# Pauli-blocking in the quasideuteron model of photoabsorption

M.B. Chadwick
*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

P. Obložinský* and P. E. Hodgson
*Nuclear Physics Laboratory, University of Oxford, Oxford OX1 3RH, United Kingdom*

G. Reffo
*Ente Nazionale Energie Alternative, 40138 Bologna, Italy*
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We calculate the effects of Pauli-blocking in the quasideuteron model of hard photon-absorption. Our approach is based upon phase space considerations and uses Fermi-gas state densities that conserve linear momentum. The Pauli-blocking function that we obtain differs from Levinger's phenomenological exponential factor and yields nuclear photoabsorption cross sections that are in good agreement with experimental data for photon energies up to the pion threshold. The temperature dependence of the nuclear photoabsorption cross section in the quasideuteron regime is also investigated. The model that we use to describe nuclear photoabsorption is particularly suited for application to preequilibrium models of hard photon emission.

## I. INTRODUCTION

The quasideuteron model [1–3] describes the dominant mechanism for nuclear photoabsorption for incident photon energies in the range $40 \lesssim \epsilon_\gamma \lesssim 140$ MeV. This model was first proposed by Levinger [1,3] and has subsequently been applied extensively to analyze nuclear photoabsorption cross sections [4–7]. The model does, however, contain two free parameters and treats the effects of the Pauli exclusion principle in an entirely phenomenological manner. In this work we develop a quasideuteron model of photoabsorption which includes Pauli-blocking effects theoretically and does not contain any free parameters. Recent interest in quasideuteron photoabsorption stems from attempts to describe hard photon emission by applying detailed balance [8–10]. For this purpose one needs a good knowledge of photoabsorption cross sections, and, in addition, when applied to preequilibrium reaction models, one needs a thorough description of the photoabsorption process in terms of particle and hole excitations. In a subsequent paper we intend to show how our model of photoabsorption can be extended to describe preequilibrium photon emission in proton-induced reactions. The temperature dependence of the nuclear photoabsorption cross section is of considerable importance since it is needed for calculating photon emission rates from hot nuclei [8,9] as well as in dipole sum-rule considerations [11]. In the present work we therefore calculate the temperature dependence of Pauli-blocking effects and show its impact on photoabsorption cross sections. In addition to the above applications, the quasideuteron model has also been used to investigate the swelling of nucleons in nuclei in the European Muon Collaboration effect [12], and antinucleon annihilation processes in antinucleon-nucleus reactions [13].

In the quasideuteron model it is assumed that photoabsorption takes place on correlated neutron-proton pairs within a nucleus. The relatively small photon wavelengths ensure that the interaction takes place with a nucleon-nucleon pair, rather than with the nucleus as a whole, and the predominantly electric-dipole nature of the interaction implies photoabsorption only by neutron-proton pairs. Levinger showed [1, 3] that the nuclear photoabsorption cross section $\sigma_{qd}(\epsilon_\gamma)$ can be expressed in terms of the free deuteron photodisintegration cross section $\sigma_d(\epsilon_\gamma)$,

$$\sigma_{qd}(\epsilon_\gamma) = \frac{L}{A} NZ \sigma_d(\epsilon_\gamma) f(\epsilon_\gamma), \tag{1}$$

where $L$ is the Levinger parameter and $f(\epsilon_\gamma)$ is the Pauli-blocking function. The factor $NZ$ is the total number of neutron-proton pairs inside the nucleus, which is multiplied by a reduction factor $L/A$ to account for the fact that it is only correlated pairs that can be considered to be quasideuterons [13, 14]. In addition, the function $f(\epsilon_\gamma)$ accounts for those excitations of neutron-proton pairs that cannot occur since the Pauli-exclusion principle allows only final particle states which lie above the Fermi level. This effect is particularly important for low photon energies, and Levinger suggested that it can be represented by an exponential Pauli-blocking function[3]

 

$$f_{\rm Lev}(\epsilon_\gamma) = e^{-D/\epsilon_\gamma}, \tag{2}$$

where $D$ is a constant. Although a theoretical estimate for the Levinger parameter is well known [1], no theoretical derivation for the Pauli-blocking function has been given. In practice, $L$ and $D$ are treated as free parameters to fit the photoabsorption data. The difficulty in separating the effects of the Levinger parameter and the Pauli-blocking function in Eq. (1) has resulted in a substantial ambiguity in the $L$ and $D$ values used by different groups; they range from $L = 4.9$ and $D = 60$ MeV (Ref. [9]) to $L = 10$ and $D = 80$ MeV (Ref. [15]).

The motivation for the present work came from an attempt to describe the quasideuteron photoabsorption process in a framework that lends itself to application within a preequilibrium description of hard photon emission. Such an attempt is facilitated by the observation that hard photon absorption in the nucleus leads to the excitation of a neutron-proton pair. This can be viewed as the creation of a two-particle–two-hole state [15, 16] of which the two particles, for a given hole pair, can be treated using techniques that have been devised for preequilibrium nuclear reaction models. To this end we use a Fermi-gas model of the nucleus and require that the accessible state density includes only states that can be reached by momentum, as well as energy, conservation.

In Sec. II we give a derivation of the nuclear photoabsorption cross section in the quasideuteron model in which the Pauli-blocking function and the Levinger parameter are calculated in a consistent way. An essential part of our approach is concerned with the calculation of the state densities of excited neutron-proton pairs. We show in Sec. III how the temperature dependence of the nuclear photoabsorption cross section can be calculated. In Sec. IV we compare our Pauli-blocking function with previous phenomenological exponential parametrizations. The nuclear photoabsorption cross section that we obtain in the quasideuteron model is then compared with experimental photoabsorption data for a number of nuclei and some conclusions are given.

## II. CALCULATION OF THE PAULI-BLOCKING FUNCTION

### A. Formalism

In Levinger's original derivation of the nuclear photoabsorption cross section, effective-range theory was used to show that the wave function of a neutron-proton pair inside a nucleus (a quasideuteron) is proportional to that of a free deuteron for close neutron-proton separations. This allows the quasideuteron photoabsorption cross section on a pair with relative momentum $k$, $\sigma_{qd}(k,\epsilon_\gamma)$, to be written in terms of the deuteron photodisintegration cross section as [1]

$$\sigma_{qd}(k,\epsilon_\gamma) = \sigma_d(\epsilon_\gamma)\frac{2\pi(1-\alpha r_0)}{V\alpha}\frac{1}{\alpha^2+(k/\hbar)^2}, \tag{3}$$

where $k = \frac{1}{2}|\mathbf{k}_\nu - \mathbf{k}_\pi|$ is the initial relative momentum of a neutron-proton pair. $\alpha^{-1} = \hbar/(2.23m)^{\frac{1}{2}}$ is related to the neutron-proton scattering length [17], $m$ being the nucleon mass, and $r_0$ is the effective range. The nuclear volume in the above expression is $V = \frac{4}{3}\pi 1.2^3 A$ fm$^3$.

