Numerical quasiparticle representation of the nucleon-nucleon T matrix

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In this paper we develop a practical "hybrid" numerical representation of the nucleon-nucleon T matrix. Part of the expression contains nonseparable terms which are easily calculated, and the rest consists of a separable representation of small rank in terms of Weinberg states. The method rests on the observation that when a set of positive-energy Weinberg states is used to obtain a separable representation of the potential V then the residue ΔV, due to the basis-set truncation, has very special properties: (1) The contribution ΔT to the T matrix due to ΔV is identical to the undistorted T matrix for ΔV alone, T_Δ, i.e., the usual Moeller distortion factors in the two-potential formula are unity in this case. (2) A perturbative-iterative treatment of T_Δ in powers of ΔV is found to be equivalent to the finite-rank representation of operators of the type T, $T-V$, $T-V-VG_0V$, and so on. This equivalence has both practical and theoretical implications. On the one hand, it provides a reliable method for calculating the T matrix and for analyzing the corresponding accuracy properties. On the other hand, a connection is established between each order of the quasiparticle method and the different variational principles which underlie the finite-rank representation of operators such as T, $T-V$, $T-V-VG_0V$, etc. Numerical examples are provided for two different nucleon-nucleon singlet potentials (Reid soft core and Malfliet-Tjon). In the Malfliet-Tjon case, for instance, two Weinberg states are found to be sufficient in order to give an accuracy of 0.1% for the calculation of $T-V-VG_0V$, while for $T-V$ and T the same two states give an accuracy of 1% and 10%, respectively, in an interval of 6 fm^{-1} around the on-shell point.

I. INTRODUCTION

Positive-energy Weinberg states [1] (PEWS) are a useful basis for solving scattering problems because these functions asymptotically obey the appropriate outgoing wave boundary condition. They have been used for the solution of the Schrödinger wave function (coupled or uncoupled) [1,2], for the solution of a many-particle shell-model system with one nucleon in the continuum [3], for obtaining a representation of the two-nucleon T matrix [4,5], and for obtaining a nonlocal dynamic polarization potential which expresses the effect of channel coupling [6]. Negative-energy Weinberg states (NEWS) have been similarly used with good success for generating a separable representation of the two-body T matrix for many years [7]. Renewed activity with NEWS has more recently provided a separable representation for the T matrix for modern potentials [8], and also for obtaining the energy dependence of the dynamic polarization potential due to channel coupling at the threshold of the opening of a new channel [9]. NEWS are also used in atomic scattering situations [10], or for including the effects of breakup in deuteron-nucleus transfer reactions [11].

Many applications, however, require good accuracy with low rank, while the convergence with the number of Weinberg basis states can be rather slow, especially if a tensor interaction is present. Moreover, for strong interactions the T matrix may be several orders of magnitude smaller than the potential, in which case the T matrix expansion requires higher precision than the corresponding expansion of the potential [12]. For cases in which a strict separable expansion of the T matrix is required, other representations have become popular, such as the one based on the Ernst, Shakin, and Thaler (EST) method [13] or the W-matrix method [14].

When a purely separable expansion is not required, a two-potential approach was advocated [12], in which the high-rank terms of the representation are replaced by nonseparable expressions. This procedure is particularly simple when PEWS are used for a basis of the separable representation. The procedure is as follows. The potential is first expressed by a separable representation in terms of the PEWS. This representation is then truncated at any rank S, giving rise to the separable potential V_S and to the remainder $\Delta V = V - V_S$. Under these conditions [12] the total T matrix is the sum of the separate T matrices which correspond to V_S (called T_S), and to ΔV (called T_Δ), respectively, all distortion factors which usually occur in the two-potential formula [15] being rigorously equal to unity in this case.

In this article we illustrate this method for the particular case in which ΔV is sufficiently small so that T_Δ can be obtained perturbatively. This method is a form of the quasiparticle method [1], also called the Hilbert-Schmidt [1] method. From a theoretical point of view this approach is more adequate than the fully separable representation of T since it is known [16] that one cannot rigorously express a local potential as a sum over a finite number of separable terms. We give a numerical illustration of the method for the Malfliet-Tjon (MT-I) and the Reid soft core (RSC) nucleon-nucleon potentials. For simplicity, we restrict ourselves to the singlet case $^1S_0$. For each of these potentials we compare the numerical

convergence properties of three different expressions for T, containing, respectively, the zero-, the first-, and second-order quasiparticle contributions. We find that the accuracy properties and convergence rate of these different expressions are dramatically different, and we show how this can be understood within the Weinberg state formalism.

