RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 44, NUMBER 6 DECEMBER 1991

Systematic behavior of the neutron-deficient molybdenum nuclei

C. J. Gross,[(1)] W. Gelletly,[(2)] M. A. Bentley,[(2)] H. G. Price,[(2)] J. Simpson,[(2)]
K. P. Lieb,[(1)] D. Rudolph,[(1)] J. L. Durell,[(3)] B. J. Varley,[(3)] and S. Rastikerdar[(4)]

[(1)]*II. Physikalisches Institut, Universität Göttingen, D-3400 Göttingen, Federal Republic of Germany*
[(2)]*Science and Engineering Research Council, Daresbury Laboratory, Warrington WA4 4AD, United Kingdom*
[(3)]*Schuster Laboratory, University of Manchester, Manchester M13 9PL, United Kingdom*
[(4)]*Physics Department, Isfahan University, Isfahan, Iran*

(Received 18 July 1991)

Excited states in ^{86,88}Mo have been identified using recoil-γ and $\gamma\gamma$ coincidences. Energy-level systematics of the molybdenum isotopes from the $N=Z$ line to the $N=50$ shell closure are compared to their strontium and zirconium neighbors. This comparison, an estimate of the $B(E2)$ strengths, and a Woods-Saxon cranking model have been used to suggest that the molybdenum isotopes undergo a transition from spherical to moderately deformed shapes at ^{86}Mo.

It is now well established [1–4] that there is a region of strongly deformed nuclei centered on the self-conjugate nucleus ^{76}Sr which has a quadrupole deformation $|\beta_2| \approx 0.4$. Macroscopic-microscopic calculations [5–7] of the nuclear total Routhian surfaces (TRS) reproduce the observed deformation at the center of this region very well. Such calculations combine the classical collective nuclear properties of the liquid drop model with Strutinsky corrections for the effects of pairing and shell structure. The input to such calculations comes from the measured properties of nuclei closer to stability. For the $A>80$ nuclei in this region one expects a transition from strongly deformed shapes to spherical nuclei at the $N=Z=50$ closed shells. Such a transition has been observed [8–11] experimentally in nuclei with $Z\leq 40$ at $N\approx 45$. Our recent investigations [12–14] of the light Nb isotopes ($Z=41$) have indicated similar results. An abrupt increase [15] in collectivity in the three-quasiparticle (3qp) band as compared to the 1qp band in ^{87}Mo has been observed. The properties of these transitional nuclei should provide an excellent testing ground for the TRS calculations. In this context we report here the first observation of the decay of excited states in the even-even nuclei ^{86,88}Mo with in-beam γ-ray spectroscopy techniques.

The nuclei with $N\approx Z$ lie a long way from the line of stability for $A\approx 80$. As long as we are confined to stable beams and targets they can only be produced in fusion-evaporation reactions with small cross sections. In the present experiments the prompt γ rays from ^{86,88}Mo produced in fusion-evaporation reactions were identified by detecting them in coincidence with the recoiling nuclei using the Daresbury recoil separator [16] (RS) and POLYTESSA array [17]. The following reactions were employed: (i) ^{40}Ca(^{50}Cr,$2p2n$)^{86}Mo and (ii) ^{40}Ca(^{50}Cr,$2p$)^{88}Mo at 170 MeV, (iii) ^{58}Ni(^{32}S,$2p2n$)^{86}Mo and (iv) ^{58}Ni(^{32}S,$2p$)^{88}Mo at 110 MeV. It should be noted that inverse reactions are preferred as the higher recoil velocity enhances the efficiency of the RS and the Z resolution of the ionization chamber. Reactions (iii) and (iv) are not inverse, however, since a ^{32}S target which could withstand the ^{58}Ni beam could not be found. Identification of the molybdenum isotopes was obtained with reactions (i) and (ii).