Following Levinger, we assume that if all the possible final neutron and proton states after photoabsorption are not Pauli-blocked, the photoabsorption cross section on a quasideuteron is given by Eq. (3). However, if the available phase space for the neutron and proton after photoabsorption is reduced by Pauli-blocking, we assume that the quasideuteron photoabsorption cross section is also reduced by the same amount. Thus we suppose that the cross section for photoabsorption is proportional to the available phase space. This is reasonable since Fermi's golden rule ought to be applicable as the electromagnetic perturbation is small compared to the nuclear interactions. The problem of determining the nuclear photoabsorption cross section, and hence the Pauli-blocking function, requires an integration of Eq. (3) over all possible initial neutron and proton momenta, in which the effects of Pauli-blocking are taken into account. This integral, for a Fermi-gas nucleus, can be expressed as

$$\sigma_{qd}(\epsilon_\gamma) = \int_0^{k_F}\int_0^{k_F} d^3\mathbf{k}_\nu d^3\mathbf{k}_\pi \rho(1p,\mathbf{k}_\nu)\rho(1p,\mathbf{k}_\pi)\sigma_{qd}(k,\epsilon_\gamma)F(\mathbf{k}_\nu,\mathbf{k}_\pi,\mathbf{k}_\gamma), \tag{4}$$

where $\mathbf{k}_\nu$, $\mathbf{k}_\pi$, and $\mathbf{k}_\gamma$ are the neutron, proton, and photon momenta, respectively, before photoabsorption, and $k_F$ is the Fermi momentum. The density of single-particle neutron and proton states in momentum space, $\rho(1p,\mathbf{k}_\nu)$ and $\rho(1p,\mathbf{k}_\pi)$, must reproduce the number of neutrons and protons in the nucleus, so

$$\rho(1p,\mathbf{k}_\nu) = \frac{N}{\frac{4}{3}\pi k_F^3} \equiv \kappa_\nu,$$
$$\rho(1p,\mathbf{k}_\pi) = \frac{Z}{\frac{4}{3}\pi k_F^3} \equiv \kappa_\pi. \tag{5}$$

The adopted normalization is important since the quasideuteron photoabsorption is of a volume nature [6, 7]. The blocking factor $F(\mathbf{k}_\nu,\mathbf{k}_\pi,\mathbf{k}_\gamma)$ reduces the cross section for photoabsorption on a particular neutron-proton pair, $\sigma_{qd}(k,\epsilon_\gamma)$, and is discussed in detail below.

After photoabsorption both the neutron and proton upon which the absorption takes place must have momenta above the Fermi momentum. Photoabsorption processes where either one, or both, of these nucleons do not satisfy this requirement are forbidden by the exclusion principle, and since the quasideuteron photoabsorption cross section is proportional to the available phase space, the cross section for photoabsorption on a particular neutron- proton pair in Eq. (3) should be reduced. This reduction can be calculated by determining the ratio of the phase space available upon photoabsorption to the "full" phase space that would be available if the exclusion principle did not apply.

Figure 1 shows, in momentum space, the absorption of a hard photon upon a neutron-proton pair within the nucleus. The initial and final linear momenta, and the corresponding energies, are related by

$$\mathbf{k}_\nu + \mathbf{k}_\pi + \mathbf{k}_\gamma = \mathbf{K} = \mathbf{k}'_\nu + \mathbf{k}'_\pi,$$
$$\epsilon_\nu + \epsilon_\pi + \epsilon_\gamma = E = \epsilon'_\nu + \epsilon'_\pi, \tag{6}$$

where primes refer to final states after photoabsorption, and $\mathbf{K}$ and $E$ are the total linear momentum and total energy, respectively. If the exclusion principle is obeyed, the vectors $\mathbf{k}'_\nu$ and $\mathbf{k}'_\pi$ must extend beyond the Fermi sphere as defined by the radius $k_F$. Considering all accessible final states that are consistent with the above constraints, one obtains the neutron-proton state density which depends on $\mathbf{K}$ and $E$ only. This neutron-proton state density is written as $\rho^P(2p, E, \mathbf{K})$, where the superscript $P$ indicates that the Pauli exclusion principle has been taken into account. The blocking factor $F(\mathbf{k}_\nu, \mathbf{k}_\pi, \mathbf{k}_\gamma)$ can then be expressed as the ratio of this two-particle state density to the one that is obtained when the Pauli exclusion principle is excluded,

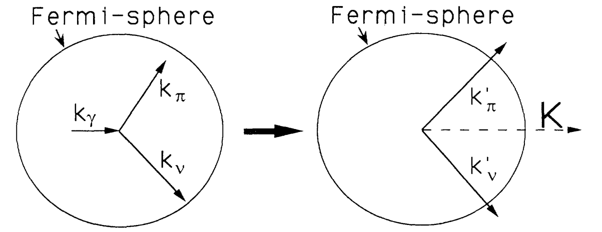


FIG. 1. Absorption of a hard photon by a quasideuteron in the nucleus. The initial linear momenta of the photon, neutron, and proton are $\mathbf{k}_\gamma$, $\mathbf{k}_\nu$, and $\mathbf{k}_\pi$, respectively, and the final linear momenta are $\mathbf{k}'_\nu$ and $\mathbf{k}'_\pi$. The total momentum is $\mathbf{K}$.

$$F(\mathbf{k}_\nu, \mathbf{k}_\pi, \mathbf{k}_\gamma) = \frac{\rho^P(2p, E, \mathbf{K})}{\rho(2p, E, \mathbf{K})}. \tag{7}$$

Considering the structure of Eq. (1), the Pauli-blocking function can be finally expressed as

$$f(\epsilon_\gamma) = \frac{\int_0^{k_F}\int_0^{k_F} d^3\mathbf{k}_\nu d^3\mathbf{k}_\pi \rho(1p, \mathbf{k}_\nu)\rho(1p, \mathbf{k}_\pi)\sigma_{qd}(k, \epsilon_\gamma) F(\mathbf{k}_\nu, \mathbf{k}_\pi, \mathbf{k}_\gamma)}{\int_0^{k_F}\int_0^{k_F} d^3\mathbf{k}_\nu d^3\mathbf{k}_\pi \rho(1p, \mathbf{k}_\nu)\rho(1p, \mathbf{k}_\pi)\sigma_{qd}(k, \epsilon_\gamma)}. \tag{8}$$