In Sec. II we review the properties of the Weinberg states which are relevant to the present study; in Sec. III we present the connection between the perturbative treatment of T_Δ and the high-rank part of a Weinberg expansion; Sec. IV contains the numerical results for the MT-I and the RSC potentials; and Sec. V contains the conclusions.

II. THE WEINBERG STATE REPRESENTATION

The T matrix which corresponds to a given potential V obeys the Lippmann-Schwinger equation $T(E)=V+VG_0(E)T(E)$, which has the solution

$$T=(1-VG_0)^{-1}V=V\Omega_V \ , \tag{2.1}$$

where Ω_V is the distortion operator associated to the potential V,

$$\Omega_V=1+G_VV=1+G_0T \ . \tag{2.2}$$

In the above G_0 and G_V are, respectively, the undistorted and the V-distorted Green's functions calculated at the energy E. When the potential V is decomposed into two parts V_S and $\Delta V=V-V_S$, then the two-potential formula for the T matrix can be written [12,15] as

$$T=T_S+(1+T_SG_0)\tau_\Delta(1+G_0T_S) \ . \tag{2.3}$$

Here T_S is the T matrix which corresponds to V_S and τ_Δ is the T matrix corresponding to ΔV in the presence of the Green's function $\mathcal{G}_S$ distorted by V_S, $\tau_\Delta=\Delta V+\Delta V\mathcal{G}_S\tau_\Delta$, with $\mathcal{G}_S=G_0+G_0V_S\mathcal{G}_S$. Since V_S is assumed to be separable, T_S also is, but τ_Δ is not separable.

The above equation shows that the additional term in T due to the effect of ΔV in the presence of V_S is given by τ_Δ modified by the distortion factors Ω_S' and Ω_S. Since ΔV is small, τ_Δ can be obtained as a perturbative series in ΔV, but in view of the presence of the distortion factors Ω_S, it is not easy to know the size of the second term in Eq. (2.3). According to Eq. (2.1), these distortion factors have a large effect when they act on V_S, since T_S is much less than V_S, at least when the energy is far away from any pole of the T matrix. However, when they act on τ_Δ, which to first order is equal to ΔV, these distortion factors may have a much smaller effect. This is shown [12] to be the case when V_S is constructed from negative-energy Weinberg functions for the example of the Reid soft core singlet potential. It was also shown in Ref. 12 that when V_S is constructed with positive-energy Weinberg functions, then τ_Δ is rigorously equal to the T matrix T_Δ which is associated to V_Δ alone;

$$T_\Delta=\Delta V+\Delta VG_0T_\Delta \ , \tag{2.4}$$

and in addition the distortion factors Ω_S have no effect on T_Δ. Therefore, in the case that positive-energy Weinberg states are used for the construction of the separable expression for V_S or T_S, one has

$$T=T_S+T_\Delta \ ; \tag{2.5}$$

i.e., the correction ΔT to T_S is itself a T-matrix for the potential ΔV. The derivation of this result is included in the Appendix in order to clearly show why it holds for the case of positive-energy Weinberg states (PEWS). Below we review the properties of the PEWS which are relevant to the connection between the expansion of ΔT in terms of PEWS and the calculation of ΔT through perturbation theory.

The PEWS $|\Gamma_s(E)\rangle$ are defined by the eigenvalue equation

$$G_0(E)V|\Gamma_s(E)\rangle=\gamma_s|\Gamma_s(E)\rangle \ , \tag{2.6}$$

where $G_0(E)$ denotes the free Green's function with incident energy E, orbital momentum l, and outgoing boundary condition. The orthogonality of these states is expressed by the equation

$$\langle\Gamma_s|V|\Gamma_{s'}\rangle=\delta_{ss'} \ , \tag{2.7a}$$

which defines also the normalization here assumed. The eigenvalues γ_s are complex dimensionless quantities which, for usual well-behaved potentials, have only one accumulation point at zero [1]; i.e., $|\gamma_s|\to 0$ as $s\to\infty$. As is well known, these functions allow one to represent both the potential V and the scattering matrix $T=V+VG_0T$ in the form

$$V=\sum_{s=1}^{\infty}|\chi_s\rangle\langle\chi_s| \ , \tag{2.7b}$$

$$T=\sum_{s=1}^{\infty}|\chi_s\rangle\frac{1}{1-\gamma_s}\langle\chi_s| \ , \tag{2.7c}$$

