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ARTICLES

Decays of the K^-p atom and the $\Lambda(1405)$

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To investigate differences between some recent calculations on the decays of the K^-p atom, we calculate the pionic decays $K^-p \to \Lambda\pi^0$, $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Sigma^-\pi^+$ in an isobar model and compare them with experimental rates determined from measured branching ratios together with the imaginary part of the $\bar{K}N$ scattering amplitude. The comparison shows that the coupling constants at the strong-interaction vertices are reduced significantly from the on-shell values. These coupling constants are then applied to an isobar-model calculation of the radiative transition rates. Comparing these with results from a recent experiment gives values for the radiative decay widths of the $\Lambda(1405)$: $\Gamma_{\Lambda(1405)\to\Lambda\gamma}=27\pm8$ keV and $\Gamma_{\Lambda(1405)\to\Sigma^0\gamma}=10\pm4$ or 23 ± 7 keV.

I. INTRODUCTION

The K^-p atom has for some time been regarded [1–8] as a useful source of information on the $\Lambda(1405)$. The mass of the K^-p system at threshold (1432 MeV) is close to that of the $\Lambda(1405)$, and since the $\Lambda(1405)$ has a width [9] of 40 ± 10 MeV, it is expected to have a strong influence on the properties of the K^-p atom.

The $\Lambda(1405)$ itself is a poorly understood object. In most quark-model calculations published to date, it is described as a p-state q^3 baryon with mainly SU(3) singlet structure [10–14]. Most recently, a chiral-bag-model calculation by Umino and Myhrer [15,16] treated the $\Lambda(1405)$ as a q^3 structure, but finds that it has a substantial contribution of SU(3) octet symmetry, rather than mainly singlet, as in most earlier calculations. However, its rather low mass has sometimes given problems in these calculations, and it was suggested over 25 years ago [7,17–21] that it may be more straightforwardly explained as a bound state of the $\bar{K}N$ system and a resonance in $\pi\Sigma$ scattering and, hence, of $q^4\bar{q}$ structure. These two pictures form the basis of many currently popular models of the $\Lambda(1405)$. For example, the nonrelativistic quark model of Isgur and Karl [4,14] describes the $\Lambda(1405)$ as a q^3 hadron, whereas in the cloudy-bag model [7,8,19] it has the structure of a $\bar{K}N$ bound state. A definitive experiment to distinguish between these models has not been identified.

Radiative widths play an important part in understanding baryon structure and the nature of the quark confinement mechanism. Two of the three possible radiative decays of the $\Lambda(1405)$, to $\Lambda\gamma$ and to $\Sigma^0\gamma$, have been studied experimentally by measurements of the branching ratios for the corresponding decays of the K^-p atom (see, e.g., Refs. [22,23] and the reviews by Roberts *et al.* [24], Barrett [7], and Lowe [6]). Several experimental results have been published, but the most definitive are the recent results from Brookhaven experiment 811 (Whitehouse *et al.* [22,23]):

$$R_{\Lambda\gamma}=\frac{K^-p\to\Lambda\gamma}{K^-p\to\text{anything}}=(0.86\pm0.12)\times10^{-3}$$

and

$$R_{\Sigma\gamma}=\frac{K^-p\to\Sigma^0\gamma}{K^-p\to\text{anything}}=(1.44\pm0.23)\times10^{-3}\ .$$

The other available radiative decay channels, $K^-p\to\Sigma(1385)\gamma$ and $\Lambda(1405)\gamma$, have not yet been observed experimentally.

The problem arises as to how to relate the measured branching ratios for the K^-p atom to the radiative widths for the $\Lambda(1405)$. Several calculations have been published which attempt to do this (these are reviewed in Refs. [6,7]). In many of these, an isobar-model approach is used to relate the properties of the K^-p atom to the parameters of nearby hadrons. Although this method is not the most sophisticated approach, the K^-p system at this energy is strongly dominated by the $\Lambda(1405)$, while the treatment of the other remaining amplitudes as a contribution from nearby poles is a useful approximation. Two of the published calculations [3,5] have shown that the contributions from the relatively remote N^* and K^* resonances are negligible, and so the problem reduces to a rather small number of graphs. In the first calculation of this type, by Burkhardt, Lowe, and Rosenthal [3], an isobar model was used to calculate all s-, t-, and u-channel exchanges shown in Fig. 1. In addition, some graphs involving exchanged K^* and N^* were calculated, but their contributions were found to be small. The denominator of the branching ratio was not calculated in an isobar model, but was derived from the experimental scattering amplitude at threshold. The branching ratio for $K^- p \rightarrow \Lambda\gamma$ was calculated as a function of the transition moment for the $\Lambda(1405) \rightarrow \Lambda\gamma$ vertex, and the radiative width for $\Lambda(1405) \rightarrow \Lambda\gamma$ was deduced by comparison of the calculated branching ratio with experiment. It was found that the $K^- p$ atom decays were indeed dominated by the $\Lambda(1405)$ contribution [Fig. 1(b)], but that other terms were not negligible. The publication by Burkhardt, Lowe, and Rosenthal contains a numerical error; this is corrected in a paper by Workman and Fearing [5], which presents a calculation which is otherwise similar to that of Burkhardt, Lowe, and Rosenthal.