The identification procedures used in the present work have been described previously [1–4,12]. The mass and Z gated spectra shown in Fig. 1 are "pure," i.e., they contain only decays from ^{86}Mo and ^{88}Mo. The inset in Fig. 1 con-

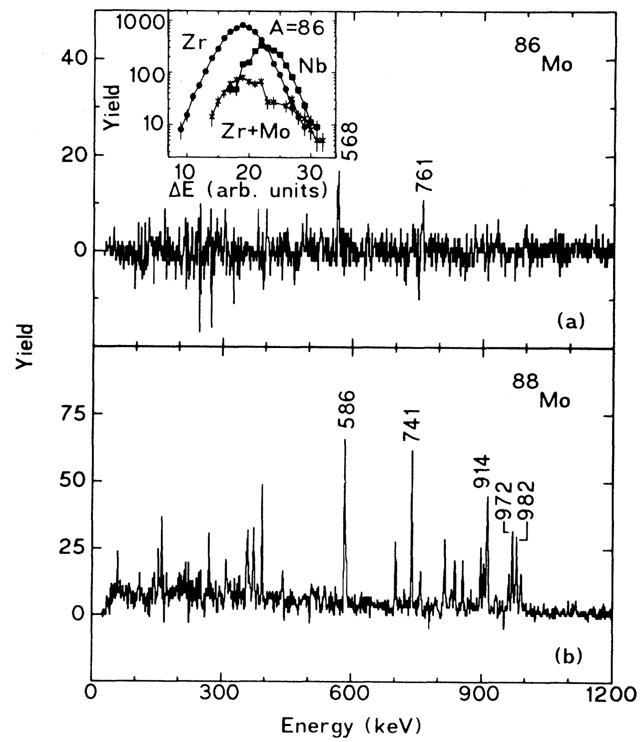

FIG. 1. Spectra gated by mass and Z resulting in "pure" spectra for (a) ^{86}Mo and (b) ^{88}Mo. The inset contains the γ-ray yield distributions of the $A=86$ nuclei as a function of energy loss in the ionization chamber located at the end of the recoil separator. The energy loss is Z dependent. The transitions used are 234 keV (^{86}Zr [9]), 248 keV (^{86}Nb [14]), and 567 keV doublet (566 keV, ^{86}Zr [19] and 568 keV, ^{86}Mo).

tains the γ-ray intensity distributions for the $A=86$ nuclei with different atomic numbers as a function of recoil energy loss in the ionization chamber. Despite the contamination of the 568-keV ^{86}Mo transition by the 566-keV γ ray [9] from ^{86}Zr, unambiguous identification of two transitions in ^{86}Mo was possible. The pure ^{88}Mo spectrum was not difficult to obtain as the ^{88}Tc$+pn$ channel [18] was much more weakly populated. Despite the many transitions identified, we restrict our discussion to the four transitions shown in Fig. 2. $\gamma\gamma$ coincidences have been used to construct the ^{86}Mo and ^{88}Mo level schemes.

The positive-parity states in the Mo isotope chain [4,19,20] from the $N=Z$ line to the $N=50$ spherical shell closure are shown in Fig. 2. The lowering of the 2^+ state as N decreases is indicative of an increase in collectivity. This and similar trends in the Mo chain can be compared with the more well-known Sr [3,10,11,20–22] and Zr [1,8,9,20,23,24] isotopes in Fig. 3. Another indication of collectivity is the ratio of the energies of the 4^+ and 2^+ yrast states: A value of 2.0 is expected for vibrational excitations and 3.3 in the rotational limit. In the $A=80$ region, despite the presence of some of the largest ground-state deformations known, the largest ratio attained [1,25] is 2.86 in ^{80}Zr. It has been observed in the $N=44$ isotones ^{82}Sr and ^{84}Zr that rotational bands are found despite $E(4^+)/E(2^+)$ ratios [8,10] of 2.32 and 2.34, respectively. The ratio for ^{86}Mo is 2.34.