where the functions $|\chi_s\rangle$ are defined as

$$|\chi_s\rangle=V|\Gamma_s\rangle \ . \tag{2.8}$$

In view of the factor $(1-\gamma_s)^{-1}$, the contribution to T from the states whose eigenvalues are much greater than unity is suppressed, while the contribution from the states whose value of $|1-\gamma_s|$ is close to zero, as in the case of resonances and bound states, are enhanced. Of course, the same is not the case for the potential, since the factor $(1-\gamma_s)^{-1}$ is absent. Hence, if the spectrum of the γ eigenvalues contains values much larger than unity, T can be much smaller than V, as is the case for the Reid or the Malfleit-Tjon potentials. On the other hand, the states whose eigenvalues are much less than unity contribute nearly in the same amount to both T and V. Thus, if the sums over s in Eqs. (2.7) are cut off at an upper limit S, as has to be done for practical calculations, then the remainder ΔV and the corresponding quantity ΔT are

$$\Delta V=\sum_{t=S+1}^{\infty}|\chi_t\rangle\langle\chi_t| \ , \tag{2.9a}$$

TABLE I. Weinberg eigenvalues γ_s in the space $l=0$ for $E=5$ MeV. These results are obtained in the model space of Ref. 5, as described in the text. For other model spaces the numerical results can be different, as is discussed in Ref. [20].

	$^1S_0$				$^3S_1$			
	MT-I		RSC		MT-III		RSC	
s	Re	Im	Re	Im	Re	Im	Re	Im
1	−2.60	−0.11	−14.91	−0.96	−1.90	−0.04	−20.90	−1.44
2	0.68	0.49	−1.86	−0.04	1.09	0.75	−1.81	−0.02
3	−0.37	−0.00	0.77	0.38	0.18	0.04	1.15	0.63
4	0.11	0.03	−0.19	0.00	−0.22	0.00	0.14	0.01
5	−0.05	0.00	0.11	0.03	0.07	0.01	0.10	0.03
6	0.04	0.01	0.04	0.01	0.03	0.00	−0.10	0.00

$$\Delta T=\sum_{t=S+1}^{\infty}|\chi_t\rangle\frac{1}{1-\gamma_t}\langle\chi_t|\ , \tag{2.9b}$$

which represents the errors in V and T, respectively, and are of the same order of magnitude. However, ΔT is also the solution of Eq. (2.4), since ΔT is identical to T_Δ for the PEWS expansion discussed here. If the eigenvalues γ_t, $t=S+1, S+2, \ldots$, contained in ΔV are sufficiently small compared to unity, then the solution of Eq. (2.4) can also be obtained by a rapidly converging [1] Born series, and hence the slowly converging sum over the many small eigenvalues in Eq. (2.9b) can be avoided.

This very simple analysis shows that the accuracy and the convergence rapidity of the separable PEWS representation of T are very sensitive to the properties of the Weinberg eigenvalue spectrum. As an example the first six eigenvalues γ for the Reid soft core (RSC) and the Malfliet-Tjon (MT) potentials are listed in Table I for a c.m. energy of 5 MeV. The eigenvalues with negative-(positive-) real part correspond to the repulsive (attractive) terms of the potential. The results of the RSC triplet state are taken from Ref. [5], where they are calculated by solving the s and d coupled equations, and then projecting the result on the s channel. The RSC potentials have two large repulsive eigenvalues, while the MT potentials have only one, which shows that the former has a substantially more repulsive core than the latter. This is offset to some extent by a difference in the momentum representation of the corresponding weighted form factors $\langle\chi|k\rangle/(1-\gamma)^{1/2}$, shown in Fig. 1, so that the final T matrices for the two potentials are not too different. The first attractive eigenvalues are approximately the same for the two potentials, since they give similar results for the low-energy virtual or bound states. The convergence to zero of the eigenvalues occurs approximately at the same rate for the two potentials. Due to the differences in the eigenvalue spectrum of the two potentials, the rank of the PEWS separable representation required to obtain a given accuracy for T is different for the two cases, as is shown in Sec. 4.

III. THE PERTURBATIVE EXPRESSIONS

We shall derive now in the frame of the Weinberg representation expressions which replace Eq. (2.9b) for ΔT. These expressions are based on a perturbative-iterative treatment of the solution of Eq. (2.4) for ΔT in terms of ΔV and lead to more accurate, although only partially separable, representations for T.