A rather different approach is presented by Darewych, Koniuk, and Isgur [4], which differs in three respects from the above two treatments. First, their calculation was an application of the nonrelativistic quark model of Isgur and Karl to the $K^- p$ atom. Thus all amplitudes, for both radiative and pionic decays, were calculated on the Isgur-Karl model. Second, they included some quark-exchange graphs [25]. The contribution from these was found to be small, but not negligible; in fact, the decay $K^- p \rightarrow \Lambda\pi^0$ proceeds entirely through these terms. Third and most importantly for the present work, Darewych, Koniuk, and Isgur argue that the simple isobar model, when on-shell values are used for all coupling constants, considerably overestimates all terms except the $\Lambda(1405)$ graph contribution. The reasons behind this are partly that in the s-channel graphs with even-parity exchange, such as those of Fig. 1(a), the first vertex is parity

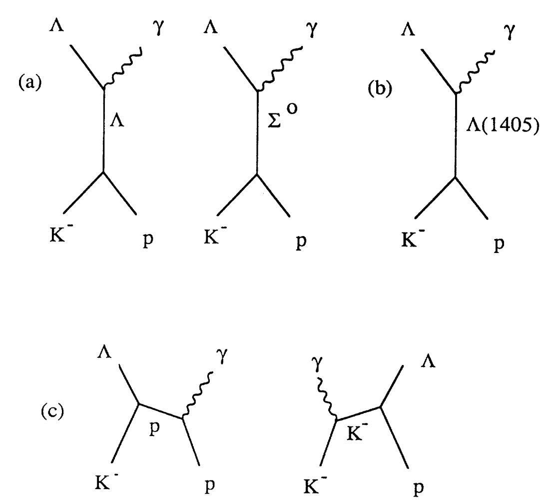

FIG. 1. Contributions to the radiative decays of the $K^- p$ atom.

forbidden, so that the only contribution in fact comes from the "Z graphs," involving a $\bar{\Lambda}$ or $\bar{\Sigma}$. Consequently, these vertices are actually much further off shell than would appear at first sight from Fig. 1. In addition, Darewych, Koniuk, and Isgur point out that the intermediate state has three extended baryons in close proximity and the simple isobar model is probably a bad approximation in dealing with this situation. Following this argument to its limit, they drop all contributions except that of the $\Lambda(1405)$ graph (which is parity allowed) and their quark-exchange terms. Both radiative and pionic decays were then calculated using just these two contributions. An isobar-model calculation with the same basic assumption of $\Lambda(1405)$ dominance for both radiative and pionic decays has been reported by Lowe and Burkhardt [26]. Not all authors agree with the arguments presented by Darewych, Koniuk, and Isgur. For example, Workman and Fearing [5] quote examples of pion-nucleon interactions in which Z graphs are known to make an important contribution.

A crucial point, therefore, is the amplitude of some of the strong-interaction vertices in the graphs shown in Fig. 1 and, in particular, whether, within the framework of the isobar model, these are suppressed by form factors to values well below what one would deduce from the use of on-shell coupling constants. In this paper we investigate this problem by calculating the *pionic* decay modes of the $K^- p$ atom in an isobar model, using the same methods that were used for *radiative* decay calculations by Burkhardt, Lowe, and Rosenthal [3] and by Workman and Fearing [5]. The rates for $K^- p \rightarrow \Lambda\pi^0$, $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Sigma^-\pi^+$ are well known. Further, these processes involve many of the same vertices, at the same momentum transfers, as in the radiative decays. The pionic decays therefore provide a direct experimental measure of the strengths of these vertices as they contribute to the radiative decays and, hence, also a test of the validity of the arguments advanced for or against their suppression. Of course, the suppression of these vertices follows naturally from any model which treats the hadron internal structure at the quark level. However, the aim of this paper is to give a more phenomenological, and hence less model-dependent, treatment and to deduce as much as possible about the contribution of these terms from experiment.

II. RATES FOR $K^- p$ PIONIC DECAYS

The branching ratios for these pionic decays are well measured. Recent values are [22]

$$R_{\Lambda\pi^0} = 0.075, \quad R_{\Sigma^+\pi^-} = 0.196 ,$$

$$R_{\Sigma^0\pi^0} = 0.261, \quad R_{\Sigma^-\pi^+} = 0.467 .$$

To obtain partial decay widths from these, we use the method of Bardeen and Torigoe [27], which relates the total decay width of the $K^- p$ atom to the imaginary part of the $\bar{K}N$ scattering amplitude at threshold. They define a pseudopotential, the imaginary part of which is given by

$$W = (4\pi/2m)\,\mathrm{Im} f_{K^- p} ,$$

where f_{K^-p} is the K^-p scattering amplitude at threshold and m is the reduced mass of the K^-p system. The total width of the K^-p atom is then

$$\Gamma_{K^-p\to\text{anything}}=2W|\psi(0)|^2$$
$$=(4\pi/m)\text{Im}f_{K^-p}|\psi(0)|^2 ,$$

where $\psi(0)$ is the kaon atomic wave function at the origin. For f_{K^-p} we use the relation

