Weak proton capture reactions on ^{1}H and ^{3}He and tritium β decay

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The cross sections for the weak proton capture reactions $p+p \to d+e^+ + \nu_e$ and $p+{}^3\text{He} \to {}^4\text{He}+e^+ + \nu_e$ have been calculated with realistic wave functions determined by using the Argonne v_{14} nucleon-nucleon interaction. To minimize the uncertainty in the axial exchange current operator, its matrix element has been adjusted so as to reproduce the measured Gamow-Teller matrix element for β decay of tritium. The exchange current contribution enhances the calculated rate of the *pp* capture reaction by 1.5% and decreases that of the $p\,{}^3$He reaction by almost a factor of 5.

I. INTRODUCTION

The continuing unsettled state of the solar-neutrino problem [1] has stimulated repeated attempts to improve the reliability of the calculated cross sections for those solar-neutrino reactions that cannot be measured in the laboratory [2–11]. We here contribute to this effort by a simultaneous calculation of the weak proton capture reactions $p+p \to d+e^+ + \nu_e$ and $p+{}^3\text{He} \to {}^4\text{He}+e^+ + \nu_e$ using realistic wave functions that correspond to the Argonne v_{14} model for the nucleon-nucleon interaction [12], complemented by the Urbana VII model for the three-nucleon interaction [13]. The wave functions of the *pp* continuum state and the deuteron are obtained by solving the Schrödinger equation, while variational wave functions are used for the $p\,{}^3$He and ^{4}He states. Both the Coulomb interaction as well as the (strong) initial-state interactions are taken into account. Since the wave functions used give very satisfactory binding energies [14], and realistic electromagnetic form factors for all the nuclei involved [15–18], the calculated impulse approximation cross-section values should be reliable.

The main source of uncertainty in the cross sections for the weak proton capture reactions on the bound few-nucleon systems is the contribution from the axial exchange current operator. Although the presence of such a term in the Gamow-Teller operator is well known from the analysis of β decay of tritium [19–23], its form depends completely on the dynamical model used to construct it, and it is not even partly constrained by the nucleon-nucleon force as is the corresponding electromagnetic exchange current operator [24]. To address this uncertainty we adopt the phenomenological approach of adjusting the—in any case poorly known—cutoff masses in the meson-nucleon vertices in the conventional model for the exchange-current operator [21] so as to obtain agreement with the experimental value for the Gamow-Teller matrix element in tritium β decay. The required adjustment is in fact very slight, as is obvious from the recent demonstration [23] that good agreement with the empirical matrix element can be obtained with three-body wave functions that correspond to realistic nucleon-nucleon potentials, and the conventional model for the meson-exchange-current operator [21].

With the strength of the exchange-current operator determined in this way we find that the predicted cross section for the *pp* capture reaction is enhanced by only 1.5% by the exchange-current contribution. This is considerably less than the enhancement found in earlier studies [4,6]. The present smaller value is due to the strong canceling effect on the main pion axial exchange-current mechanism—the Δ_{33} excitation current—that is caused by ρ-meson exchange, which was not considered in Refs. [4,6]. Another reason for the diminished exchange current effect is the introduction of form factors to regulate the high-momentum behavior of the hadronic vertices in the axial exchange-current operators, which were also not considered in Refs. [4,6].

In the case of the $p\,{}^3$He capture reaction we find that the net effect of the exchange current operator is to reduce the cross section predicted in the impulse approximation by almost a factor of 5. The reduction is caused by the destructive interference between the matrix elements of the one- and two-body currents in the reaction $p+{}^3\text{He} \to {}^4\text{He}+e^+ + \nu_e$. This sign difference is due to the correlations in the initial scattering state wave function, and does not appear when schematic wave functions are used as in Refs. [7,11]. The same sign difference is present in the case of the electromagnetic capture reaction $n+{}^3\text{He} \to {}^4\text{He}+\gamma$ [25].

This paper is divided into five sections. In Sec. II we describe the exchange-current operator and the determination of its strength by the empirical Gamow-Teller matrix element in β-decay of ^{3}H. In Secs. III and IV we discuss the calculation of the pp and $p\,^3$He weak capture reactions, while in Sec. V we summarize our conclusions.

