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Linear momentum in the exciton model: A consistent way to obtain angular distributions

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We present an exciton model for preequilibrium emission in nucleon-induced reactions in which linear-momentum conservation is included. The particle emission contributions from the first two preequilibrium stages are calculated by determining exact particle-hole state densities with a specific energy and linear momentum in a Fermi-gas model of the nucleus. Angular distributions arise naturally from our treatment and do not have to be added in an *ad hoc* way. The angular distributions that we obtain from the first two preequilibrium stages are identical to those found using the Kikuchi-Kawai quasifree scattering kernel.

It has been established that nonequilibrium processes play an important role in nuclear reactions induced by light projectiles with incident energies above about 10 MeV. The characteristic features of particle emission from the composite nucleus before equilibrium has been reached (preequilibrium emission) are an excess of high-energy particles, and a forward peaking in the observed angular distributions. The overabundance of high-energy particles is due to the nuclear excitation energy being shared among only a few degrees of freedom in the early stages of the reaction when preequilibrium emission occurs, and the forward peaking is indicative of the incident projectile's direction being partially preserved. Both quantum mechanical and semiclassical theories have been developed to account for preequilibrium emission. Quantum mechanical approaches such as that of Feshbach, Kerman, and Koonin (FKK) [1] and Tamura *et al.* [2] have been able to successfully describe the spectral shape and angular distribution of emitted particles, though the calculations are rather involved and their predictive power is limited [3]. The semiclassical exciton [4] and hybrid [5] preequilibrium models, on the other hand, are able to describe the angle-integrated spectral shapes successfully, though in their usual formulation they cannot yield angular distributions directly. In this paper we shall show that by modifying the exciton model to include linear-momentum effects it yields angular distributions in a natural and consistent way. We shall not explicitly discuss the hybrid model, though the modifications needed in it for the inclusion of linear-momentum effects should be similar to those that we present for the exciton model.

In the exciton model the particle emission rates from the preequilibrium stages of the reaction are calculated by invoking microscopic reversibility and applying phase-space arguments. In its usual formulation it does not conserve linear momentum in the various intranuclear transitions and cannot yield information concerning the angular distribution of emitted particles. In order to obtain such information, it has become commonplace to include in the model, in an *ad hoc* manner, a nucleon-nucleon scattering kernel obtained either from free nucleon-nucleon scattering [6, 7] or, more realistically, from quasifree scattering in nuclear matter using the Kikuchi-Kawai (KK) expression [8–12]; for reviews see Refs. [13, 14]. While the inclusion of a nucleon-nucleon scattering kernel within an exciton model is a physically plausible way to obtain angular distributions, no formal theoretical connection has been made between the exciton model and quasifree scattering descriptions. We shall show that by conserving linear momentum in the exciton model and by developing state densities with linear momentum, the angular distributions obtained are identical to those found using KK quasifree scattering. We do not make use of the *fast particle* approximation, as in Ref. [6], but treat the excited particles and holes for a given preequilibrium stage statistically. The forward-peaked angular distributions that we obtain arise purely from phase-space factors, and possible dynamical effects are disregarded.

An exciton model which does conserve linear momentum and yields angular distributions directly has been proposed by Mädler and Reif [15]. They used a partition function technique, which is only accurate for large numbers of excited particles and holes, to determine state densities with linear momentum in an equidistant single-particle level model of the nucleus. But since the preequilibrium spectrum from nucleon-induced reactions is dominated by emission from simple particle-hole excitations, their approach is of limited value, though it can be applied in heavy-ion reaction calculations [16]. Below we present a method for exactly determining state densities with linear momentum, which follows from our previous work on photoabsorption [17]. Since our approach involves convoluting single-particle and -hole states in a Fermi-gas nucleus, the complexity of the integrals in-

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creases rapidly for more complex preequilibrium stages. We are able, however, to determine the state densities with linear momentum needed for the calculation of first and second stage preequilibrium emission in nucleon-induced reactions. We assume, following Chiang and Hüfner [18], that preequilibrium emission beyond the second stage can be ignored before equilibrium emission occurs.