$$f_{K^-p}=\tfrac{1}{2}(f^{(0)}_{\bar{K}N}+f^{(1)}_{\bar{K}N}) ,$$

relating the K^-p scattering amplitude to those for the $I=0$ and 1 states. For the latter we take the average of all values listed in the tabulation of Nagels *et al.* [28], giving

$$\text{Im}f_{K^-p}=0.75\pm0.17 \text{ fm} ,$$

which agrees well with the analysis of Martin and Sakitt [29]. Thus

$$\Gamma_{K^-p\to\text{anything}}=(1130\pm256)|\psi(0)|^2 \text{ MeV fm}^3 .$$

The factor $|\psi(0)|^2$ cannot readily be calculated. The capture takes place from higher atomic states with a range of n values. These atomic states are likely to be s states with greater than 99% probability [30], but are dominated [31] by principal quantum numbers in the range 5–10. In any case, the kaon wave function is distorted from the Coulomb form by the K^-p strong interaction. However, $|\psi(0)|^2$ is also a factor in the individual pionic decay rates and so, to the extent that its variation over the interaction region can be ignored, cancels when calculating branching ratios. The experimental rates then follow by combining this total width with the above branching ratios:

$$\Gamma_{\Lambda\pi^0}=85|\psi(0)|^2 \text{ MeV},\quad \Gamma_{\Sigma^+\pi^-}=227|\psi(0)|^2 \text{ MeV} ,$$
$$\Gamma_{\Sigma^0\pi^0}=295|\psi(0)|^2 \text{ MeV},\quad \Gamma_{\Sigma^-\pi^+}=523|\psi(0)|^2 \text{ MeV} .$$

In calculating the individual rates, all graphs of Fig. 2 were included. Here we are guided by the results of Refs. [3] and [5] for the radiative processes, where the contributions from the more remote N^* and K^* isobars were found to be small. The amplitudes for the first graphs of Figs. 2(a), (b), and (c) are, respectively,

$$\mathcal{M}=\bar{u}(\Lambda)G_{\Lambda\Sigma\pi}\gamma^5\frac{i(\not{p}+m_\Sigma)}{p^2-m_\Sigma^2}G_{N\Sigma K}\gamma^5u(p)$$
$$=-i\frac{G_{\Lambda\Sigma\pi}G_{N\Sigma K}}{m_{Kp}+m_\Sigma}\bar{u}(\Lambda)u(p) ,$$

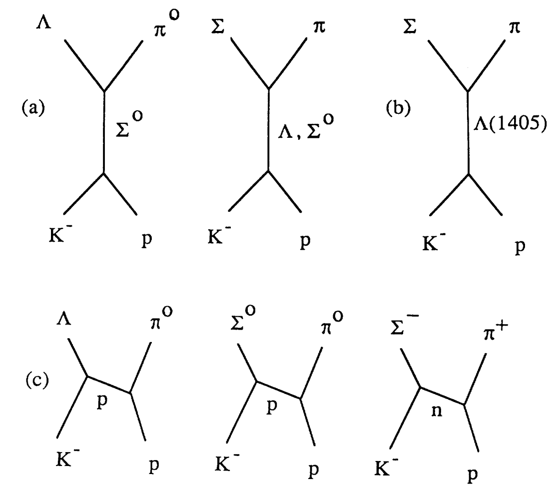

FIG. 2. Contributions to the pionic decays of the K^-p atom.

$$\mathcal{M}=\bar{u}(\Sigma)G_{\Lambda(1405)\Sigma\pi}\frac{i(\not{p}+m_{\Lambda(1405)})}{p^2-m^2_{\Lambda(1405)}}G_{N\Lambda(1405)K}u(p)$$
$$=i\frac{G_{\Lambda(1405)\Sigma\pi}G_{N\Lambda(1405)K}}{m_{Kp}-m_{\Lambda(1405)}}\bar{u}(\Sigma^+)u(p) ,$$

and

$$\mathcal{M}=-\bar{u}(\Lambda)\gamma^5G_{N\Lambda K}\frac{i(\not{p}+m_p)}{p^2-m_p^2}\gamma^5Gu(p)$$
$$=-i\frac{G_{N\Lambda K}G}{2m_pE_\pi-m_\pi^2}\bar{u}(\Lambda)\not{p}_\pi u(p) ,$$

with analogous expressions for the remaining graphs. These expressions are written in terms of the I-spin reduced coupling constants as defined, for example, by Dumbrajs *et al.* [32]. We denote the ratio $G_{N\Sigma K}/G_{N\Lambda K}$ by a. All symbols and conventions are as defined in Bjorken and Drell [33].

The K^-p atom decay rates for pionic decays are then given by

$$\Gamma=\frac{m_Hp_\pi}{4\pi m_K(m_p+m_K)}\bar{\Sigma}_{\text{spins}}|\mathcal{M}|^2|\psi(0)|^2 ,$$

where m_H denotes the mass of the final-state hyperon and $\bar{\Sigma}_{\text{spins}}$ represents the appropriate average and sum over initial and final spins.

