Distribution of single-particle strength due to short-range and tensor correlations

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The distribution of single-particle strength in nuclear matter is calculated for a realistic nucleon-nucleon interaction. The influence of the short-range repulsion and the tensor component of the nuclear force on the spectral functions is to move approximately 13% of the total strength for all single-particle states beyond 100 MeV into the particle domain. This result is related to the abundantly observed quenching phenomena in nuclei which include the reduction of spectroscopic factors observed in $(e,e'p)$ reactions and the missing strength in low-energy response functions.

A fundamental new insight into the structure of the nucleus is emerging from the results of recent $(e,e'p)$ experiments [1]. These experiments reveal that it is no longer possible to associate a simple mean-field picture with the nucleus. First, only part of the mean-field single-particle (sp) strength for hole states is observed in the experimentally accessible energy region. Second, this strength becomes more strongly fragmented when the mean-field sp energy of the orbital is further removed from the Fermi energy. This suggests the validity of Landau's quasiparticle picture for the finite nucleus [2,3]. Using the so-called CERES method [4] an occupation number of 0.75 for the $3s\frac{1}{2}$ orbital in ^{208}Pb has been obtained [5] together with a spectroscopic factor of 0.65 [6].

A microscopic calculation of the distribution of sp strength in lighter nuclei is reported in Ref. [3] where the Dyson equation is solved for the finite system employing an energy-dependent self-energy. Only "low-energy" excitations are included in Ref. [3] which allow the coupling of sp degrees of freedom to two-particle–one-hole $(2p1h)$ and two-hole–one-particle $(2h1p)$ excitations within a range of about 100 MeV around the Fermi energy for the nuclei ^{48}Ca and ^{90}Zr. As a typical result, the total strength in the experimentally accessible domain in the $(e,e'p)$ reaction is overestimated by about 10%–15% for these nuclei, although the strength distribution is well described. This calculation also indicates that the background contribution to the hole strength is of the order of 10% in agreement with the experimental result for the $3s\frac{1}{2}$ in ^{208}Pb and other theoretical approaches [7]. It should also be mentioned that calculations of the low-energy response with inclusion of two-particle–two-hole $(2p2h)$ excitations up to about 100 MeV of excitation energy indicate that the shapes of the strength distributions for all excitation modes are well described, although the overall strength is systematically overestimated by about 20%–30% [8].

The calculations reported in Refs. [3] and [8] assume that all sp strength of the orbitals in the considered configuration space is available. Therefore it is natural to seek an explanation for the remaining discrepancies by focusing on the distribution of sp strength. This will be relevant for an explanation of both the 10%–15%

discrepancy between theoretical and experimental sp strength discussed in Ref. [3] and for the additional quenching that is required for the description of low-energy response functions [8].

The study of the nucleon spectral function has been made for nuclear matter in order to accurately include the influence of the short-range repulsion and the tensor force. Calculations for the hole part of the spectral function have been performed in the context of the correlated basis functions (CBF) method [9]. Both hole and particle parts of the spectral function were studied in Ref. [10] by applying the self-consistent Green's function (SCGF) method to a semirealistic central interaction. A discussion was given in Ref. [10] indicating that the impact of the short-range correlations is clearly seen in the particle spectral function, although the presence of high-momentum components in the ground state is reflected in the hole spectral function. We present here results of detailed calculations of the particle spectral function for a realistic interaction [11] including the effect of the nuclear tensor force.

The SCGF method and all details necessary for performing the calculations presented here for a realistic interaction are discussed in Ref. [10] where a semirealistic interaction, not including the nuclear tensor force, was used. The basic physical idea underlying this method is that the properties of the particles are determined by its interaction with the other particles. This interaction is in first approximation determined by the most dominant physical correlation in the system. In the nuclear case this is the short-range repulsion of the basic interaction. Therefore it is necessary to sum the ladder diagrams into the interaction with the inclusion of hole-hole propagation. The hole-hole propagation is required since it provides the information for the calculation of the hole spectral function. The resulting interaction is used to calculate the self-energy which then is used to solve Dyson's equation which provides the spectral functions. The interaction should then be recalculated since the particles are dressed by their interactions with the medium. A nonlinear formulation of the many-particle problem is thus obtained which requires the self-consistent determination of the sp propagator. Here we present results of a calculation in which the sp spectrum is determined self-

consistently from

$$\varepsilon(k) = \frac{\int d\omega\, \omega S_h(k,\omega)}{\int d\omega\, S_h(k,\omega)}, \quad k < k_F \tag{1}$$

and

$$\varepsilon(k) = \frac{\int d\omega\, \omega S_{QP}(k,\omega)}{\int d\omega\, S_{QP}(k,\omega)}, \quad k > k_F. \tag{2}$$