II. THE AXIAL EXCHANGE CURRENT AND TRITIUM β DECAY

For the axial exchange current operator we use a slightly expanded version of the "conventional" pion and ρ-meson exchange model first described in a systematic way by Chemtob and Rho [21]. These are the two-body currents associated with the axial contact πNN and ρNN interactions [Fig. 1(a)], with excitation of virtual intermediate Δ_{33} resonances [Figs. 1(b) and 1(c)], and the $\pi\rho$ exchange-current mechanism [Fig. 1(d)]. The differences between the set of exchange-current operators considered here and the original one [21] are (a) the inclusion of the important ρ-meson exchange contribution to the Δ_{33} excitation mechanism, (b) the inclusion of high-momentum cutoff factors at the hadronic vertices in the exchange current operators, and (c) the complete retention of the nonlocal momentum-dependent terms in the exchange current operators.

It was pointed out in Ref. [22] that contributions of similar magnitude to those associated with the ρ-meson exchange currents can arise from exchange of A_1 mesons. In principle, the exchange-current operator associated with the A_1 meson is important in effective chiral Lagrangian models, as the A_1 meson is the chiral partner of the ρ meson. We here do not consider the additional exchange-current operators that arise once the A_1-meson field is systematically taken into account, because of our phenomenological approach to the exchange-current operator: to use the simplest possible operator that gives a proper description of the longest-range mechanism and which can be adjusted so as to reproduce the empirical ^{3}H β-decay Gamow-Teller matrix element. Thus we also do not consider the renormalization of the exchange-current correction that in effect is caused when one takes into account the explicit Δ_{33} configurations in the wave functions [26].

As the derivation of the axial exchange-current operators can be found in the literature, e.g., in the review of the Towner [27], we shall here be content to list the relevant exchange-current expressions in the Appendix, where the numerical values for the various coupling constants are also given.

The matrix element of the axial current operator for the Gamow-Teller part of the transition $^3\mathrm{H}\rightarrow{}^3\mathrm{He}+e^-+\bar{\nu}_e$ has been evaluated with two different wave functions in the present work. The first one is a 34-channel Faddeev wave function of the Los Alamos–Iowa group [28] obtained for the Argonne v_{14} nucleon-nucleon interaction [12] and a version of the Urbana VII three-nucleon interaction [13], in which the strengths of the repulsive and attractive parts have been modified so as to yield the experimental value for the binding energy of ^{3}H. The second one is a variational wave function for the Argonne v_{14}+Urbana VII interaction analogous to the ones we use for the $p\,^3$He and ^{4}He wave functions below.

In order to obtain the empirical value 0.961 ± 0.003 [29] for the Gamow-Teller matrix element with the 34-channel Faddeev wave functions we have used monopole forms for the πN and ρN vertex form factors $f_\pi(k)$ and $f_\rho(k)$ in the exchange-current expressions in the Appendix with the mass scales $\Lambda_\pi=0.9$ GeV/c^2 and $\Lambda_\rho=1.35$ GeV/c^2. Furthermore, we have assumed that the $N\Delta$ transition form factors are the same as the NN vertex factors, i.e., $f_{\pi\Delta}(k)=f_\pi(k)$, $f_{\rho\Delta}(k)=f_\rho(k)$. The values for the cutoff masses Λ_π and Λ_ρ fall within the conventional range of values, but are somewhat larger than those used in the recent similar calculation in Ref. [23] ($\Lambda_\pi=0.8$ GeV/c^2, $\Lambda_\rho=1.0$ GeV/c^2).

In Table I we quote the contributions to the Gamow-Teller matrix elements obtained from the individual components of the axial current operator listed in the Appendix for both the Faddeev and variational calculations. The two calculations are in good agreement. For comparsion, we also list the corresponding values obtained in Ref. [23] with a similar wave function and the same model for the two-nucleon interaction, but a different three-nucleon interaction. Note that in Ref. [23] the nonlocal parts of the exchange-current operators were dropped as was the ρ-meson exchange axial seagull current. The matrix element of the single-nucleon current is in agreement with that obtained in Ref. [23], and the total predicted value 0.961 reproduces the empirical datum by construction.