The expressions for the pionic widths then become

$$\Gamma_{\Lambda\pi^0}=|-0.1899aG_{\Lambda\Sigma\pi}G_{N\Lambda K}+0.3022G_{N\Lambda K}G|^2|\psi(0)|^2 ,$$

$$\Gamma_{\Sigma^+\pi^-}=\left|-0.1640aG_{\Sigma\Sigma\pi}G_{N\Lambda K}-0.1690G_{\Lambda\Sigma\pi}G_{N\Lambda K}+\frac{11\,600-215.2i\Gamma_{\Lambda(1405)}}{726.3+\frac{1}{4}\Gamma^2_{\Lambda(1405)}}G_{\Lambda(1405)\Sigma\pi}G_{N\Lambda(1405)K}\right|^2|\psi(0)|^2 ,$$

$$\Gamma_{\Sigma^0\pi^0}=\left|0.1690G_{\Lambda\Sigma\pi}G_{N\Lambda K}-\frac{11\,600-215.2i\Gamma_{\Lambda(1405)}}{726.3+\frac{1}{4}\Gamma^2_{\Lambda(1405)}}G_{\Lambda(1405)\Sigma\pi}G_{N\Lambda(1405)K}-0.2541aG_{N\Lambda K}G\right|^2|\psi(0)|^2 ,$$

$$\Gamma_{\Sigma^-\pi^+}=\left|0.1640aG_{\Sigma\Sigma\pi}G_{N\Lambda K}-0.1690G_{\Lambda\Sigma\pi}G_{N\Lambda K}+\frac{11\,600-215.2i\Gamma_{\Lambda(1405)}}{726.3+\frac{1}{4}\Gamma^2_{\Lambda(1405)}}G_{\Lambda(1405)\Sigma\pi}G_{N\Lambda(1405)K}+0.5082G_{N\Sigma K}G\right|^2|\psi(0)|^2 .$$

In the calculation we assume that all terms are real except for the $\Lambda(1405)$ term. The mass of the $\Lambda(1405)$ was fixed at 1405 MeV, but its width is less well known and therefore was treated as an adjustable parameter.

III. PIONIC DECAYS: COMPARISON WITH EXPERIMENT

The experimental values for the pionic widths were equated to the theoretical expressions to yield equations which can be solved for the coupling constants. There are two problems in deducing coupling constants from this procedure. First, the equations do not yield individual coupling constants directly, but rather products of pairs of these. Second, experiments do not determine the signs of the amplitudes, and so there are three sign ambiguities in the above equations. We simply take each possible combination of signs and look for an acceptable solution to the equations. In deciding on an acceptable solution, we were guided by the SU(3) and SU(6) values for signs of the coupling constants. These are given, for example, by Dumbrajs *et al.* [32], and we find from their tabulation that three products $G_{N\Lambda K}G_{\Lambda\Sigma\pi}$, $G_{N\Lambda K}G$, and $G_{N\Lambda K}G_{\Sigma\Sigma\pi}$ are all predicted to be negative, while the ratio $a=G_{N\Sigma K}/G_{N\Lambda K}$ is positive. The remaining product is $G_{N\Lambda(1405)K}G_{\Lambda(1405)\Sigma\pi}$. Of course, SU(3) and SU(6) make no unambiguous prediction for the coupling constants involving the $\Lambda(1405)$ since, as discussed above, the structure of this hyperon is not understood well on these models.

If a and $\Gamma_{\Lambda(1405)}$ are regarded as known parameters, then the equations can be solved for these four products. In fact, each of these quantities is known with only limited accuracy from experiment. Therefore, the equations were solved for a range of values of both a and $\Gamma_{\Lambda(1405)}$, covering the experimentally reasonable range of each.

Only one set of sign choices for the amplitudes gives a solution with signs of the coupling constant products that is consistent with the SU(3) and SU(6) predictions quoted above. For the "best" values of a and $\Gamma_{\Lambda(1405)}$, i.e., $a=0.44$ and $\Gamma_{\Lambda(1405)}=40$ MeV, the solution is given in the second row of Table I. For comparison, the first row of Table I shows the on-shell values for these quantities. These were mostly taken from Dumbrajs *et al.* [32], with just two exceptions: (i) The quantity $G_{N\Sigma K}$ is rather poorly determined. At the time of our earlier paper [3], the best value seemed to be 5.8, a value deduced from the effective coupling constant G_{NYK} through the relation

$$G^2_{NYK}=G^2_{N\Lambda K}+0.84G^2_{N\Sigma K} ,$$

using a value [32,34,35] of 14 for $G^2_{NYK}/4\pi$. Several lower values of $G_{N\Sigma K}$ are listed by Dumbrajs *et al.*, but the most recent determination [36] of $G^2_{NYK}/4\pi$ gives rather high values of 16.9 or 17.9. In view of this uncertainty, the same value that was used in our earlier work is retained here. (ii) The coupling constant $G_{\Lambda(1405)\Sigma\pi}$ was determined directly from the measured width [9] of the $\Lambda(1405)$ through the relation

$$\Gamma_{\Lambda(1405)}=3\frac{G^2_{\Lambda(1405)\Sigma\pi}p_\pi(E_\Sigma+m_\Sigma)}{4\pi m_{\Lambda(1405)}} .$$

