Nucleon binding corrections to lepton-nucleus deep inelastic scattering: Use of a realistic spectral function

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Nuclear spectral functions computed with realistic nuclear forces are used to compute mean separation energies and to estimate the binding corrections to lepton-nucleus deep inelastic scattering. The separation energies are large and significant binding effects are obtained.

I. INTRODUCTION

Lepton-nucleus deep inelastic scattering (DIS) experiments have shown that the nuclear structure function $F_2^A(x)$ is different from the one for a free nucleon, $F_2^N(x)$ [1–3]. A substantial part of the observed deviation of $R(x)=F_2^A(x)/AF_2^N(x)$ from unity ["EMC (European Muon Collaboration) effect"] can be ascribed to effects of nuclear binding and Fermi motion (4–7]. It is clearly necessary to understand these conventional nuclear effects very well. Only then can one verify the presence of other physics.

Quantitative estimates of such conventional nuclear effects begin with (but require more than) knowledge of the nonrelativistic nuclear spectral function $S(\mathbf{p},E)=\langle\Psi|a_p^\dagger\delta(E-H+E_A)a_p|\Psi\rangle$, where Ψ is the ground-state nuclear wave function, a_p destroys a nucleon with momentum $\mathbf{p}$, and E is the separation energy. Until recently, detailed information about $S(\mathbf{p},E)$ was absent, and therefore only qualitative estimates of these nuclear effects could be obtained.

Recently Benhar *et al.* [8] have obtained an accurate approximation to the spectral function for nuclear matter using the realistic Urbana $V_{14}+$ three-nucleon interaction [9]. Their method is based upon the use of a ground-state variational wave function containing all the possible spin-isospin and tensor correlations. The intermediate $A-1$ states $a_p|\Psi\rangle$ are approximated by orthogonalized correlated basis states.

A striking feature of the spectral function of Ref. (8) is that the correlations cause large deviations from the mean-field picture. There are significant probabilities to find nucleons with high momentum and with large separation energies. In particular, we find below that the mean separation energy, $<E>\sim 70$ MeV, is much larger than the values adopted in the past which were in the range $<E>\sim 26$–40 MeV. Recent results [10] using the Reid soft-core potential give $<E>=85$ MeV. The large value of $<E>$ and high-momentum components obtained from a realistic $S(\mathbf{p},E)$ may have significant consequences for nuclear DIS.

The importance of including the effects of nucleon-nucleon correlations and the resulting large mean removal energies in calculating the EMC ratio has been emphasized by several groups [11–13]. In particular, Ciofi degli Atti and Liuti [11] used microscopic spectral functions for light systems ($A=2,3,4$) to demonstrate that including correlations enhances the computer EMC effect in the direction of the experimental data for values of $x\approx 0.5$. Ciofi degli Atti and Liuti [12] also considered the effects of correlations in heavier nuclei, using an approximation to the spectral function. We do not use this approximation, but we find results similar to those of Ref. [12]. Shlomo and Vagradov [13] showed that, for heavier nuclei, the use of empirical removal energies improved the agreement with experiment. Other works [14] focus on the role of the flux factor. Roznyek and Birse [15] studied the influence of correlations for $x>1$. They used the Jastrow method, so their wave functions are phenomenological.

The nuclear spectral function for nuclear matter was computed from a nuclear force yielding good nuclear matter saturation properties in Ref. [8]. This nice technical development makes it possible to quantitatively investigate the role of correlations for heavy nuclei. In the present paper we use this spectral function to study the nucleon binding and Fermi motion effects for DIS on nuclear matter.

II. THE CONVOLUTION MODEL

The most popular approach to the EMC effect is the convolution formalism, first discussed in detail by Jaffe [5]. This method is based upon the following assumptions: (i) only incoherent contributions are kept (the photon is emitted and absorbed by the same quark in the same nucleon), (ii) final-state interactions of the hit nucleon are neglected, and (iii) the structure function $F_2^N(x)$ of the bound nucleon is the same as that for a free nucleon, i.e., off-shell effects are neglected. Assumption (i) seems well justified, but assumptions (ii) and (iii) are much more open to criticism.

Starting with the convolution formula for the nuclear hadronic tensor

$$W_{\mu\nu}^A(P_A,q)=\int d^4p\, S(p)W_{\mu\nu}^N(p,q)\ , \qquad (1)$$

the structure function per nucleon $F_2^A(x)$ is given by

$$F_2^A(x)=\int_x^A dz\, f_A^N(z)F_2^N(x/z) \qquad (2)$$

in the Bjorken limit. Here $x = Q^2/2m_N\nu$ and the convolution function f_A^N defines the light-cone momentum distribution of the nucleon

$$f_A^N(z) = \int d^4p\, S(p) z \delta \left[z - \frac{p^0+p^3}{m_N} \right] , \quad (3)$$

where $S(p)$ is the relativistic spectral function, which is normalized as $\int d^4p\,(p^0+p^3)/m_N S(p)=1$. Hence, $f_A^N(z)$ satisfies the normalization $\int_0^\infty dz\, f_A^N(z)=1$. The factor z in Eq. (3) is also known as the flux factor.

