PHYSICAL REVIEW C VOLUME 44, NUMBER 1 JULY 1991

Comment on "Comparison of realistic and symmetry-determined S and D pairs for ^{156}Gd"

Jin-Quan Chen,[1] Da Hsuan Feng,[1] Mike W. Guidry,[2,3] and Cheng-Li Wu[1,2,3]
[1]*Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104*
[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37990*
[3]*Joint Institute of Heavy-Ion Research, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831*
(Received 4 October 1990)

This paper comments on the arguments of the paper by P. Halse [Phys. Rev. C **39**, 1104 (1989)].

Recently, Halse published a paper entitled "Comparison of Realistic and Symmetry-Determined S and D Pairs for ^{156}Gd [1]. The main points of this paper are (1) the S and D pair wave functions for ^{156}Gd were *extracted* from a Hartree-Fock-Bogoliubov (HFB) type calculation [2]; in his opinion, these pairs are realistic; (2) overlaps of these pair of wave functions with the various symmetry-dictated fermion S and D pairs were computed. From the overlaps and the "realistic" nature of the HFB-type S and D pairs, the author concluded that "there is evidence for the validity of a pseudospin $\frac{1}{2}$ decomposition (i.e., pseudo-SU(3) scheme [3])" and that ". . . the realistic pairs are not well described by the other symmetries discussed."

By "the other symmetries," the author meant primarily those obtained from the fermion dynamical symmetry model (FDSM) [4]. The author concluded from his computed pair overlaps that the symmetry-dictated S and D pairs of the FDSM for heavy nuclei (rare earths and beyond) are *woefully inadequate,* thereby casting serious doubt on the validity of the model.

We suggest that Halse's paper is flawed at a fundamental level, thus rendering the conclusions essentially irrelevant to the important discussion of the microscopic validity of these theories. The following are our specific comments.

(1) *The overlap for differently truncated many-body wave functions is not a meaningful quantity to assess the validity of a microscopic model.* According to many-body theory, it is well known that for two different model spaces, the wave functions may be quite different. Yet, by properly choosing effective operators for the various physical quantities in the respective spaces, two truncation schemes can meaningfully describe the same system irrespective of how little their wave functions overlap (except for states which are completely out of the model space). Thus, it is our opinion that little useful information can be extracted from the overlap of wave functions in different model spaces. After all, for the FDSM, the chosen model space is a severely truncated valence (k-i) space, while the ^{156}Gd HFB calculation utilizes a mean-field approximation within the valence shells. These are two quite *different* spaces, and therefore one should not be surprised if the wave function overlap is small.

(2) *To calculate a pair overlap is even less meaningful.* In Ref. [1], the author has computed the overlap between the S (or D) pair as defined in Ref. [2], and the symmetry-dictated S (or D) pair of the FDSM. For fermions, having a large pair overlap by no means implies that the corresponding many-body wave function overlap must also be large (as we have previously stressed the many-body wave function overlap *itself* is already not a meaningful quantity if the two wave functions belong to different model spaces). In fact, the many-body wave function overlap may even vanish since one can construct orthogonal many-body wave functions from the *same* S and D pairs. Likewise, it is not inconceivable that while the many-body wave function overlap is unity, the pair overlap may be zero. This is because one is always free to recouple the fermion pairs by a unitary transformation in a many-body wave function. This point can be transparently illustrated by the following "trivial" example. Consider a wave function for four identical particles $F_1=|(j_1,j_1)L_a(j_2j_2)L_b;J\rangle$, in the standard angular momentum coupling notation. By a unitary transformation, this can be rewritten as

$$F_2=-\sum \langle (j_1,j_2)L_1(j_1,j_2)L_2;J|(j_1,j_1)L_a(j_2,j_2)L_b;J\rangle |(j_1,j_2)L_1(j_1,j_2)L_2;J\rangle \ , \tag{1}$$

where the

$$\langle (j_1,j_2)L_1(j_1,j_2)L_2;J|(j_1,j_1)L_a(j_2,j_2)L_b;J\rangle$$

are $9j$ coefficients. Obviously, the many-body wave function overlap is $\langle F_1|F_2\rangle=1$, and yet the pair overlaps of F_1 and F_2 are *zero* since $|(j_1,j_2)L\rangle$'s are orthogonal to $|(j_1,j_1)L\rangle$'s and $|(j_2,j_2)L\rangle$'s. This simple example vividly illustrates that *the pair overlap has little or no physical significance* and cannot be the guiding principle for validating a many-body wave function. This misconception is based on a failure to appreciate the difference between a system approximated by (structureless) bosons and the realistic nuclear many-body problem evaluated in a basis of *fermion* pairs.