Of course, the sign remains undetermined by this procedure. Table II summarizes the values adopted for these parameters, together with signs that are consistent with the SU(6) conventions of Dumbrajs *et al.*

Two features emerge immediately from inspection of

TABLE I. Parameters relevant to the pionic decays of the K^-p atom. Row 1 gives the values for a and $\Gamma_{\Lambda(1405)}$ from Ref. [9], together with on-shell values for the coupling constants (see text). Row 2 gives the values of products of coupling constants deduced from the present analysis, using values of a and $\Gamma_{\Lambda(1405)}$ from row 1. Rows 3–5 show the effect on these results of small changes in the parameters a and $\Gamma_{\Lambda(1405)}$.

	a	$\Gamma_{\Lambda(1405)}$ (MeV)	$G_{\Lambda(1405)\Sigma\pi}\times G_{N\Lambda(1405)K}$	$G_{N\Lambda K}\times G$	$G_{N\Lambda K}\times G_{\Lambda\Sigma\pi}$	$G_{N\Lambda K}\times G_{\Sigma\Sigma\pi}$
On-shell values	0.44	40±10	±2.6	−174	−161	−169
Present analysis; parameters of row 1	(0.44)	(40)	−1.9	−60	−108	−30
Present analysis; adjusted parameters	0.44	60	−1.9	−47	−59	−51
Present analysis; adjusted parameters	0.42	60	−1.9	−46	−57	−59
Present analysis; adjusted parameters	0.40	60	−1.9	−44	−55	−68

TABLE II. Coupling constants and other parameters used in the present analysis.

Parameter	Value	Source
G	-13.2	Dumbrajs *et al.* [32]
$G_{N\Lambda K}$	13.2	Dumbrajs *et al.* [32]
$G_{\Lambda\Sigma\pi}$	-12.2	Dumbrajs *et al.* [32]
$G_{\Sigma\Sigma\pi}$	-12.8	Dumbrajs *et al.* [32]
$G_{N\Sigma K}$	5.8	see text
$G_{\Lambda(1405),K^-p}$	$\pm(3.2\pm0.6)$	Dumbrajs *et al.* [32]
$G_{\Lambda(1405),\Sigma\pi}$	±0.805	From $\Lambda(1405)$ width; see text
$\mathrm{Im} f_{K^-p}$	0.75 ± 0.17 fm	Nagels *et al.* [28]

the first two rows of Table I. First, the coupling constants involving the $\Lambda(1405)$ are reasonably close to the on-shell values, particularly in view of the uncertainty in the experimental value of $G_{N\Lambda(1405)K}$. By contrast, the other three products are well below the experimental on-shell values. This picture is quite consistent with the picture of Darewych *et al.*; both vertices involving the $\Lambda(1405)$ are parity allowed, while their parity suppression argument would imply a reduction below the on-shell value of, if nothing else, the quantities $G_{N\Lambda K}$ and $G_{N\Sigma K}$, the first of which is a common factor in the other three products. If $G_{N\Lambda K}$ and $G_{N\Sigma K}$ are the only two coupling constants affected, then one would expect the three products involving them to be similar in magnitude, a condition which is not particularly well satisfied by row 2 of Table I. However, minor adjustments to the parameters a and $\Gamma_{\Lambda(1405)}$ can readily change this situation as is shown in rows 3–5 of Table I, where the values found are consistent with a reduction in $G_{N\Lambda K}$ and, since a is close to the on-shell value, in $G_{N\Sigma K}$ also.

However, the parity suppression mentioned above is not the only possible interpretation of the values found for these products of coupling constants. Quite apart from any argument invoking Z graphs, it is clear that the vertices involving the $\Lambda(1405)$ are much more nearly on-shell than are any other of the vertices in Fig. 2. Thus the contribution from vertices other than the $\Lambda(1405)$ graphs is expected to be reduced if only by virtue of the finite size of the hadrons involved. As an illustration, we calculate the form factors expected for the K-p-Λ and K-p-$\Lambda(1405)$ vertices which would result from the usual expression for the form factor in low-q^2 elastic scattering, assuming that the baryons have a rms radius of $R=1.0$ fm. We use four-momentum transfers of $q^2=(m_{Kp}-m_\Lambda)^2$ and $q^2=(m_{Kp}-m_{\Lambda(1405)})^2$, respectively, which one would deduce from the time-ordered graphs of Fig. 2 *without* any additional contribution that would be implied if Z graphs alone are considered. The expression [37,38]

$$F(q^2)=\frac{1}{1+q^2R^2/6}$$

then gives form factors of $F(q^2)=0.70$ and 0.97 for the K-p-Λ and K-p-$\Lambda(1405)$ vertices, respectively. While quantitative agreement is not expected from this simple picture, these values are at least qualitatively consistent with the behavior of the coupling constants shown in Table I.