An additional fingerprint of collectivity is the lifetime of the first 2^+ state. A recent [26] global parametrization of the $B(E2,2^+\rightarrow 0^+)$ strengths for approximately 280 nuclei found that a rather good estimate of the collectivity of the nucleus can be obtained on the basis of the energy of the 2^+ state:

$$B(E2,2^+\rightarrow 0^+)=\frac{(94\pm 17)Z^2}{A^2E(2^+)}\ \text{W.u.}, \qquad (1)$$

where $E(2^+)$ is given in MeV. This estimation and measured $B(E2,2^+\rightarrow 0^+)$ values in the Mo, Zr, and Sr isotopes are also shown in Fig. 3. The estimates seem reasonably good in the transition region while overpredicting and underpredicting the measured values at the extremes. This is to be expected [26] near the closed shell. As many very collective nuclei lie far from stability their

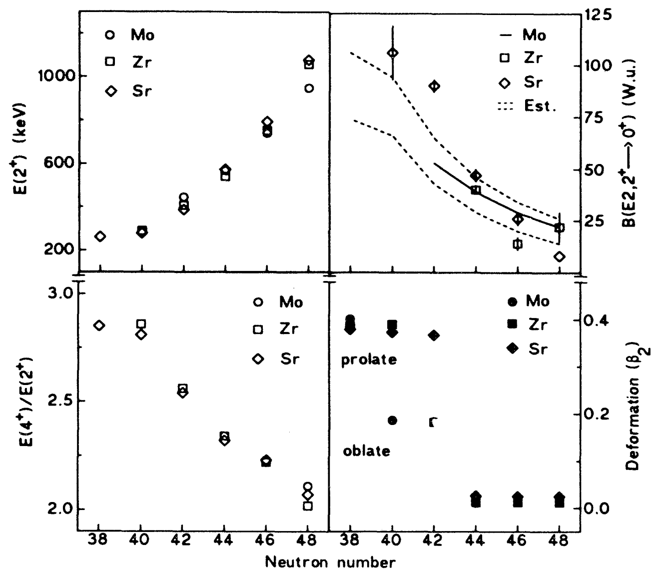

FIG. 3. A comparison of various experimental quantities and theoretical predictions for the Mo, Zr, and Sr nuclei with $N<50$. The quantities are the 2^+ energy levels (upper left), the ratio of the 4^+ yrast energy to that of the 2^+ (lower left), experimental and estimated (see text) $B(E2,2^+\rightarrow 0^+)$ strengths (upper right), and the predicted ground-state deformations from Ref. [7] (lower right). The Mo $B(E2)$ strengths are derived from Eq. (1) as are those labeled "Est." which is the envelope corresponding to the 2^+ energies from the Sr and Zr isotopes.

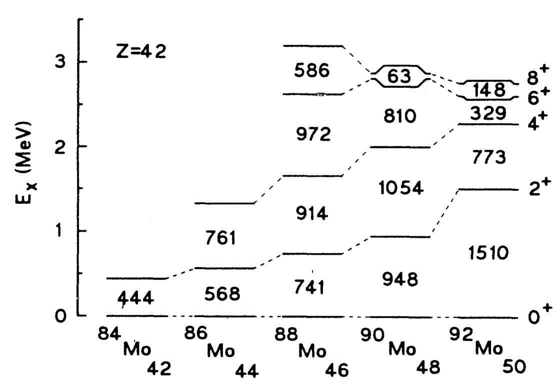

FIG. 2. Positive-parity yrast energy levels in the Mo isotopes from the $N=Z$ line to the $N=50$ spherical shell closure.

lifetimes are difficult to measure. The transitional nuclei which usually lie closer to stability may have been over represented in the data set used to generate Eq. (1). Nevertheless, ^{86}Mo and ^{88}Mo do lie in the shape transition region between the large single-particle energy gaps at particle number 38 (prolate) and 50 (spherical). It seems appropriate to conclude that ^{84}Mo and ^{86}Mo are more collective than ^{88}Mo and ^{90}Mo; the latter [19] has been described using a spherical shell-model approach. But are they permanently deformed?