The denominator can be evaluated analytically by observing that the six-dimensional integral reduces to a one-dimensional integral when it is transformed into relative-momentum space, the distribution of relative momenta of two nucleons in a Fermi gas being [18]

$$M(k) = 24\frac{k^2}{k_F^3}\left[1 - \frac{3k}{2k_F} + \frac{1}{2}\left(\frac{k}{k_F}\right)^3\right]. \tag{9}$$

This yields the denominator of Eq. (8) in the form $(L/A)NZ\sigma_d(\epsilon_\gamma)$, where $L = 6.5$, obtained using the standard values of $\alpha = 0.232$ fm$^{-1}$, $r_0 = 1.761$ fm, and a Fermi energy of 35 MeV used throughout this paper. This differs from Levinger's result [1] of $L = 6.4$ only because of the numerical values of the constants adopted. The numerator of Eq. (8) has to be evaluated numerically using neutron-proton state densities that are derived below.

### B. State densities with linear momentum

When a hard photon is absorbed by a quasideuteron, a neutron-proton pair becomes excited within the nucleus and this can be viewed as the creation of a two-particle–two-hole state. We are interested in processes that are shown in Fig. 1 and described by Eq. (6). We consider specific neutron and proton initial states (which become holes after photoabsorption) and wish to determine the density of accessible final states for the two excited particles. We do not determine the accessible two-particle–two-hole state density, as is done in Refs. [10,16], since we assume a quasideuteron photoabsorption cross section that is $k$ dependent and therefore different for different quasideuterons. Hence we calculate, for each hole pair, the density of accessible two-particle states that are characterized by a total energy $E$ and a total momentum $\mathbf{K}$. We note that these quantities are measured from the bottom of the nuclear potential well.

Techniques for calculating accessible particle-hole state densities for various types of transitions have been developed for use in preequilibrium reaction models [19]. The procedure that is adopted for the Fermi-gas model is to convolute single-particle state densities with an energy-conserving delta function. We have found, however, that for the present purpose this may not be sufficiently accurate since the role of linear momentum cannot be neglected. Since the photoabsorption cross section on a quasideuteron is dependent upon the neutron and proton momenta, it is appropriate to include momentum effects in the accessible state density calculations. For this reason we consider only states that can be reached by momentum and energy conservation, and hence we derive the density of two-particle states with a specific total momentum as well as total energy. To do this, we include a momentum-conserving delta function into the state density convolution integral. When Pauli-blocking is considered, this state density may be written as

$$\rho^P(2p, E, \mathbf{K}) = \int\int d^3\mathbf{k}'_\nu d^3\mathbf{k}'_\pi \rho(1p, \mathbf{k}'_\nu)\rho(1p, \mathbf{k}'_\pi)\delta(E - {k'_\pi}^2/2m - {k'_\nu}^2/2m)\delta(\mathbf{K} - \mathbf{k}'_\pi - \mathbf{k}'_\nu)$$
$$\times\Theta(k'_\pi - k_F)\Theta(k'_\nu - k_F). \tag{10}$$

The delta functions impose both energy and momentum conservation and Pauli-blocking is accounted for by the unit step function, $\Theta$ (1 if argument is greater than zero, 0 otherwise). The integration over $\mathbf{k}'_\nu$ can be done immediately, with momentum and energy conservation both being accounted for in one delta function,

$$\rho^P(2p,E,\mathbf{K}) = \kappa_\nu \kappa_\pi \int d^3\mathbf{k}'_\pi \delta(E - {k'_\pi}^2/m - K^2/2m + (Kk'_\pi/m)\cos\theta_\pi)\Theta(k'_\pi - k_F) \times\Theta(({k'_\pi}^2 + K^2 - 2k'_\pi K \cos\theta_\pi)^{\frac{1}{2}} - k_F), \quad (11)$$

where $\theta_\pi$ represents the angle between $\mathbf{k}'_\pi$ and $\mathbf{K}$. The integration over $\phi_\pi$ implicit in the above equation may be performed immediately yielding

$$\rho^P(2p,E,\mathbf{K}) = \frac{2\pi m\kappa_\nu\kappa_\pi}{K} \int k'_\pi dk'_\pi \int \sin\theta_\pi d\theta_\pi \delta\left[\frac{m}{Kk'_\pi}\left(E - \frac{{k'_\pi}^2}{m} - \frac{K^2}{2m}\right) + \cos\theta_\pi\right] \times\Theta(k'_\pi - k_F)\Theta(({k'_\pi}^2 + K^2 - 2k'_\pi K \cos\theta_\pi)^{\frac{1}{2}} - k_F). \quad (12)$$

This integral can be solved by substituting $t = \cos\theta_\pi$, and determining the range of $k'_\pi$ for which the second integral (over $t$) gives unity. The $\Theta$ functions can be eliminated by beginning the integral over $k'_\pi$ at $k_F$, and restricting the $t$ integral over values of $t$ satisfying

$$-1 \le t \le \min\left(1, \frac{K^2 + {k'_\pi}^2 - k_F^2}{2k'_\pi K}\right). \quad (13)$$

The resulting state density has dimensions MeV$^{-1}$ (MeV/$c$)$^{-3}$ and is given by

$$\rho^P(2p,E,\mathbf{K}) = \begin{cases} 2\pi m\kappa_\nu\kappa_\pi\sqrt{mE - \frac{K^2}{4}} & \text{if } \begin{cases} mE \ge k_F^2 + k_F K + \frac{1}{2}K^2 \\ \text{or } K \ge 2k_F \text{ and } \frac{1}{4}K^2 \le mE \le k_F^2 - k_F K + \frac{1}{2}K^2, \end{cases} \\ 2\pi m\kappa_\nu\kappa_\pi\left(\frac{mE - k_F^2}{K}\right) & \text{if } \begin{cases} K \ge 2k_F \text{ and } k_F^2 - k_F K + \frac{1}{2}K^2 \le mE \le k_F^2 + k_F K + \frac{1}{2}K^2 \\ \text{or } K \le 2k_F \text{ and } k_F^2 \le mE \le k_F^2 + k_F K + \frac{1}{2}K^2, \end{cases} \\ 0 \text{ if } \begin{cases} K \le 2k_F \text{ and } mE \le k_F^2 \\ \text{or } K \ge 2k_F \text{ and } mE \le \frac{1}{4}K^2. \end{cases} \end{cases} \quad (14)$$

As expected from symmetry arguments, the two-particle state density is dependent only upon the magnitude of the total momentum and not its direction. The two-particle state density that includes all transitions (including those that violate the Pauli principle) can be obtained from the results in Eq. (14) in the limit of $k_F \to 0$, yielding

$$\rho(2p,E,\mathbf{K}) = \begin{cases} 2\pi m\kappa_\nu\kappa_\pi\sqrt{mE - \frac{K^2}{4}} & \text{if } mE \ge \frac{1}{4}K^2, \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

Equations (14) and (15) are used to evaluate expression (7) and thus also the numerator of the Pauli-blocking function in Eq. (8).

