Reply to "Comment on 'Comparison of realistic and symmetry-determined S and D pairs for ^{156}Gd' "

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I reply to the preceding Comment.

The preceding Comment [1] by Chen *et al.* discusses two separate tests [2,3] of the so-called "fermion dynamical symmetry model" [4,5] (FDSM), one for an isospin-invariant version [6–8], in the *sd* shell [2] (see also Ref. [9]), the other [3] using an HFB calculation [10] for ^{156}Gd. In both cases, by directly examining the assumption that ". . . coherent S and D pairs are the most important building blocks in low-energy collective states," [11], the FDSM was found to have no microscopic justification. Chen *et al.* [1] seek to contest this conclusion; in this reply their comments are answered.

The opinions expressed by Chen *et al.* [1] may be summarized as follows.

1. The basic assumption of the FDSM, that low-energy collective shell model states fall into a particular subspace [11,12] (a statement concerning wave functions), should not be investigated directly. The existence of effective operators capable of reproducing data should be the criterion by which the validity of this supposedly microscopic [13] model is assessed.

2. A comparison of pairs gives no indication of the overlap of many-pair states.

3. The $\delta+Q\cdot Q$ Hamiltonian is not a sufficiently reasonable approximation to a realistic full shell model effective interaction, even for low-energy nuclear structure physics. In addition, the particular mean field solution of this [10] may not be an accurate approximation.

4. The possible FDSM symmetries in the *sd* shell are expected not to be realized there.

Opinion 1 applies to both investigations [2,3] and is the central point, referring to any such future test. Opinions 2 and 3 apply to the ^{156}Gd test [3] only, opinion 4 to the *sd* shell investigation [2].

Each point will be answered in turn.

1. The existence of effective operators is the appropriate criterion for the validity of a *phenomenological* model; it cannot be sufficient for a supposedly realistic *microscopic* model, whose necessarily stronger statement should satisfy some additional criteria. For the FDSM, the microscopic structure employs the usual full shell model valence spaces, for which a realistic description is widely accepted as involving only a small selection of known effective interactions. (A brief discussion on the range of Hamiltonians generally considered as realistic, in this context of low-energy levels, is given in Sec. III.) The defining assumption is then that the resulting eigenfunctions can be at least approximately constructed from particular symmetry-determined S and D pairs [5] vis. "... coherent S and D pairs are the most important building blocks in low-energy collective states" [11], "The FDSM . . . is, in fact, a prescription for solution of the shell model through a radical symmetry dictated truncation" [12], ". . . fully microscopic connections between these dynamical symmetries and the underlying shell structure" [13], and ". . . any dynamical symmetries relevant to nuclear structure should manifest themselves directly from the fermion degrees of freedom without explicitly introducing bosons" [14]. That the FDSM is intended for use with its own effective interactions [15], as in fact are all models whether or not they have a valid microscopic structure, does not allow the question of the microscopic validity to be evaded.

The results of Refs. [2,3] do not concern the weaker, *phenomenological* interpretation of the model, which in fact appears to be that implied in the original presentation of the FDSM as the "schematic monopole and quadrupole pairing model" of Ginocchio [16], involving only the fitting of data with *ad hoc* effective operators. The microscopic structure central to the later, supposedly realistic, interpretation [11–13] plays no role in determining these operators (in contrast to a derivation from those of the underlying shell model) and so cannot be supported by such an approach. Indeed, the phenomenological similarity to the successful interacting boson model [17] (IBM) is alone sufficient to ensure that suitable effective operators can be found.

The approach under discussion concerns only the strong, *microscopic* interpretation [11–13]. As noted above, this view of the FDSM, like other truncation schemes for the full shell model, assumes that a particular symmetry determined subspace includes the important components of collective states; if the symmetry determined states are *not* similar to the eigenstates of a realistic effective Hamiltonian appropriate to that shell model space, then the model does *not* provide a realistic description of the latter. Thus overlaps of low-energy eigenstates and the symmetry determined subspace quantify the validity of this assumption directly. If the overlaps between the assumed states and the full eigenvectors are low, then the former must instead give other, more highly excited levels, and it is the presumably different "physics" of these that is incorporated by the assumed degrees of freedom.

Chen *et al.* [1] state that overlaps between different spaces are meaningless. However, in the context of shell model truncation schemes such as the FDSM [12], the designation "different" is misleading; the relation between the FDSM and full shell model spaces is that the former is a *subspace* of the latter. Moreover, as discussed above, the supposed sufficiency of this subspace for describing collective states is itself the central tenet [11] of the microscopic interpretation of the FDSM.