It should be noted that the calculation of the spectral functions requires knowledge of the self-energy at all energies for which its imaginary part does not vanish. This range of energies is very large due to the strong short-range repulsion in the Reid potential as is further discussed in Ref. [10]. As a result, the calculations are computationally quite involved. Once the spectrum given by Eqs. (1) and (2) has been determined self-consistently, the final spectral functions are obtained from the Dyson equation. For the hole spectral function this yields the relation to the self-energy given by

$$S_h(k,\omega) = \frac{1}{\pi} \frac{\mathrm{Im}\Sigma(k,\omega)}{[\omega - k^2/2m - \mathrm{Re}\Sigma(k,\omega)]^2 + [\mathrm{Im}\Sigma(k,\omega)]^2},$$
$$\omega < \varepsilon_F^-. \tag{3}$$

The quasiparticle contribution to the spectral function, $S_{QP}(k,\omega)$, describes a Lorentzian approximation to the peak of the spectral function. It can be obtained from the self-energy by calculating

$$\varepsilon_{QP}(k) = \frac{k^2}{2m} + \mathrm{Re}\Sigma(k,\varepsilon_{QP}(k)), \tag{4}$$

the quasiparticle energy, the imaginary part of the self-energy at the quasiparticle energy, $w(k) = \mathrm{Im}\Sigma(k, \varepsilon_{QP}(k))$, and the strength under the peak which is given by

$$z(k) = \left[1 - \left(\frac{\partial \mathrm{Re}\Sigma(k,\omega)}{\partial\omega}\right)_{\omega=\varepsilon_{QP}(k)}\right]^{-1}. \tag{5}$$

As a result

$$S_{QP}(k,\omega) = \frac{1}{\pi} \frac{z^2(k)|w(k)|}{[\omega - \varepsilon_{QP}(k)]^2 + [z(k)w(k)]^2}. \tag{6}$$

For momenta above k_F, this quasiparticle contribution [10], $S_{QP}(k,\omega)$, to the particle spectral function, $S_p(k,\omega)$, was used to determine the sp energy in Eq. (2). This choice is dictated by the actual shape of S_p (discussed below), which would otherwise result in an unrealistically large gap at k_F. Equation (2) now, instead, generates the quasiparticle energy which is also given by Eq. (4). In practice, therefore, Eq. (4) is used to determine the spectrum above k_F. Distinguishing between states below and above k_F introduces a gap in the sp energy which is necessary to avoid pairing instabilities and to properly take into account the collective bound pair states that result from solving the ladder equation [12,13]. The choice of the sp spectrum below k_F has hardly any influence on the particle spectral functions [14]. Any sensible choice of the sp spectrum above k_F will also lead to

very similar particle spectral functions. The specific choice given in Eqs. (1) and (2), therefore, has no consequences for the discussion of the particle spectral functions below. Ideally, the full spectral distribution should be determined self-consistently and in that case no sp spectrum needs to be chosen.

The integral appearing in the denominator of Eq. (1) generates the occupation number of the sp state with momentum k,