In the exciton model it is assumed that an incident nucleon interacts with the target nucleus to form a two-particle-one-hole ($2p1h$) state, and in subsequent two-body nucleon-nucleon interactions the excited system may pass through more complex particle-hole configurations towards equilibrium. Particle emission can occur from the early preequilibrium stages and these particles typically contribute to the high-energy part of the emission spectrum. The double-differential cross section for the emission of a particle with energy ϵ and direction Ω can be written as

$$\frac{d^2\sigma}{d\epsilon\, d\Omega} = \sigma_R \sum_{\substack{\Delta n=+2 \\ n=3}} \frac{\lambda_n(\epsilon,\Omega)}{\Lambda_n^+ + \Lambda_n} D_n , \tag{1}$$

where the number of excitons is $n = p + h$. The reaction cross section of the incident particle on the target nucleus is σ_R, and D_n is the depletion factor, representing the probability that the system reaches the n-exciton configuration without preequilibrium decay. Λ_n^+ and Λ_n are the total rates for decay to more complex exciton configurations and for particle emission, respectively, and $\lambda_n(\epsilon,\Omega)$ is the double-differential emission rate for a given type of particle. This is found from microscopic reversibility to be

$$\lambda_n(\epsilon,\Omega) = \frac{m\epsilon\, \sigma_{\rm inv}(\epsilon) R(p)}{2\pi^3\hbar^3} \, \frac{\rho(p-1,h,E-\epsilon_\Omega,\mathbf{K}-\mathbf{k}_\Omega)}{\rho(p,h,E,\mathbf{K})} , \tag{2}$$

where the reaction cross section for the inverse process of nucleon absorption on the residual nucleus is $\sigma_{\rm inv}(\epsilon)$. The composite system total energy and momentum before particle emission are E and $\mathbf{K}$ respectively, and the residual nucleus energy and momentum after emission are $E-\epsilon_\Omega$ and $\mathbf{K}-\mathbf{k}_\Omega$ respectively, all these quantities being measured relative to the bottom of the nuclear well. The energy and momentum of the emitted particle relative to the bottom of the nuclear well are $\epsilon_\Omega = \epsilon + B + \epsilon_F$ and $\mathbf{k}_\Omega$, where $|\mathbf{k}_\Omega| = \sqrt{2m(\epsilon + B + \epsilon_F)}$, B being the binding energy and ϵ_F the Fermi energy. $R(p)$ is a correction factor to account for neutron-proton distinguishability, and is discussed below. In the above expression state densities with linear momentum are shown, though the state densities that are used in the original exciton model are a function of energy only. Since we also wish to compare angle-integrated emission spectra predicted by the exciton model both with and without linear-momentum conservation, we indicate below how to calculate Fermi-gas state densities with and without linear momentum. Furthermore, our method for determining state densities with linear momentum will be rendered more transparent if we first indicate how state densities without linear momentum can be calculated.

The state density of a p-particle h-hole system can be obtained by convoluting single-particle and -hole densities with an energy-conserving delta function. When linear-momentum effects are not accounted for, this can be expressed as

$$\rho(p,h,E) = \frac{1}{p!h!} \int_{i=1} \cdots \int_{i=p} \int_{j=1} \cdots \int_{j=h} \delta\left(E - \sum_{i=1}^{p} \epsilon_i + \sum_{j=1}^{h} \epsilon_j\right) \times \prod_{i=1}^{p} \rho(1p,\epsilon_i)\, \theta(\epsilon_i - \epsilon_F)\, d\epsilon_i \prod_{j=1}^{h} \rho(1h,\epsilon_j)\, \theta(\epsilon_F - \epsilon_j)\, d\epsilon_j , \tag{3}$$

where i labels the particles and j the holes. The theta functions are unity if their argument is greater than zero and zero otherwise, accounting for Pauli blocking. The densities of single particles and holes in energy space are represented by $\rho(1p,\epsilon_i)$ and $\rho(1h,\epsilon_j)$, with the energies $\epsilon_{i,j}$ measured relative to the bottom of the nuclear well. The factorials $p!$ and $h!$ account for the indistinguishability of the particles and holes. If an equidistant single-particle model of the nucleus is used, the above expression would yield the Ericson state density expression, corrected to include finite nuclear well depth restrictions [19]. In a Fermi-gas model of the nucleus the single-particle and -hole densities in energy space are given by $\rho(1p,\epsilon_i) = 3A\sqrt{\epsilon_i}/2\epsilon_F^{3/2}$ and $\rho(1h,\epsilon_j) = 3A\sqrt{\epsilon_j}/2\epsilon_F^{3/2}$, where A is the nuclear mass number.