IV. CALCULATION OF THE RADIATIVE DECAYS

The next step is to apply the information on coupling constants obtained in the above analysis to the problem of radiative transitions, i.e., to calculate the contribution of the graphs shown in Fig. 1 to $K^-p\to\Lambda\gamma$ and $\Sigma^0\gamma$. Unfortunately, in these graphs the coupling constants do not occur as products of pairs as in the pionic decays, and so it is necessary to decide on values of certain individual coupling constants, specifically $G_{N\Lambda K}$, $G_{N\Sigma K}$, and $G_{N\Lambda(1405)K}$. Table I implies that $G_{N\Lambda(1405)K}$ is quite close to the on-shell strength of 3.2, and so we use this value in calculating the radiative decays. For $G_{N\Lambda K}$ and $G_{N\Sigma K}$, the products of either of these with one other coupling constant seem to be about 0.3 times the on-shell strength. The essential problem is to decide whether to attribute this factor of 0.3 to either one of the coupling constants or whether to assume a factor of $\sqrt{0.3}$ for each, or any other division of the factor between the two coupling constants. There is no model-independent way of selecting between these alternatives, but there is no compelling reason to believe that any one of these coupling constants behaves in a different fashion from the others. Therefore, we make the simplest assumption, namely, that each coupling constant is subject to a form factor of $\sqrt{0.3}$. Since there is no evidence that $a=G_{N\Lambda K}/G_{N\Sigma K}$ differs from the on-shell value of 0.44, we assume that $G_{N\Lambda K}$ and $G_{N\Sigma K}$ are reduced by the same factor. Inevitably, the calculation from this point onwards becomes of only qualitative validity, but we believe that this may be preferable to a more precise calculation in one of the two extreme approaches.

We now repeat the calculation of the radiative widths in the same way as those published by Burkhardt, Lowe, and Rosenthal, [3] and by Workman and Fearing [5]. The contributions to the amplitude from the first graphs of Figs. 1(a), 1(b), and 1(c) are

$$\mathcal{M}=i\frac{eG_{N\Lambda K}\kappa_\Lambda}{2m_p}\bar{u}(\Lambda)\epsilon_\mu\sigma^{\mu\nu}k_\nu\frac{\not{p}+m_\Lambda}{p^2-m_\Lambda^2}\gamma^5u(p)\ ,$$

$$\mathcal{M}=i\frac{eG_{N\Lambda(1405)K}\kappa_{\Lambda(1405)\Lambda}}{2m_p}\times\bar{u}(\Lambda)\epsilon_\mu\sigma^{\mu\nu}k_\nu\gamma^5\frac{\not{p}+m_{\Lambda(1405)}}{p^2-m_{\Lambda(1405)}^2}u(p)\ ,$$

and

$$\mathcal{M}=iG_{N\Lambda K}\bar{u}(\Lambda)\gamma^5\frac{\not{p}+m_p}{p^2-m_p^2}\left[ie\gamma^\mu+\frac{e\kappa_p}{2m_p}\sigma^{\mu\nu}k_\nu\right]\epsilon_\mu u(p) ,$$

with analogous expressions for the other graphs. Here κ_Λ, κ_p, and $\kappa_{\Lambda(1405)\Lambda}$ are, respectively, the Λ magnetic moment, the proton anomalous magnetic moment, and the transition moment for $\Lambda(1405)\rightarrow\Lambda\gamma$.

The radiative branching ratio is given by

$$\Gamma_{K^-p\rightarrow\Lambda\gamma}=\frac{m_\Lambda k}{4pm_K m_{Kp}}\overline{\Sigma}|\mathcal{M}|^2|\psi(0)|^2 ,$$

where k is the photon momentum and the sum in $\overline{\Sigma}$ now includes photon polarizations. Analogous equations apply to the process $K^-p\rightarrow\Sigma^0\gamma$. These expressions correct a sign error in the calculation and a typographical error in the paper of Burkhardt, Lowe, and Rosenthal [3].

We employ the same analysis method as in both the earlier papers; i.e., the calculated branching ratio is plotted as a function of the relevant transition moment for the $\Lambda(1405)$, $\kappa_{\Lambda\gamma}$, or $\kappa_{\Sigma\gamma}$, and the results are compared with the experimental values. For the latter we use the recent results [22,23] from Brookhaven experiment 811, quoted in Sec. I of this paper. All parameters are taken to have the same values as listed in Burkhardt, Lowe, and Rosenthal, except that we use a recent result [39] for the $\Sigma^0\rightarrow\Lambda\gamma$ transition moment of $\kappa_{\Sigma\Lambda}=1.59$.

First, in Fig. 3 we show, for comparison, the results of calculating with the *on-shell* coupling constants, as in both previous publications. Comparison with experiment suggests that the allowed range of values for the transition moments are