We have also performed a numerical evaluation of the two-particle state density with linear momentum $\mathbf{K}$ and energy $E$ in Eq. (10) to check our analytic results, in the specific case of $\kappa_\pi = \kappa_\nu = \kappa$. Our procedure was to set up a three-dimensional grid of points in momentum space, representing allowed momentum states, and to define the total two-particle momentum $\mathbf{K}$ as a vector in this space. The proton momentum $\mathbf{k}'_\pi$ was then allowed to scan all points on this grid, and for each particular point taken the neutron momentum vector $\mathbf{k}'_\nu$ was defined so that the total momentum gave $\mathbf{K}$. For each particular pair, energy conservation could then be checked and the number of two-particle states per MeV is given by the number of proton and neutron configurations that have energies lying within a 1 MeV bin around $E$, not counting any configuration in which either the neutron or proton momentum is less than the Fermi momentum. This gives the density of states with momentum exactly $\mathbf{K}$, and the number of states per unit energy space per unit momentum space lying around $\mathbf{K}$ is this density multiplied by $\kappa$. Excellent agreement was obtained between our numerical results and the above analytic expressions.

In the specific case of $\mathbf{K} = 0$ the above relation (15) can be understood in a simple physical way. The neutron and proton after photoabsorption now move in opposite directions with the same absolute values of momenta. Hence the motion of one nucleon essentially defines that of the other, and the two-particle state density has a square-root energy dependence typical of a one-particle density in energy space in a Fermi gas. We also note that Eq. (15) with $\mathbf{K} = 0$, after a simple transformation, agrees with the expression that Bethe and Peierls [20] used for the phase space available after the photodisintegration of a deuteron.

In order to demonstrate the influence of linear momentum on the Pauli-blocking function as calculated by Eq. (8) we also need two-particle neutron-proton state densities where only energy is considered. Such state densities can be obtained straightforwardly from Eq. (14) by integrating over all possible momenta which are consistent with a given energy $E$,

$$\rho^P(2p,E) = \int_0^{\sqrt{4mE}} \rho^P(2p,E,\mathbf{K})4\pi K^2 dK. \quad (16)$$

This state density can also be obtained directly by convoluting neutron and proton single-particle state densities in energy space within the Fermi-gas model,

$$\rho^P(2p,E) = \int\int \rho(1p,\epsilon_\pi)\rho(1p,\epsilon_\nu)\delta(E-\epsilon_\pi-\epsilon_\nu) \times\Theta(\epsilon_\pi-\epsilon_F)\Theta(\epsilon_\nu-\epsilon_F)d\epsilon_\pi d\epsilon_\nu, \quad (17)$$

where $\epsilon_F$ is the Fermi energy and the densities of neutron and proton single-particle states in energy space are given by

$$\rho(1p,\epsilon_\nu) = \frac{3N}{2\epsilon_F}\sqrt{\frac{\epsilon_\nu}{\epsilon_F}}, \qquad \rho(1p,\epsilon_\pi) = \frac{3Z}{2\epsilon_F}\sqrt{\frac{\epsilon_\pi}{\epsilon_F}}, \quad (18)$$

yielding the analytic solution

$$\rho^P(2p,E) = \frac{9NZ\,\Theta(E-2\epsilon_F)}{4\epsilon_F^3}\left[\frac{1}{2}(E-2\epsilon_F)\sqrt{E\epsilon_F-\epsilon_F^2}+\frac{1}{4}E^2\sin^{-1}\left(\frac{E-2\epsilon_F}{E}\right)\right]. \quad (19)$$

Mutual agreement between Eqs. (16) and (19) provides a stringent test on our state densities with linear momentum, and this was carefully checked. A similar procedure may be applied to obtain $\rho(2p,E)$, the total state density that is obtained when the Pauli exclusion principle is not obeyed, yielding

$$\rho(2p,E) = \frac{9\pi NZ}{32\epsilon_F^3}E^2. \quad (20)$$

These last two equations should replace Eqs. (14) and (15) if one wants to calculate the Pauli-blocking function when momentum conservation is ignored.

Figure 2 shows the two-particle state densities of Eqs. (14) and (15) as a function of total energy for two distinct values of $K$, for the case of $^{208}$Pb. As expected, state densities for $K = 0$ display a square-root energy dependence, and when Pauli-blocking is included this density is cut to zero at twice the Fermi energy (70 MeV). This threshold corresponds to configurations in which the neutron and proton move in opposite directions to each other with momenta $k_F$ in magnitude. In the case of $K = 600$ MeV/$c$ the threshold appears at about 96 MeV. This corresponds to a configuration in which the two particles are aligned with identical momenta, and in this case the threshold is the same for both the state densities in which Pauli-blocking is included and ignored. The small magnitude of the densities shown in Fig. 2 should not concern the reader since this reflects the small magnitude of $\kappa_\pi$ and $\kappa_\nu$. Thus, for instance, even though we find that $\rho^P(2p, E = 100 \text{ MeV}, K = 0) = 3.74 \times 10^{-6}$ MeV$^{-1}$(MeV/$c$)$^{-3}$, when the state density with linear momentum is integrated over all possible values of momenta [see Eq. (19)] the total density is given by $\rho^P(2p, E = 100 \text{ MeV}) = 801$ MeV$^{-1}$. In this example, the excitation energy $E = 100$ MeV corresponds to an excitation energy of 30 MeV relative to the Fermi level.