$$n(k) = \int_{-\infty}^{\varepsilon_F^-} d\omega\, S_h(k,\omega). \tag{7}$$

The result for the occupation number for $k = 0$ calculated according to Eq. (7) for the Reid soft-core potential at $k_F = 1.36$ fm^{-1} is 0.83. Results from other methods such as Brueckner theory [15,16] and CBF theory [17,18] for other realistic interactions for the occupation of $k = 0$ are very similar. In Refs. [15] and [16] 0.82 is reported for the Paris potential [19]. Older CBF calculations [17] for the Urbana v_{14} interaction [20] give 0.87, whereas more recent CBF results [9,18] give 0.83. All recent calculations for different interactions using different methods give a strikingly similar result for $n(0)$. This should be contrasted to the result at k_F which is most easily expressed in terms of $z(k_F) = n(k_F^-) - n(k_F^+)$. For the Reid interaction we find $z(k_F) = 0.72$. In Refs. [17] and [18] $z(k_F) = 0.7$ is reported for the Urbana v_{14} interaction. However, for the Paris interaction $z(k_F) = 0.35$ has been obtained in Ref. [15] and 0.47 in Ref. [16]. These results suggest that for an orbital which is far removed from the Fermi energy ($k = 0$) the depletion is rather uniquely pinned down and one has reason to expect a similar depletion for deeply bound orbitals in nuclei. In contrast to the $k = 0$ case, the depletion and occupation of momenta around k_F is very sensitive to the low-lying excitation modes of nuclear matter which are anyway very different from those of real nuclei where shell effects and collective low-energy surface vibrations dominate the low-energy excitation modes. Occupation numbers around k_F, therefore, should be considered less relevant in comparing with finite nuclei [18,21].

The importance of the particle spectral function is to exhibit where the unoccupied sp strength is located in energy. It is obtained by solving the Dyson equation and is related to the self-energy by

$$S_p(k,\omega) = -\frac{1}{\pi}$$
$$\times \frac{\mathrm{Im}\Sigma(k,\omega)}{[\omega - k^2/2m - \mathrm{Re}\Sigma(k,\omega)]^2 + [\mathrm{Im}\Sigma(k,\omega)]^2},$$
$$\omega > \varepsilon_F^+. \tag{8}$$

The striking feature of the particle spectral function is that it has a high-energy tail that can be directly related to the presence of repulsion at short distances as shown in Ref. [10]. Here the first detailed results for the particle spectral function are presented for a realistic interaction. In Fig. 1 the particle spectral function is plotted for three different momenta, $k = 0.79$, 1.74, and 5.04 fm^{-1}, as a function of energy. All momenta below k_F have the same high-energy tail as the dotted curve for $k = 0.79$ fm^{-1} in
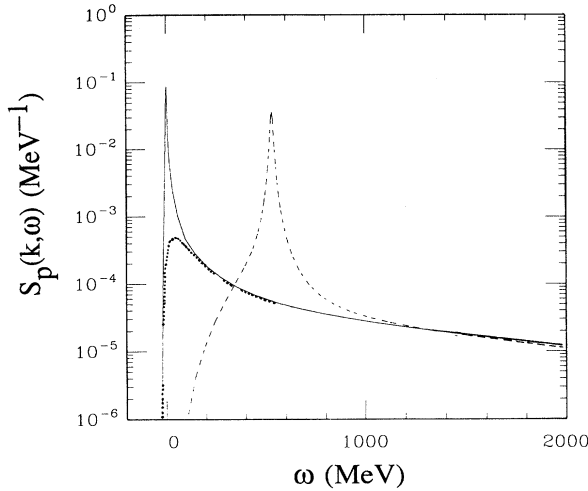
FIG. 1. Particle function for three different momenta; $k = 0.79$ (dotted line), 1.74 (solid line), and 5.04 fm^{-1} (dashed line). The high-energy tail is identical for all momenta below 5 fm^{-1} and is related to the short-range repulsion in the nucleon-nucleon interaction.

FIG. 2. Particle spectral function for $k = 0.79$ fm^{-1}. The results without the inclusion of the tensor force are given by the dashed line. The solid curve is the same as the dotted curve in Fig. 1. The difference between the curves indicates the energy domain to which tensor correlations move sp strength.

Fig. 1. For momenta larger than k_F a quasiparticle peak, which broadens with increasing momentum, can be observed on top of the same high-energy tail. Therefore the results display a common, essentially momentum-independent high-energy tail. The location of sp strength at high energy simply means that the interaction has sufficiently large matrix elements to compensate energy denominators encountered in the ladder equation. For this particular interaction a significant amount of strength is found at high energy. This result was, of course, already anticipated a long time ago [22]. The characteristic tail of the particle spectral function also precludes the choice of a sp spectrum for momenta above k_F analogous to Eq. (1). Such a choice would result in a particle spectrum which starts several hundred MeV higher than the highest sp energy for the hole states.

