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Correlation between $E2$ and $M1$ transition strength in even-even vibrational, transitional, and deformed nuclei

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We show that the linear energy weighted $M1$ sum rule is proportional to the summed $E2$ strength in the region of vibrational, transitional, and strongly deformed even-even nuclei using the proton-neutron interacting boson model. This general relation is compared with extensive data on $M1$ and $E2$ transition strength in rare-earth nuclei truncating the $M1$ sum rule to states below an excitation energy of 4 MeV. The above observation also implies a quadratic dependence on deformation (δ) for the summed orbital $M1$ strength.

Since the discovery of a strong isovector $M1$ excitation in inelastic electron-scattering experiments on ^{156}Gd, performed at Darmstadt [1], many attempts have been made, both experimentally and theoretically, to understand this low-lying 1^+ mode of motion [2,3]. In particular, starting from microscopic calculations in a deformed shell model, the quasiparticle random-phase approximation (QRPA) has been shown to exhibit many properties of the experimentally observed data [3–6] which are now available for nuclei throughout the whole rare-earth region, as well as for many light nuclei. Both the excitation energies, the fragmentation, and strong orbital character of a particular class of 1^+ states could be described in a qualitative way. More recently, (p,p') experiments at TRIUMF [7] pointed towards the existence of spin-flip $M1$ strength located at higher energies ($5 \leq E_x \leq 9$ MeV), results that can be obtained from the QRPA studies [5,8,9]. In all the above studies, a picture of rather important fragmentation of a "scissorlike" 1^+ mode results.

The first calculations, relating to the possible observation of a new collective 1^+ mode, though, were obtained from collective model approaches [two-rotor model, interacting boson model (IBM-2), ...]. These latter studies [10–13] concentrated in all cases on a single, rather strong collective orbital excitation mode, corresponding to rotational oscillations of protons relative to the neutrons.

Very recently, the now extensive set of data on $M1$ strength in rare-earth nuclei and its mainly orbital nature pointed towards the existence of a strong correlation between the functional dependence of the summed $M1$ strength (summed up to $E_x \leq 4$ MeV) and the $B(E2;0_1^+ \rightarrow 2_1^+)$ value on proton and neutron number [14]. This general relationship points towards some deeper connection between these two quantities and, because of the smooth behavior in both $\sum_f B(M1;0_1^+ \rightarrow 1_f^+)$ and $B(E2;0_1^+ \rightarrow 2_1^+)$ on Z and N, suggests a collective model approach.

One can express the linear energy weighted $M1$ sum rule in terms of a double commutator of the $M1$ operator with the nuclear Hamiltonian as

$$\sum_f B(M1;0_1^+ \rightarrow 1_f^+) E_x(1_f^+) = (\sqrt{3}/2) \langle 0_1^+ | [[\hat{H}, \hat{T}(M1)], \hat{T}(M1)]^{(0)} | 0_1^+ \rangle . \quad (1)$$

Starting from the IBM-2 Hamiltonian describing proton-neutron mixed-symmetry modes of motion [11–13]

$$\hat{H}_1 = \epsilon_{d_\pi} \hat{n}_{d_\pi} + \epsilon_{d_\nu} \hat{n}_{d_\nu} + \kappa_{\pi\nu} \hat{Q}_\pi \cdot \hat{Q}_\nu , \quad (2)$$

with the following expressions for the boson number and quadrupole operator,

$$\hat{n}_{d_\rho} = (d^\dagger \tilde{d})_\rho^{(0)} ;$$
$$\hat{Q}_\rho = (s^\dagger \tilde{d} + d^\dagger s)_\rho^{(2)} + \chi_\rho (d^\dagger \tilde{d})_\rho^{(2)} , \quad (3)$$

one can evaluate the double commutator, using the IBM-2 $M1$ operator,

$$\hat{T}(M1) = \sqrt{3/4\pi} (g_\pi \hat{L}_\pi + g_\nu \hat{L}_\nu) ;$$
$$\hat{L}_\rho = \sqrt{10} (d^\dagger \tilde{d})_\rho^{(1)} . \quad (4)$$