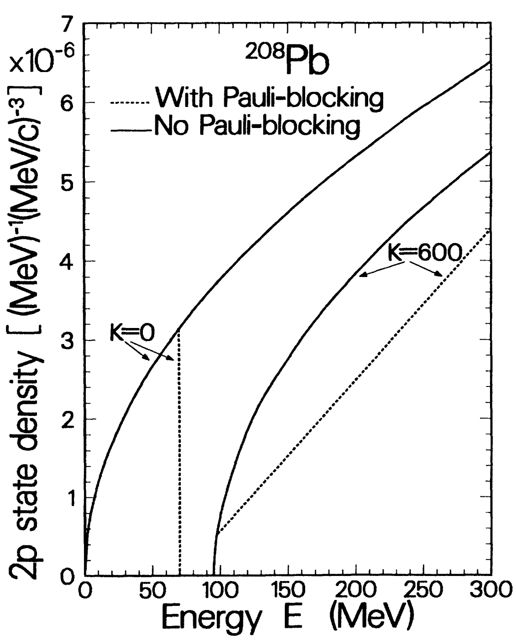


FIG. 2. The two-particle state density for $^{208}$Pb as a function of the excitation energy $E$ for total momenta $K = 0$, 600 MeV/$c$. The two-particle state densities including, as well as neglecting, Pauli-blocking are shown; see Eqs. (14) and (15).

## III. TEMPERATURE DEPENDENCE

There has been some recent interest in evaluating photon emission rates from hot nuclei which have been produced in heavy-ion collisions, by applying the principle of detailed balance [8,9]. For such an approach one needs to know the photoabsorption cross section on an excited nucleus, and in the absence of any experimental measurements one has to rely on theoretical estimates. It is a straightforward matter to determine the temperature dependence of the nuclear photoabsorption cross section in our model.

Prakash *et al.* [9] calculated the temperature dependence of the photoabsorption cross section in the quasideuteron model, and expressed their results in terms of a "correction factor." In fact they considered only the relative temperature variation of the Levinger parameter and completely neglected any possible variation of the Pauli-blocking function with temperature. But Herrmann *et al.* [8], when using detailed balance to study the emission of photons following the reaction $^{92}$Mo+$^{92}$Mo at an incident energy of 19.5$A$ MeV, pointed out that the Pauli-blocking effects should decrease (and hence the Pauli-blocking function should increase) for photoabsorption on hot nuclei. However, no quantitative estimate of this effect was presented. In the quasideuteron model, the temperature dependence of the nuclear photoabsorption cross section is given by the product of the dependences of the Levinger parameter *and* the Pauli-blocking function on the nuclear temperature. We indicate below how these effects can be calculated.

In an excited nucleus at an equilibrium temperature $T$, the probability of occupation of a single-particle state by a nucleon follows a Fermi-Dirac distribution. This should be taken into account when the procedure for calculating $L$ and $f(\epsilon_\gamma)$, described in Sec. II, is followed. Thus, the expressions for the single-particle densities in momentum space, $\rho(1p,\mathbf{k}_\nu)$ and $\rho(1p,\mathbf{k}_\pi)$, should be modified for finite temperatures to

$$\rho(1p,\mathbf{k}_\nu,T)=\frac{\kappa_\nu^T}{\exp[(\epsilon_\nu-\epsilon_F)/T]+1},$$
$$\rho(1p,\mathbf{k}_\pi,T)=\frac{\kappa_\pi^T}{\exp[(\epsilon_\pi-\epsilon_F)/T]+1}. \tag{21}$$

The constants $\kappa_\nu^T$ and $\kappa_\pi^T$ are obtained from the requirement that the integrated densities over all momentum space must reproduce the number of neutrons and protons, so

$$\kappa_\nu^T=\frac{N}{\int d^3\mathbf{k}_\nu/\{\exp[(\epsilon_\nu-\epsilon_F)/T]+1\}}, \tag{22}$$

and a similar expression applies for $\kappa_\pi^T$. The accessible $2p$ state density, including Pauli-blocking, is also different at finite temperatures. It can be evaluated in an approach analogous to Eq. (10), but using a distribution of occupied states rather than the sharp cutoffs that are descibed by the $\Theta$ functions. The $2p$ state density is now given by

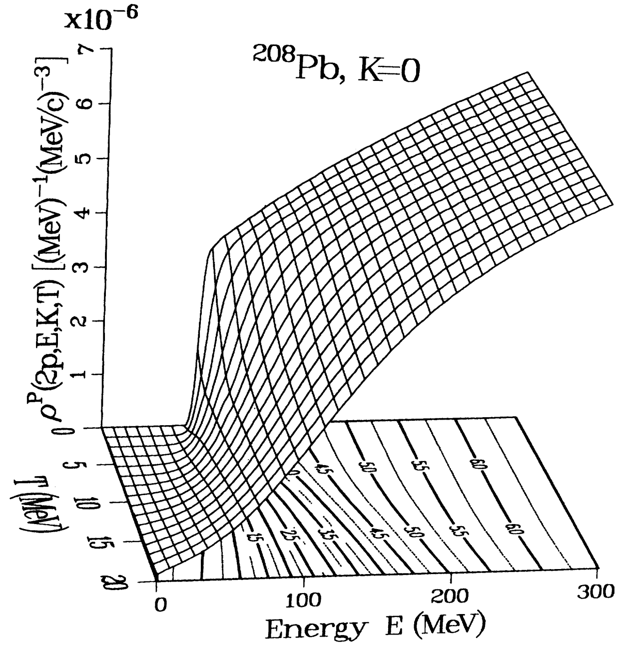


FIG. 3. A three-dimensional representation of the temperature-dependent $2p$ state density for $^{208}$Pb as a function of temperature $T$ and energy $E$ for $K=0$. The numerical values on the contour lines should be multiplied by $10^{-6}$. The discontinuity due to Pauli-blocking at 70 MeV for $T=0$ is smoothed out with increasing nuclear temperature.

$$\rho^P(2p,E,\mathbf{K},T)=\int\int d^3\mathbf{k}'_\nu d^3\mathbf{k}'_\pi\left(\kappa_\nu-\frac{\kappa_\nu^T}{\exp[(\epsilon_\nu-\epsilon_F)/T]+1}\right)\left(\kappa_\pi-\frac{\kappa_\pi^T}{\exp[(\epsilon_\pi-\epsilon_F)/T]+1}\right)$$
$$\times\delta(E-k_\pi'^2/2m-k_\nu'^2/2m)\delta(\mathbf{K}-\mathbf{k}'_\pi-\mathbf{k}'_\nu), \tag{23}$$

where the expressions inside the large parentheses are the densities of single-particle states that are unoccupied in momentum space. The high dimensionality of this integral can be reduced analytically to give

$$\rho^P(2p,E,\mathbf{K},T)=\frac{2\pi m}{K}\int k'_\pi dk'_\pi\left(\kappa_\nu-\frac{\kappa_\nu^T}{\exp[(E-\epsilon_\pi-\epsilon_F)/T]+1}\right)\left(\kappa_\pi-\frac{\kappa_\pi^T}{\exp[(\epsilon_\pi-\epsilon_F)/T]+1}\right).$$
$$\times\int\sin\theta_\pi d\theta_\pi\delta\left[\frac{m}{Kk'_\pi}\left(E-\frac{k_\pi'^2}{m}-\frac{K^2}{2m}\right)+\cos\theta_\pi\right], \tag{24}$$