Figure 3 also summarizes the predicted ground-state deformations for the Mo, Zr, and Sr isotopes as calculated in Ref. [7]. A sharp transition from deformed to spherical shapes is predicted to occur at $N=44$. Such a drastic loss of collectivity would reveal itself in the variation with N of the 2^+ energies and $B(E2,2^+\rightarrow 0^+)$ strengths which has not been observed experimentally. However, a sudden change in the character of the excited states does occur at $N=46$. Excited states in the $N=44$ isotones with $36\leq Z\leq 40$ are organized into rotational bands and are connected by fast $E2$ transitions with relatively constant $B(E2)$ strengths [8,27]. The $N=46$ isotones, however, have $B(E2)$ values which vary widely and cannot be described as rotational [9,11]. The striking similarities between the Mo, Zr, and Sr isotopes suggest that the states in ^{84}Mo and ^{86}Mo are at least moderately deformed.

In order to explore this further, we have calculated TRS maps using a Hartree-Fock-Bogolyubov cranking calculation (including pairing) with a nonaxial Woods-

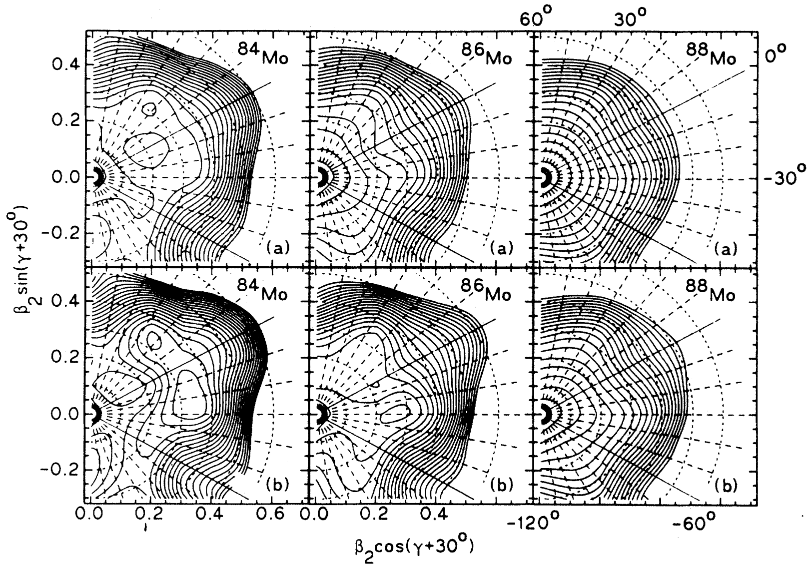

FIG. 4. The total Routhian surfaces calculated for ^{84,86,88}Mo (a) in their ground states and (b) at rotational frequency $\hbar\omega \approx 0.3$ MeV. The energy separation between contour lines is approximately 250 keV.

Saxon potential. A description of the model, potential, and procedures used for these calculations can be found in Refs. [5,28–30]. The TRS maps of the ground states in ^{84,86,88}Mo are shown in the upper part of Fig. 4. The lower part of the figure contains maps calculated at a rotational frequency $\hbar\omega \approx 0.3$ MeV which is characteristic of the states shown in Fig. 2. The results for ^{84}Mo indicate a very soft core with a wide and flat minimum. Two deformed minima can be distinguished at $(\beta_2,\gamma) = (0.19,-62°)$ and $(0.32,+20°)$ which are 400-500 keV above the spherical minimum. Upon cranking, two minima become better defined and straddle the prolate axis, $(0.33,-25°)$ and $(0.35,+22°)$, while the spherical minimum has disappeared. The ground states of ^{86}Mo and ^{88}Mo are predicted to be spherical. The inclusion of cranking has little effect on the spherical minimum in ^{88}Mo (although the cranked shell model may not give correct results in the case of spherical nuclei). On the other hand, a triaxial minimum at $(0.27,-30°)$ is only 160 keV above the spherical minimum in ^{86}Mo at $\hbar\omega \approx 0.3$ MeV. Similar calculations have yielded comparable results for the Sr and Zr isotones [5,6,31]. It seems likely, therefore, that the Mo isotopes follow a similar trend which is dependent on N: $^{84}Mo_{42}$ is permanently deformed, $^{86}Mo_{44}$ is beta and gamma soft, and $^{88}Mo_{46}$ is weakly deformed. Clearly, an extension of the levels and lifetime measurements are necessary to confirm this analysis. Such measurements are in progress [32].