We now generalize the above expression to allow state densities with a specific linear momentum to be determined. The convolution of the single-particle and -hole states is now performed in momentum space, and a linear-momentum-conserving delta function is included in the integration,

$$\rho(p,h,E,\mathbf{K}) = \frac{1}{p!h!} \int_{i=1} \cdots \int_{i=p} \int_{j=1} \cdots \int_{j=h} \delta\left(E - \sum_{i=1}^{p} \epsilon_i + \sum_{j=1}^{h} \epsilon_j\right) \delta\left(\mathbf{K} - \sum_{i=1}^{p} \mathbf{k}_i + \sum_{j=1}^{h} \mathbf{k}_j\right) \times \prod_{i=1}^{p} \rho(1p,\mathbf{k}_i)\, \theta(k_i - k_F)\, d^3\mathbf{k}_i \prod_{j=1}^{h} \rho(1h,\mathbf{k}_j)\, \theta(k_F - k_j)\, d^3\mathbf{k}_j , \tag{4}$$

where $\mathbf{k}_i$ and $\mathbf{k}_j$ are the single-particle and -hole linear momenta, and k_F is the Fermi momentum. In the Fermi-gas model the single-particle and -hole states are eigenstates of linear momentum, the density of such states in momentum-space being a constant which reproduces the number of nucleons,

$$\rho(1p,\mathbf{k}_i) = \rho(1h,\mathbf{k}_j) = \frac{A}{\frac{4}{3}\pi k_F^3} \equiv \kappa. \tag{5}$$

Neither Eq. (3) nor Eq. (4) include the possibility that some of the excited particles/holes can Pauli block other particles/holes, though for the simple particle-hole configurations that we consider this effect can be safely ignored. As expected from symmetry, the density of states with linear momentum, $\rho(p,h,E,\mathbf{K})$, is independent of the direction of the total momentum $\mathbf{K}$ and depends only upon its magnitude. The dimensions of the state densities with linear momentum are $\text{MeV}^{-1}(\text{MeV}/c)^{-3}$, and they obey the relation

$$\rho(p,h,E) = \int \rho(p,h,E,\mathbf{K})\; 4\pi K^2 dK. \tag{6}$$

For nuclear excitations with large numbers of particles and holes, the dimensionality of the state density integrals becomes too high for their practical solution. However, we shall consider particle emission from only the first two preequilibrium stages since these dominate the preequilibrium spectrum for most nucleon-induced reactions. Thus we must evaluate the state densities for $1p1n$, $2p1h$, $2p2h$, and $3p2h$ excitations. The $\rho(1p,1h,E,\mathbf{K})$ density requires the solution of a six-dimensional integration, which can be solved analytically using the techniques shown in Ref. [17], giving

$$\rho(1p,1h,E,\mathbf{K}) = \frac{\pi m \kappa^2}{K} \times \begin{cases} \left[k_F^2 - \left(\frac{mE}{K} - \frac{K}{2}\right)^2\right] & \text{if } K_{\min} < K < K_1 \text{ or } K_2 < K < K_{\max}\,, \\ 2mE & \text{if } K_1 < K < K_2\,, \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

where

$$K_{\substack{\min\\ \max}} = \sqrt{2mE + k_F^2} \mp k_F, \tag{8}$$

$$K_{\substack{1\\2}} = \sqrt{2(k_F^2 - mE) \mp 2k_F\sqrt{k_F^2 - 2mE}}. \tag{9}$$

The high dimensionality of the integrations for the evaluation of the more complex state densities can be reduced by breaking up the integrals, making use of analytic solutions for simpler configurations. For instance, the $\rho(2p,2h,E,\mathbf{K})$ requires a twelve-dimensional integration, though it can be expressed as a convolution of two $1p1h$ state densities, each of which is known analytically, so that

$$\rho(2p,2h,E,\mathbf{K}) = \frac{1}{2!\,2!}\int\int \rho(1p,1h,E_1,\mathbf{K}_1)\; \rho(1p,1h,E-E_1,\mathbf{K}-\mathbf{K}_1)\; d^3\mathbf{K}_1\; dE_1, \tag{10}$$

which, by symmetry, can be reduced to a three-dimensional integral and can be solved numerically without any difficulties. We checked that when the state densities with linear momentum are integrated over all total momenta [using Eq. (6)] they yield the Fermi-gas state densities without linear momentum of Eq. (3).