The above derivation of F_2^A follows the technique of Ref. [7], and the form is essentially the same as that of Ref. [6].

In practice, the relativistic spectral function is not available and we have to deal with a nonrelativistic approximation. The connection between these two spectral functions is not obvious. We simply replace the relativistic quantity $zS(p)$ in Eq. (3) by the nonrelativistic one $zS(\mathbf{p},E)$, with $p^0 = m_N - E$ and $S(\mathbf{p},E) = S(p,E)$. This replacement ignores relativistic effects such as terms of order $p^2|m_N^2$. Our purpose here is to investigate the consequences of the large value of $<E>$; the derivation of a fully relativistic spectral function is beyond the scope of this work.

The net result is that

$$f_N^A(z) = 2\pi m_N z \int dE \int_{p_{\min}} p\, dp\, S(p,E) , \quad (4)$$

where $p_{\min} = m_N(1-z)+E$ is used in Eq. (2) to obtain $F_2^A(x)$.

III. PROPERTIES OF THE (NONRELATIVISTIC) SPECTRAL FUNCTION

In the mean-field approximation the spectral function for nuclear matter has the simple form

$$S(p,E) \propto \theta(p_F - |p|)\delta(E - \epsilon_p) , \quad (5)$$

where ϵ_p is the single-particle energy, and p_F is the Fermi momentum. This spectral function enters if the reactions knocking out nucleons led only to two-body final states.

The calculation of Ref. [8] shows that the effect of two-body correlations leads to two types of modifications of Eq. (5). (i) For $p < p_f$, about 15% of the single-particle strength is spread over a large missing energy E region (several hundreds of MeV); this effect is caused by mixing of the one-hole (1h) state with high-lying two-hole–one-particle (2h-1p) states in the $A-1$ system. (ii) A depletion of about 15% of the strength of normally occupied states ($p < p_F$) which is spread over a large p,E region; this is due to the presence of 2p-2h admixtures in the initial state.

Numerical integration of $S(p,E)$ over all values of E lead to the single-nucleon density $n(p)$. In particular [8], the single particle occupancies near the Fermi surface are given by

$$n(p) = \int dE\, S(p,E) \approx \begin{cases} 0.79 \text{ at } p = p_F - \epsilon , \\ 0.06 \text{ at } p = p_F + \epsilon , \end{cases} \quad (6)$$

and thus the quasiparticle strength Z_F, which is the discontinuity of $n(p)$ at p_F, is $Z_F = 0.73$.

The empirical value of Z_F as found in (e,e',p) experiments on heavy nuclei [16] is generally somewhat smaller ($Z_F \sim 0.60$). This difference may be explained by surface effects in finite systems, which lead to an additional smearing of the Fermi surface.

The fraction of nucleons with momenta $p < p_F$ is [8]

$$4\pi \int_0^{p_F} n(p) p^2 dp = 0.85 . \quad (7)$$

The lowest E and p^2 moments of S are relevant for DIS. We integrate and find that the mean removal energy is given by

$$\langle E \rangle = \int dE \int d^3p\, E S(p,E) = 71 \text{ MeV} , \quad (8)$$

and the kinetic energy per nucleon by

$$\langle T \rangle = \int dE \int d^3p \frac{\hbar^2 p^2}{2m} S(p,E) = 36 \text{ MeV} . \quad (9)$$

Further investigation reveals that as expected about 50% of the contribution to $\langle T \rangle$ comes from $p > p_F$, and about 40% of the contribution to $\langle E \rangle$. The latter value is much larger than generally assumed.

An additional point to investigate is whether the Koltun sum rule [17] is satisfied. This states that

$$E_A/A = \tfrac{1}{2}(\langle T \rangle - \langle E \rangle) , \quad (10)$$

if only two-body forces are present. The left-hand side is -16 MeV, while the right-hand side is -17 MeV. The near equality indicates that the effects of three-body forces are not dramatic.

IV. RESULTS

It is useful to study the effects of the $\langle T \rangle$ and $\langle E \rangle$ moments of S on $f(z)$ separately. To do that, consider the averaged quantity [7]

$$\langle z \rangle = \int_0^{m_A/m_N} dz\, z f(z) \approx \frac{\langle p_0 \rangle}{m_N} + \frac{1}{3}\frac{\langle p^2 \rangle}{m_N^2}$$
$$= 1 - \frac{\langle E \rangle}{m_N} + \frac{2}{3}\frac{\langle T \rangle}{m_N} . \quad (11)$$

The numerical values obtained above lead to $\langle z \rangle = 0.94$. The convolution function $f(z)$ is shown in Fig. 1. Observe that $f(z)$ peaks at $z=0.95$; this net shift results from a cancellation of a negative shift due to $\langle E \rangle$ and a smaller positive shift due to $\langle T \rangle$. The peak is at a value of z slightly higher than $\langle z \rangle$.