It should be stressed that wave functions are of interest not only as a measure of any symmetry based understanding of the numerically complex shell model description of collective motion, but also, and perhaps more importantly, as the means (together with the presumed known operators appropriate for the enveloping full shell model) toward the construction of effective interactions necessary for the full implementation of the model. In cases where a given subspace wholly contains the eigenstates of the full shell model, the operators of the latter can simply be used unchanged. If the overlaps are reasonable but not unity, then perturbation theory can be used to renormalize these operators to give the effective versions appropriate to the truncated space. However, if the overlaps are low, then a new problem of drastic renormalization or even mapping is encountered, for operators obtained from a direct procedure would be appropriate for states other than those to which they would be applied, and any numerical agreement would then be entirely fortuitous. (It should be noted that in cases where perturbative renormalization is employed, such as the Kuo-Brown shell model interactions [18], the results are only applied to those levels for which the model wave functions are approximately valid, and not to those where the overlap is low. For instance, the *sd* shell interaction is not appropriate for levels that are dominated by the non-*sd* shell states [19,20], even though the overlap with *sd* shell states is nonzero [21]). This latter situation is precisely what the supposed direct "microscopic connections" [12] of the FDSM are intended to avoid [14]; unfortunately, results to date [2,3,9] show the connections to be tenuous, and the FDSM to be firmly in that final category. In other words, although there is indeed no need for a mapping between the shell model eigenstates and bosons, as there is for the IBM, there must instead be a mapping between the former and the FDSM pair states, since the two do not coincide.

In cases where the direct analysis of shell model eigenfunctions is impracticable, analyses of realistic full shell model effective interactions themselves provide an alternative and economical method for assessing the validity of symmetry schemes. It should be noted that approximate conservation of a symmetry is not necessarily a sufficient criterion; in addition to being decoupled, the chosen states must be significantly lower in the spectrum. The methods of statistical spectroscopy appear ideally suited to such an approach [22,23].

The general acceptance of the above is confirmed by a survey through existing literature. Investigations into the goodness of symmetries, or other kinematically determined constructions, for microscopic models invariably make reference to a realistic effective interaction appropriate for the full shell model, using either the interaction itself or its eigenfunctions [23]. Direct analyses using eigenfunctions have been performed where practicable; examples are SU(4) [24,25], pseudo-SU(3)[26,27], seniority [28], the OAI interpretation of the IBM [29,30], and the shell model approximation itself [21,31]. Investigation using the analysis of Hamiltonians has been performed for SU(3), pseudo-SU(3), and SU(4) [23,32,33]. In contrast, although the FDSM is a supposedly microscopic model, Chen *et al.* [1] suggest acceptance on the grounds of only phenomenological criteria.

2. That pair overlaps give only an approximate measure for the usefulness of a many-pair wave function is clearly stated in Ref. [3]. However, the example given by Chen *et al.* [1] to suggest that there is no relation is not really appropriate since its critical features are that two different L-pairs are used in one representation, and a sum over all possible L values is used in the other; neither situation is present in the case of practical interest. It is a rather contrived and unusual example. In addition, although the well-known nonorthogonality of different many-pair states [29,1] cannot be ignored, it is usually not large for the shell model spaces of interest.

Indeed, pair overlaps have been widely used to give some indication of the many-body result [30]. (The simple overlap between FDSM and SDI pairs has also been calculated [34].) An explicit example of the likely relation can be found in the *sd*-shell study [2,9], for which the D pair overlaps are around 40%, and the ^{24}Mg (four pairs) $J=4$ overlaps are around 20% for the ground band, 3% for the γ band, and 0.6% for the β band. In comparison, the ^{156}Gd D pair overlaps are 24.6% for the protons, and 9.4% for the neutrons.

A simple way of resolving this uncertainty in the present case [3] would be to compare the energies calculated for condensates of the FDSM pairs with those for the optimized pairs [10]. If the energies are similar, it would be evidence that the FDSM pairs are well suited for describing the low-energy levels; if the former are much higher, it would be evidence to counter that claim. Such a study should be very easy to perform.

It is indeed assumed that the particular mean field solution described in Ref. [10] is an accurate approximation to that of a diagonalization within the full shell model space.