From Eq. (2) it is clear that the angular distribution of emitted particles from a preequilibrium stage arises from phase-space factors. For a given particle emission energy, the various emission directions result in different total momenta being transfered to the residual nucleus, with corresponding different accessible state densities. Thus the angular distribution of emitted particles from the $n = 3$ stage (i.e., single-step scattering) is given by the variation of $\rho(1p,1h,E-\epsilon_\Omega,\mathbf{K}-\mathbf{k}_\Omega)$ with the emission angle. The angular distribution that we obtain using $1p1h$ state densities according to Eq. (7) is identical to that found by KK [8] for single-step quasifree scattering from a noninteracting Fermi-gas nucleus. An inspection of the physics involved suggests that this result is to be expected since our exciton model, and the quasifree scattering model of KK, both conserve linear momentum and energy in a Fermi-gas nucleus. Furthermore, the expression used by KK for single-step scattering uses a basic free-space nucleon-nucleon cross section which is isotropic, so that all the angular dependence arises implicitly from phase-space factors, as done explicitly in our approach. The similarity of our exciton model with KK's approach can be most clearly seen in the work of Chiang and Hüfner [18], who use the KK scattering function to calculate single- and double-step quasifree scattering. Their expressions for the single- and double-step scattering use nuclear response functions [20] for a noninteracting Fermi gas, which are directly proportional to our $1p1h$ and $2p2h$ state densities with linear momentum.

In Fig. 1 the variation of the residual nucleus $1p1h$ and $2p2h$ state densities [Eqs. (7) and (10)] with emission angle is shown for the reaction $^{184}\text{W}(n,n')$ for an incident energy of 26 MeV and emission energies of 14.5 and 18.5 MeV. These densities are strongly forward peaked due to the variation of the state density with the linear momentum deposited in the residual nucleus. This for-

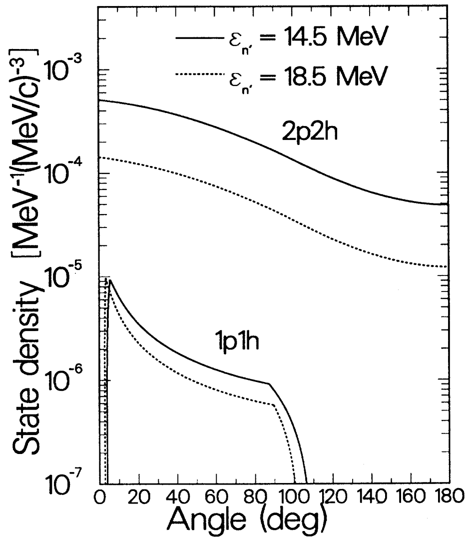

FIG. 1. The variation of the $1p1h$ and $2p2h$ state densities with emission angle for the residual nucleus in the reaction $^{184}W(n,n')$. The incident energy is 26 MeV, and the emission energies are $\epsilon_{n'}$=14.5 and 18.5 MeV.

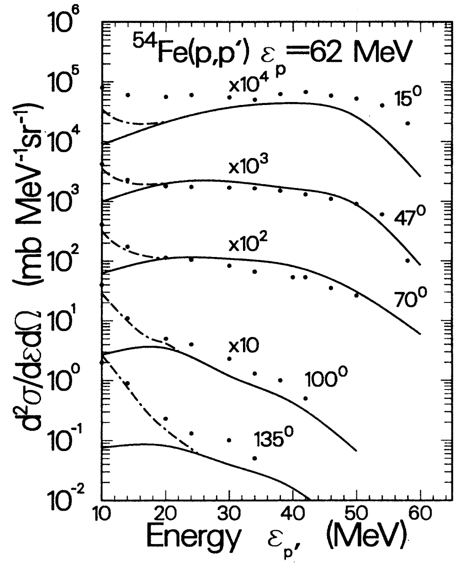

FIG. 3. Spectra of protons emitted at a number of angles in the $^{54}Fe(p,p')$ reaction induced by 62 MeV protons, compared with experimental data [25]. The full line shows the sum of n = 3 and n = 5 preequilibrium emission in our model, and the dash-dot line includes the equilibrium emission contribution.

ward peaking decreases with increasing exciton number as the linear momentum brought in by the projectile is shared among more particles and holes and the memory of the incident direction is lost. Since the angular distribution of emitted particles comes from the variation of the residual-nucleus phase space with emission angle, no preequilibrium emission from the n = 3 stage can occur for angles greater than about 110°. This is a kinematical effect resulting from the restrictions of energy and momentum conservation and is also seen in Refs. [8, 15].

