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# Λ-hypernuclei magnetic moments in a relativistic model

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We calculate the magnetic moments of several hypernuclei using the $\sigma$-$\omega$ model with the addition of a tensor term to describe the $\omega$-$\Lambda$ coupling. We find that this change in the $\Lambda$ coupling induces a modification in the structure of the hyperon sector of the baryonic current and has implications on the structure of the backflow current for finite nuclei. It manifests itself more dramatically in those cases where the $\Lambda$ is sitting in $s$ states which are precisely the states more likely to be measured experimentally. Our results indicate that, if no large deviations from the Schmidt values are observed experimentally, this might as well indicate a success and not a pitfall of the relativistic approach.

During the past few years a number of papers in the literature[1–3] have emphasized the importance of measuring the magnetic moments of $\Lambda$ and $\Sigma$ hypernuclei. The basic motivation for this interest lies on the ability of the $\Lambda$ and $\Sigma$ hyperons to sit on the lowest (innermost) shell-model orbit, hence allowing us, by carrying out this measurement on mainly heavy nuclei, to probe hadron properties deep in the nuclear environment. Thus, one can study effects such as deconfinement,[4] changes in the mass[5] and size[6] of the nucleon in the nucleus, and also possible changes in the value of the Bohr magneton.[1] The result of this type of experiment will also include contributions from effects of purely nuclear origin such as the polarization of the core, the exchange of mesons between baryons, and effects of relativistic origin. If one expects to obtain serious indications of something beyond the standard baryon picture from these experiments then it is important to pin down the nuclear contributions as precisely as possible. Of those listed above, effects of relativistic origin based on the polarization of the negative-energy core have been shown to introduce rather large deviations with respect to the standard Schmidt values, at least in the case of the magnetic moments of $\Lambda$ hypernuclei. Thus, the existing two (to our knowledge) calculations in the literature (Refs. 2 and 3) predict similar and rather sizable corrections. In Ref. 2, for example, an 8% deviation for ${}^{41}_{\Lambda}$Ca and 12% for ${}^{209}_{\Lambda}$PB are predicted, whereas in Ref. 3, for a particular set of parameters ($L2$), the same deviation is predicted for ${}^{41}_{\Lambda}$Ca and a slightly lower one for ${}^{209}_{\Lambda}$Pb. In these calculations, the bulk of the corrections are due to the contribution of the core-response current, also known as the *backflow*, to the magnetic operator. This isoscalar contribution to the baryonic current has been shown to cancel the enhancement that the *nucleon*-valence particle current exhibits in relativistic mean-field calculations.[7,8] Thus, for the *nuclear* case, the enhanced nucleon current and the vacuum response add up to bring the calculation of magnetic moments into agreement with the Schmidt values.[8] For the case of a $\Lambda$ hyperon orbiting a nucleus, this cancellation is not expected to occur to the same extent since the core response depends on the coupling of the vector meson to the valence particle ($\Lambda$) and this coupling is anticipated[9] to be different for the $\Lambda$ than for the nucleons. In fact, since the $\Lambda$ is an isoscalar, the idea was advanced[2] that one could directly probe the core-response current by measuring the deviations of the magnetic moments of hypernuclei from the Schmidt values. One would than be testing an effect of *genuine* relativistic origin. We notice that the corrections we refer to above correspond to the case of a $\Lambda$ orbiting a closed-shell core of nucleons in an $s_{1/2}$ orbit. Values have also been presented for higher $\Lambda$ orbits but only those for the $s$ shell are of experimental relevance.

In this work we show that, even in the presence of a strong renormalizing medium current, the relativistic calculation of hypernuclear magnetic moments may still give results that come very close to the standard Schmidt values. The origin for this difference in the predictions—obtained within the same model—lies on the type of vertex used to describe the coupling of the $\omega$ meson to the hyperon. Thus, in addition, to the standard electric coupling, the inclusion of a tensor term to describe the $\omega\Lambda\Lambda$ vertex—whose rationale we discuss below— and a judicious choice for the values of the coupling constants is enough to cancel the, otherwise expected, strong-core corrections. We make these points clearer in the following.