$$\kappa_{\Lambda(1405)\Lambda}=-0.41\pm0.03$$

and

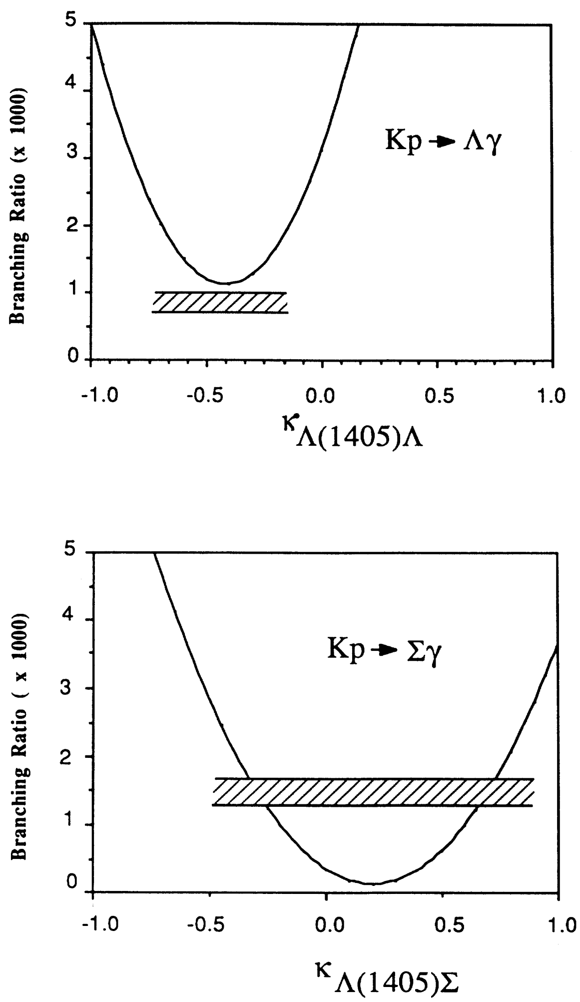

FIG. 3. Comparison of the calculated radiative decay branching ratios for the K^-p atom (solid curve) with experiment [22,23] (crosshatched area). The calculated branching ratios are plotted as a function of the relevant transition moment for the $\Lambda(1405)$ radiative decay. On-shell values are used for all coupling constants.

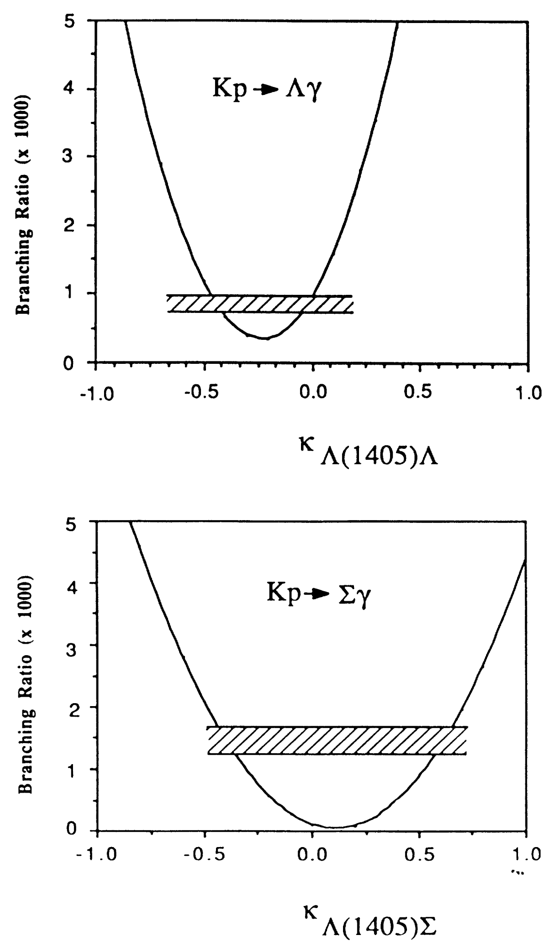

FIG. 4. Comparison of the calculated radiative decay branching ratios for the K^-p atom with experiment, as in Fig. 3, but with all strong-interaction coupling constants reduced by a factor of $\sqrt{0.3}$.

$$\kappa_{\Lambda(1405)\Sigma} = -0.29\pm0.05 \quad \text{or} \quad 0.69\pm0.05 \; .$$

Strictly, there is no solution for $\kappa_{\Lambda(1405)\Lambda}$ within the experimental errors for this choice of parameters; the above result indicates the range over which $\kappa_{\Lambda(1405)\Lambda}$ falls within about two standard deviations of the experimental number. Then, applying the above analysis of the coupling constants to the problem, Fig. 4 shows the predicted branching ratio calculated with the coupling constants $G_{N\Lambda K}$ and $G_{N\Sigma K}$ reduced by a factor of $\sqrt{0.3}$. The allowed ranges for the transition moments are now

$$\kappa_{\Lambda(1405)\Lambda} = -0.43\pm0.02 \quad \text{or} \quad -0.02\pm0.02$$

and

$$\kappa_{\Lambda(1405)\Sigma} = -0.39\pm0.05 \quad \text{or} \quad 0.61\pm0.04 \; .$$

From these values we calculate the radiative widths of the $\Lambda(1405)$ from the relation

$$\Gamma_{\Lambda(1405)\to Y\gamma} = \frac{\kappa^2_{\Lambda(1405)Y} e^2 k^3}{4\pi m_p^2} \; .$$

Again, for comparison, we show first the results that follow from the use of on-shell coupling constants, which would give

$$\Gamma_{\Lambda(1405)\to\Lambda\gamma} = 24\pm4 \text{ keV}$$

and

$$\Gamma_{\Lambda(1405)\to\Sigma^0\gamma} = 5.2\pm1.6 \quad \text{or} \quad 30\pm4 \text{ keV} \; .$$