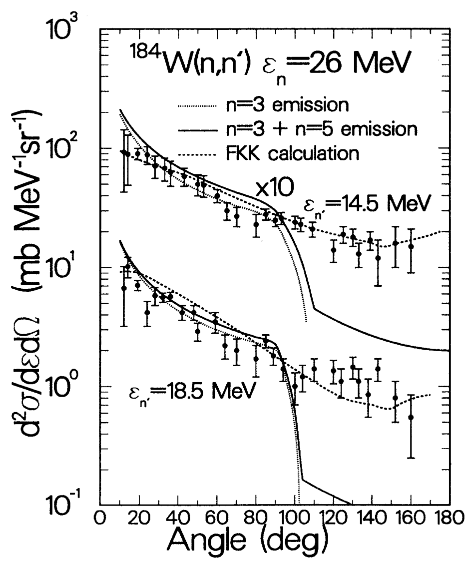

FIG. 2. Angular distributions for 14.5 and 18.5 MeV neutrons emitted in the reaction $^{184}W(n,n')$, induced by 26 MeV neutrons. At these emission energies the equilibrium emission contributions were found to be negligible. Shown for comparison are quantum mechanical FKK calculations and experimental data, taken from Marcinkowski *et al.* [3].

For the calculation of nucleon emission cross sections we used Kalbach's parametrization [21] for the transition rates to more complex configurations, Λ_n^+, which was originally determined without linear-momentum considerations. This is reasonable since we found that Λ_n, obtained by integrating Eq. (2) over all angles and energies for neutrons and protons, agreed to within 5% with the value obtained when linear-momentum effects were not included. Also, this integral did not differ significantly from its value obtained using the traditional Ericson equidistant single-particle level state densities, corrected for a finite nuclear well depth. The neutron-proton distinguishability factor $R(p)$ [22] in Eq. (2) is consistent with the above parametrization [23]. The reaction cross sections in Eqs. (1) and (2) were determined using the Becchetti-Greenlees optical potential [24], and we took the Fermi energy to be 35 MeV.

We have determined angular distributions for 14.5 and 18.5 MeV emitted neutrons in the reaction $^{184}W(n,n')$ induced by 26 MeV neutrons. Our results are shown in Fig. 2 and it is evident that the observed forward peaking in the data is accounted for in our model, though we underpredict the data at backward angles. Neutron emission from the n = 3 stage dominates scattering in the forward direction but does not contribute beyond 110°, whereas n = 5 emission covers all directions but is too weak to account for the backward-angle data. This underprediction was also seen in Refs. [9–11] where the KK quasifree scattering kernel was used in semiclassical preequilibrium models, and results from the absence in our model of effects such as diffraction of the nucleons in the mean-field nuclear potential [10, 11]. It is beyond the scope of the present work to include such effects, which really require a quantum mechanical treatment. The dashed line shows a quantum mechanical calculation of the neutron scattering cross section using the FKK theory [3] which uses the distorted-wave Born approximation, and with single- and double-step scattering the theory describes the angular distributions well. In Fig. 3 we show the proton emission spectra at five different angles for the reaction $^{54}Fe(p,p')$ induced by 62 MeV protons. For low emission energies we have included the equilibrium emission contribution (reduced due to the reaction flux lost through $n = 3$ and $n = 5$ preequilibrium emission), determined with the Hauser-Feshbach code GNASH [26]. The shapes of the spectra generally agree fairly well with experiment, but again we underpredict the backward-angle data. We also determined the angle-integrated spectrum and found that it describes the data well (since the backward-angle cross section is a minor fraction of the total preequilibrium cross section), and have compared it with an exciton model calculation using Fermi-gas state densities which do not include linear momentum, from Eq. (3). We found differences of less than 5%, indicating that it is not necessary to include linear-momentum effects when determining angle-integrated spectra.

In summary, the inclusion of linear momentum effects in an exciton model is able to explain the forward-peaked angular distributions observed in preequilibrium decay. The state densities with linear momentum that we develop can be calculated simply for the preequilibrium emission processes of physical importance, and the angular distributions which they yield are identical to those seen in KK quasifree scattering. We have, therefore, provided a link between exciton model and quasifree scattering descriptions of nuclear reactions, and have provided further justification for the commonly adopted procedure of using a KK scattering kernel in an exciton model. Although most groups have used such a kernel with an exciton model based on equidistant rather than Fermi-gas states, we expect the errors introduced by this procedure to be small.

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