(3) *Is there a realistic S or D pair?* As was discussed in (2), fermion wave functions can be expressed in terms of different pair structures. Hence, there is no inherent advantage in a many-body fermionic system to seek a realistic pair structure since they lack definite meaning. For example, the wave function for a state with two S pairs $|SS\rangle$; the overlaps with two D pairs, two G pairs ($\langle SS|DD\rangle$, $\langle SS|GG\rangle$) etc., are generally *nonvanishing.* This means that, to a certain extent, higher angular momentum correlations for a fermionic many-body wave function are always present, even though the wave function is constructed purely from the S (fermion) pairs. This is a very important difference between a fermion pair and a boson: when the fermion pairs are replaced directly by bosons, the overlaps between two s bosons and two higher angular momentum bosons (e.g., $\langle ss|dd\rangle$, $\langle ss|gg\rangle$) must vanish. Therefore, the higher angular momentum correlations for fermions associated with the exchange effect are lost by such a simple replacement.

(4) *What is a realistic interaction?* It was argued in Ref. [1] that the surface delta interaction plus the quadruple *n-p* interaction is a "realistic interaction." This statement requires close scrutiny. To be precise, as far as bare nucleon-nucleon interaction is concerned, this statement is certainly untrue; potentials on the market like the Paris potential is far more realistic. On the other hand, for effective interactions, it is well known that different truncation schemes require different effective interactions and therefore any interaction which *matches* a given model space can be considered as effective or "realistic." In this sense, even the bare nucleon-nucleon interaction, which should only be used for the exact, and therefore, infinite Hilbert space, is unrealistic if the space has been truncated. Thus, it is simply a *misconception* to claim that one effective interaction is more realistic than another without careful study of the corresponding model spaces. In this context, although the pairing plus quadruple interaction has been extensively used for many studies in the heavy-mass region, it is at best a reasonable effective interaction for some limited regions (say vibrational) or for other approaches (say RPA or various mean-field approximations in which the Hilbert space is different from the spherical shell model). It should not, and cannot, be regarded as the only "realistic interaction" to be used everywhere in nuclear structure.

(5) *What constitutes a realistic nuclear structure calculation?* One of the central themes of Ref. [1] is that the HFB calculation (in this case for ^{156}Gd) is a "realistic calculation." With this criteria, Halse proceeded to ascertain the *correctness* of other theories by comparing with the "truth" (via the pair overlap). Hence, it may be appropriate at this point to examine what one means by a "realistic" calculation in nuclear structure physics. From the outset, it should be stressed that we find the results of Ref. [2] reasonable as far as HFB calculations are concerned. These authors did not employ the term "realistic" to describe their results.

Presently, there are two main microscopic approaches in nuclear structure physics which may be considered as realistic: one is the spherical shell model, the other the HFB method. The shell model simplifies the many-body problem by truncating the (infinite) Hilbert space to a tractable model space, and then attempts to establish the appropriate effective operators and interactions within this space. This approach requires one to abandon the bare nucleon-nucleon interaction, and is now a standard approach for light systems [5]. For heavy nuclei, the large dimensionality of the model space renders intractable the straightforward application of this approach. The HFB approach avoids the large dimensionality problem by a mean-field approximation. However, to obtain a self-consistent mean field without any space truncation is also nontrivial, therefore some form of effective interaction is still a necessity in practical work. In spite of the fact that two-body or many-body correlations may not be fully taken into account, the success of the HFB method in studying deformed nuclei are significant. Recently, great strides have been made in carrying out such calculations [6]. Hence, these two approaches, shell model and HFB, are quite complementary in nature.

The main difficulty of the HFB approach is how to project the states with good particle number and angular momentum. Although in principle such projections can be carried out, in practice this is complicated and time consuming. Hence, for many practical calculations, additional approximations were introduced. Typical approximations include the following:

(i) use a phenomenological deformed potential with average pairing, like the pairing plus Nilsson, or pairing plus Woods-Saxon potential, to replace the self-consistent mean field (which, in principle, should only be obtained by solving self-consistently the HFB equation, starting from an effective nucleon-nucleon interaction without *a priori* deformation;

(ii) use a cranked HFB scheme, or projection after variation instead of the exact projection before variation;

(iii) limit the Hilbert space to a shell model space. With these additional approximations, such calculations can no longer be viewed as realistic HFB calculations. In particular, if the space has been truncated to the shell-model space as in (iii), then one deals essentially with a shell-model calculation with the diagonalization procedure replaced by a mean-field approximation. The ^{156}Gd HFB calculation [2] mentioned by Halse belongs to this category.