In summary, the lowest excited states of the ground bands in ^{86,88}Mo have been identified using recoil-γ techniques. A comparison of the energy-level systematics between the Mo isotopes and their Sr and Zr neighbors suggests a transition from moderately deformed shapes to near spherical shapes occurs between ^{86}Mo and ^{88}Mo. A phenomological collectivity estimate and Hartree-Fock cranking calculations support this interpretation.

The authors would like to acknowledge the assistance given by the crew and staff at Daresbury Laboratory. We would like to thank M. Kabadiyski and M. Weiszflog for helpful discussions and C. J. Lister and D. J. Blumenthal for help with the experiments. This work was supported by the Deutsches Bundesministerium für Forschung und Technologie under Contract No. 06GÖ141/I, grants from the Science and Engineering Research Council (United Kingdom), and Isfahan University (Iran).

[1] C. J. Lister, M. Campbell, A. A. Chishti, W. Gelletly, L. Goettig, R. Moscrop, B. J. Varley, A. N. James, T. Morrison, H. G. Price, J. Simpson, K. Connel, and O. Skeppstedt, Phys. Rev. Lett. **59**, 1270 (1987).

[2] B. J. Varley, M. Campbell, A.A. Chishti, W. Gelletly, L. Goettig, C. J. Lister, A. N. James, and O. Skeppstedt, Phys. Lett. B **194**, 463 (1987).

[3] C. J. Lister, P. J. Ennis, A. A. Chishti, B. J. Varley, W. Gelletly, H. G. Price, and A. N. James, Phys. Rev. C **42**, R1191 (1990).

[4] W. Gelletly, M. A. Bentley, H. G. Price, J. Simpson, C. J. Gross, J. L. Durell, B. J. Varley, Ö. Skeppstedt, and S. Rastikerdar, Phys. Lett. B **253**, 287 (1991).

[5] W. Nazarewicz, J. Dudek, R. Bengtsson, T. Bengtsson, and I. Ragnarsson, Nucl. Phys. A **435**, 397 (1985).

[6] P. Bonche, H. Flocard, P. H. Heenen, S. J. Krieger, and M. S. Weiss, Nucl. Phys. A **443**, 39 (1985).

[7] P. Möller and J. R. Nix, At. Data Nucl. Data Tables **26**, 165 (1981).

[8] H. G. Price, C. J. Lister, B. J. Varley, W. Gelletly, and J. W. Olness, Phys. Rev. Lett. **51**, 1842 (1983).

[9] E. K. Warburton, C. J. Lister, J. W. Olness, P. E. Hau-

stein, S. K. Saha, D. E. Alburger, J. A. Becker, R. A. Dewberry, and R. A. Naumann, Phys. Rev. C **31**, 1211 (1985).

[10] C. Baktash, G. Garcia-Bermudez, D. G. Sarantites, W. Nazarewicz, V. Abenante, J. R. Beene, H. C. Griffin, M. L. Halbert, D. C. Hensley, N. R. Johnson, I. Y. Lee, F. K. McGowan, M. A. Riley, D. W. Stracener, T. M. Semkow, and A. Virtanen, Phys. Lett. B **255**, 174 (1991).

[11] A. Dewald, U. Kaup, W. Gast, A. Gelberg, H.-W. Schuh, K. O. Zell, and P. von Brentano, Phys. Rev. C **25**, 226 (1982).

[12] C. J. Gross, W. Gelletly, M. A. Bentley, J. Simpson, B. J. Varley, J. L. Durell, Ö. Skeppstedt, and S. Rastikerdar, Phys. Rev. C **43**, R5 (1991).

[13] A. Jungclaus, K. P. Lieb, C. J. Gross, J. Heese, D. Rudolph, D. J. Blumenthal, P. Chowdhury, P. J. Ennis, C. J. Lister, Ch. Winter, J. Eberth, S. Skoda, M. A. Bentley, W. Gelletly, and B. J. Varley, Z. Phys. A **340**, 125 (1991).

