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Spin determination and quantized alignment in the superdeformed bands in ^{152}Dy, ^{151}Tb, and ^{150}Gd

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The spin of the lowest level observed in the superdeformed band in ^{152}Dy is determined to be $I_0=25$ rather than the previously assigned $I_0=22$. As a result, we have $\mathcal{J}^{(1)} > \mathcal{J}^{(2)}$ and both $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ decrease very slowly with rotational frequency. From the spin determination of the two pairs of identical bands in ^{152}Dy and ^{151}Tb*, ^{151}Tb and ^{150}Gd*, evidence has been obtained to support the assumption of quantized alignments in units of $\hbar/2$.

The discovery of superdeformed (SD) rotational band in ^{152}Dy was one of the significant advances in nuclear structure physics in recent years [1,2]. Since then numerous SD bands have been observed in the mass $A\sim150$, 130, 190 regions [3]. The experimental data on SD bands are in the form of a series of gamma-ray energies, E_γ's, with almost constant difference ΔE_γ. It has been found that there exist great regularities in the SD bands, which have been the subject of theoretical and experimental efforts. One of the striking features of the SD bands is the astounding similarities of transition energies in different nuclei [4–7]. For example, the excited SD bands in ^{151}Tb and ^{150}Gd have almost identical gamma-ray energies to those in the yrast SD bands in neighboring $Z+1$ nuclei, ^{152}Dy and ^{151}Tb, respectively [4]. Therefore, it was argued that there may exist some underlying symmetry [5] and this behavior may be characterized by a quantized spin alignment [6,7]. A most useful concept to understand the SD band is the moment of inertia, which is related to the observed gamma-ray energies. The kinematic moment of inertia $\mathcal{J}^{(1)}=\hbar^2 I_x(dE/dI_x)^{-1}$ is usually estimated by the difference quotient

$$\mathcal{J}^{(1)}(I-1)=(2I-1)\hbar^2/E_\gamma(I\to I-2)\,,$$

while the dynamic moment of inertia $\mathcal{J}^{(2)}=\hbar^2(d^2E/dI_x^2)^{-1}$ is roughly given by

$$\mathcal{J}^{(2)}(I)=4\hbar^2/\Delta E_\gamma=4\hbar^2/[E_\gamma(I+2\to I)-E_\gamma(I\to I-2)]\,.$$

The exact determination of the kinematic moment of inertia requires an unambiguous assignment of the angular momentum. Therefore, the level spin determination is crucial for understanding the physics of SD band. However, in all cases in the $A\sim150$ region the exact angular momenta have not been measured [3]. In some cases, the angular momenta are estimated from the known spin values at which the SD bands deexcite into the yrast states. These assignments are probably often accurate to a few $\hbar$. In Ref. [1], the spin of the lowest level observed in the SD band of ^{152}Dy was assigned as $I_0=22$. For the SD bands in ^{151}Tb and ^{150}Gd, no spin assignment was given [3].

In Ref. [9], an effective and reliable method to determine the spins of the SD bands from the observed transition energies was proposed. It was found that the observed SD rotational spectra can be reproduced extremely well by the two-parameter closed expression [10]

$$E(I)-E(I_0)=a\{[1+bI(I+1)]^{1/2}-[1+bI_0(I_0+1]^{1/2}\}\,, \qquad (1)$$

which is derived from the Bohr Hamiltonian for a well-deformed nucleus with small axial asymmetry. From Eq. (1), the transition energy from level I to level $(I-2)$ is

$$E_\gamma(I)\equiv E_\gamma(I\to I-2)=a[\sqrt{1+bI(I+1)}-\sqrt{1+b(I-1)(I-2)}]\,. \qquad (2)$$

For an SD cascade $I_0+2n\to I_0+2n-2\to\cdots\to I_0+4\to I_0+2\to I_0$, the observed transition energies $E_\gamma(I_0+2n)$, $E_\gamma(I_0+2n-2),\ldots,E_\gamma(I_0+4)$, and $E_\gamma(I_0+2)$ can be least-squares fit to Eq. (2) with fitting parameters a and b. It was found that the agreement between the calculated and observed transition energies depends sensitively on the prescribed level spins. When a correct I_0 value is assigned, the calculated energies coincide with the observed results incredibly well. However, if I_0 is shifted away from the correct one, even merely by ±1, the root-mean-square (rms) deviation,

$$\sigma=\left\{\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_\gamma^{\text{calc}}(I_i)-E_\gamma^{\text{expt.}}(I_i)}{E_\gamma^{\text{expt.}}(I_i)}\right|^2\right\}^{1/2}, \qquad (3)$$

will increase drastically (see Fig. 1). Therefore, the spin value of the lowest level I_0, hence all the spin values of the SD states, can be determined unambiguously.