A quantitative discussion of the location of the missing sp strength can be given using Fig. 2. The solid line represents the particle spectral function for $k = 0.79$ fm^{-1}. the integrated strength accounts for 17% of the sp strength. This is in agreement with the sum rule since the integrated hole strength [see Eq. (7)] provides 83% of sp strength. The strength in the interval from 100 MeV above the Fermi energy to infinity amounts to 13% with 7% residing above 500 MeV. To understand the influence of the tensor force on this distribution, a calculation of the ladder equation was performed in which the tensor coupling in the $^3S_1$-$^3D_1$ coupled channel was switched off. The resulting particle spectral function is given by the dashed line in Fig. 2 (also for $k = 0.79$ fm^{-1}). The integrated sp strength now amounts to 10.5% and should be regarded as resulting from pure short-range correlations. Figure 2 shows that the tensor force moves an additional 6.5% of strength to the first 1000 MeV above the Fermi energy. This is consistent with CBF calculations of the momentum distribution which show depletions of a similar size
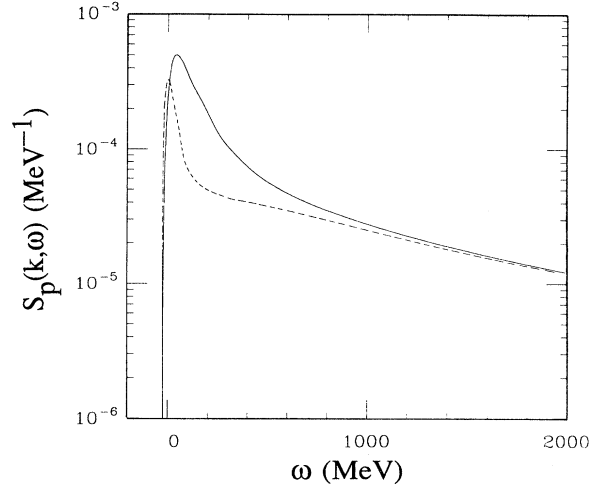
due to tensor correlations [17].

The implications of these nuclear matter results for nuclear structure are manifold. Making the very plausible assumption that the role of short-range and tensor correlations will be identical in a heavy nucleus, one can expect that all sp states will have about 15% of their strength removed to high energy. This is about the amount necessary to bring the results of Ref. [3] into quantitative agreement with $(e,e'p)$ results for ^{48}Ca and ^{90}Zr. The present experimental information on the $3s\frac{1}{2}$ proton in ^{208}Pb shows that this orbital is occupied for 75% (with an error of 9%) with 10% residing in the background [5]. This is consistent with the notion that deep-lying orbitals in a heavy nucleus should have occupation numbers similar to those in nuclear matter, i.e., about 80%. The theoretical results obtained here suggest that 15% of the missing 25% of sp strength for the $3s\frac{1}{2}$ orbital is removed to very high energy. The remaining 10% should then be found in the first 100 MeV above the Fermi energy in the particle domain. Calculations for lighter nuclei [3] show that this is indeed possible although this strength is expected to be highly fragmented as well. For the $3s\frac{1}{2}$ proton the strength of the quasihole pole, z_h, is 65%, 10% additional occupation is obtained as background, and 25% of the strength is depleted and spread to very high energies.

The implications of these results to the low-energy response are also important. Extending this picture to the mostly empty low-lying particle states one can expect a similar 65% strength of the quasiparticle pole, z_p, with an identical distribution of the background. It is then clear that for ph excitations at low energy for which the influence of the ph interactions is expected to be small, like $M12$ and $M14$ excitations [23] in ^{208}Pb only the fraction z_p*z_h of the simple shell-model estimate will be found experimentally. A similar conclusion holds also for the description of low-lying Gamow-Teller strength and

other excitation modes which can now be understood quantitatively when the present results for particle spectral functions are included [8].

The presence of considerable *sp* strength at high energy, therefore, provides a simple explanation for the appearance of quenching phenomena in nuclear physics. Short-range and tensor correlations lead to a depletion of *sp* strength, which can explain much if not everything that

has been discussed under the heading of quenching phenomena in nuclear physics.

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