724	0.141	4.603
	$Y_\pi tt$	0.980	1.271	1.693	0.018	4.741
	Ctt	0.189	0.243	0.331	0.011	4.869
	tt	0.000	0.000	0.000	0.000	4.899
	$\delta\sigma\sigma tt$	1.214	1.584	1.835	0.085	−2.130
	$Y_\rho\sigma\sigma tt$	1.210	1.564	1.830	0.070	−2.121
	$Y_\pi\sigma\sigma tt$	1.076	1.433	1.870	0.019	−2.050
	$C\sigma\sigma tt$	0.665	1.387	2.195	0.281	−2.016
	$\sigma\sigma tt$	0.374	1.386	2.456	0.525	−2.011

four lowest 0^+, $T=0$ states and for the DIAS ($T=2$). The normalization constants $\mathcal{N}_1, \ldots, \mathcal{N}_8$ are fixed by requiring that the transition amplitudes in the case of the simplest configuration be the same for different operators, i.e., $\langle \text{DIAS}|Xtt|(f_{7/2}^{14}p_{3/2}^{2})0^+, T=2\rangle$ and $\langle \text{DIAS}|X\sigma\sigma tt|(f_{7/2}^{14}p_{3/2}^{2})0^+, T=2\rangle$ be independent of X. The excitation energies of the calculated 0^+ states and of the measured ones are not in good agreement, probably because of the smallness of the model space that is used. It is not our intention here to try to fit the energy spectrum. Of course, also the transition operators we use are not the actual ones appearing in the pion DCX reaction. Our calculation, however, demonstrates that for a certain class of transition operators, namely, those having a short range and/or spin dependence, one indeed can obtain substantial transition strength to the lowest three 0^+ ($T=0$) states in addition to the DIAS. In fact, operators of the type δtt or $Y_\rho tt$ give about the correct cross-section ratios for the lowest 0^+ states and the DIAS.

C. Ni→Zn transitions

As already mentioned, in the Ni isotopes configuration mixing is very strong. It is of interest to explore DCX-type transitions to nonanalog states. In Table VI we show transitions to the lowest five 0^+ states with isospin $T=|T_i-2|$ (where T_i refers to the parent state). For comparison we include also the transitions to the analog

TABLE VI. $^{56+n}\text{Ni}(0^+_{\text{g.s.}})\rightarrow{}^{56+n}\text{Zn}(0^+_f, T_f=|T_i-2|)$ transition amplitudes for various types of operators in the full $(f_{5/2}p_{3/2}p_{1/2})$ space shell-model calculations using the "ASDI" interaction. The excitation energies of the final 0^+ states are also shown. The transition amplitudes to the DIAS are also shown for comparison.

n	Final state	0_1^+	0_2^+	0_3^+	0_4^+	0_5^+	DIAS
2	E_x	0.000	2.974	4.463			0.000
	δtt	3.611	0.348	0.624			3.611
	$Y_\rho tt$	3.597	0.403	0.577			3.597
	$Y_\pi tt$	3.077	0.386	0.362			3.077
	Ctt	2.290	0.102	0.123			2.290
	tt	2.000	0.000	0.000			2.000
	$\delta\sigma\sigma tt$	−6.018	0.580	1.041			−6.018
	$Y_\rho\sigma\sigma tt$	−6.050	0.578	1.053			−6.050
	$Y_\pi\sigma\sigma tt$	−5.536	0.024	1.389			−5.536
	$C\sigma\sigma tt$	−4.650	0.707	1.929			−4.650
	$\sigma\sigma tt$	−4.239	1.021	2.220			−4.239
4	E_x	0.000	3.238	4.057	4.765	6.426	7.623
	δtt	2.709	0.998	1.706	1.248	0.444	2.969
	$Y_\rho tt$	2.629	0.963	1.703	1.238	0.381	3.099
	$Y_\pi tt$	1.632	0.672	1.221	0.892	0.124	3.847
	Ctt	0.438	0.201	0.343	0.258	0.032	4.589
	tt	0.000	0.000	0.000	0.000	0.000	4.899
	$\delta\sigma\sigma tt$	4.515	1.663	2.844	2.080	0.741	−4.948
	$Y_\rho\sigma\sigma tt$	4.481	1.725	2.915	2.135	0.801	−4.973
	$Y_\pi\sigma\sigma tt$	3.603	2.043	3.005	2.240	1.519	−4.488
	$C\sigma\sigma tt$	2.457	2.046	3.153	2.070	3.019	−3.688
	$\sigma\sigma tt$	1.923	1.988	3.320	1.959	3.925	−3.322
6	E_x	0.000	1.999	3.102	3.407	3.674	11.976
	δtt	3.079	0.597	0.385	1.188	0.972	2.835
	$Y_\rho tt$	2.981	0.588	0.361	1.232	0.949	3.075
	$Y_\pi tt$	1.808	0.379	0.273	0.973	0.646	4.858
	Ctt	0.477	0.100	0.088	0.273	0.184	6.912
	tt	0.000	0.000	0.000	0.000	0.000	7.746
	$\delta\sigma\sigma tt$	5.132	0.996	0.641	1.980	1.620	−4.724
	$Y_\rho\sigma\sigma tt$	5.074	1.014	0.687	1.997	1.630	−4.747
	$Y_\pi\sigma\sigma tt$	3.929	0.943	1.036	1.629	1.532	−4.251
	$C\sigma\sigma tt$	2.531	0.784	1.111	1.181	1.321	−3.455
	$\sigma\sigma tt$	1.889	0.719	1.081	1.026	1.221	−3.094

TABLE VI. (*Continued*).