From Fig. 1 and Table I it is seen that the spin of the lowest level in the SD band in ^{152}Dy is $I_0=25$ [i.e., $E_\gamma(27\to25)=602.3$ keV], which is larger than the previously assigned value by $3\hbar$. However, as pointed out in Ref. [11], the previous spin assignment for the SD band in ^{152}Dy was not unambiguous and the uncertainty in spin

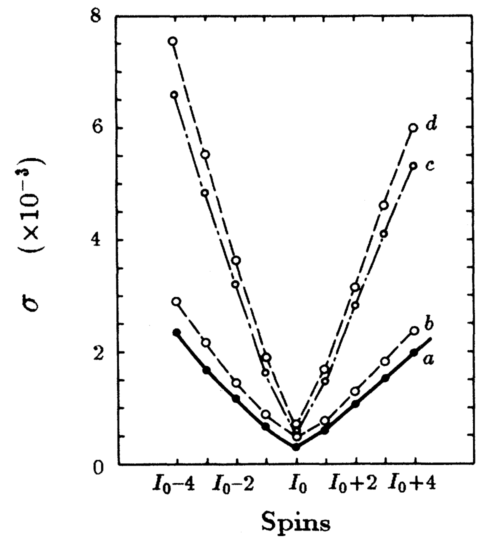

FIG. 1. The rms deviations for various spin assignments of the SD bands in ^{152}Dy, ^{151}Tb, and ^{150}Gd (see text). Curves a ^{150}Gd* (excited SD band), $I_0=38$; b ^{151}Tb (yrast SD bands), $I_0=\frac{73}{2}$; c ^{152}Dy (yrast SD bands), $I_0=25$; d ^{151}Tb* excited SD bands), $I_0=\frac{55}{2}$.

assignments was $2\hbar$ under the even spin assumption. Moreover, it was also emphasized [11] that the correct spin of the lowest level observed cannot be less than $I_0=22$, as this would mean that no angular momentum is removed by the transitions between the bottom of the SD band and the oblate single-particle states. Therefore, our results do not conflict with the above statement [11] drawn from the analysis of the experiment. The similar analyses (see Fig. 1 and Tables I and II) show $I_0=\frac{55}{2}$, for the SD band in ^{151}Tb* (excited), $[E_\gamma(\frac{59}{2}\rightarrow\frac{55}{2})=647.0$ keV]; $I_0=\frac{73}{2}$, for the SD band in ^{151}Tb (yrast), $[E_\gamma(\frac{77}{2}\rightarrow\frac{73}{2})=728.0$ keV)]; $I_0=38$, for the SD band in ^{150}Gd (excited), $[E_\gamma(40\rightarrow 38)=770$ keV].

Once a correct spin assignment is made, using the corresponding parameters a and b the precise values of kinematic moment of inertia $\mathcal{J}^{(1)}$ and dynamic moment of inertia $\mathcal{J}^{(2)}$ can be calculated from the following analytic expressions [9]:

$$\mathcal{J}^{(1)}=\frac{\hbar I_x}{\omega}=\hbar^2 I_x\left(\frac{dE}{dI_x}\right)^{-1}=\mathcal{J}_0\left(1-\frac{(\hbar\omega)^2}{a^2b}\right)^{-1/2}, \tag{4}$$

$$\mathcal{J}^{(2)}=\hbar^2\left(\frac{d^2E}{dI_x^2}\right)^{-1}=\mathcal{J}_0\left(1-\frac{(\hbar\omega)^2}{a^2b}\right)^{-3/2}, \tag{5}$$