We will denote the separable expression for V, truncated at the upper limit S, and the corresponding separable representation of T as V_S and $T_S^{(0)}$, respectively. According to Eqs. (2.7) they are given by

$$V_S=\sum_{s=1}^{S}|\chi_s\rangle\langle\chi_s|\ , \tag{3.1}$$

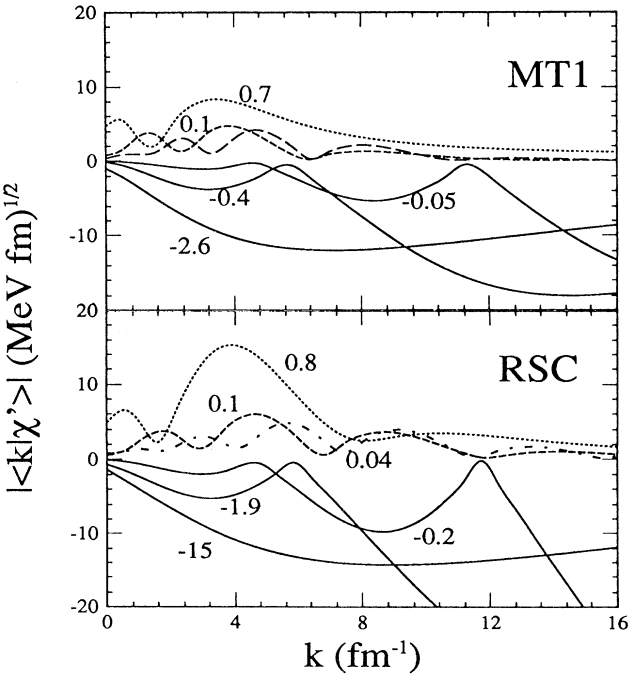

FIG. 1. Momentum dependence of the absolute value of the Weinberg form factors χ'_s, for the $^1S_0$ Malfliet-Tjon I and the Reid soft core potentials. The numbers next to each curve are the value of the real part of the corresponding Weinberg eigenvalue γ_s, listed in Table I. The form factors which correspond to a repulsive eigenvalue ($\gamma<0$) were multiplied by -1, so as to distinguish them clearly from the attractive ones. The form factor χ'_s is equal to the quantity χ_s, defined in Eq. (2.8), divided by $(1-\gamma_s)^{1/2}$.

$$T_S^{(0)}=\sum_{s=1}^{S}|\chi_s\rangle\frac{1}{1-\gamma_s}\langle\chi_s| \ . \tag{3.2}$$

The corresponding errors ΔV and $\Delta T_S^{(0)}$ are given by

$$\Delta V=V-V_S \quad \text{and} \quad \Delta T_S^{(0)}=T-T_S^{(0)} \ . \tag{3.3}$$

These errors are also given by Eqs. (2.7), but we will avoid using them for numerical purposes, since the contribution to the sums in Eq. (2.7) from the large values of s requires an accurate numerical knowledge of many small eigenvalues, which is difficult to achieve with the presently available numerical diagonalization subroutines.

The basis of the quasiparticle method consists in solving Eq. (2.4) by iterations, which converge since ΔV contains only eigenvalues smaller than unity. The first iteration for $\Delta T=T_\Delta$ is ΔV, the second one is $\Delta V G_0 \Delta V$, and so on. If we add these corrections to $T_S^{(0)}$, given by Eq. (3.2), we obtain

$$T_S^{(1)}=T_S^{(0)}+\Delta V \ , \tag{3.4a}$$

$$\Delta T_S^{(1)}=T-T_S^{(1)} \ , \tag{3.4b}$$

$$T_S^{(2)}=T_S^{(1)}+\Delta V G_0 \Delta V \ , \tag{3.5a}$$

$$\Delta T_S^{(2)}=T-T_S^{(2)} \ , \tag{3.5b}$$

for the T matrix and the error, respectively. Now, using Eqs. (3.2), (3.3), and (3.1), we have

$$T_S^{(1)}=V+\sum_{s=1}^{S}|\chi_s\rangle\frac{\gamma_s}{1-\gamma_s}\langle\chi_s| \ , \tag{3.6a}$$

$$\Delta T_S^{(1)}=\sum_{t=S+1}^{\infty}|\chi_t\rangle\frac{\gamma_t}{1-\gamma_t}\langle\chi_t| \ . \tag{3.6b}$$

The same manipulations, when applied to Eqs. (3.5), with $T_S^{(1)}$ given by Eq. (3.6a), yield

$$T_S^{(2)}=V+VG_0V+\sum_{s=1}^{S}|\chi_s\rangle\frac{\gamma_s^2}{1-\gamma_s}\langle\chi_s| \ , \tag{3.7a}$$

$$\Delta T_S^{(2)}=\sum_{t=S+1}^{\infty}|\chi_t\rangle\frac{\gamma_t^2}{1-\gamma_t}\langle\chi_t| \ . \tag{3.7b}$$

In a similar way, analogous equations for the general terms $T_S^{(n)}$ and $\Delta T_S^{(n)}$ can be easily derived.