3. Clearly, several candidates for "realistic" effective shell model interactions (that is, supported, to some extent, by some basic "physics") are available. In particular there are those derived from the underlying process of meson exchange [35], or from observed nucleon scattering [18,36], which can be perturbatively renormalized to account for small admixtures to the assumed wave functions; others are caricatures embodying qualitative features, such as a δ interaction as an approximation to the known short-range nature of the nucleon-nucleon interaction [37,38], or $-Q\cdot Q$ (where Q is r^2Y_2, not the generally different quadrupole operator of the FDSM) to reflect the known quadrupole deformation of the nuclear surface [39]. In contrast, the interactions necessary for the FDSM classifications to be realized have no motivation apart from this goal. These may or may not be a reasonable approximation to some or all of the "physi-

cal" interactions; the resolution of this question requires some direct comparison, not unconstrained fitting within a model space.

Rather than perform *no* comparisons because of not knowing which *one* realistic interaction to choose, it would be better to compare against a selection to delineate the range of possible conclusions. Particularly in the event of comparable degrees of similarity, the problem of which one to choose would become entirely unimportant.

Indeed, although the several interactions available are certainly different in some respects, their low-energy eigenfunctions, which are the important aspect in this context (the FDSM S,D condensates are not used to describe the higher energy states [5]), are quite possibly similar. An explicit example of this is provided by ^{24}Mg in the *sd* shell [40], where the lowest levels calculated with the bare or renormalized Kuo interactions, which are themselves almost identical with squared overlaps of about 99%, are close to those arising from both a $-Q\cdot Q$ Hamiltonian [the SU(3) scheme], with squared overlaps about 70%, and from the MSDI interaction [38], to about 85%.

4. The existence of more than one possible symmetry is not a reason for none to be conserved. (In passing, it is noted that in Ref. [4] the possible *sd* shell symmetries are *all* considered.) That the authors' opinion is unreasonable can be appreciated by noting that nearly *all* spaces allow more than one possible symmetry, for instance, the shell model can have *jj* or *L-S* coupling, the *i*-active FDSM can have SU(4), SU(2)$\times$Sp(4), and SO(7), and the FDSM in an *sd* or pseudo-*sd* shell can indeed have Sp(6), SO(8), or SU(3)$\times$SU(4) [4]; in only the latter case, and only recently [1], has the plurality been used to suggest that *no* symmetries are in fact expected to be realized.

The most suitable phenomenological picture emerged from the *k*-active scheme, which provides a reasonable qualitative description of the observed spectra [7]; however, none of the FDSM symmetries were found to be valid at the microscopic level [2,9]. (Chen *et al.* [1] repeat the result [9] that SU(3)$\times$SU(4) was found to be good for ^{20}Ne, but omit to mention the severe breakdown for the other nuclei investigated, ^{22}Ne and ^{24}Mg.)

To summarize: The results [2,3] discussed in this Comment do not refer to the question of whether the FDSM *SD* subspace can be used for a merely *phenomenological* description, using *ad hoc* effective operators, of some collective levels [5]. Rather, the issue addressed is whether the distinguishing *microscopic* feature of the FDSM, the assumption that low-energy collective shell model states can be constructed from particular symmetry-determined S and D "building blocks" [11], an assumption concerning wave functions, is valid. Thus it is this means of truncating the full shell model space [12] that should be tested with respect to a realistic effective interaction appropriate for the full shell model, either directly by analyzing the eigenstates of interest, or approximately by statistical analysis of the interaction itself. *If* the FDSM states were close to those favored by realistic nuclear interactions, then a new conserved symmetry would be revealed, as well as a means of quantitatively describing low-energy levels by directly taking over known operators, perhaps with some renormalization, from the full shell model to a tractable space. If, in contrast but as present results [2,3] indicate, these symmetry-determined states do not well approximate low-energy levels but instead then superpositions of other, higher energy states, it would be better (since it is not *intended* to describe those higher energy states) to invest in an alternative approach. All other symmetry or otherwise kinematically based truncation schemes, with which the FDSM is in implicit competition, have been subjected to such tests [23–33], the results usually being satisfactory. Meaningful information on the *microscopic* validity of the FDSM will only be obtained if it is treated in a similar way, as in Refs. [2,3]; the mere existence of effective operators, suggested as a criterion by Chen *et al.* [1], can only give information on the phenomenological validity, which is almost ensured by the similarity to the successful IBM.

Some simple tests for estimating the probable goodness of the FDSM symmetries, without resorting to full shell model calculations, have been suggested. The several full shell model effective interactions that are believed to be, to some extent, realistic can be analyzed using the methods of statistical spectroscopy [21,22]. With particular reference to the ^{156}Gd calculation [10,3], energies calculated using the FDSM pairs could be compared to those for the optimized pairs.

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