where $\mathcal{J}_0=(\hbar^2/ab)(1+bK^2)^{1/2}\approx\hbar^2/ab$ is the bandhead moment of inertia. It is very important to note that for different assingments of angular momenta, the magnitude, and behavior of $\mathcal{J}^{(1)}$ may be quite different (see Fig. 2). For the SD band in ^{152}Dy, according to the previously assigned $I_0=22$, we would have $\mathcal{J}^{(1)}<\mathcal{J}^{(2)}$ and $\mathcal{J}^{(2)}$ decreases monotonically with ω, while $\mathcal{J}^{(1)}$ increases monotonically with ω, which seems hard to understand. On the contrary, for $I_0=25$, we have $\mathcal{J}^{(1)}>\mathcal{J}^{(2)}$, and both $\mathcal{J}^{(1)}$

TABLE I. Comparison of the identical SD bands in ^{152}Dy and ^{151}Tb* (excited).

^{152}Dy Assigned I	$E_\gamma(I)$ Expt.[a]	Calc.[b]	^{151}Tb* (excited) Assigned I	$E_\gamma(I)$ Expt.[c]	Calc.[d]
63	1449.4	1452.0	$\frac{127}{2}$		1453.5
61	1401.7	1402.6	$\frac{123}{2}$		1403.4
59	1353.0	1353.5	$\frac{119}{2}$	1353.0	1353.8
57	1304.7	1304.7	$\frac{115}{2}$	1305.0	1304.5
55	1256.6	1256.2	$\frac{111}{2}$	1256.0	1255.6
53	1208.7	1207.9	$\frac{107}{2}$	1207.0	1207.0
51	1160.8	1159.9	$\frac{103}{2}$	1158.0	1158.8
49	1112.7	1112.2	$\frac{99}{2}$	1112.0	1110.9
47	1064.8	1064.7	$\frac{95}{2}$	1063.0	1063.3
45	1017.0	1017.3	$\frac{91}{2}$	1016.0	1016.0
43	970.0	970.2	$\frac{87}{2}$	970.0	969.0
41	923.1	923.3	$\frac{83}{2}$	922.0	922.3
39	876.1	876.6	$\frac{79}{2}$	876.0	875.8
37	829.2	830.1	$\frac{75}{2}$	828.0	829.5
35	783.5	783.7	$\frac{71}{2}$	783.0	783.5
33	737.5	737.5	$\frac{67}{2}$	738.0	737.6
31	692.2	691.5	$\frac{63}{2}$	692.0	692.0
29	647.2	645.6	$\frac{59}{2}$	647.0	646.5
27	602.2	599.7			

[a]References [1,2] and [11].
[b]The transitions 2-17 are involved in the least-squares fitting. $a=-7.1543\times10^5$ keV, $b=-1.5729\times10^{-5}$, $\mathcal{J}_0=88.86\hbar^2$ MeV^{-1}.
[c]Reference [4].
[d]All the transitions observed are involved in the least-squares fitting. $a=-5.3019\times10^5$ keV, $b=-2.0839\times10^{-5}$, $\mathcal{J}_0=90.51\hbar^2$ MeV^{-1}.

and $\mathcal{J}^{(2)}$ decrease very slowly with ω. In the frequency range observed, $\hbar\omega\sim(0.30\text{–}0.70)$ MeV, $\mathcal{J}^{(1)}$ increases only by $\sim1\%$, while $\mathcal{J}^{(2)}$ by $\sim3\%$, which is expected by Eqs. (4) and (5), because $d\ln\mathcal{J}^{(2)}/d\omega=3d\ln\mathcal{J}^{(1)}/d\omega$. The moments of inertia of the four SD bands in ^{152}Dy, ^{151}Tb, and ^{150}Gd are plotted against ω in Fig. 3. It is worthwhile to note that the variations of $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ with ω are in excellent agreement with the results of the microscopic calculation [12]. Microscopically, the variation of $\mathcal{J}$ with frequency depends critically on the number of high-N intruder orbit occupied. For ^{152}Dy ($\pi6^4\nu7^2$), four protons occupying the $N=6$ shell and two neutrons occupying the $N=7$ shell, the total proton contribution to $\mathcal{J}$ shows little variation with ω and a very slight decrease in $\mathcal{J}$ comes mostly from the two high-N neutrons. The virtually constant $\mathcal{J}^{(2)}$ is particular to ^{152}Dy and a much greater variation with frequency is expected for the neighboring nuclei with different proton configurations. Indeed, the extracted $\mathcal{J}^{(2)}$ for the SD bands in ^{151}Tb and ^{150}Gd* shows a much sharper decrease with ω (see Fig. 3).