which can then be solved numerically without any difficulties. In the limit of $T\to 0$ this state density reduces to Eq. (14). In Fig. 3 we show a three-dimensional representation of the temperature-dependent state density for $K=0$. At a nuclear temperature $T=0$ MeV the state density as a function of the two-particle energy $E$ has a discontinuous cut at $E=70$ MeV, as is also shown in Fig. 2. As mentioned above, this occurs because the two particles must move in opposite directions with equal absolute linear momenta, and for total energies below $2\epsilon_F$ they will be blocked by the Pauli principle. As the nuclear temperature increases, particles below the Fermi level become excited, which leads to a reduction in Pauli-blocking effects below the Fermi energy and an increase in blocking above the Fermi energy. This is clearly seen in Fig. 3, where the sharp discontinuity evident at $T=0$ MeV becomes smoothed out for higher nuclear temperatures.

The above expressions allow the temperature dependences of the Levinger parameter and Pauli-blocking function (and hence the nuclear photoabsorption cross section) to be determined. Our results are shown in the following section.

## IV. RESULTS AND DISCUSSION

Using the state densities with linear momentum described in Sec. II B a full evaluation of the Pauli-blocking function $f(\epsilon_\gamma)$ was obtained. The integral in the numerator of Eq. (8) for the Pauli-blocking function was evaluated numerically. Symmetry allowed this six-dimensional integral to be reduced to a five-dimensional integral, but the integration still required 2 h of CPU time on a Cray computer for each photon energy. We have also calculated the Pauli-blocking function that is obtained when linear momentum conservation is ignored, by using the state densities (19) and (20) in relation (7). The results are shown in Fig. 4, where we plot them along with Levinger's exponential function (2) using $D = 60$ and 80 MeV.

It can be seen from Fig. 4 that imposing momentum conservation has the effect of increasing the Pauli-blocking function for high photon energies. This can be understood by realizing that the contributions to quasideuteron photoabsorption are greatest for quasideuterons with small relative momenta, which correspond, on average, to quasideuterons which yield small total momenta **K**. To illustrate this point, we show in Fig. 5 the distribution of relative momenta in a Fermi gas; see Eq. (9). However, the probability of photoabsorption on a quasideuteron with relative momentum $k$ is, from Eq. (3), also proportional to $1/(\hbar^2\alpha^2 + k^2)$, and so the overall probability of photoabsorption on a quasideuteron with relative momenum $k$ is proportional to the product of Eqs. (3) and (9). This probability distribution is also shown in Fig. 5, and it is evident that photoabsorption on quasideuterons with low relative momenta is most likely. For low photon energies the total momentum **K** then remains rather small, resulting in the emitted neutron and proton tending to move apart with equal absolute momenta in opposite directions. For high-energy photons, however, the total momentum becomes larger and the effects of momentum conservation are more pronounced. In the limiting case of the total momentum $\mathbf{K} = 0$ it is clear from Eq. (14) and Fig. 2 that no Pauli-blocking occurs above an energy of $2\epsilon_F = 70$ MeV if momentum conservation is taken into account. However, since we treat the kinematics more accurately, we do still find Pauli-blocking effects persisting above 70 MeV, though the effects are smaller than if we had neglected momentum conservation. It is, therefore, clear why Prakash *et al.* [9], using the results of Reitan [21], obtain a Pauli-blocking function which is unity above $2\epsilon_F$. They make the approximation that the total momentum of the emitted neutron and proton is zero, and hence that the emitted particles always move apart in opposite directions with identical absolute momenta. We, on the other hand, do not make such an assumption. We treat the kinematics of the photoabsorption process correctly

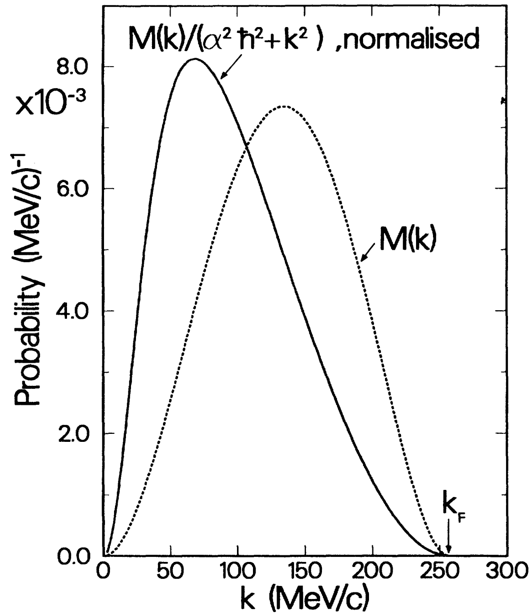


FIG. 5. The product of $M(k)$ and $1/(\alpha^2\hbar^2 + k^2)$ (normalized), shown by the full curve, represents the probability of photoabsorption on a quasideuteron with relative momentum $k$. The distribution of relative momentum in a Fermi gas, $M(k)$, is shown by the dotted curve, and it is evident that the quasideuteron mechanism leads to an enhancement of photoabsorption processes on neutron-proton pairs with low relative momentum.

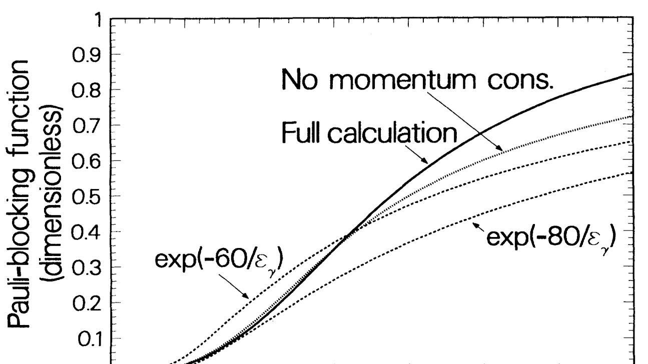


FIG. 4. The Pauli-blocking functions, $f(\epsilon_\gamma)$, calculated with the full formalism as well as with the effects of linear momentum ignored, are shown as functions of the incident photon energy. Shown for comparison are Levinger exponential functions, $e^{-D/\epsilon_\gamma}$, with $D = 60$ and 80 MeV.

within a Fermi-gas model of the nucleus, and therefore obtain more general and accurate results.

