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Total cross section for $p+p \rightarrow p+p+\pi^0$ near threshold

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We compare a phenomenology of π^0 production in p-p scattering with recent Indiana University Cyclotron Facility data for energies near threshold. The computed magnitude of the cross section is too small by a factor of about 5, even though the energy dependence is satisfactorily described.

Reference [1] reports the first study of the reaction $p+p \rightarrow p+p+\pi^0$ close to the pion-production threshold. The experiment uses the new Indiana University Cyclotron Facility (IUCF) cooler facility to measure the four momenta of both outgoing protons, a truly impressive feat. The abstract of Ref. [1] states that the measured energy dependence of the total cross section is not compatible with that predicted by models of s-wave pion production and rescattering. Reference [1] describes the data with phase-space factors and an effective-range treatment of the Coulomb interaction between the two protons in the final state.

The s-wave pion-production and rescattering model of Koltun and Reitan [2] serves as a prototype for essentially all later treatments of meson-production and -absorption reactions and—due to its simplicity—also provides physical insight. It is therefore interesting to see if a proper evaluation of such a model can reproduce the observed energy dependence and magnitude of the total cross section. In this paper we compute the low-energy total cross section according to the model of Ref. [2]: The model is defined by the graphs of Fig. 1. The pion is produced at one of the nucleons and then either emitted directly as in Fig. 1(a) or rescattered in an s-wave interaction with the second nucleon as in Fig. 1(b). The initial $^3P_0$ and final $^1S_0$ interactions between the two protons are taken into account. Those are the only proton-proton partial waves included, the outgoing pion is necessarily in an s-wave state. The pion-nucleon s-wave scattering operators have the forms ϕ^2 or $\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\boldsymbol{\pi}$, where $\boldsymbol{\phi}$ is the pion field operator and $\boldsymbol{\pi}$ its canonical momentum. The coefficients of these operators are determined from s-wave pion-nucleon scattering phase shifts. In their original evaluation, Koltun and Reitan [2] used the Hamada-Johnston potential *without* the Coulomb interaction to describe the interaction between the protons. We employ the Reid soft-core potential *with* the Coulomb interaction taken into account.

The evaluation of the two terms of Fig. 1 yields the transition amplitude of Eqs. (14) and (15) of Ref. [2]. The total cross section is obtained by multiplying the summed absolute square of the transition amplitude by the appropriate phase space and flux factors, i.e., $\rho(E_f)$ and v, along with the usual average over initial spins and

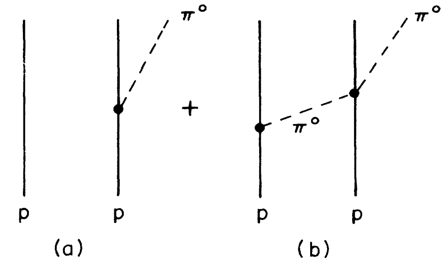

FIG. 1. The reaction mechanisms for the process $pp \rightarrow pp\pi^0$. Process (a) describes the direct term, process (b) the rescattering term.

with the sum over final spins. The result is

$$\sigma_{tot} = 2\pi/v \int dE_f |T(E_f)|^2 \rho(E_f) , \tag{1}$$

where E_f is the center-of-mass energy of the protons in the final ${}^1S_0$ state and $T(E_f)$ is the amplitude corresponding to the sum of the terms of Fig. 1 evaluated between initial and final states with the inclusion of the full two-proton interaction. Our computational procedure is to calculate $T(E_f)$ as a function of energy and to numerically evaluate the integral of Eq. (1). The procedure is straightforward.

Our calculations for the total cross section are shown in Fig. 2. The data are to be compared with the scale on the left and the predictions with the scale on the right. One sees immediately that the theoretical prediction is too small by a factor of about 5, and that the energy dependence is well reproduced. Furthermore, the Coulomb interaction between the two protons in the final state cannot be omitted at energies close to pion-production threshold, i.e., for $\eta \leq 0.5$.

As noted above, the pion-production model we use does not reproduce the magnitude of the measured total cross section. However, if one adjusts the overall strength of the pion-production operator in order to reproduce the total cross section at $\eta = 0.4$, the *energy dependence* of the total cross section is rather satisfactorily accounted for. Therefore, it is possible to obtain a dynamical *s*-wave rescattering model with one parameter that reproduces the data.

Our results are in conflict with those of Ref. [2]. Whereas we find a sizable energy dependence for σ_{tot}/η^2 according to Fig. 2, Ref. [2] gets a constant value of rather different magnitude. There are three reasons for that difference: First, the Coulomb interaction in the final state reduces the computed value of the cross section as seen in Fig. 2. Second, our use of the Reid soft-core potential leads to a 30% reduction in the computed transition matrix elements—*before* rescaling; that reduction reflects the sensitivity of computed pion-production amplitudes to different parametrizations of the short-range part in the proton-proton wave functions. Third, and most importantly, Ref. [2] employs an effective-range approximation for the energy dependence of the transition matrix $T(E_f)$ based on an assumed energy dependence for the final ${}^1S_0$ two-proton states $|\psi(p_f)\rangle$, i.e.,

$$|\psi(p_f)\rangle \simeq \frac{\sin\delta(p_f)}{p_f} \left[-\frac{|\psi(0)\rangle}{a} \right] , \tag{2}$$

$$\frac{\sin\delta(p_f)^2}{p_f^2} \simeq \frac{1}{[-a^{-1} + \frac{1}{2} r_0 p_f^2 - P r_0^3 p_f^4]^2 + p_f^2} , \tag{3}$$

where $\delta(p_f)$ is the ${}^1S_0$ phase shift, p_f the relative momentum in the final two-proton state, and a, r_0, and P are the Coulomb-subtracted effective-range parameters. [The approximation is written in Eqs. (2) and (3), omitting the Coulomb force. The Coulomb-modified effective-range approximation is not investigated here.]