With coupling constants set to $\sqrt{0.3}$ of the on-shell values, we get

$$\Gamma_{\Lambda(1405)\to\Lambda\gamma} = 27\pm3 \quad \text{or} \quad 0.1\pm0.1 \text{ keV}$$

and

$$\Gamma_{\Lambda(1405)\to\Sigma^0\gamma} = 10\pm3 \quad \text{or} \quad 23\pm3 \text{ keV} \; .$$

The second solution for $\Lambda(1405)\to\Lambda\gamma$, of ~ 0.1 keV, is almost certainly wrong, as it is well below any of the wide range of published calculations. Thus we take the first solution, 27 ± 3 keV, to be the correct choice. Either solution for $\Lambda(1405)\to\Sigma^0\gamma$ seems equally likely. The errors quoted here refer to the range allowed by the experimental branching ratios and do not include errors in the input parameters or uncertainties inherent in the method itself. The two input parameters with significant uncertainties are f_{K^-p} ($\pm23\%$) and $G_{\Lambda(1405)Kp}$ ($\pm19\%$). These errors are incorporated below.

V. DISCUSSION

The present paper gives an internally consistent description of the decays of the K^-p atom. It is clear from examination of the radiative widths derived in Sec. IV that neither extreme treatment is really adequate; the terms seem to be reduced significantly below the on-shell values, but not to the point where their contribution to the radiative decays is negligible. Since our analysis provides only values for products of pairs of coupling constants, there is some degree of arbitrariness in assigning values to the individual coupling constants required to calculate radiative decays. Nevertheless, the consistency with the form factors expected on simple finite-size arguments suggests that the values derived here are at least qualitatively correct. Of course, form factors will arise naturally in any composite model.

The radiative widths found for the $\Lambda(1405)$ as described in Sec. IV, after incorporating errors due to uncertainties in f_{K^-p} and $g_{\Lambda(1405),K^-p}$, are

$$\Gamma_{\Lambda(1405)\to\Lambda\gamma} = 27\pm8 \text{ keV}$$

and

$$\Gamma_{\Lambda(1405)\to\Sigma^0\gamma} = 10\pm4 \quad \text{or} \quad 23\pm7 \text{ keV} \; .$$

These widths are of the same order of magnitude as those found in several bag-model calculations. Calculations carried out so far using the Isgur-Karl model [14] predict higher values, i.e., $\Gamma_{\Lambda(1405)\to\Lambda\gamma} = 143$ keV and $\Gamma_{\Lambda(1405)\to\Sigma^0\gamma} = 91$ keV. Some calculations of radiative widths have been reported in which $q^4\bar{q}$ terms are included in the $\Lambda(1405)$; results from these are discussed in Refs. [6], [18], and [19]. Most of these predict radiative decay rates very different from those found here. However, we note that the cloudy-bag model [7,8,19], in which the $\Lambda(1405)$ has mainly a $q^4\bar{q}$ structure, comes close to fitting the K^-p atomic branching ratios. In fact, these cloudy-bag-model calculations already incorporate form factors, which are inherent in the model and which have values similar to those found here.

In contrast to the Isgur-Karl calculations, a recent chiral-bag-model calculation by Umino and Myhrer [15,16] predicts somewhat smaller values for the radiative widths, $\Gamma_{\Lambda(1405)\to\Lambda\gamma} = 75$ keV and $\Gamma_{\Lambda(1405)\to\Sigma^0\gamma} = 2.4$ keV. The small value for the second of these numbers is a result of a delicate cancellation between singlet and octet contributions, and a rather small change in Umino and Myhrer's wave function gives a prediction in good agreement with our result. The discrepancy in the $\Lambda(1405)\to\Lambda\gamma$ widths is not so easy to understand. However, an essential feature of their calculation is that the $\Lambda\gamma$ width of the $\Lambda(1405)$ is greater than the $\Sigma^0\gamma$ width, as it is also in the Isgur-Karl model predictions. This is the case in our analysis, even though the $\Lambda\gamma$ branching ratio of the K^-p atom is the lower of the two. The interference with other terms causes a first inspection of the atomic branching ratios to give a misleading impression.

An extensive analysis of the related process $\gamma p\to K^+\Lambda$ has been published by Adelseck and Saghai [40] using a similar model to that employed here. They extract values for $G_{N\Sigma K}$ and $G_{N\Lambda K}$. Their value for the former is consistent with our value, though the latter is higher and is more similar to the on-shell value. By contrast, a recent crossing-consistent analysis of $\gamma p\to K^+ Y$ and $K^-p\to\gamma Y$ by Williams, Ji, and Cotanch [41] derived a value of $G_{N\Lambda K}$, which is well below the on-shell value and is qualitatively consistent with ours, but found $G_{N\Sigma K}$ to be substantially lower than our value and that of Ref. [40].

However, the momentum range covered in these analyses is quite different from that for the threshold Kp system considered here. Also, Adelseck and Saghai note that an attempt to extend their calculation to the Kp atom is not consistent with recent experimental data.

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