The one-body term $\epsilon_{d_\rho} \hat{n}_{d_\rho}$ ($\rho = \pi, \nu$) results in a vanishing contribution to the right-hand side of Eq. (1). The quadrupole interaction term on the other hand gives rise to the following expression for the double commutator, i.e.,

$$[[\hat{H}_1, \hat{T}(M1)], \hat{T}(M1)]^{(0)} = (30/4\pi) \Bigg(g_\pi^2 \kappa_{\pi\nu} \Bigg[-(\sqrt{3}/5)(d^\dagger s + s^\dagger \tilde{d})^{(2)} - 2\sqrt{3} \left(\frac{1}{5} + \begin{Bmatrix} 2 & 2 & 2 \\ 2 & 2 & 1 \end{Bmatrix} \right) \chi_\pi (d^\dagger \tilde{d})^{(2)} \Bigg] \cdot \hat{Q}_\nu + (\pi \rightleftarrows \nu) + 2 g_\pi g_\nu \kappa_{\pi\nu} (\sqrt{3}/5) \hat{Q}_\pi \cdot \hat{Q}_\nu \Bigg) , \quad (5)$$

$$= -3(\sqrt{3}/2\pi)(g_\pi - g_\nu)^2 \kappa_{\pi\nu} \hat{Q}_\pi \cdot \hat{Q}_\nu . \quad (6)$$

Starting from the more general F-spin invariant IBM-2 Hamiltonian [11–13],

$$\hat{H}_2=\epsilon_{d_\pi}\hat{n}_{d_\pi}+\epsilon_{d_\nu}\hat{n}_{d_\nu}+\kappa(\hat{Q}_\pi+\hat{Q}_\nu)\cdot(\hat{Q}_\pi+\hat{Q}_\nu)\,, \tag{7}$$

where $\epsilon_{d_\pi}=\epsilon_{d_\nu}$ the evaluation of the double commutator is somewhat more involved but results in the particularly simple expression

$$[[\hat{H}_2,\hat{T}(M1)],\hat{T}(M1)]^{(0)}=-3(\sqrt{3}/\pi)\kappa(g_\pi-g_\nu)^2\hat{Q}_\pi\cdot\hat{Q}_\nu\,. \tag{8}$$

So, in both cases, we obtain for the energy weighted $M1$ sum rule

$$\sum_f B(M1;0_1^+\rightarrow 1_f^+)E_x(1_f^+)=(9/2\pi)\begin{Bmatrix}-\kappa_{\pi\nu}/2\\ -\kappa\end{Bmatrix}(g_\pi-g_\nu)^2\langle 0_1^+|\hat{Q}_\pi\cdot\hat{Q}_\nu|0_1^+\rangle\,, \tag{9}$$

for the Hamiltonian $\hat{H}_1$ and $\hat{H}_2$ in the upper line and lower line, respectively.

From expression (9) one observes a strong correlation between the $M1$ energy weighted sum rule and the strength of the deformation driving quadrupole proton-neutron force. The expectation value in the 0^+ ground state of the latter force is a measure for the quadrupole deformation energy which can also be related to the corresponding binding energy in a quadrupole deformed mean field such as the Nilsson model [15].

We now evaluate the ground-state expectation value appearing on the right-hand side of expression (9). Inserting a complete set of intermediate 2^+ states, called $|2_f^+\rangle$, this expectation value also becomes

$$\sum_f\langle 2_f^+||\hat{Q}_\pi||0_1^+\rangle\langle 2_f^+||\hat{Q}_\nu||0_1^+\rangle\,. \tag{10}$$

If we make the tacid assumption that the states describing the 0_1^+ and 2_f^+ have good F-spin quantum number $F=F_{\max}=(N_\pi+N_\nu)/2$, the reduced matrix elements can be related to the one for the full $\hat{Q}_\pi+\hat{Q}_\nu$ operator [12], since

$$\frac{\langle F||\hat{Q}_\rho||F\rangle}{\langle F||\hat{Q}_\pi+\hat{Q}_\nu||F\rangle}=\frac{N_\rho}{N_\pi+N_\nu}\,. \tag{11}$$