The present results for the Δ excitation exchange-current components are in good agreement with those obtained in Ref. [23], if one takes into account the different

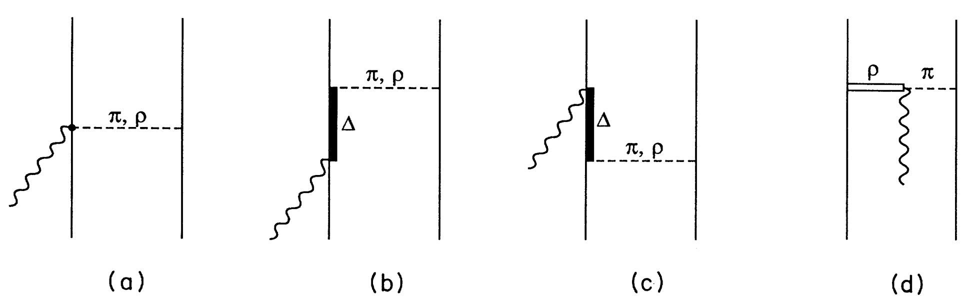

FIG. 1. Axial exchange-current mechanisms: (a) π- and ρ-meson exchange seagull or pair currents, (b), (c) π- and ρ-meson exchange Δ_{33} excitation currents, and (d) $\pi\rho$ exchange current.

TABLE I. Contributions to the Gamow-Teller matrix element of the tritium β decay from the individual components of the z projection of the axial current operator. The results quoted below have been divided by g_A.

	Faddeev	Variational	Ref. [23]
IA	0.923	0.926	0.923
$\Delta\pi$	0.054	0.053	0.044
$\Delta\rho$	−0.022	−0.022	−0.009
$\pi\rho$	0.0069	0.0067	0.001[a]
πS	0.0038	0.0034	0.003[a]
ρS	−0.0043	−0.0042	
Total	0.961	0.963	0.962

[a]Obtained from the local part of the operator.

cutoff masses used in the hadronic form factors. We find, however, that in general the nonlocal parts of the exchange-current operators give contributions of the same order of magnitude as the local parts, and that leaving them out as was done in Ref. [23] is a poor approximation.

III. THE REACTION $p+p\to d+e^++\nu_e$

The theoretical description of the solar-burning reaction $p+p\to d+e^++\nu_e$ was first given by Bethe and Critchfield [30]. The calculation of the cross section requires the matrix element of the axial current operator between the pp continuum state and the deuteron, where in the continuum state both the nuclear and the Coulomb interactions are included. Since the main uncertainty in the prediction is associated with the magnitude of the exchange-current contribution, a variety of methods have been tried in the literature to tie it to some related observable reaction rate. The most obvious is to test the model for the axial current operator on muon capture in deuterium, but the observed total capture rates have large experimental uncertainties and therefore do not provide any detailed constraints on the exchange-current operator [6,31]. Another one is the possibility of linking the axial exchange-current operator to the effective operator that describes P-wave pion production in pp collisions [32], but this possibility would require high-quality experimental data on pion production near threshold, of hitherto unattainable precision [6]. We therefore find that determining the strength of the exchange-current operator by means of the empirically well-known Gamow-Teller matrix element in tritium β decay, as done above, should be the most reliable method at the present time.

For the calculation of the matrix element of the axial current operator we have slightly modified the strength of the central $S=0$, $T=1$ component of the Argonne v_{14} potential (which was fit to np data) so as to obtain agreement with the experimental pp scattering length -7.823 ± 0.01 fm when the Coulomb potential is taken into account. The resulting value for the effective range 2.771 fm is also in acceptable agreement with the empirical value 2.794 ± 0.015 fm.

The spin-averaged total cross section for the reaction $p+p\to d+e^++\nu_e$ can be written in the form

$$\sigma=\frac{1}{(2\pi)^3}\frac{G_V^2 m_e^5 f_{pp}}{v_{pp}}\sum_{m_1m_2m}|\langle d,m|\mathbf{A}_-|pp,m_1m_2\rangle|^2\,, \tag{3.1}$$

where G_V is the vector coupling constant ($G_V=1.151\times10^{-5}$ GeV^{-2} [33]), m_e the electron mass, and v_{pp} the relative pp velocity. The (dimensionless) integrated Fermi function f_{pp} for the proton weak capture on p is parametrized as $f_{pp}=0.142[1+9.04E$ (MeV)] [9,34], E being the pp relative energy. The deuteron and pp wave functions are written as

$$\psi_m(\mathbf{r})=\left[\frac{u(r)}{r}Y_{011}^m+\frac{w(r)}{r}Y_{211}^m\right]\eta_0^0\,, \tag{3.2}$$