(6) *What about the s-d shell?* In Ref. [1], a comment was made about an earlier test of the FDSM in the *s-d* shell [7] in which the conclusion was "negative." In Ref. [7] the overlaps between the FDSM many-body wave functions and Wildenthal *s-d* shell wave functions were computed [5]. the FDSM wave functions were obtained by assuming the k-active [Sp(6) symmetry], i-active [SO(8) symmetry], and $SU^k(3)\times SO^i(6)$ symmetry, respectively, and the FDSM effective interactions were determined by fitting the spectra of some selected *s-d* shell nuclei. Since neutrons and protons are in the same shell, isospin degrees of freedom were also taken into account. Generally speaking, the FDSM symmetry $SU^k(3)\times SO^i(6)$ was found to be in better accord with the data than the FDSM SO(8) or Sp(6) symmetries. For ex-

ample, a reasonable description of well-defined nuclei near ^{20}Ne was found [8] for the former symmetry (in this paper, Halse chose not to refer to this as an FDSM calculation; however, it had already been established in Ref. [4] that $SU^k(3)\times SO^i(6)$ was a possible FDSM symmetry in the *s-d* shell.) However, less attention was paid to the fitting of data. Instead, Ref. [7] concentrated on the overlaps of the wave functions. Since the overlaps are small for these wave functions, Halse concluded that this undermined the microscopic basis of the FDSM. In view of what we have said earlier in comment (1), such a conclusion seems to have little justification or physical meaning. The only relevant comparison between different model spaces is between *matrix elements,* not wave functions.

We wish to point out here that *none* of the FDSM dynamical symmetries is expected to fit the data well in the *s-d* shell. It is a prediction of the FDSM that both k and i are active in the *s-d* shell; therefore, there may be no reason to prefer one to the other. A logical conclusion is to consider k and i active pairs simultaneously; this will immediately lead to the result that the smallest closed algebra to include both k and i active pairs is SO(24), which is simply the entire *s-d* shell. Following this line of argument, it is consistent with the FDSM to doubt that a smaller subspace in the *s-d* shell is utilized by nature to describe the physics.

Finally, we would like to point out that the $SU^k(3)\times SO^i(6)$ symmetry is the only dynamical symmetry in the *s-d* shell which involves both k and i active pairs. However, it exists only when pairing can be neglected in the Hamiltonian, which is more likely to be valid for the well-deformed nuclei. This would provide a plausible physical reason for why in Halse's FDSM calculation the $SU^k(3)\times SO^i(6)$ dynamical symmetry limit produces a reasonable fit for the deformed nuclei.

This work was supported in part by the United States National Science Foundation and the United States Department of Energy.

[1] P. Halse, Phys. Rev. C **39**, 1104 (1989).
[2] L. C. deWinters, N. R. Walet, P. J. Brussard, K. Allert, and A. E. L. Dieperink, Phys. Lett. B **179**, 322 (1986).
[3] K. T. Hecht and A. Adler, Nucl. Phys. **A137**, 129 (1969); A. Arima, K. Shimizu, and M. Harvey, Phys. Lett. **30B**, 517 (1969).
[4] C.-L. Wu, D. H. Feng, X.-G. Chen, J.-Q. Chen, and M. W. Guidry, Phys. Lett. **168B**, 313 (1986); *ibid.* Phys. Rev. C **36**, 1157 (1987).
[5] See, for example, B. H. Wildenthal, in *Nuclear Shell Model,* edited by M. Vallieres and B. H. Wildenthal (World Scientific, Singapore, 1985).
[6] A. Faessler, R. Hitlin, and K. Devi, Phys. Lett. **61B**, 133 (1976); J. L. Egido, P. Ring, and H. Mang, Nucl. Phys. **A451**, 77 (1986); A. Goodman, Phys. Rev. C **33**, 2212 (1986); K. W. Schmid and F. Grummer, Rep. Prog. Phys. **50**, 731 (1987); K. W. Schmid, F. Grummer, M. Kyotoku, and A. Faessler, Nucl. Phys. **A452**, 493 (1986).
[7] P. Halse, Phys. Lett. B **189**, 119 (1987).
[8] P. Halse and J. N. Ginocchio, Phys. Rev. C **36**, 2611 (1987).