Let us start from the mean-field-theory (MFT) Lagrangian density of nucleons and $\Lambda$ particles in the presence of scalar ($\sigma$) and vector fields ($\omega_\mu$),

$$\mathcal{L}_{\rm MFT}=\bar{\psi}_N[\gamma_\mu(i\partial^\mu-g_{\omega NN}\omega^\mu)-(M_N-g_{\sigma NN}\sigma)]\psi_N +\bar{\psi}_\Lambda[\gamma_\mu i\partial^\mu-(M_\Lambda-g_{\sigma\Lambda\Lambda}\sigma)]\psi_\Lambda+\mathcal{L}_{\omega\Lambda\Lambda} +\text{mesonic terms}\ , \quad (1)$$

 

and let us take, for $\mathcal{L}_{\omega\Lambda\Lambda}$, the form

$$\mathcal{L}_{\omega\Lambda\Lambda}=-g_{\omega\Lambda\Lambda}\bar{\psi}_{\Lambda}\gamma^{\mu}\omega_{\mu}\psi_{\Lambda}+\frac{f_{\omega\Lambda\Lambda}}{2M_{\Lambda}}\bar{\psi}_{\Lambda}\sigma^{\mu\nu}\psi_{\Lambda}\partial_{\nu}\omega_{\mu} . \quad (2)$$

The second (tensor) term in this expression is the natural extension to the standard coupling term (proportional to $\gamma_{\mu}$) and its introduction can be motivated by invoking the vector-dominance model (VDM), for example. However, in the quark picture to which we will often turn, this would require us to include the $\phi$ meson in order to preserve the OZI rule. Instead, here we adopt the ideas introduced in Ref. 10 and further elaborated on in Ref. 11. We invoke the quark model [which means using SU(6) relations, ideal mixing for the vector mesons and the OZI rule] to motivate the choice of Eq. (2). Thus, in the quark model (QM), one expects the spin of the $\Lambda$ to be carried exclusively by the strange quark which does not couple to the $\omega$ meson. Therefore, this vertex must be different from the $\omega$-nucleon vertex and this difference is taken into account by introducing the tensor term in (2).

As for the values of the coupling constants we use the following. For the electric coupling constant $g_{\omega\Lambda\Lambda}$, we take the ratio $g_{\omega\Lambda\Lambda}/g_{\omega NN}=2/3$, since in the QM the nucleon contains no $s$ quarks and the $\Lambda$ contains one. For the magnetic coupling $G_m$ ($G_m=g_{\omega\Lambda\Lambda}+f_{\omega\Lambda\Lambda}$), the QM predicts a value of $G_m=0$ (see Ref. 9). This result is obtained as above assuming ideal mixing for the $\omega$ and $\phi$ vector mesons, using SU(6) relations and a vanishing $\phi NN$ coupling. The same value for $G_m$, which will be relevant for our later arguments, can also be derived using a different, and perhaps more illustrative, argument in the spirit of the ideas in Ref. 10. Thus, we introduce the Gordon decomposition[12]

$$i\bar{\psi}_{\Lambda}\sigma^{\mu\nu}\psi_{\Lambda}\frac{(p'-p)_{\nu}}{2M_{\Lambda}}=\bar{\psi}_{\Lambda}\gamma^{\mu}\psi-\bar{\psi}_{\Lambda}\psi_{\Lambda}\frac{(p'-p)^{\mu}}{2M_{\Lambda}} \quad (3)$$

to transform the vertex of Eq. (2) to

$$\mathcal{L}_{\omega\Lambda\Lambda}=-(g_{\omega\Lambda\Lambda}+f_{\omega\Lambda\Lambda})\bar{\psi}_{\Lambda}\gamma^{\mu}\psi_{\Lambda}\omega_{\mu}+f_{\omega}\bar{\psi}_{\Lambda}\psi_{\Lambda}\frac{(p'+p)^{\mu}}{2M_{\Lambda}}\omega_{\mu} , \quad (4)$$

with $p$ and $p'$ the momenta of the initial and final $\Lambda$. In fact, the relation is only valid if the $\Lambda$ is on shell. Use could be made of a more accurate decomposition taking into account the meson fields in the Dirac equation but, for our purposes, the conclusions based on the above still hold. Now, since we have argued that the $\omega$ meson does not couple directly to the strange quark (which carries all the spin of the hyperon), then the magnetic coupling must be zero and consequently, $f_{\omega\Lambda\Lambda}=-g_{\omega\Lambda\Lambda}$, which is the result we presented above. In the VDM all the magnetic moment of the $\Lambda$ would come from the $\phi$ meson which couples only to the strange quark. In what follows, the coupling of the $\phi$ meson to the nucleus (and hence the meson itself) will be neglected—a valid assumption if the OZI rule is not strongly violated. With this choice of $f_{\omega\Lambda\Lambda}$, the vertex, Eq. (4) becomes

$$\mathcal{L}_{\omega\Lambda\Lambda}=-g_{\omega\Lambda\Lambda}\bar{\phi}_{\Lambda}\phi_{\Lambda}(p'+p)^{\mu}\omega_{\mu}/2M_{\Lambda} . \quad (5)$$