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r})=4\pi\frac{1}{\sqrt{2}}\sum_{lm_l}i^l[1+(-1)][Y_{lm_l}(\hat{k})]^*\frac{1}{r}\times u_l^{(+)}(r;k)Y_{lm_l}(\hat{r})\chi_0^0\eta_1^1\,, \tag{3.3}$$

where $\mathbf{k}$ is the relative pp momentum, and $\chi_{m_s}^s$ and $\eta_{m_t}^t$ are two-nucleon spin and isospin state vectors, respectively. Because of the axial vector character of the transition operator, only even l waves in the initial pp scattering state contribute. The radial functions $u_l^{(+)}$ with outgoing-wave boundary conditions behave asymptotically as

$$u_l^{(+)}(r;k)|_{r\to\infty}\sim\frac{1}{2k}[h_l^{(2)}(kr)+e^{2i\delta_l(k)}h_l^{(1)}(kr)]\,. \tag{3.4}$$

Here the functions $h_l^{(1,2)}(kr)$ are defined as $e^{\mp i\sigma_l(k)}[F_l(kr)\mp iG_l(kr)]$, σ_l being the Coulomb phase shift, and F_l and G_l the regular and irregular Coulomb functions, respectively. If only the Coulomb interaction is included, then $\delta_l\to\sigma_l$ and $u_l^{(+)}\to k^{-1}e^{i\sigma_l}F_l$.

As the calculation of the matrix element of the axial current $\mathbf{A}_-$ is carried out in configuration space the expressions of the different contributions to $\mathbf{A}_-$ listed in the Appendix are Fourier transformed to r space. The dependence upon the momentum transfer $\mathbf{q}$ in the resulting operator is neglected, which is an approximation that is well justified in the energy range of relevance for the solar-burning reactions under consideration.

It is conventional to express the cross section in terms of the astrophysical S factor,

$$S(E)=E\sigma(E)e^{2\pi\eta}\,, \tag{3.5}$$

and its derivative at zero energy [1]. In Eq. (3.5) $\eta=\alpha/v_{pp}$, α being the fine-structure constant.

We obtain the values $S(E=0)=4.00\times10^{-25}$ MeV b and $dS(E)/dE|_{E=0}=4.67\times10^{-24}$ b, which are close to the values $S(E=0)=4.07\,(1\pm0.051)\times10^{-25}$ MeV b and $dS(E)/dE|_{E=0}=4.52\times10^{-24}$ b quoted in Ref. [1]. The above results include the exchange-current contribution, which in the case of $S(E=0)$ amounts to enhancement of only 1.5%.

The fact that the present result for the exchange-current contribution is considerably smaller than the original estimates [4,6] is mainly due to the important (negative) contribution from the ρ-meson exchange Δ_{33} excitation mechanism, which was not taken into account in Refs. [4,6]. In Table II we show the cumulative contributions to the cross section from the components of the axial current operators in the Appendix at 1.0-, 2.5-, and 5.0-keV kinetic energy in the initial pp state (center-of-mass system). The results in the table show that the exchange-current contribution represents at most a 1.5% enhancement of the cross section calculated in the impulse approximation over the whole low-energy range considered.

The value we obtain for $S(E=0)$ in the impulse approximation is about 5% smaller than that obtained in Ref. [9]. However, in the present calculation the same potential model is used to determine both the initial scattering state as well as the deuteron wave function. We find that the contributions associated with waves $l \geq 2$ in the initial scattering state are too small to be visible on the numbers in Table II in the present energy range (few keV). Finally, we have also investigated the numerical significance of the hadronic vertex form factors in the exchange-current operators on the calculated cross sections. If one replaces all those form factors by unity the net exchange current contribution would be roughly doubled, but then of course the agreement with the Gamow-Teller matrix element of tritium β decay would not be maintained.

IV. THE REACTION $p+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+e^++\nu_e$

The single-nucleon term in the axial current operator cannot connect the main spatially symmetric S-state components of the initial and final bound states in the reaction $p+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+e^++\nu_e$. The cross section therefore is due to transitions between the small mixed symmetry S- and D-state components, which are mediated by the one-body current, and transitions mediated by the exchange-current operator. In this respect this reaction is similar to the corresponding electromagnetic capture reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$, although the axial and electromagnetic exchange-current operators are quite dissimilar [25,35].

In the calculation of the matrix element of the axial current operator in the Appendix for the weak $p+{}^3\mathrm{He}$ capture reaction we use ${}^3\mathrm{He}$ and ${}^4\mathrm{He}$ wave functions that have been constructed by the variational method using the Argonne v_{14} potential [12] with the Urbana VII three-nucleon interaction [13]. Because of the relatively strong tensor component in the v_{14} interaction and the two-pion-exchange part of the three-nucleon potential, the ${}^3\mathrm{He}$ and ${}^4\mathrm{He}$ ground states have large D-state probabilities -9.2% and 17.5%, respectively.