Equations (3.6) and (3.7) give the desired hybrid expression for the T matrix, one part of which is separable, the other nonseparable. The separable part of each of the expressions for $T^{(n)}$ is expected to converge more rapidly the higher the order (n), since the corresponding error $\Delta T^{(n)}$ has a higher power of γ_s in the numerator, which, in view of the smallness of the absolute values of γ_t for $t>S$, is correspondingly reduced. Indeed, the numerical results described in the next section show differences in accuracy of orders of magnitude between Eqs. (3.2), (3.6), and (3.7). By contrast, the denominators are the same in all the expressions for $T^{(n)}$. Since the denominators contain the bound-state poles and resonances, the singularity properties of T are contained equally well in each of the expressions for T.

It is worthwhile to note that one can look at the expressions $T_S^{(0)}$, $T_S^{(1)}$, and $T_S^{(2)}$ from quite a different point of view: These expressions are indeed separable representations for the operators T, $T-V$, and $T-V-VG_0V$, respectively. The different accuracy which one obtains for the separable representation of T and $T-V$ has already been emphasized in the literature [17], which in turn depends on the difference between the various variational principles [17,18] which underlie the separable representations. Indeed, a variational principle of the Schwinger type has to be considered for the separable representation of T, a variational principle of the Hulthén-Kohn type for $T-V$, and yet another different variational principle [19] for $T-V-VG_0V$. It must, however, be emphasized that, while the variational principles hold for any basis set, the equivalence between the perturbative treatment of the correlation ΔV and the separable representations of T, $T-V$, and $T-V-VG_0V$, given by Eqs. (3.2), (3.6), and (3.7), hold only with the use of the PEWS, since in this case the distortion terms Ω are unity. For other basis states, one has to analyze in detail the effect of the Ω's on the perturbation expressions [19].

IV. NUMERICAL RESULTS

In the previous section two hybrid expressions for the T matrix were presented in which the contributions which ordinarily arise from the high-rank parts of the separable representation are replaced by nonseparable terms. The purely separable representation $T^{(0)}$ is given by Eq. (3.2), while the hybrid expressions $T^{(1)}$ and $T^{(2)}$ and their respective errors are given by Eqs. (3.6) and (3.7), respectively.

The expressions $T^{(0)}$, $T^{(1)}$, and $T^{(2)}$ for T are evaluated numerically for the Reid soft core and the Malfliet-Tjon nucleon-nucleon potentials, and the nature of the convergence of the result as a function of the rank S of the separable part is examined below. Since the Weinberg eigenvalue spectrum of these potentials is different, insight on the connection between the nature of the spectrum and the convergence properties can thereby be obtained. The c.m. energy is 5 MeV, the corresponding on-shell momentum is $\simeq 0.35\ \text{fm}^{-1}$.

The numerical calculations are performed by the method described in Ref. 5; however, the functions Γ defined in the present paper differ from the ones in Ref. 5 by a normalization factor $(\gamma)^{-1/2}$. In this method the Weinberg eigenfunctions Γ_s are expanded in a basis of "primitive" Sturmian functions ϕ_j, which are given in terms of spherical Bessel functions of complex wave numbers. The size M of this basis, 75 in this case, and the value of the matching radius $R=15$ fm are chosen large enough so that the resulting T matrix is stable to three significant figures, as is discussed in connection to Table I of that reference. This method is approximately equivalent to a calculation in complex momentum space with an upper momentum limit of 15 fm^{-1} and momentum steps which are approximately equally spaced and are of size 0.2 fm^{-1}. In this model space the eigenvalues are not as stable as the resulting value of the T matrix. A new method is being developed to obtain the Weinberg

eigenstates entirely in real momentum space [20], which avoids the need to introduce a primitive basis or a matching radius. The T matrices for the two methods agree to better than 0.5% for momenta below 6 fm^{-1}, but there are differences between the two eigenvalue spectra. The attractive eigenvalues in the two methods agree very well, and the repulsive ones are more stable in the new method. In addition, a few of the old repulsive eigenvalues are split into two different new ones. This is understandable, since the repulsive form factors extend out to larger momenta than the attractive ones, as is illustrated in Fig. 1. Since the new method encompasses a larger momentum space, it is expected that the repulsive eigenquantities also differ from those in the old, more restricted, method. These differences and similarities are discussed further in Ref. [20].