Notice that the spacelike vector field couples to the hyperon convection current exclusively. This is natural since we deliberately killed all spin dependence with our choice of coupling constants.

Two comments are in order. The first one relates to the scalar coupling to the hyperon $g_{\sigma\Lambda\Lambda}$. We use for its value the same argument as above and assume that the $\sigma$ meson couples only to the $u$ and $d$ quarks. In this case, $g_{\sigma\Lambda\Lambda}$ and $g_{\sigma NN}$ should also be in the ratio 2/3. The second comment addresses the coupling of the $\omega$ meson to the nucleons. Since we motivated the tensor term in (2) by resorting to the quark structure of the baryon, the question arises whether the same vertex should be used for the $\omega NN$ coupling. In this case the QM, and also the VDM, predict a coupling for the $\omega$-tensor term that relates to the isoscalar anomalous magnetic moment of the nucleon ($\kappa_s=-0.12$ nm) and is therefore small. Consequently, no tensor terms are considered for the meson-nucleon coupling in our calculations.

We now explore the implications that the structure of the Lagrangian (2) has on the determination of magnetic moments. For the nuclear matter case, Furnstahl and Serot[8] have shown that the *backflow* current has its origin in the spacelike vector field generated by the *valence particle*. The other generated fields (namely, the scalar and timelike vector) have vanishing matrix elements between all possible combinations of particle and hole (below the Fermi level and in the Dirac sea) states in the *static, uniform* case. Furthermore, the contribution of the core response to the baryonic current is proportional to the *valence-particle* current [see Eq. (4.16) in Ref. 8]. In the nucleon case this is the standard baryonic current ($\bar{\psi}\gamma_{\mu}\psi$) which contains the convection and the Dirac magnetization currents. For a valence $\Lambda$ particle, we have shown that the spacelike vector field couples only to the convection current, hence, core response to an extra $\Lambda$ will be proportional to the convection part exclusively ($\bar{\psi}_{\Lambda}\mathbf{k}/M_{\Lambda}\psi_{\Lambda}$). Whereas this subtle difference is irrelevant for nuclear matter, it turns out to be important for finite nuclei. Following Refs. 2 and 8, we derive the final expression for the total baryon currents in nuclear matter (per unit volume),

$$\mathbf{j}_B=\bar{\mathcal{U}}_{\Lambda}(\mathbf{t},\lambda)\frac{\mathbf{t}}{M_{\Lambda}}\mathcal{U}_{\Lambda}(\mathbf{t},\lambda)\left[1-\alpha\frac{\lambda_{\omega}\rho_N/E^*_{k_F}}{1+\lambda_{\omega}\rho_N/E^*_{k_F}}\right] , \quad (6)$$

where $\lambda_{\omega}=g^2_{\omega NN}/m^2_{\omega}$ and $\alpha=g_{\omega\Lambda\Lambda}/g_{\omega NN}$. In Eq. (6), $\rho_N=2k_F^3/3\pi^2$ and $E_{k_F}=(k_F^2+M_N^{*2})^{1/2}$. To calculate magnetic moments, we adopt the effective electromagnetic current $\mathbf{J}$ of Ref. 13. Since the $\Lambda$ carries no electric charge, there is no convection current contribution from the valence particle and we only need to take into account the anomalous magnetic moment ($\mu_{\Lambda}^{\mathrm{an}}=-0.613\mu_N$). The response of the core gives a convection contribution of the form

$$\langle \mathbf{j}_b(\mathbf{x})\rangle=-\frac{\alpha}{2}\frac{1}{M_{\Lambda}}\bar{\psi}_{\Lambda}\frac{\nabla}{i}\psi_{\Lambda}\left[1+\frac{E^*_{k_F}}{\lambda_{\omega}\rho_N(\mathbf{x})}\right]^{-1} , \quad (7)$$