In the asymptotic region, the initial spin-triplet $p\,{}^3\mathrm{He}$ scattering state has the form

$$\psi_{1m}(p+{}^3\mathrm{He})=\frac{1}{\sqrt{4}}[\,|\phi_p(1)\psi_{{}^3\mathrm{He}}(234)\rangle_{1m} - |\phi_p(2)\psi_{{}^3\mathrm{He}}(341)\rangle_{1m} + |\phi_p(3)\psi_{{}^3\mathrm{He}}(412)\rangle_{1m} - |\phi_p(4)\psi_{{}^3\mathrm{He}}(123)\rangle_{1m}\,]\ . \quad (4.1)$$

Here $\phi_p(i)$ represents the asymptotic scattering state of the proton, and is a function of the distance between nucleon i and the center of mass of the remaining cluster of nucleons. The ground-state ${}^3\mathrm{He}$ wave function of nucleons i, j, and k is represented by $\psi_{{}^3\mathrm{He}}(ijk)$.

The scattering length is obtained microscopically by a variational method. A variational calculation is performed for a fixed boundary condition, in this case a specific logarithmic derivative for ϕ at a radius R. This radius must be large enough so that the proton and ${}^3\mathrm{He}$ do not interact except through the Coulomb interaction; we have used $R=15$ fm. By varying the logarithmic derivative until the total energy within the volume is equal to the ground-state energy of ${}^3\mathrm{He}$, the scattering length can be determined. The effective range can be calculated from the derivative of the total energy with respect to the change in logarithmic derivative.

The full variational wave function for the $p\,{}^3\mathrm{He}$ system has the general form

$$\psi_{1m}(1234)=S\left[\prod_{i<j}F_{ij}\right]\Phi\ , \quad (4.2)$$

where S represents the symmetrization operator, F_{ij} are two-body correlation operators, and Φ is a Slater determinant:

$$\Phi=A\left[\left[\frac{\phi(1)}{\prod_{j\neq 1}f^c(\tilde r_{1j})}\uparrow_1 p_1\right](\uparrow_2 p_2)(\downarrow_3 p_3)(\uparrow_4 n_4)\right]\ . \quad (4.3)$$

In the expression for Φ, A is the antisymmetrization operator and the long-range spatial dependence is incorporated through the function $\phi(1)$, which is an S-wave scattering state in the relative coordinate $\mathbf{r}_1-\frac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)$. The inverse product of central correlations in Φ cancels, at large distances, the central correlations in the symmetrized product that act between the scattered and bound-state nucleons. Hence, the correct asymptotic scattering wave function [Eq. (4.1)] is recovered.

For this cancellation to occur, it is necessary that $\tilde r(r)\rightarrow r$ at large distances. However, we must preserve

TABLE II. Cumulative total cross-section values for the reaction $p+p\rightarrow d+e^++\nu_e$.

Current operator	$E=1.0$ keV σ 10^{-30} fm^2	2.5 keV σ 10^{-26} fm^2	5.0 keV σ 10^{-25} fm^2
IA	9.054	1.291	4.061
$+\pi S$	9.068	1.293	4.067
$+\rho S$	9.055	1.291	4.061
$+\Delta\pi$	9.220	1.315	4.135
$+\Delta\rho$	9.157	1.306	4.107
$+\pi\rho$	9.188	1.310	4.121
Total	9.188	1.310	4.121

the short-distance properties of the central correlations in the numerator of Eq. (4.2). We choose $\tilde{r}=[r^2+r_0^2\exp(-r^2/r_0^2)]^{1/2}$, where r_0 is a variational parameter with a typical value of 1.5–2 fm. The function $\tilde{r}$, and hence the correlations in the denominator, go to a constant for distances much smaller than r_0. This expression for the wave function is equivalent to introducing different central correlations for the bound and scattered nucleons; the former go to a constant asymptotically while the latter decay as appropriate for the bound state of three nucleons.

The full pair correlation operators F_{ij} depend upon the spin and isospin of the pair ij, and are obtained as solutions of two-body Schrödinger-like equations with parametrized potential terms:

$$(-\hbar^2\nabla^2/m+v_{ij}+\lambda_{ij})F_{ij}=0 \ . \tag{4.4}$$

The function v_{ij} is the two-body potential in a given spin-isospin channel, and λ_{ij} is a parametrized function used to fix the asymptotic properties of F and minimize the Hamiltonian's expectation value. At large distances, the noncentral parts of F are cut off; this cutoff does not affect the ground-state energy of the three-body system significantly.