TABLE II. Comparison of the identical SD bands in ^{151}Tb (yrast) and ^{150}Gd* (excited).

^{151}Tb (yrast)			^{150}Gd* (excited)		
Assigned	$E_\gamma(I)$		Assigned	$E_\gamma(I)$	
I	Expt. [a]	Calc. [b]	I	Expt. [c]	Calc. [d]
$\frac{137}{2}$	1432.5	1444.1	68		1440.4
$\frac{133}{2}$	1380.7	1388.1	66	1378.0	1385.6
$\frac{129}{2}$	1330.0	1333.8	64	1337.0	1332.2
$\frac{125}{2}$	1278.5	1280.9	62	1277.0	1280.0
$\frac{121}{2}$	1228.5	1229.3	60	1229.0	1229.1
$\frac{117}{2}$	1178.9	1179.0	58	1180.0	1179.2
$\frac{113}{2}$	1130.2	1129.8	56	1130.0	1130.5
$\frac{109}{2}$	1082.5	1081.7	54	1082.0	1082.7
$\frac{105}{2}$	1034.9	1034.5	52	1036.0	1035.7
$\frac{101}{2}$	988.7	988.3	50	990.0	989.7
$\frac{97}{2}$	942.8	943.0	48	944.0	944.4
$\frac{93}{2}$	898.0	898.5	46	900.0	899.9
$\frac{89}{2}$	854.0	854.7	44	856.0	856.0
$\frac{85}{2}$	811.3	811.6	42	813.0	812.8
$\frac{81}{2}$	769.2	769.2	40	770.0	770.1
$\frac{77}{2}$	728.0	727.4			

[a] Reference [8].
[b] The transitions 1-12 are involved in the least-squares fitting. $a=-1.6907\times10^5$ keV, $b=-5.4341\times10^{-5}$, $\mathcal{J}_0=108.85\hbar^2$ MeV^{-1}.
[c] Reference [4].
[d] The transitions 1-11 are involved in the least-squares fitting. $a=-1.8425\times10^5$ keV, $b=-5.0768\times10^{-5}$, $\mathcal{J}_0=106.91\hbar^2$ MeV^{-1}.

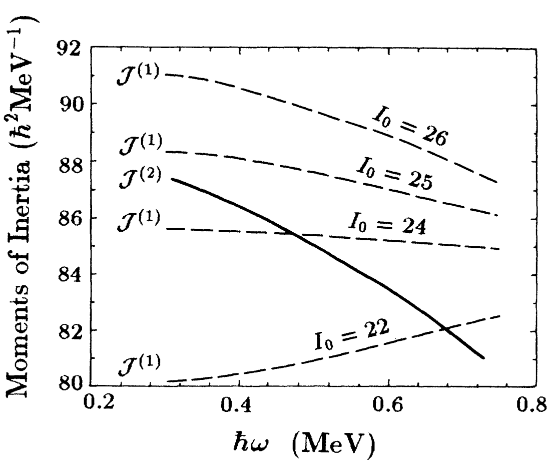

FIG. 2. Comparison of $\mathcal{J}^{(2)}$ and $\mathcal{J}^{(1)}$ for various I_0 in ^{152}Dy. The extracted $\mathcal{J}^{(1)}$'s using Eq. (4) for $I_0=22$, 24, 25, and 26 are denoted by dashed lines, while $\mathcal{J}^{(2)}$'s using Eq. (5) for $I_0=25$ are denoted by the solid line. In the experimental estimate, $\mathcal{J}^{(1)}(I-1)=(2I-1)\hbar^2/E_\gamma(I\to I-2)$ which is sensitive to the spin assignment, while $\mathcal{J}^{(2)}(I)=4\hbar^2/[E_\gamma(I+2\to I)-E_\gamma(I\to I-2)]$, independent of the spin assignment.