In the numerical evaluation of Ref. [2], the above approximation is made even more severe by omitting the square bracket in Eq. (3). Doing that allows the integral over E_f for the total cross section in Eq. (1) to be performed analytically. That additional approximation is a significant underestimation of the denominator leading to an unrealistically large constant value for σ_{tot}/η^2. We obtain 10.6 μb, which is more than twice as much as our peak value. The result of Ref. [2], stated as 17 μb there, is still larger. This is due to the difference of the computed transition matrix elements between the Hamada-Johnston and the Reid soft-core potentials.

We do want to stress that the full effective-range approximation according to Eqs. (2) and (3) appears to be reliable within about 5% for a description of the total cross section at energies corresponding to $\eta \leq 0.4$. At higher energies it can create sizable inaccuracies up to 30%. This fact is demonstrated in Fig. 3.

Our calculations yield a disagreement between the com-

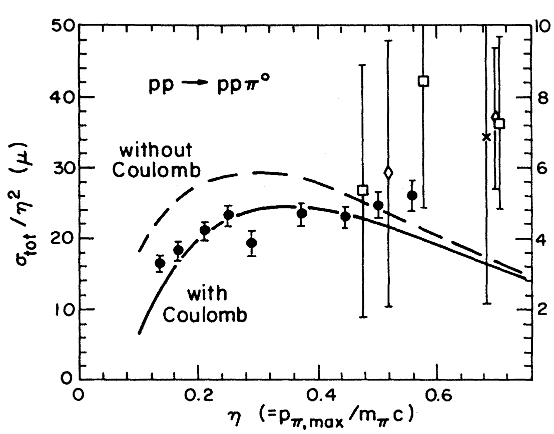

FIG. 2. The energy dependence of the total cross section for the reaction $pp \to pp\pi^0$. The solid circles refer to the IUCF data of Ref. [1], whereas the older experimental data are from Refs. [8,9,11] as quoted in Ref. [1]. The solid curve shows the full calculation and the dashed curve shows the effect of omitting the Coulomb interactions. The data are to be evaluated using the scale on the left, while the theory uses the scale on the right.

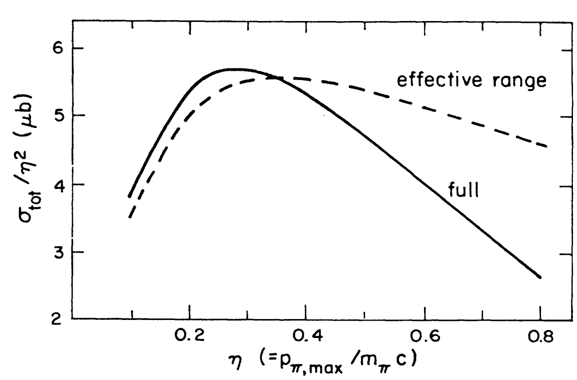

FIG. 3. The energy dependence of the total cross section $pp \to pp\pi^0$. The solid curve shows the proper calculation without Coulomb interaction (dashed curve of Fig. 2). The dashed curve results from using the effective-range approximation according to Eqs. (2) and (3). The source of experimental data is described in Fig. 2.

puted and measured *absolute* values for the total cross section by a factor of 5. Thus the theory is missing something important. We suspect that the problem originates from the use of an over-simplified pion-nucleon interaction [3,4]. The on-shell s-wave pion-nucleon interaction is constrained to be small by the requirements of chiral symmetry. In the sigma model, for example, the large pair terms are canceled by the exchange of a sigma meson. For pion-production reactions the pion and nucleon are, however, not on shell. Indeed, either a pion or a nucleon must be off shell for the reaction to proceed. The cancellation between the pair and sigma-exchange terms is not complete for off-shell pions or nucleons. That means that the π-nucleon amplitude relevant for pion production in many-nucleon systems must be larger than the theoretical one for on-shell particles. Including such an effect is likely to enhance the value of the amplitudes computed here. Thus, the observed disagreement between the present calculation and the experimental data for *absolute* values of cross sections is expected. We note that the high-momentum-transfer pion-production process is rather sensitive to many terms important at small separations between nucleons that would otherwise have small effects [5]. In this case, the IUCF data offers a new opportunity to investigate the off-shell pion-nucleon scattering amplitude.

We shall now argue that the correct description of the energy dependence is not an accident. Using an improved pion-nucleon amplitude or some other two-body mechanism would yield a cross section with a very similar energy dependence for the IUCF experiment. This is because the energy dependence is controlled by phase-space factors, Coulomb penetration factors, and the p-p phase shifts. For example, we can reproduce the data by multiplying the pion-production operator by a factor in two different ways: (i) One may multiply both strength parameters of the process described in Fig. 1 by one scale factor. (ii) One may argue that the single-nucleon term is rather reliable and, therefore, rescale the two-nucleon rescattering term only. Both methods of rescaling lead to essentially indistinguishable results.

The main results of this paper are the failure to reproduce the magnitude of the cross section, the importance of the Coulomb interaction for the energy dependence of the total π^0-production cross section near threshold, and the limited validity of the effective-range approximation for the higher-energy data of Ref. [1]. We think the latter two results have general significance, though the conclusions are derived from a rather unsophisticated calculation. Reference [1] also observes the importance of the Coulomb interaction for the description of the data, but—in contrast to the present dynamical model—fully relies on the validity of the effective-range approximation for the conclusion without further numerical check.

The recent IUCF experiment [1] has ushered in a new era for pion-production data. Theories appear above to reproduce the observed energy dependence of the total cross section. This paper points out some of the requirements which a full future theoretical description has to include.

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