where $\psi_\Lambda$ now describes the Hartree solutions for a closed-shell nucleus,

$$\psi_\Lambda = \begin{bmatrix} F_{lj}\Phi_{\kappa,m} \\ iG_{lj}\Phi_{-\kappa,m} \end{bmatrix} \tag{8}$$

and where we made use of the local-density approximation (LDA) to incorporate the results of nuclear matter [Eq. (6)] to the finite nucleus case. Using as our definition of magnetic moment the expression[14]

$$|\mu| = \lim_{q\to 0} \frac{2M_N}{iq} \langle \bar{\psi}_{\lambda,\uparrow} e^{-iqy} J_x(0) \psi_{\lambda,\downarrow} \rangle = \mu_{an} + \mu_b \ , \tag{9}$$

where $\sigma \| \hat{\mathbf{z}}$ and $\mathbf{q} \| \hat{\mathbf{y}}$, we obtain, for the anomalous contribution,

$$\mu_{an} = 2\mu_\Lambda j \lambda_\kappa \int r^2 dr \left[ \frac{F_{lj}^2(r)}{2l_\kappa+1} + \frac{G_{lj}^2(r)}{2l_{-\kappa}+1} \right] \tag{10}$$

and for the core-response contribution (backflow),

$$\mu_b = -\frac{j}{2} \int r^2 dr (F_{lj}^2 - G_{lj}^2) B_f(r) + \lambda_\kappa \frac{j}{2} \int r^2 dr \left[ \frac{F_{lj}^2(r)}{2l_\kappa+1} + \frac{G_{lj}^2(r)}{2l_{-\kappa}+1} \right] B_f(r) \ . \tag{11}$$

In Eqs. (10) and (11), $j$ is the total angular momentum of the $\Lambda$ orbital ($j = l \pm \frac{1}{2}$) and $\kappa = (l-j)(2j+1)$. Also,

$$\lambda_\kappa = \begin{cases} 1 & \text{if } \kappa < 0 \ , \\ -1 & \text{if } \kappa > 0 \end{cases}$$

and

$$l_\kappa = \begin{cases} \kappa & \text{if } \kappa > 0 \ , \\ -(\kappa+1) & \text{if } \kappa < 0 \ . \end{cases}$$

The function $B_f(r)$ is given by

$$B_f(r) = \alpha \left[ 1 + \frac{E_{k_F}^*}{\lambda_\omega \rho_N(r)} \right]^{-1} \frac{M_N}{M_\Lambda} \ . \tag{12}$$

For the calculations presented here, the $\Lambda$ wave functions were obtained using the global set of parameters for the $\Lambda$-nucleus potential reported in Ref. 11. Thus, the Dirac equation was solved assuming Woods-Saxon shapes for the scalar and vector potentials with depths of $-268.6$ and $238.5$ MeV, respectively. Both radii were taken to be of the form $R = r_0(A) A^{1/3}$ with $r_0(A)$ given by $r_0(A) = 1.19 - 0.45 A^{-2/3}$. The nucleon-nucleus potentials were accordingly scaled to satisfy the relationship

$$g_{\omega NN}/g_{\omega\Lambda\Lambda} = g_{\sigma NN}/g_{\sigma\Lambda\Lambda} = 3/2$$

TABLE I. Magnetic moments of $\Lambda$ hypernuclei. All units are in Bohr magnetons. The blanks in our results correspond to states that are unbound. The data taken from Ref. 3 correspond to the case in which the parameter set $L2$ is used (values missing from the table were not reported in the quoted reference).