We finally obtain the result that

$$\sum_f B(M1;0_1^+\rightarrow 1_f^+)E_x(1_f^+)=\sum_f B(E2;0_1^+\rightarrow 2_f^+)\,, \tag{12}$$

where an effective charge (in units e b) is to be used and is given as

$$e_{\rm eff}=\left[(9/2\pi)\begin{Bmatrix}-\kappa_{\pi\nu}/2\\ -\kappa\end{Bmatrix}(g_\pi-g_\nu)^2N_\pi N_\nu/(N_\pi+N_\nu)^2\right]^{1/2}, \tag{13}$$

with the gyromagnetic factors expressed in units μ_N and $\kappa_{\pi\nu},\kappa$ (<0) in MeV. This has to be done in order to bring the different dimensions of the $M1$ and $E2$ part of expression (12) in accordance with each other.

We remark that the Majorana term in the IBM-2 Hamiltonian is necessary for an adequate description of mixed-symmetry states [12,13]. This term will also give rise to an extra contribution to the energy weighted $M1$ sum rule. In the case of an F-spin invariant form of the Majorana term ($\xi_1=\xi_2=\xi_3=\xi$)

$$\hat{M}=\xi\left[(s_\pi^\dagger d_\nu^\dagger-s_\nu^\dagger d_\pi^\dagger)^{(2)}\cdot(s_\pi\tilde{d}_\nu-s_\nu\tilde{d}_\pi)^{(2)}-2\sum_{k=1,3}(d_\pi^\dagger d_\nu^\dagger)^{(k)}\cdot(\tilde{d}_\pi\tilde{d}_\nu)^{(k)}\right], \tag{14}$$

one obtains

$$[[\hat{M},\hat{T}(M1)],\hat{T}(M1)]^{(0)}=\xi(g_\pi-g_\nu)^2(\sqrt{3}/20\pi)[(d_\pi^\dagger\tilde{d}_\pi)^{(1)}\cdot(d_\nu^\dagger\tilde{d}_\nu)^{(1)}+3(d_\pi^\dagger\tilde{d}_\pi)^{(2)}\cdot(d_\nu^\dagger\tilde{d}_\nu)^{(2)}$$
$$+6(d_\pi^\dagger\tilde{d}_\pi)^{(3)}\cdot(d_\nu^\dagger\tilde{d}_\nu)^{(3)}+10(d_\pi^\dagger\tilde{d}_\pi)^{(4)}\cdot(d_\nu^\dagger\tilde{d}_\nu)^{(4)}+3[(s_\pi^\dagger\tilde{d}_\pi)^{(2)}\cdot(d_\nu^\dagger s_\nu)^{(2)}+(d_\pi^\dagger s_\pi)^{(2)}\cdot(s_\nu^\dagger\tilde{d}_\nu)^{(2)}]]\,. \tag{15}$$

When applying relationship (12) for realistic cases, one should take this destructive contribution into account.

Here now, we incorporate a number of approximations to make the exact relation (12) more useful.

(i) In the energy weighted sum rule, since we discuss the IBM-2 and as such, collective orbital scissorslike states are considered, most $M1$ strength is coming from a very restricted interval near $E_x\simeq 3$ MeV [14,16]. So, an average energy $\overline{E_x}(1^+,\text{coll})$ can be taken out of the sum on the left-hand side of Eq. (12). We note that, when plotting the energy weighted as well as the nonenergy weighted $M1$ sum rule, using the experimental $B(M1)$ values and the corresponding excitation energies versus the experimental $B(E2)\uparrow$ values, one obtains approximately a straight line in both cases. The slopes, however, exhibit a ratio of about 3. These results support the above approximation.

(ii) In transitional and, in particular, in strongly deformed nuclei, by far most of the summed $E2$ strength resides in the first 2^+ state. So, within a good approximation, the sum on the right-hand side in Eq. (12) can be restricted to the 2_1^+ level only, and for which state $F=F_{\max}$ holds to a very good approximation [12].

