

Three-body force effect in triton dipole sum rules

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We calculated the two- and three-body enhancement factors κ_2 and κ_3 of the dipole sum rule of the triton, arising, respectively, from the two- and three-body interactions. We performed consistent calculations taking the same interactions for the triton wave function and for the sum rule. The magnitudes of κ_2 and κ_3 are of the order of 0.80–0.86 and 0.03–0.04, respectively. The calculated integrated cross section and the bremsstrahlung-weighted cross section are linearly correlated to the calculated triton binding energy.

After much effort in few-nucleon physics, the importance of the three-body force effect has been well recognized not only for the triton binding energy [1–4], but also for low-energy physical quantities, such as the asymptotic normalization constant [5] and the charge radius [4] of the triton, through linear relationships between these quantities and the binding energy. In addition to the static three-body force effect, the importance of the three-body exchange mechanisms in the ${}^3\text{He}(\gamma, p){}^2\text{H}$ reaction [6] has been pointed out together with the three-body final-state interaction [7]. With this background, we intend to study the three-body current effect in the present paper. For this purpose, we have chosen the triton dipole photoabsorption, because its energy-integrated cross section is free from the final-state interaction.

We calculate the integrated cross section (σ_0) of the

triton electric dipole photoabsorption, i.e., Thomas-Reiche-Kuhn (TRK) sum rule [8],

$$\sigma_0 = \frac{2\pi^2 e^2}{\hbar c} \langle \psi_{3\text{H}} | [D_z, [H, D_z]] | \psi_{3\text{H}} \rangle, \quad (1)$$

where H is the total Hamiltonian, expressed as a sum of the kinetic-energy operator K and the two- and three-nucleon interactions V and W :

$$H = K + V + W. \quad (2)$$

Denoting the nucleon mass by M , and defining the dipole operator D_z by

$$D_z = \sum_{i=1}^3 \frac{1 + \tau_3(i)}{2} r_z(i), \quad (3)$$

TABLE I. The results of the triton sum rules. The first column shows the interactions used for calculating the wave functions and the sum rules. TM 700 (800) means the pion cutoff mass is taken to be 700 (800) MeV for the TM three-body force.

Wave function	Channel	BE (MeV)	σ_0 (mb MeV)	κ_2	κ_3	σ_{-1} (mb)
AV14	5	7.45	72.4	0.818	...	2.73
AV14	18	7.58	72.5	0.821	...	2.71
AV14	34	7.68	72.6	0.823	...	2.66
AV14+TM 700	34	8.42	75.3	0.855	0.035	2.47
AV14+TM 800	34	9.29	78.3	0.899	0.067	2.28
RSC	18	7.24	70.5	0.771	...	2.73
RSC+TM 700	18	8.21	73.8	0.819	0.034	2.46
PARIS	34	7.64	70.6	0.772	...	2.63
PARIS+TM 700	34	8.32	72.9	0.798	0.031	2.45
TRS	34	7.55	72.3	0.817	...	2.66
TRS+TM 700	34	8.47	75.6	0.859	0.039	2.42

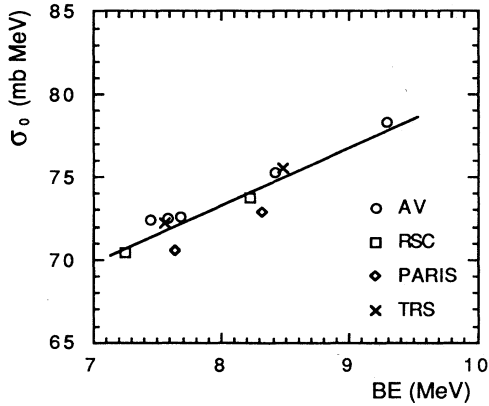


FIG. 1. The relationship between σ_0 and BE, as shown in Table I.