The first six eigenvalues for the two potentials, obtained in the model space of Ref. [5] described above, are listed in Table I, and the momentum representation of the corresponding form factors is illustrated in Fig. 1. The absolute values of the form factors for the repulsive eigenvalues were multiplied by -1, so as to separate them clearly from the attractive form factors. The figure shows that the repulsive form factors have much larger-momentum components than the attractive ones, and further, the MT form factors are smaller than the RSC ones. These differences reflect the fact that the RSC potential has a stronger repulsive core, and has a more complicated spatial dependence than the MT potential. This behavior of the form factors affects the convergence properties of $T_S^{(0)}$, $T_S^{(1)}$, and $T_S^{(2)}$ as a function of the rank S, as will now be examined. The values of the $\langle k|T^{(n)}|k'\rangle$ are calculated numerically using Eqs. (3.2), (3.6a), and (3.7a), respectively. The sums over s are carried out using the form factors $\langle k|\chi_s\rangle$ and eigenvalues γ_s discussed above. The order of the states is such that the ones with the highest absolute value of γ_s occur first. The matrix elements involving the Green's function are evaluated making use of the identity $G_0=\sum|\phi_j\rangle\langle\phi_j|$, where the ϕ's are the primitives mentioned above, and the sum over j is cut off at the same upper limit M as the upper limit which is used for the calculation of the Weinberg states in terms of the primitives.

The errors $\langle k|\Delta T_S^{(n)}|k'\rangle$, $n=0,1,2$, are obtained from the numerical difference $T-T_S^{(n)}$. The "exact" value of T is calculated from Eq. (3.6a) in which the sum over s is taken to the maximum value M. The results for the errors are illustrated in Figs. 2 and 3. It is clear that the fastest convergence is achieved for $T^{(2)}$, as was expected since the expression in Eq. (3.7b) has the highest power of γ_s in the numerator. Convergence for the RSC case sets in only after the first two states are included in the sums over s, because their respective eigenvalues γ_s are larger than unity. For the MT case only one eigenvalue lies outside the unit circle (it also is repulsive), and hence convergence sets in already after one state. Also, the larger the momenta k or k', the slower is the convergence of T_S towards T. The repulsive form factors slow down the convergence more than the attractive ones because the former have stronger momentum components.

The nonseparable component T_Δ of the T matrix,

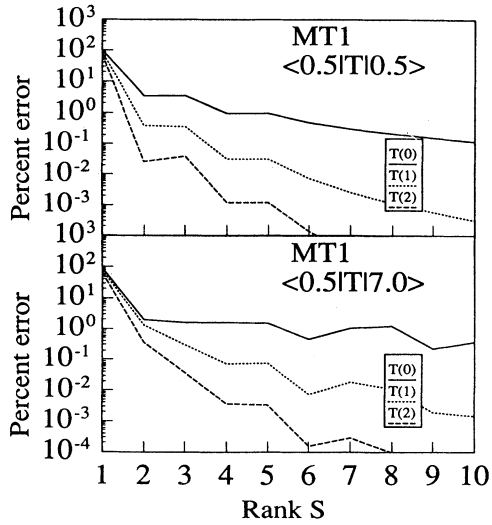

FIG. 2. Absolute value of the error for three approximations to the T matrix as a function of the rank S of the separable part V_S of the Malfliet-Tjon potential. For $T_S^{(0)}$, defined in Eq. (3.2), the correction ΔT due to the remainder ΔV is completely neglected, and $T_S^{(0)}$ is purely separable of rank S, while $T^{(1)}$ and $T^{(2)}$ are obtained by approximating ΔT by ΔV and by $\Delta V+\Delta V G_0 \Delta V$, respectively, and adding the result to $T^{(0)}$. The values of the momenta k and k', in fm^{-1}, are given in the figure.