(iii) In applications to rare-earth nuclei, the product $\kappa_{\pi\nu}N_\pi N_\nu/(N_\pi+N_\nu)^2$ is very smoothly varying and stays near to a value of -0.02 for the region of nuclei $^{146\text{-}152}$Nd, $^{148\text{-}154}$Sm, $^{150\text{-}156}$Gd. Indeed, the value $\kappa_{\pi\nu}$ increases for N and/or Z approaching the closed shell [17] since $\kappa=\kappa_0[(\Omega_\pi-N_\pi)(\Omega_\nu-N_\nu)]^{1/2}$ and the product $(N_\pi/N)\cdot(N_\nu/N)$ decreases for N or Z approaching the closed shell, compensating quite well to an almost constant value. Using the parameter values obtained from fitting to the nuclear properties in the Nd, Sm, and Gd isotopes [17], one can determine the value of $e_{\rm eff}$. The results are presented in Table I. One indeed finds typical values for the effective charge used in standard IBM-2 calculations for the rare-earth mass region [17,18]. Moreover, performing a numerical calculation with parameters taken from Ref. [17], also taking into account the Majorana contribution of Eq. (15) with $\xi = 0.17$ MeV, for ^{154}Sm, a deviation on the effective charge of not more than 5% is introduced.

Using the above three quite reasonable assumptions, we finally obtain an approximate relation connecting the summed $M1$ strength and the $B(E2;0_1^+ \rightarrow 2_1^+)$ reduced transition probability as

$$\sum_f B(M1;0_1^+ \rightarrow 1_f^+)\overline{E_x}(1^+,\text{coll}) \simeq B(E2;0_1^+ \rightarrow 2_1^+) . \tag{16}$$

Since experimental data indicate that most orbital strength is exhausted below $E_x \leq 4$ MeV, the summed $M1$ strength should follow very closely the behavior of the $B(E2;0_1^+ \rightarrow 2_1^+)$ value. This relation, as described in Eq. (16), is born out by experimental $B(M1)$ and $B(E2)$ values and is illustrated in Fig. 1, taken from Ref. [14], and comes about from the precise structure of the Hamiltonian (using a quadrupole-quadrupole force) and the form of the IBM-2 $M1$ operator. Similar relations could also be obtained within the nuclear shell model [19] and point towards the general value of relations like (12) and the more approximate one in Eq. (16). There remains a problem, though , with the large $B(M1)$ value measured in ^{164}Dy. The smooth variation in the summed $M1$ strength cannot accommodate such a large and sudden change in a single nucleus. Since the sum rule equality, expressed in Eq. (12), mainly accounts for the orbital part of the $M1$ sum rule at lower energies [the derivation of relation (12) was carried out within the IBM-2], the deviation might hint towards important $M1$ spin-flip contributions. At the same time, it should be noted that the possible observation of a doublet in the nucleus ^{164}Dy using (γ,γ') experiments [3] might lead to too large a value for the experimental summed $M1$ strength.

We have also carried out more detailed QRPA calculations in the rare-earth region [3,8] for which most data are available at present, concentrating on 1^+ states and the corresponding $B(M1;0_1^+ \rightarrow 1_f^+)$ values. In plotting the summed orbital $M1$ strength versus the P factor [20] [equal to $N_pN_n/(N_p+N_n)$] in Fig. 2, a behavior which is rather similar to the experimental summed strengths results, indicating saturation for a number of nuclei near the value of $P \simeq 7-8$ and with a summed $\sum B_{\rm orbital}(M1)$ strength near $2.5\mu_N^2$. We summed the $M1$ strength up to 4 MeV (to comply with the data points obtained and also expressing the fact that most $M1$ strength in this energy region is mainly of orbital character). The theoretical QRPA values, though, exhibit a much slower rise with P than the data in Fig. 1 indicate. This might be due to the use of the Nilsson model description as the underlying basis for a microscopic description of 1^+ states and the corresponding $B(M1)$ strength, even in the region of transitional nuclei in between vibrational and strongly deformed nuclei.