Our Pauli-blocking function (obtained with state densities that conserve linear momentum) has the same general energy dependence as that of Levinger's *ad hoc* exponential function, i.e., at low incident energies it tends to zero and at high energies to unity, as it must, but the exact energy variation is rather different (see Fig. 4). We note that it is not possible to reproduce the steep energy dependence of $f(\epsilon_\gamma)$ which we obtain with a phenomenological exponential function. Since the Pauli-blocking function calculation requires such a large amount of CPU time, we have found a polynomial fit to our results to facilitate future uses of our Pauli-blocking function in nuclear reaction calculations and data evaluations. Our results can be well approximated in the photon energy range 20–140 MeV by the polynomial

$$f(\epsilon_\gamma) = 8.3714 \times 10^{-2} - 9.8343 \times 10^{-3}\epsilon_\gamma + 4.1222 \times 10^{-4}\epsilon_\gamma^2 - 3.4762 \times 10^{-6}\epsilon_\gamma^3 + 9.3537 \times 10^{-9}\epsilon_\gamma^4. \quad (25)$$

Our approach for calculating the Pauli-blocking function does not allow us to treat the Levinger number $L$ as a free parameter. Rather, we obtain a value of $L = 6.5$ directly from our model, and it is consistent with the range of values from $L = 4.9$ through 10.0 which have been used in previous works [9–10,12–15]. With the Pauli-blocking function and Levinger parameter that we have determined it is possible to evaluate nuclear photoabsorption cross sections using Eq. (1). The free deuteron photodisintegration cross section was taken as [16]

$$\sigma_d(\epsilon_\gamma) = 61.2\frac{(\epsilon_\gamma - 2.224)^{3/2}}{\epsilon_\gamma^3} \text{ mb}, \quad (26)$$

with $\epsilon_\gamma$ measured in MeV. This parametrization fits the experimental data well below 100 MeV. However, there is a large scatter in the experimental free deuteron photodisintegration cross sections in the range 100 MeV $< \epsilon_\gamma <$ 140 MeV (Ref. [22]) and the parametrization seems to fit the lower values rather than the average in this energy range. This may lead to a certain underprediction of the corresponding nuclear photoabsorption cross section at the highest energies. In Fig. 6 we show the quasideuteron contribution to the nuclear photoabsorption cross section compared with data for the nuclei Pb, Ta, Sn, and Ce. We also show the tails of the giant dipole resonances (GDR) which may contribute even at these high photon energies. The data as well as the GDR tails are taken from Ref. [4]. It is seen that the sum of these two contributions describes the data fairly well. The comparison with data that we obtain seems to be better than that obtained with a phenomenological exponential Pauli-blocking function (see, for instance, Leprêtre *et al.* [4]). If their quasideuteron component is added to the GDR component they significantly overestimate the data below a photon energy of 40 MeV).

We now discuss the temperature dependence of the

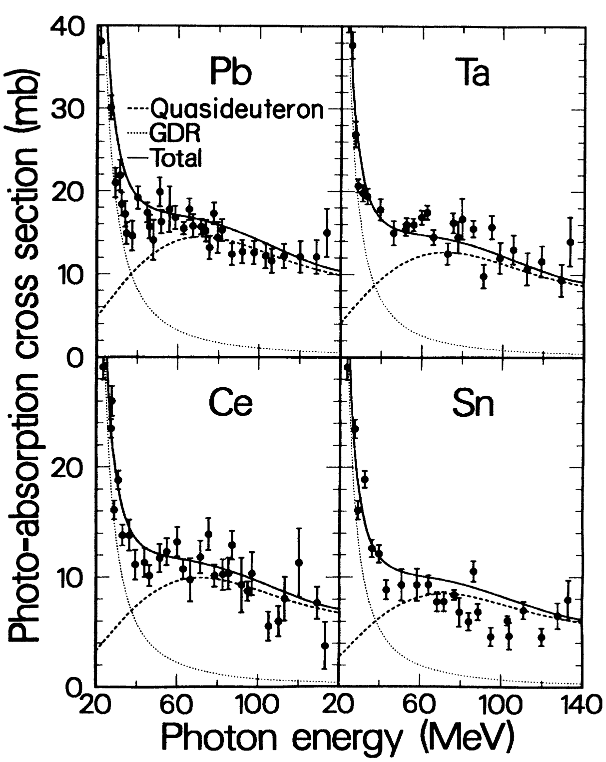


FIG. 6. The calculated quasideuteron component of the nuclear photoabsorption cross section as a function of photon energy is compared with experimental data for Pb, Ta, Sn, and Ce. The full curve is the sum of the quasideuteron and GDR contributions. The tails of the GDR as well as experimental data are taken from Ref. [4].

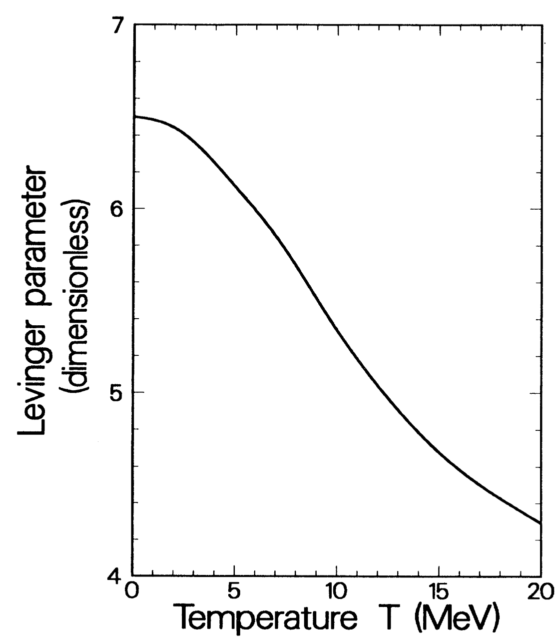


FIG. 7. Variation of the Levinger parameter with nuclear temperature.