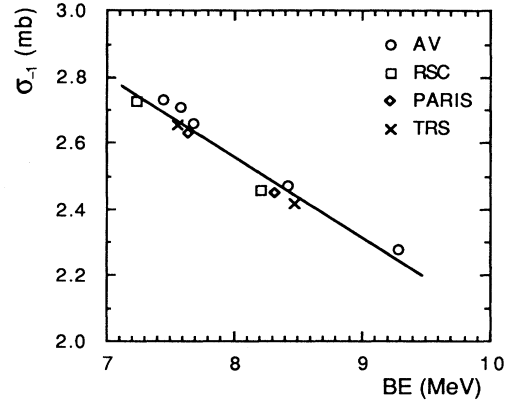


FIG. 2. The relationship between σ_{-1} and BE, as shown in Table I.

we express Eq. (1) as

$$\sigma_0 = \frac{4\pi^2}{3} \frac{e^2 \hbar c}{Mc^2} (1 + \kappa_2 + \kappa_3), \quad (4)$$

where the enhancement factors κ_2 and κ_3 are defined by

$$\kappa_2 = \frac{3}{2} \frac{Mc^2}{(\hbar c)^2} \langle \psi_{3H} | [D_z, [V, D_z]] | \psi_{3H} \rangle \quad (5)$$

and

$$\kappa_3 = \frac{3}{2} \frac{Mc^2}{(\hbar c)^2} \langle \psi_{3H} | [D_z, [W, D_z]] | \psi_{3H} \rangle. \quad (6)$$

The results of calculations have been reported from time to time limited to κ_2 . The enhancement factor κ_3 , representing the three-body force effect, is reported for the first time in the present paper.

The commutation relations in Eqs. (5) and (6) are due to the Seigert theorem [9]:

$$\nabla \cdot \mathbf{j} = -\frac{i}{\hbar} [H, \rho]. \quad (7)$$

Since the two-body part of the charge density (ρ) is $\mathcal{O}(g_\pi^2/M^3)$ against $\mathcal{O}(1)$ for the one-body part [10], we keep the one-body operator in ρ . Then, the one-, two-, and three-body parts of the current (\mathbf{j}) are given in terms of the commutation relations $[K, \rho]$, $[V, \rho]$, and $[W, \rho]$, respectively, each of which contributes 1, κ_2 , and κ_3 in Eq. (4).

It has been reported earlier that the triton sum rule is quite sensitive to its wave functions [11]. Therefore we take care of the consistency of the calculations, i.e., the interactions in H of Eq. (1) are the same as those used to calculate the triton wave function. The consistency is in conformity with the gauge invariance. In the calculation, we took four realistic two-body interactions, RSC [12], AV14 [13], PARIS [14], and TRS [15], without or with the Tucson-Melbourne (TM) three-body force [16]. We took account of 18 or 34 angular momentum states (channels).

Our results are listed in Table I. The wave functions used were obtained in Ref. [4], which were used also in Ref. [17]. The experimental value of σ_0 is 29.0 ± 3.4 mb MeV [18], which was obtained by integrating the photon energy up to 30.0 MeV. Another estimate shows $\sigma_0 \approx 50$ mb MeV [19] by integrating up to 100 MeV. In any case, the ambiguities are too large to be compared with the theory. The binding energy (BE) dependence of σ_0 is shown in Fig. 1. σ_0 increases along with the increase of the calculated BE. It seems that σ_0 is correlated with BE linearly as other physical quantities stated in the first paragraph. The most plausible value of σ_0 is 75.1 mb MeV corresponding to the experimental BE (8.48 MeV). The three-body force makes increases not only in κ_3 but also in κ_2 (cf. Table I). Our results are larger than the results given by Ref. [20]. This difference may indicate the importance of using a good wave function as well as consistent calculations. From this table, we see that $\kappa_2 \approx 0.80$ – 0.86 and $\kappa_3 \approx 0.03$ – 0.04 (for the two-body interaction + TM 700), which show that the two-body contribution to the total current is about 45% while the three-body one is about 2%–3% in the photonuclear reactions.

In addition, the bremsstrahlung-weighted cross section [20] (σ_{-1}) is listed in Table I. Its BE dependence is shown in Fig. 2. σ_{-1} decreases as BE increases. In this figure we expect the theoretical value of $\sigma_{-1} \approx 2.44$ mb, which is larger than the value ($\sigma_{-1} \approx 1.937$ – 1.957 mb) given in Ref. [20]. The experimental value is 2.76 ± 0.12 mb in Ref. [19], which almost agrees with our result, and 1.753 ± 0.176 mb in Ref. [18].

In conclusion, (1) we have performed a consistent calculation of the triton sum rule. (2) Each of the integrated dipole photoabsorption cross sections and the bremsstrahlung-weighted cross sections are linearly correlated to the calculated binding energy. (3) We found that the two- and three-body current contributions are about 45% and 2%–3%, respectively, below the pion threshold energy.

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