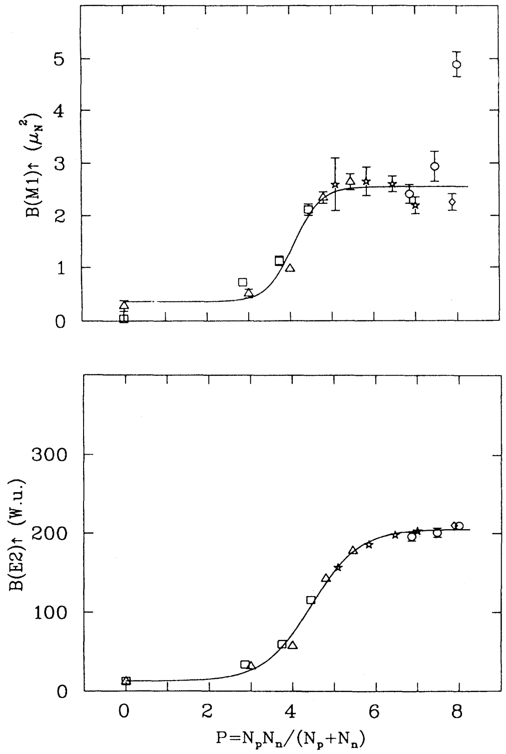

FIG. 1. A comparison between the functional forms of the summed $B(M1)$ strength, up to $E_x \leq 4$ MeV (upper part) and of the $B(E2;0_1^+ \rightarrow 2_1^+)$ values (lower part). The data (taken from Ref. [14]) correspond to the following nuclei: ◇, ^{168}Er; ○, $^{160-164}$Dy; ☆, $^{154-160}$Gd; △, $^{144,148-154}$Sm; and □, $^{142,146-150}$Nd.

A QRPA study for 2^+ collective states and the corresponding $E2$ decay using a deformed shell model (Nilsson model) has not been carried out. It was pointed out, though, by Casten, Heyde, and Wolf [21], that the $B(E2;0_1^+ \rightarrow 2_1^+)$ values exhibit a saturation when approaching the region of strongly deformed nuclei. There the $B(E2)$ saturation was derived implicitly from an evaluation of the quadrupole proton-neutron ground-state expectation value (using the Nilsson model) and an effective $N_{\rm eff}$ boson number was deduced. Starting from the right-hand side of Eq. (9), we have shown that the proton-neutron quadrupole ground-state expectation value is indeed proportional to $\sum_f B(E2;0_1^+ \rightarrow 2_f^+)$ and so, the $E2$ saturation property can most probably be traced back to a

TABLE I. The boson effective charge $e_{\rm eff}$, derived from Eq. (13), using the $\kappa_{\pi\nu}$ parameters from Scholten [17]. The g factors are chosen as $g_\pi = 1\mu_N$ and $g_\nu = 0\mu_N$ [12,13].

	N			
	86	88	90	92
Nd	0.148	0.130	0.111	0.126
Sm	0.127	0.113	0.114	0.120
Gd	0.106	0.103	0.115	0.119

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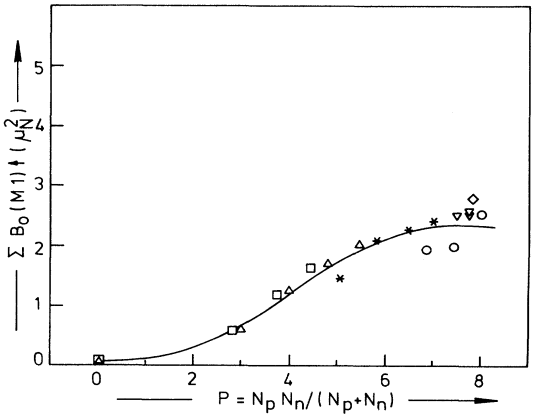

FIG. 2. The summed orbital $M1$ strength, calculated using the QRPA method for rare-earth nuclei, as outlined in Refs. [2,3] and [8]. The symbols denote the calculated QRPA values for the various nuclei obtained for the Er, Dy, Gd, Sm, and Nd isotopes, and are identical with the ones used in Fig. 1. Here, we also give results for the $^{172\text{-}176}$Yb nuclei (denoted by the symbol ∇). The solid line is drawn just to guide the eye.