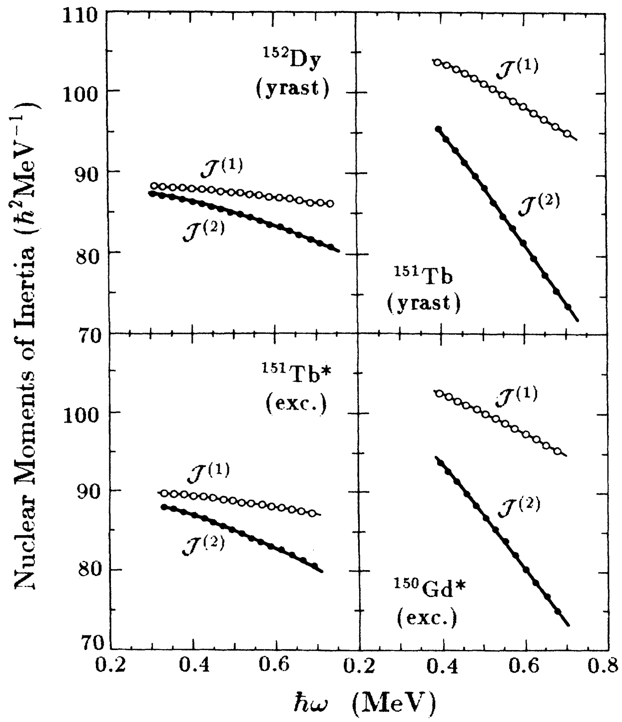

FIG. 3. The extracted $\mathcal{J}^{(1)}$ vs ω plot and $\mathcal{J}^{(2)}$ vs ω plot for the two pairs of identical SD bands in ^{152}Dy and ^{151}Tb*, and ^{151}Tb and ^{150}Gd*.

Having determined the spins of the SD bands, we can compare the perspective values of the alignments of the identical bands. It is very interesting to note that the spin differences in the two identical bands, ^{151}Tb* and ^{152}Dy, are just $\frac{1}{2}$ (see Table I) and the difference in angular momentum alignment $i=\Delta I_x$ is very close to $\frac{1}{2}$ over the whole range of observed frequency (see Fig. 4). Similarly, $i\sim-\frac{1}{2}$ for the two identical SD bands, ^{151}Tb and ^{150}Gd*. These results provide evidence supporting the assumption of quantized alignments in units of $\hbar/2$ suggested by Stephens *et al.* [6,7]. Several attempts have been made to understand the identical bands in neighboring nu-

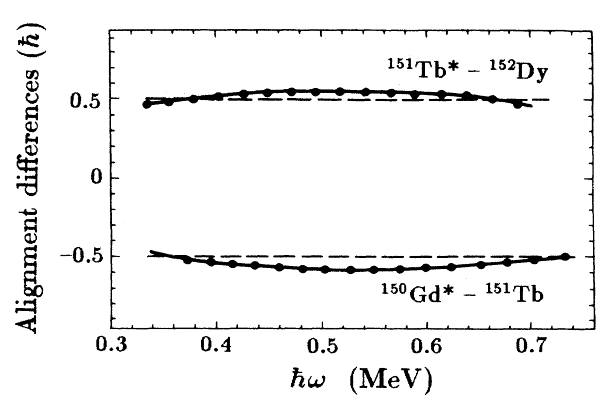

FIG. 4. The difference in spin alignments in the two pairs of identical SD bands.

clei. Nazarewicz [5] attributed the origin of these intriguing bands to the presence of pseudospin decoupled orbits near the Fermi surface for large deformation and certain particle numbers. However, the spins of the SD bands in ^{152}Dy, ^{151}Tb (yrast and excited), and ^{150}Gd (excited) were not known then. Moreover, the analyses there depend on the evenness or oddness of the spin of the core. From the present spin assignment, it seems hard to understand the pair of identical SD bands in ^{152}Dy and ^{151}Tb* (excited) in terms of the pseudospin decoupled orbit $[\tilde{2}\tilde{2}\tilde{0}\,\frac{1}{2}]$. However, a likely explanation may be provided in the framework of the particle rotor model with the coupling of certain pseudospin doublets [13]. Systematic analysis of the spin alignments for all the SD bands in $A\sim190$ and 150 regions will be published in a forthcoming paper.

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