Levinger parameter and Pauli-blocking function that is obtained in our model, using the results of Sec. III. Since the photoabsorption cross section on a quasideuteron is proportional to $1/(\hbar^2\alpha^2 + k^2)$ and with increasing temperatures the average relative momentum increases, the Levinger parameter should decrease for increasing temperatures. This is shown in Fig. 7 and it agrees with the results of Prakash *et al.* [9]. Our completely new result, however, is the temperature dependence of the Pauli-blocking function as shown for $T$ = 0, 10, and 20 MeV in Fig. 8. It is evident that, except for the highest incident photon energies, Pauli-blocking effects decrease with increasing temperature, in agreement with the suggestion by Herrmann *et al.* [8], and this work provides a quantitative estimate of the effect. The Pauli-blocking function increases significantly with increasing temperature for low incident photon energies due to a depletion of the number of occupied states below the Fermi energy for high nuclear temperatures. From the figure it can be seen that an inversion of this effect occurs for very high incident energies, where the Pauli-blocking function actually decreases with increasing temperature. In this case the effects of Pauli-blocking due to excited nucleons in a hot nucleus lead to a decrease in the Pauli-blocking function. The overall temperature dependence of the nuclear photoabsorption cross section in our model is given by the combined effects of the temperature dependences of $L$ and $f(\epsilon_\gamma)$. In Fig. 9 we show the quasideuteron photoabsorption cross section on $^{208}$Pb as a function of the incident energy for temperatures $T$ = 0, 10, and 20 MeV. For incident photon energies below about 65 MeV the decreasing effects of Pauli-blocking with increasing temperature lead to a dramatic increase in the cross section, while for energies above this value the cross section decreases with increasing temperature. These results are quite different to those of Prakash *et al.*, who find that the nuclear photoabsorption cross section decreases with increasing temperatures for all photon energies. As mentioned earlier, this discrepancy results from their neglect of temperature effects on the Pauli-blocking function.

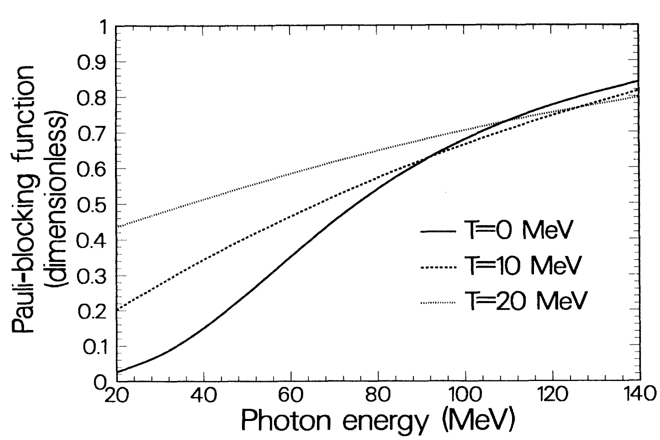


FIG. 8. The Pauli-blocking function for three different nuclear temperatures. Except for the highest energies, an increasing nuclear temperature leads to a reduction in the effects of Pauli-blocking.

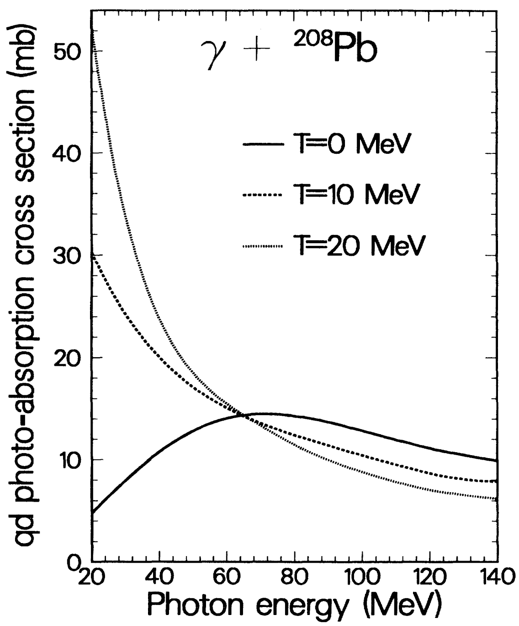


FIG. 9. The quasideuteron photoabsorption cross section for $^{208}$Pb as a function of incident photon energy for three different nuclear temperatures. The decreased effects of Pauli-blocking at high temperatures result in a dramatic increase in the cross section for low incident photon energies.

It is of interest to compare our results with those obtained by Knoll and Guet [11] using dipole sum-rule considerations. Knoll and Guet suggest that the nuclear photoabsorption should increase with increasing temperature, and this is (in contrast to the results of Prakash *et al.*) now supported by our calculations for photon energies below 65 MeV. The discrepancy between the quasideuteron model and the sum-rule prediction persists at higher energies. This suggests that Levinger's Eq. (3), which describes the extent to which a neutron-proton pair is correlated in a deuteron-like structure, may need modifying for high temperatures where other correlations become important. This presumably may show up in a reduction of the decrease of $L$ with increasing temperature.

In an alternative explanation of the quasideuteron nuclear photoabsorption cross sections, Leprêtre *et al.* [4] and Bergère [5], following the ideas of Laget [23], proposed that a quasideuteron within a nucleus can only undergo photoabsorption via exchange processes, and so only a fraction of the free deuteron photodisintegration cross section should be used. It seems to us that even if this is so, Pauli-blocking effects would still damp the photoabsorption cross sections, and then their explanation would not be consistent with the data. In our model we provide a theoretical basis for Pauli-blocking effects which is supported by the observed photo-absorption cross sections, and therefore suggests the importance of nonexchange terms in the absorption of a photon by a quasideuteron.

In this work we have developed expressions for state densities in which we only consider states that can be reached by both energy and momentum conservation. This approach could be easily extended for use in preequilibrium reaction calculations at energies where the semiclassical Fermi-gas model of the nucleus is valid. In the preequilibrium model the neutron or proton emission spectrum for nucleon-induced reactions is dominated by the emission from the first preequilibrium stage and the theoretical expression for this first-stage emission rate is given by the ratio of a one-particle–one-hole to two-particle–one-hole state density [24]. For these simple configurations one could evaluate these state densities using our approach and obtain nucleon emission spectra within this more accurate application of detailed balance.

To summarize, we have presented a model for quasideuteron photoabsorption which provides a theoretical basis for the effects of Pauli-blocking. We used a Fermi-gas model of the nucleus and in the phase-space expressions we included only states that can be reached by momentum and energy conservation. The Pauli-blocking function and Levinger parameter were evaluated in a consistent way, and when used to calculate nuclear photoabsorption cross sections, good agreement with the data was obtained for a range of medium-heavy target nuclei. We have also shown that in order to correctly determine the temperature dependence of the quasideuteron photoabsorption cross section, one must include the variation of Pauli-blocking effects with temperature. Our results for this temperature dependence were in agreement with the qualitative suggestion of Herrmann *et al.* and differ significantly from the results of Prakash *et al.* where the temperature variation of the Pauli-blocking function was neglected.

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* Permanent address: Institute of Physics, Slovak Academy of Sciences, 842 28 Bratislava, Czechoslovakia.

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