The proton scattering state ϕ is obtained from scattering solutions in a Woods-Saxon plus Coulomb potential well with a given logarithmic derivative. The function ϕ is a linear combination of several radial excitations within the well. The coefficients of the various states are determined by minimizing the expectation value of the full Hamiltonian. The procedure is very similar to that used previously for the study of the radiative neutron capture reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ [25].

Using this variational wave function, we obtain a scattering length of 10.1±0.5 fm. This result agrees well with the value 10.2±1.4 fm extracted from the $p\,{}^3$He phase shifts [36,37] in Ref. [8]. The Monte Carlo subtraction techniques discussed in Ref. [38] are required in order to obtain reasonable statistical errors. The effective range is difficult to determine accurately with this method, but is roughly consistent with the value of 1.3 fm reported in Ref. [8].

The expression for the spin-averaged total cross section is obtained from that given in Eq. (3.1) by replacing v_{pp} and f_{pp} with the corresponding $p\,{}^3$He relative velocity and integrated Fermi function, for which we use the value $f_{p\,{}^3\mathrm{He}}=2.544\times10^6$ at zero energy. The numerical evaluation of the matrix element of the axial current operator given in the Appendix was again carried out using Monte Carlo integration techniques.

In the impulse approximation we obtain for $S(E=0)$ the value 5.8×10^{-23} MeV b, which is about 20% smaller than the value 8×10^{-23} MeV b quoted in Ref. [1]. In this case the effect of the exchange-current operator is, however, to reduce the prediction value by almost a factor of 5 to 1.3×10^{-23} MeV b. This strong reduction is due to the fact that the exchange-current matrix element in this reaction has a sign opposite to that of the impulse approximation. This result is similar to that recently found in the case of the corresponding electromagnetic capture reaction $n+{}^3\mathrm{He}\rightarrow{}^3\mathrm{He}+\gamma$ [25]. Therefore the expected solar-neutrino flux from the $p\,{}^3$He reaction should be about five times smaller than the value 7.6×10^{-3} cm^{-2} s^{-1} given in Ref. [1].

TABLE III. Contributions to the matrix element $\langle{}^4\mathrm{He}|A_{-,x}|p\,{}^3\mathrm{He},\,1m=1\rangle$ in fm$^{3/2}$ from the individual components of the x projection of the axial current operator. The matrix elements have been multiplied by $[(e^{2\pi\eta}-1)/2\pi\eta]^{1/2}$, with $\eta=2\alpha/v_{p\,{}^3\mathrm{He}}$. The statistical errors associated with the Monte Carlo integration are given in parentheses.

IA	0.37(1)
πS	0.001(5)
ρS	0.035(5)
$\Delta\pi$	−0.40(1)
$\Delta\rho$	0.18(1)
$\pi\rho$	−0.023(5)
Total	0.17(1)

In Table III we show the cumulative contribution to the matrix element of the axial current from its individual components that are listed in the Appendix. The present conclusion that the exchange-current contribution reduces the predicted cross section is at variance with the earlier findings of Refs. [8,11] that it should increase the cross section. The difference can be traced to the use of schematic wave functions in Refs. [8,11] which lead to an overprediction of the one-body contribution, and to the neglect of the interaction effects in the initial $p\,{}^3$He scattering state, which modify the relative wave function strongly at intermediate range.

We find that the cross section is not as sensitive to the scattering length as that obtained for the corresponding radiative capture reaction [25]. Changing the scattering length from 11 to 9 fm results in a 50% increase in the impulse-approximation cross section, with a slightly smaller increase ($\approx$35%) in the full cross section. We find that the impulse and exhange contributions are of comparable magnitude, and a large cancellation exists between the two. In the radiative-capture case, the cross section was in contrast largely dominated by the exchange terms.