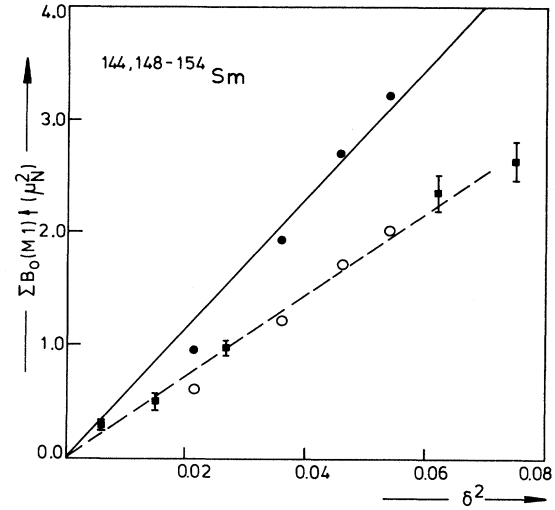

FIG. 3. Comparison between the calculated summed $M1$ strength (with $E_x \leq 4$ MeV) using the QRPA and the experimentally summed $M1$ strength [16] as a function of the square of the nuclear deformation δ^2. We present the total summed $M1$ strength (solid circles and solid line) and the summed orbital $M1$ strength (open circles and dashed line). The solid and dashed lines are fits to the theoretical values.

saturation in ground-state proton-neutron binding energy (or in a corresponding saturation for the ground-state quadrupole deformation value for strongly deformed nuclei, as calculated by Möller and Nix [22]).

Finally, an interesting observation results from Eq. (16). In the region of deformed nuclei, the $B(E2;0_1^+ \rightarrow 2_1^+)$ varies quadratically with the nuclear quadrupole deformation δ. Thus, Eq. (16) implies that the summed $M1$ orbital strength [this is realized in the (γ,γ') experiment on $^{148\text{-}154}$Sm by summing $M1$ strength up to $E_x \simeq 4$ MeV [16]] will also vary quadratically with the ground-state equilibrium deformation when extracting the value of δ from the experimental $B(E2;0_1^+ \rightarrow 2_1^+)$ expression (see Fig. 3). On the same figure, we indicate the QRPA summed orbital $M1$ strength (summed up to the excitation energy of $E_x \leqq 4$ MeV) which corroborates the data as well as the relation, given in Eq. (16). The differences in δ_{th}^2 and δ_{exp}^2 occur since the theoretical values are obtained from ground-state equilibrium value calculations carried out by Möller and Nix [22], whereas the experimental values were extracted from corresponding $E2$ transitions and using the rotational expression.

Concluding, we have derived, starting from the proton-neutron interacting boson model (IBM-2), a general expression relating the linear energy weighted $M1$ sum rule to the summed $E2$ strength in a given nucleus. One can approximate this relation for orbital $M1$ strength, located in the energy region around $E_x \simeq 3$ MeV, into an expression showing a proportionality between the summed $M1$ strength below 4 MeV and the $B(E2;0_1^+ \rightarrow 2_1^+)$ value. In this particular energy region, spin-flip excitations that mainly contribute to the $M1$ sum rule at higher excitation energies (5 MeV $\leq E_x \leq$ 9 MeV) are expected to contribute only in a minor way. Such a relation is born out by the experimental data on $M1$ and $E2$ strengths in rare-earth nuclei. Moreover, a quadratic dependence on nuclear deformation with the summed orbital $M1$ strength below $E_x \leq 4$ MeV is implied. Microscopic quasiparticle RPA (QRPA) calculations are in line with the data on summed $M1$ strength in most of the rare-earth nuclei.

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