[14] C. J. Gross, K. P. Lieb, D. Rudolph, M. A. Bentley, W. Gelletly, H. G. Price, J. Simpson, D. J. Blumenthal, P. J. Ennis, C. J. Lister, Ch. Winter, J. L. Durell, B. J. Varley, Ö. Skeppstedt, and S. Rastikerdar, Nucl. Phys. A (in press).

[15] Ch. Winter, D. J. Blumenthal, P. Chowdhury, B. Crowell, P. J. Ennis, C. J. Lister, C. J. Gross, J. Heese, A. Jungclaus, K. P. Lieb, J. Eberth, and S. Skoda, Phys. Lett. B **258**, 289 (1991); Ch. Winter, D. J. Blumenthal, P. Chowdhury, B. Crowell, B. J. Ennis, S. J. Freeman, C. J. Lister, C. J. Gross, J. Heese, A. Jungclaus, K. P. Lieb, D. Rubolph, M. A. Bentley, W. Gelletly, J. Simpson, J. L. Durell, and B. J. Varley, Nucl. Phys. A (in press).

[16] A. N. James, T. P. Morrison, K. L. Ying, K. A. Connell, H. G. Price, and J. Simpson, Nucl. Instrum. Methods Phys. Res. Sect. A **267**, 144 (1988).

[17] P. J. Nolan, D. W. Gifford, and P. J. Twin, Nucl. Instrum. Methods Phys. Res. Sect. A **236**, 95 (1985); J. Simpson, M. Bates, C. Brookes, and P. J. Nolan, *ibid.* **269**, 209 (1988).

[18] D. Rudolph, F. Cristancho, C. J. Gross, A. Jungclaus, K. P. Lieb, M. A. Bentley, W. Gelletly, J. Simpson, H. Grawe, J. Heese, K.-H. Maier, J. Eberth, S. Skoda, J. L. Durell, B. J. Varley, D. J. Blumenthal, C. J. Lister, and S. Rastikerdar, J. Phys. G **17**, L113 (1991).

[19] F. W. N. de Boer, C.A. Fields, L. E. Samuelson, and J. Sau, Nucl. Phys. A **388**, 303 (1982).

[20] *Table of Isotopes,* 7th ed., edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).

[21] C. J. Lister, B. J. Varley, H. G. Price, and J. W. Olness, Phys. Rev. Lett. **49**, 308 (1982).

[22] R. F. Davie, D. Sinclair, S. S. L. Ooi, N. Poffé, A. E. Smith, H. G. Price, C. J. Lister, B. J. Varley, and I. F. Wright, Nucl. Phys. A **463**, 683 (1987).

[23] B. J. Varley, C. J. Lister, A. J. Irving, H. G. Price, and J. W. Olness, in *Proceedings of the International Conference on High Angular Momentum Properties of Nuclei, Oak Ridge, 1982,* edited by N. R. Johnson (Harwood, New York, 1982), p. 64.

[24] H.-W. Müller, Nucl. Data Sheets **54**, 1 (1988).

[25] S. L. Tabor, Phys. Rev. C **34**, 311 (1986).

[26] S. Raman, C. W. Nestor, Jr., S. Kahane, and K. H. Bhatt, Phys. Rev. C **43**, 556 (1991).

[27] A. I. Kucharska, J. Billowes, and C. J. Lister, J. Phys. G **15**, 1039 (1989).

[28] E. F. Moore, P. D. Cottle, C. J. Gross, D. M. Headly, U. J. Hüttmeier, S. L. Tabor, and W. Nazarewicz, Phys. Rev. C **38**, 696 (1988).

[29] U. J. Hüttmeier, C. J. Gross, D. M. Headly, E. F. Moore, S. L. Tabor, T. M. Cormier, P. M. Stwertka, and W. Nazarewicz, Phys. Rev. C **37**, 118 (1988).

[30] W. Nazarewicz, in *Proceedings of the International Conference on High Spin Physics and Gamma-Soft Nuclei,* edited by J. X. Saladin, R. A. Sorensen, and C. M. Vincent (World Scientific, Singapore, 1991), p. 406.

[31] J. Dudek, W. Nazarewicz, and N. Rowley, Phys. Rev. C **35**, 1489 (1987).

[32] M. Weiszflog (private communication).