The fact that there is a large cancellation between the matrix elements of the one- and two-body parts of the axial current operator implies that the predicted value of the cross section for the $p\,{}^3$He capture reaction is exceptionally sensitive to the model for the axial exchange-current operator. While the overall strength of the exchange-current contribution in the three-nucleon system is known from the empirical Gamow-Teller matrix element in tritium β decay, there may still be an appreciable uncertainty as to the magnitude of the contributions from the various components of the exchange-current operator. This uncertainty would be magnified in the case of the $p\,{}^3$He capture reaction because of the cancellation between the impulse and exchange-current contributions, as the relative size of the different exchange-current contributions varies among different reactions. Thus a 20% uncertainty in the Δ_{33} exchange-current contribution (with a compensating uncertainty in the other exchange-current contributions that maintains the predicted value of the Gamow-Teller matrix element for tritium β decay) implies an uncertainty of a factor of 2 in the calculated cross section for the $p\,^3$He capture reaction. An uncertainty of that size in the strength of the contribution due to the Δ_{33} exchange-current mechanism is by no means unrealistic, given the schematic model that we use to describe the corresponding exchange-current operator, we have to associate a sizable uncertainty margin with our predicted value for the cross section of the $p\,^3$He capture reaction.

V. DISCUSSION

The present calculation of the solar-neutrino reactions and tritium β decay with realistic wave functions obtained with the same nuclear interaction model in all reactions shows that (a) the exchange current contribution to the pp reaction is smaller than previously thought [4, 6] and (b) the exchange-current contribution to the weak $p\,^3$He capture cross section reduces the predicted value by almost a factor of 5. While the former result corresponds to expectations from studies of the Gamow-Teller matrix element of tritium β decay [23], the latter was less expected. It shows that the study of a reaction as delicate as $p+{}^3\text{He}\to{}^4\text{He}+e^++\nu_e$, in which the pseudo-orthogonality between the main components of the bound-state wave functions prevents the one-body operator from dominating the transition rate, puts exceptional demands on the quality of the wave-function model. In particular, it shows that the interaction effects in the initial $p\,^3$He scattering state have to be treated in full.

The present result that there is a destructive interference between the matrix elements of the one- and two-body current operators in the $p\,^3$He capture reaction corresponds to our previous finding of a similar interference in the corresponding radiative capture reaction $n+{}^3\text{He}\to{}^4\text{He}+\gamma$ [25]. In that reaction the exchange-current operator does, however, completely dominate the capture rate because of the important model-independent exchange-current operator that is associated with the nucleon-nucleon interaction, but which does not contribute an axial current operator. Because of the common attempt to estimate the relative magnitude of the axial exchange-current correction from the corresponding electromagnetic reaction [2,8,11], we wish to reemphasize that "it should be clear to everyone at this point that the exchange currents must be basically different between β-decay and the magnetic moments" [21]. Because of this, and the importance of the (different) initial-state interactions in the weak $p\,^3$He and radiative $n\,^3$He capture reactions, attempts to infer the relative importance of the exchange-current contribution in one from the other are bound to be misleading.

The model for the exchange-current operator here was constrained by fitting the matrix element $\langle {}^3\text{He}|\,\mathbf{A}_+|{}^3\text{H}\rangle$ to the empirical Gamow-Teller matrix element of tritium β decay. This obviously does not altogether eliminate the uncertainty in the exchange-current contribution. Although its contribution to the pp reaction should be fairly well determined in this way because of the similarity between the radial matrix elements of the ${}^1S_0\to{}^3S_1$-${}^3D_1$ transition in the two-nucleon system to those in tritium β decay, the predicted exchange-current contribution to the $p\,^3$He capture reaction should still have a considerable theoretical uncertainty. This is because the radial matrix elements in this reaction are quite different from those appearing in the two other reactions considered, and because of the sensitivity to the initial scattering state wave function. This sensitivity is obvious from the relatively large ρ-meson exchange contributions in Table III. The less-well-known short-range part of the wave function has a large influence, and thus a considerable uncertainty should be expected.

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APPENDIX

The axial exchange-current operators are given here. The single-particle current is

$$\mathbf{A}_\pm=-g_A\boldsymbol{\sigma}\tau_\pm\ ,\quad \tau_\pm=\tfrac{1}{2}(\tau_x\pm i\tau_y)\ . \tag{A1}$$

We use $g_A=1.262$.