| $\Lambda$ single-particle state | Ref. | Magnetic moment ${}^{13}_{\Lambda}$C | ${}^{17}_{\Lambda}$O | ${}^{41}_{\Lambda}$Ca | ${}^{91}_{\Lambda}$Zr |
|---|---|---|---|---|---|
| $1s_{1/2}$ | Ref. 2 | −0.650 | −0.648 | −0.665 | −0.676 |
| | Ref. 3 | −0.658 | −0.643 | −0.656 | −0.658 |
| | Schmidt | −0.613 | −0.613 | −0.613 | −0.613 |
| | This work | −0.611 | −0.611 | −0.611 | −0.612 |
| | $\mu_{an}$ | −0.611 | −0.611 | −0.611 | −0.612 |
| | $\mu_b$ | $1.6\times10^{-3}$ | $1.7\times10^{-3}$ | $2.0\times10^{-3}$ | $1.6\times10^{-3}$ |
| $1p_{3/2}$ | Ref. 2 | −0.633 | −0.644 | −0.690 | −0.725 |
| | Ref. 3 | | | −0.689 | −0.709 |
| | Schmidt | −0.613 | −0.613 | −0.613 | −0.613 |
| | This work | −0.647 | −0.655 | −0.684 | −0.700 |
| | $\mu_{an}$ | −0.612 | −0.612 | −0.611 | −0.612 |
| | $\mu_b$ | −0.035 | −0.044 | −0.073 | −0.089 |
| $1p_{1/2}$ | Ref. 2 | 0.190 | 0.179 | 0.163 | 0.164 |
| | Ref. 3 | | | 0.165 | 0.155 |
| | Schmidt | 0.204 | 0.204 | 0.204 | 0.204 |
| | This work | 0.187 | 0.180 | 0.158 | 0.147 |
| | $\mu_{an}$ | 0.206 | 0.206 | 0.207 | 0.207 |
| | $\mu_b$ | −0.019 | −0.026 | −0.049 | −0.060 |
| $1d_{5/2}$ | Ref. 2 | −0.616 | −0.616 | −0.681 | −0.792 |
| | Ref. 3 | | | −0.685 | −0.731 |
| | Schmidt | −0.613 | −0.613 | −0.613 | −0.613 |
| | This work | | | −0.709 | −0.768 |
| | $\mu_{an}$ | | | −0.612 | −0.612 |
| | $\mu_b$ | | | −0.097 | −0.157 |

so that their depths were −394.1 MeV for the scalar and 347.7 MeV for the vector. The fit of these Λ parameters to available experimental hypernuclear spectroscopic data has met with considerable success in reproducing binding energies and spin-orbit splittings.

In Table I we present the results of our calculation of magnetic moments for the hypernuclei, ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O, ${}^{41}_{\Lambda}$Ca, and ${}^{91}_{\Lambda}$Zr. We also include for comparison the nonrelativistic Schmidt values. First, we note that, for a Λ in an $s$ state, the backflow current is, for all practical purposes, negligible and the resulting magnetic moments come very close to the standard Schmidt values. This can be understood as follows. The structure of Eq. (11) shows that, for $s$ states ($\kappa=-1$, $\lambda_\kappa=1$, $l_\kappa=0$, and $l_{-\kappa}=1$), there is a cancellation between the two terms proportional to the squares of the upper components and only a residual contribution left, due to the small lower components. This cancellation is due to the fact that, with all spin dependence eliminated—except for that contained in the lower components—and with the Λ moving a spherically symmetric orbit, there is no preferred direction in space and hence no contribution coming from the backflow current. These results are different from those presented in Refs. 2 and 3 (see Table I), and they are a direct consequence of the coupling chosen for the Λ hyperon. Since the experimental measurement will involve hypernuclei with the Λ orbiting the lowest $s$ state, it is possible that the absence of large deviations from the Schmidt value may not indicate a failure of the relativistic approach. On the contrary, since the ideas adopted here have been shown to place the $\sigma$-$\omega$ model on more consistent grounds regarding its extension to the Λ hyperons, a missing deviation might as well indicate a success and not a failure of the model.

We include also in the table the results for the magnetic moments of hypernuclei with the Λ in higher orbitals. It is interesting to note in this latter case that, despite the different structure of the backflow current, the magnetic moments are all very close to one another. In fact, this is to be expected since the currents in the three cases are similar and only their particular spin structure makes their contribution for $s$ states different.

In summary, we have calculated the magnetic moments of several hypernuclei using the $\sigma$-$\omega$ model with the modifications for the Λ-meson coupling introduced in Ref. 10. The core response was calculated following the procedures of Refs. 2 and 8. We found that the change in the Λ coupling induces a modification in the structure of the hyperon sector of the baryonic current. This change also has implications on the structure of the backflow current for finite nuclei. It manifests itself more dramatically in those cases where the Λ is sitting in $s$ states which are precisely the states more likely to be measured experimentally. Our results indicate that, in case no large deviations from the Schmidt values are observed experimentally, this might as well signal a success and not a pitfall of the relativistic approach.

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