defined in Eqs. (2.4) and (2.5), will be discussed next. This term is small, and is of interest because it can be included in three-body calculations by a perturbative method [21]. If $T^{(2)}$, given by Eq. (3.7a), is assumed to be a suitable approximation to T, then the separable part of T is given by $T_S^{(0)}$, Eq. (3.2), and the nonseparable part T_Δ is approximated by

$$T_\Delta^{(2)} \equiv \Delta V + \Delta V G_0 \Delta V \tag{4.1}$$

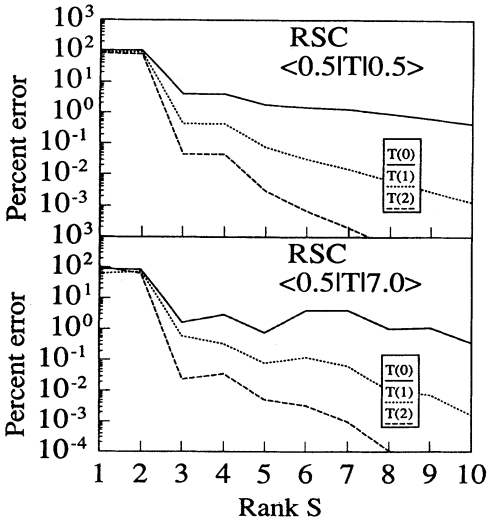

FIG. 3. Same as Fig. 2, for the Reid soft core potential.

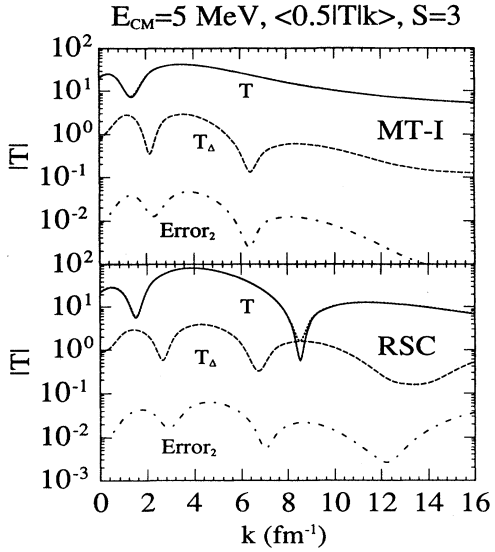

FIG. 4. Various approximations to the T matrix, as a function of the momentum k, for the MT and the RSC potentials. Only the absolute values are shown. The solid curve represents the exact value of T. The dotted curve represents the purely separable approximation, $T_S^{(0)}$. The second-order approximation $T_\Delta^{(2)}$ to ΔT, given by Eq. (4.2), is shown by the dashed curves. When the latter is added to the former, the error $\Delta T_S^{(2)}$, illustrated by the dash-dotted curve remains. The rank of the separable part of the potential is equal to 3 in both cases. The momentum k' is equal to 0.5 fm^{-1}.

to second order in ΔV. It is useful to note that $T_\Delta^{(2)}$ can also be expressed in terms of the same form factors which enter into the expression for $T_S^{(0)}$ as

$$T_\Delta^{(2)} = V + VG_0V - \sum_{s=1}^{S} |\chi_s\rangle(1+\gamma_s)\langle\chi_s| \ . \tag{4.2}$$

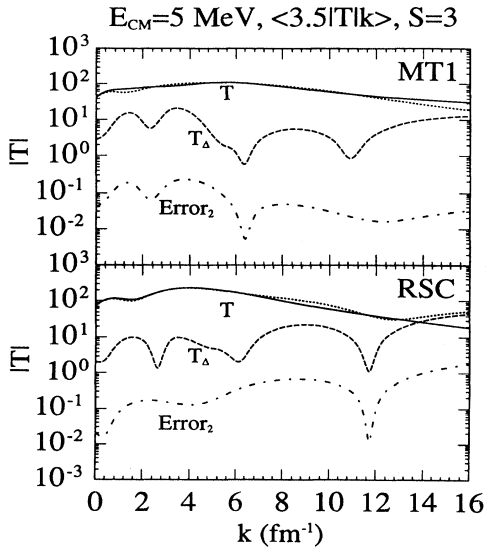

FIG. 5. Same as Fig. 4 for a value of $k'=3.5$ fm^{-1}.

The decomposition of T is then given by

$$T_S = T_S^{(0)} + T_\Delta^{(2)} + \Delta T_S^{(2)} \ , \tag{4.3}$$

where the error $\Delta T_s^{(2)}$ is given by Eq. (3.7b). The condition that $T_\Delta^{(2)}$ is small compared to $T_S^{(0)}$ and that the error $\Delta T^{(2)}$ be negligible can both be achieved by choosing the value of the rank S of the separable part sufficiently large, as will be illustrated in what follows.