The pion exchange seagull (pair) current is

$$\mathbf{A}_\pm(\pi S)=\frac{g_A}{m}\frac{f_{\pi NN}^2}{m_\pi^2}\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2+k_2^2}f_\pi^2(k_2)\times\{(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\boldsymbol{\sigma}_1\times\mathbf{k}_2-\tau_{2\pm}[\mathbf{q}+i\boldsymbol{\sigma}_1\times(\mathbf{p}_1+\mathbf{p}_1')]\}+(1\rightleftarrows 2)\ . \tag{A2}$$

The notation is m_π = pion mass and m = nucleon mass; $\mathbf{q}$ total momentum transfer $=\mathbf{k}_1+\mathbf{k}_2$; $\mathbf{k}_{1(2)}$ momentum transfer to nucleon 1(2); $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_1'$, $\mathbf{p}_2'$ initial and final nucleon momenta; $f_{\pi NN}$ = pseudovector πNN coupling constant ($f_{\pi NN}^2/4\pi=0.79$); $f_\pi(k)$ = pion-nucleon monopole vertex form factor. This expression represents the conventional pair current operator given in the literature [20,25]. This is obtained with pseudoscalar pion-nucleon coupling. With pseudovector coupling the pion momentum $\mathbf{k}_2$ in the first term in brackets would be replaced by the external momentum $\mathbf{q}$ and an additional term $(\mathbf{p}_1+\mathbf{p}_1')$ would appear with the isospin structure $(\boldsymbol{\tau}^1\times\boldsymbol{\tau}^2)_\pm$.

The ρ-meson exchange (pair) seagull current is

$$\mathbf{A}_\pm(\rho S)=-g_A\frac{g_\rho^2(1+\kappa)^2}{4m^3}\frac{f_\rho^2(k_2)}{m_\rho^2+k_2^2}$$
$$\times(\tau_{2\pm}\{(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2-i[\boldsymbol{\sigma}_1\times(\boldsymbol{\sigma}_2\times\mathbf{k}_2)]\times(\mathbf{p}_1+\mathbf{p}_1')\}$$
$$\times(\tau_1\times\tau_2)_\pm\{\mathbf{q}\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\mathbf{k}_2)+i(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times(\mathbf{p}_1+\mathbf{p}_1')-[\boldsymbol{\sigma}_1\times(\boldsymbol{\sigma}_2\times\mathbf{k}_2)]\times\mathbf{k}_2\})+(1\rightleftarrows 2)\ . \tag{A3}$$

The operator includes only those terms which are proportional to $(1+\kappa)^2$; g_ρ is the ρNN vector ($g_\rho^2/4\pi=0.5$), and κ the tensor coupling constant ($\kappa=6.6$). The ρNN monopole vertex form factor is denoted $f_\rho(k)$.

The pion exchange Δ_{33} excitation current is

$$\mathbf{A}_\pm(\pi\Delta)=-\frac{16}{25}g_A\frac{f_{\pi NN}^2}{m_\pi^2(m_\Delta-m)}\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2+k_2^2}f_{\pi\Delta}^2(k_2)[4\tau_{2\pm}\mathbf{k}_2-(\tau_1\times\tau_2)_\pm\boldsymbol{\sigma}_1\times\mathbf{k}_2]+(1\rightleftarrows 2)\ . \tag{A4}$$

The quark model relations $g_{AN\Delta}=(6\sqrt{2}/5)/g_A$, $g_{\pi N\Delta}=(6\sqrt{2}/5)f_{\pi NN}$ have been used to express the $N\Delta$ transition couplings in terms of nucleon parameters. The $\pi N\Delta$ monopole vertex form factor is denoted $f_{\pi\Delta}(k)$.

The ρ-meson exchange Δ_{33} excitation current is

$$\mathbf{A}_\pm(\rho\Delta)=\frac{4}{25}g_A\frac{g_\rho^2(1+\kappa)^2}{m^2(m_\Delta-m)}\frac{f_{\rho\Delta}^2(k_2)}{m_\rho^2+k_2^2}\{4\tau_{2\pm}(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2-(\tau_1\times\tau_2)_\pm\boldsymbol{\sigma}_1\times[(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2]\}+(1\rightleftarrows 2)\ . \tag{A5}$$

The quark model relations have been used to express the $\rho\Delta$ coupling constants in terms of ρNN coupling constants. The $\rho\Delta$ monopole vertex form factor is denoted $f_{\rho\Delta}(k)$.

The $\pi\rho$ exchange current is

$$\mathbf{A}_\pm(\pi\rho)=-2g_A\frac{g_\rho^2}{m}\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{(m_\rho^2+k_1^2)(m_\pi^2+k_2^2)}f_\rho(k_1)f_\pi(k_2)(\tau_1\times\tau_2)_\pm[(1+\kappa)\boldsymbol{\sigma}_1\times\mathbf{k}_1-i(\mathbf{p}_1+\mathbf{p}_1')]+(1\rightleftarrows 2)\ . \tag{A6}$$

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