n	Final state	0_1^+	0_2^+	0_3^+	0_4^+	0_5^+	DIAS
8	E_x	0.000	1.741	2.392	3.162	3.389	16.039
	δtt	3.110	0.600	0.021	0.216	1.548	2.776
	$Y_\rho tt$	3.005	0.600	0.032	0.245	1.540	3.118
	$Y_\pi tt$	1.784	0.456	0.029	0.180	1.065	5.894
	Ctt	0.464	0.134	0.003	0.041	0.293	9.234
	tt	0.000	0.000	0.000	0.000	0.000	10.583
	$\delta\sigma\sigma tt$	5.183	0.999	0.035	0.360	2.581	−4.626
	$Y_\rho\sigma\sigma tt$	5.112	1.007	0.064	0.319	2.601	−4.648
	$Y_\pi\sigma\sigma tt$	3.851	0.918	0.178	0.160	2.269	−4.140
	$C\sigma\sigma tt$	2.383	0.648	0.318	0.411	1.765	−3.338
	$\sigma\sigma tt$	1.718	0.498	0.414	0.455	1.557	−2.978
10	E_x	0.000	2.885	3.032	3.181	3.473	19.898
	δtt	2.868	0.754	0.687	1.177	0.216	2.734
	$Y_\rho tt$	2.769	0.764	0.677	1.187	0.262	3.173
	$Y_\pi tt$	1.620	0.612	0.510	0.803	0.307	6.923
	Ctt	0.415	0.179	0.156	0.211	0.090	11.550
	tt	0.000	0.000	0.000	0.000	0.000	13.416
	$\delta\sigma\sigma tt$	4.780	1.257	1.145	1.961	0.360	−4.557
	$Y_\rho\sigma\sigma tt$	4.707	1.311	1.121	1.940	0.385	−4.578
	$Y_\pi\sigma\sigma tt$	3.467	1.436	0.863	1.370	0.371	−4.054
	$C\sigma\sigma tt$	2.062	1.265	0.354	0.884	0.220	−3.236
	$\sigma\sigma tt$	1.432	1.163	0.054	0.727	0.145	−2.869
12	E_x	0.000	3.033	3.234	3.687	3.964	23.611
	δtt	2.246	1.063	0.713	0.319	0.845	2.698
	$Y_\rho tt$	2.169	1.024	0.763	0.287	0.827	3.226
	$Y_\pi tt$	1.261	0.593	0.661	0.187	0.620	7.932
	Ctt	0.320	0.155	0.183	0.059	0.189	13.858
	tt	0.000	0.000	0.000	0.000	0.000	16.248
	$\delta\sigma\sigma tt$	3.743	1.772	1.189	0.532	1.409	−4.497
	$Y_\rho\sigma\sigma tt$	3.683	1.713	1.237	0.574	1.396	−4.517
	$Y_\pi\sigma\sigma tt$	2.671	1.104	1.198	0.902	1.249	−3.968
	$C\sigma\sigma tt$	1.541	0.519	0.951	0.820	0.801	−3.122
	$\sigma\sigma tt$	1.039	0.250	0.849	0.684	0.522	−2.743

ones. For $A=58$ ($n=2$) the DIAS is also the ground state.

For short-range operators and for $A=56+n$ with $n\geq 4$, some of the nonanalog 0^+ transitions are quite strong and the transition amplitudes to the ground states in the final nuclei (Zn) are actually larger than those to the DIAS. It is quite conceivable that such nonanalog transitions, in particular to the ground state, should be observed in low-energy pion DCX reactions.

V. DISCUSSION AND SUMMARY

The calculations we have presented here and the DCX transition operators we used are quite schematic compared to the complicated nature of the realistic effective DCX operators. Our results therefore should not be compared directly to detailed experimental results. Nonetheless, the variety of types of transition operators we used in this work to mimic the DCX operators provides us with some significant clues as to the nature of the effective DCX transitions, such as the range, the dependence on the explicit two-body correlations, and the dependence on two-body correlations hidden in the *effective* DCX operators. We have seen that the range of the interaction and the spin dependence determine to a large extent the $N-Z$ dependence of the cross section. For long-range transition operators, the DIAS cross section rises as $(N-Z)(N-Z-1)$, and there is no special role for the $T=1$ ($N-Z=2$) nuclei. On the other hand, very-short-range DCX operators and/or the ones that contain spin degrees of freedom will produce approximately an $(N-Z)/(N-Z-1)$ dependence which gives for the $T=1$ nuclei the largest DIAS cross section. For nonanalog transitions the distinction between long- and short-range (or having spin dependence) transition operators is straightforward, simply based on the size of the

cross section. The nonanalog transitions are strongly suppressed with respect to the analog transitions for operators that are spin independent and of long range. The separation between spin-dependent transitions and very-short-range operators (i.e., operators possessing the pairing property) is not straightforward. It is possible that in the study of nonanalog transitions, one will be able to distinguish between the very-short-range and spin-dependent transitions.

There are theoretical indications [17] that cancellations that occur between the spin-dependent part of the DCX transitions and the spin-independent short-range part lead to small B amplitudes for certain pion energies. It is a general feature of the pion DCX transitions that the range of the effective operators is a rather strong function of the pion energy [1,2].

One of the important conclusions of this work is that the formulas in Eqs. (1) and (2), initially derived for a pure j^n configuration, have actually a much wider range of applicability. The above two amplitude equations can be applied also to some cases where there is strong configuration mixing. The effective transition operators deduced from such an analysis reflect the contribution of two-body correlation by having shorter ranges than the corresponding ones obtained in a pure configuration analysis.

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