The values of $T_\Delta^{(2)}$, and the corresponding error $\Delta T_S^{(2)}$, are illustrated in Figs. 4 and 5 for the MT-I and the RSC potentials. The solid lines show the absolute value of the exact T matrix, $|\langle k'|T|k\rangle|$, as a function of k, while the dotted lines show the rank-3 approximation $T_{S=3}^{(0)}$. It can be noted that for $k'=0.5$ cm^{-1} the RSC value of T has a zero near $k=8.5$ fm^{-1}, which is not present for the MT case. The larger the value of k or k', the larger is the corresponding error in the approximation of T by $T_{S=3}^{(0)}$. As S is increased from 3 to 4 the error decreases, especially at the high momenta. The perturbative term $T_\Delta^{(2)}$ is shown by the dashed lines in Figs. 4 and 5, where it is denoted as T_Δ. The error left over after $T_\Delta^{(2)}$ is added to $T_S^{(0)}$ is shown by the dash-dotted lines. It is equal to $\Delta T_S^{(2)}$. With a rank-3 approximation to V and T, the figures show that the perturbative term is approximately 1 order of magnitude smaller than T, for both the RSC and the MT cases.

V. SUMMARY AND CONCLUSIONS

A hybrid representation for the nucleon-nucleon T matrix in terms of a separable and a nonseparable piece has been presented in this study, following a quasiparticle method [1]. The separable piece is given in terms of positive-energy Weinberg eigenstates defined for the full potential and the corresponding nonseparable remainder is rigorously equal to the T matrix T_Δ for the potential ΔV. (The latter represents the difference between the full potential and the separable part.) This unique property that $\Delta T = T_\Delta$ is rigorously valid only when the form factors for the separable part are defined in terms of positive-energy Weinberg states. In the present application we define ΔV to be sufficiently small so that the Lippmann-Schwinger equation for T_Δ can be solved by perturbative iterations in ΔV. The iterations converge rapidly because ΔV is such that the eigenvalues of $G_0\Delta V$ are well inside the unit circle.

The numerical behavior of the method is examined for the Reid soft core and the Malfliet-Tjon potentials for the $^1S_0$ nucleon-nucleon state, for a c.m. energy of 5 MeV. As long as rank of the separable part is large enough so as to include the states for which the Weinberg eigenvalues lie outside the unit circle, then for each additional order included in the perturbative expansion the accuracy of the result increases by an order of magnitude, as is illustrated in Figs. 2 and 3. This property permits one to restrict the rank of the separable part to a low value, and yet achieve high accuracy for the overall result. For the RSC and the MT-I potentials the number of such Weinberg states is 3 and 2, respectively. With only one additional term included in the separable representation, the

second-order perturbative term is more than 1 order of magnitude smaller than the full value of the T matrix, and the remaining error is 3 orders of magnitude smaller than T, for momenta less than 6 fm^{-1}, as is illustrated in Figs. 4 and 5.

We expect that our study will have a number of useful applications for those cases where an expansion in terms of Weinberg states is desirable, especially if one takes into consideration that a simple way has been found [20] for calculating the Weinberg states entirely in momentum space, which avoids the complications due to the Green's function singularity.

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APPENDIX

It will now be shown that if the separable part V_S of the potential V is defined in terms of PEWS form factors, and if T_S is the corresponding separable T matrix, then the difference ΔT between the full T matrix and T_S is itself the T matrix which corresponds to ΔV.

If one starts with the Lippmann-Schwinger equation for the full T matrix, $T=V+VG_0T$, replaces T by $T_S+\Delta T$, and V by $V_S+\Delta V$, and makes use of $T_S=V_S+V_SG_0T_S$, one obtains

$$\Delta T=\Delta V+\Delta VG_0\Delta T+\Delta VG_0T_S+V_SG_0\Delta T\ . \quad \text{(A1)}$$

According to Eq. (3.2), T_S is formed from Weinberg states χ which lie in the space $s=1\cdots S$, while ΔV is contained in the complementary space $t=S+1,\cdots\infty$, as can be seen from Eq. (2.9). Further, in view of Eqs. (2.6) and (2.7), one has

$$\langle\chi_t|G_0|\chi_s\rangle=\gamma_s\delta_{ts}\ . \quad \text{(A2)}$$

Therefore, ΔV is orthogonal to G_0T_S and hence the third term in Eq. (A1) vanishes. The fourth term also vanishes because ΔT, in view of Eq. (2.9b) is orthogonal to V_SG_0. Thus, ΔT obeys the same equation as T_Δ, Eq. (2.4), and hence $\Delta T=T_\Delta$.

Because T_SG_0 is also orthogonal to ΔV, and hence to τ_Δ, the considerations above also show that the distortion factor $(1+T_SG_0)$ has no effect when acting from the left on ΔV, and hence on τ_Δ. Similarly for $(1+G_0T_S)$ acting on ΔV or τ_Δ from the right.

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