Some qualitative insight into the effect of the convolution can be obtained by expanding $F_2^A(x)$ around $z=1$ up to $\langle E \rangle/m_N$ and $\langle p^2 \rangle/m_N^2$ (valid for $x < 0.6$)

$$\frac{1}{A}F_2^A(x) = F_2^N(x) - xF_2^{N\prime}(x)\frac{\langle E \rangle}{m_N} + \frac{x^2}{2}F_2^{N\prime\prime}(x)\frac{2\langle T \rangle}{3m_N} . \quad (12)$$

It is not obvious how to compare the present result for $F_2^A(x)$ for nuclear matter with the data for finite nuclei.

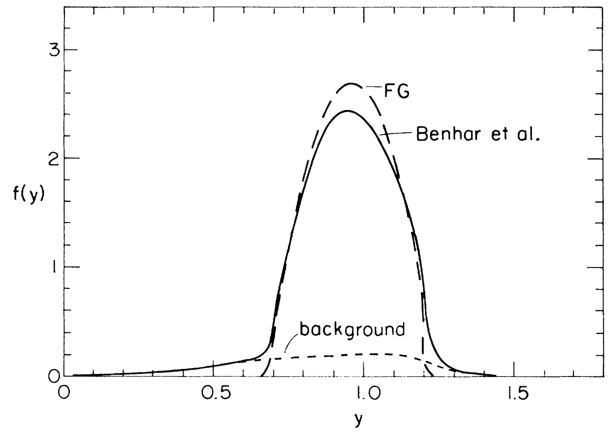

FIG. 1. The convolution function $f(z)$, from Eq. (4), solid curve (Benhar *et al.*). Fermi gas (FG) $f(z)$, long dashes. The contribution of $f(z)$ from momenta above p_F is denoted as background (short dashes).

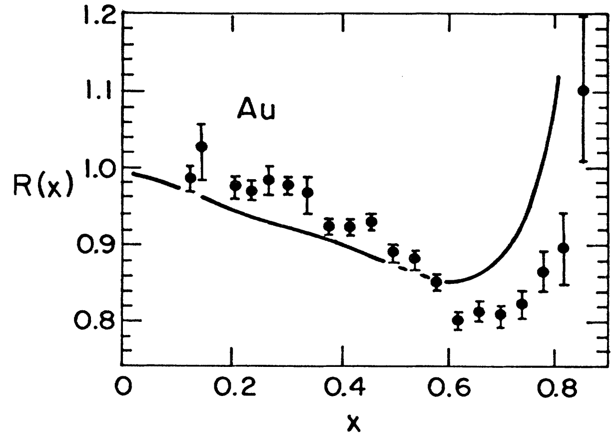

FIG. 2. Nuclear matter calculation of the ratio $R(x)$ ^{197}Au to D structure functions $F_2(x)$ compared with the data of Ref. [2].

One way of proceeding would be to assume that the A dependence of $A^{-1}F_2^A(x)$ can be decomposed into a volume part (V) and a surface contribution ($\sim A^{-1/3}$), i.e., one would parametrize the empirical $A^{-1}F_2^A(x)$ as $F_2^V(x)+A^{-1/3}F_2^S(x)$ and then extract $F_2^V(x)$ by taking $A\rightarrow\infty$. The latter would presumably correspond to the EMC effect in nuclear matter. However, in practice, this is not very meaningful because of the limited accuracy of the data. On the other hand, the global parametrization used in Refs. [2,18], $A^{-1}F_2^A(x)\sim A^{-0.044}$ (for $x=0.6$), does not have a physical $A\rightarrow\infty$ limit.

Therefore, here we compare the predicted nuclear matter $F_2(x)$ with the data for the heaviest measured nucleus, ^{197}Au. Figure 2 shows that much of the EMC effect can be explained if one uses a realistic spectral function.

It is also possible to make a rough assessment of the error made in using a nuclear matter spectral function. Recent work [19] shows that the high-momentum tail of the finite nuclear density distributions is essentially the same as that of infinite nuclear matter. Thus, one may again expect about 40% of the value of $\langle E\rangle$ (i.e., 28 MeV) to come from the region $p>p_F$. Hartree-Fock calculations give about $\langle E\rangle=30$ MeV; assuming an occupation probability for the normally occupied states of 0.85 the estimate for $\langle E\rangle$ becomes about 53 MeV. Thus, it seems that, compared to nuclear matter, the value of $\langle E\rangle$ in heavy nuclei is reduced by a factor of about 0.75. Assuming that the value of $\langle T\rangle$ is not changed, as suggested by mean-field calculations, one then finds from Eq. (11) that the value of $\langle z\rangle$ for finite nuclei should be increased from 0.94 to about 0.96. Since the depth of the minimum is roughly proportional to $\langle z\rangle$, one would then account for even less of the observed depletion around $x=0.6$. Thus, the comparison between nuclear matter theory and ^{197}Au data would be even less close. However, one cannot deny that the nucleonic contributions are very important.

V. SUMMARY

The use of the spectral functions computed with realistic nuclear forces leads to significant probabilities to excite high-lying $A-1$ intermediate states. Thus, the values of $\langle E\rangle$ and $\langle T\rangle$ are larger than those obtained from mean-field considerations. The net result is that the contributions of nuclear binding effects to lepton-nucleus deep inelastic scattering are substantial, but do not seem to account for the entire observed EMC effect.

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