Microscopic description of alpha decay of deformed nuclei

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The alpha decay of deformed nuclei is studied in the framework of the shell model. It is found that the model is able to describe the clustering of the four nucleons that eventually constitute the alpha particle. The clustering process occurs on the deformed nuclear surface and it is induced by high-lying configurations. The absolute decay width of ^{222}Rn is calculated and good agreement with experimental data is obtained.

Alpha-decay processes are among the oldest branches of microscopic physics. Their analysis, the study, and interpretation of the rich amount of data provided by them, has been fundamental since the beginning of this century to build up modern physics. Yet, many questions still remain unanswered in the understanding of the mechanisms that induce the decay of the α cluster. Thus, it is not clear whether the Pauli principle acting between the constituent nucleons in the α particle and those in the daughter nucleons has any importance.[1–3] Another question which has been only recently partially clarified, and one which is relevant for this paper, is the role played by high-lying configurations in α decay.[3–5] The absolute values of α-decay widths increase by many orders of magnitude by including a large enough number of configurations in the calculation of the mother nucleus wave function. This is necessary because on the surface of the nucleus, where the α particle is formed, the continuum part of the single-particle representation (or very high-lying shells in a bound representation) is important. But even including up to 16 major harmonic-oscillator (h.o.) shells, the absolute decay width is smaller than the experimental one in some spherical nuclei.[3,4] This deficiency was ascribed to a deficient treatment of the continuum.[4] All these studies have mainly been restricted to spherical nuclei. In deformed nuclei, microscopic treatments have been hindered by the formidable task of computing the mother nucleus wave function (including high-lying configurations) in terms of a realistic (e.g., Woods-Saxon) single-particle representation. But, with the experience gained in the study of spherical nuclei and by using modern computers, it may be time to realize such a treatment. In this Brief Report we will describe the alpha-decay process in two steps. In the first step we study, within the framework of the shell model, the behavior of the four nucleons that eventually constitute the alpha particle. This includes their clustering on the nuclear surface. In the second step we describe the penetration of the already formed α particle through the Coulomb barrier by using the Wentzel-Kramers-Brillouin (WKB) approximation.[6] The alpha formation amplitude is

$$F_L(\mathbf{R})=\int d\xi_\alpha d\xi_A[\phi_\alpha(\xi_\alpha)\phi_A(\xi_A)Y_L(\hat{R})]^*_{J_BM_B}\times\phi_B(\xi_A,\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\ , \tag{1}$$

where ξ indicates internal coordinates, B (A) labels the mother (daughter) nucleus, and $\mathbf{r}_i$ is the coordinate of the nucleon i measured from the center of the nucleus B. For the intrinsic wave function of the α particle we use the standard Gaussian form[3] with size parameter $b=0.574$ fm^{-2}. We write the wave function of the mother nucleus as

$$\phi_B(\xi_B)=\sum_{\pi\nu}X(\pi\nu;B)[\phi^\pi_{A+2}(\xi_\pi)\phi^\nu_{A+2}(\xi_\nu)]_B\phi_A(\xi_A)\ , \tag{2}$$

where π (ν) labels the proton (neutron) degrees of freedom and ϕ_A (ξ_A) is the BCS vacuum. We assume axially symmetric nuclei. Therefore, the BCS vacuum can be labeled by K_A. For simplicity in all our derivations we will also assume $K=0$ bands, i.e., $K_A=K_B=0$ and ground-state transitions (i.e., $I_i=I_f=0$). The two-quasiparticle wave function in Eq. (2) is

$$\phi_{A+2}(\xi)=\sum_{\Omega_1<\Omega_2}v^{A+2}_{\Omega_1}u^A_{\Omega_2}\mathcal{A}[\varphi_{\Omega_1}(\mathbf{r}_1)\varphi_{\Omega_2}(\mathbf{r}_2)]\ , \tag{3}$$

where Ω_i labels the single-quasiparticle states and $\mathcal{A}$ is the antisymmetrization operator We use v^{A+2} instead of v^A in the hope that it represents an improvement of the pure BCS treatment. The potential that defines our single-particle representation $\{\varphi_\Omega\}$ has an axially symmetric deformed Woods-Saxon plus spin-orbit form.[7] Within the BCS approximation that we will use here, the sum in Eq. (2) contains only one term, namely, π (ν) labels the proton (neutron) two-quasiparticle state and $X(\pi\nu;B)=1$. Expanding the single-particle deformed wave functions in terms of spherical harmonic-oscillator wave functions, the formation amplitude becomes

$$F_0(\mathbf{R}) = \sum_{N_\nu L_\nu N_\pi L_\pi} \sum_{N_\alpha L_\alpha} G_\nu(00N_\nu L_\nu;L_\nu) G_\pi(00N_\pi L_\pi;L_\pi) \times \langle L_\nu 0 L_\pi 0 | L_\alpha 0\rangle \times \langle N_\nu L_\nu N_\pi L_\pi; L_\alpha | 00 N_\alpha L_\alpha; L_\alpha \rangle \times \phi_{N_\alpha L_\alpha 0}(\mathbf{R}) , \quad (4)$$

where α labels the quantum numbers of the α particle, G is the transformation coefficient from the individual nucleon coordinates to the center of mass and relative coordinates [it includes the occupation amplitudes of Eq. (3) and the corresponding coefficients of the expansion in the spherical basis], and ϕ is the harmonic-oscillator wave function. The rest of the notation is standard. All the quantum numbers in Eq. (4) are determined by the set of single-particle states. Within the WKB formalism[6,8] one readily finds the absolute value of the decay width to be

$$\Gamma(R) = \frac{\hbar}{T_{1/2}(R)} = \hbar v \left[\frac{R}{G_0(E,R)} \right]^2 \sum_l \exp\left[-\frac{2l(l+1)}{\chi} \left[\frac{\chi}{kR} - 1 \right]^{1/2} \right] \left| \sum_\Omega (-1)^\Omega \langle I_i K_i l - \Omega | I_f K_f \rangle \sum_{l'} K_{ll'}^\Omega(B) a_{l'\Omega}(R) \right|^2 , \quad (5)$$

where G_L is the asymptotic relative wave function (Gamow function), k is the wave number, $\hbar v = 279.05\sqrt{E/\mu}$, E is the α-particle kinetic energy in MeV, and μ is the reduced mass. The dimensionless quantity χ is $\chi = 5.76(Z-2)/\hbar v$. In this formalism the quadrupole deformation is separated from the rest. The quadrupole contribution is given by the matrix K, i.e.,

$$K_{ll'}^\Omega(B) = \int_0^\pi \Theta_{l\Omega}(\vartheta) e^{BP_2(\cos\vartheta)} \Theta_{l'\Omega}(\vartheta) \sin\vartheta \, d\vartheta , \quad (6)$$

where $\Theta_{l\Omega}(\vartheta)$ is the normalized ϑ-dependent function in the spherical harmonic $Y_{l\Omega}$ and P_2 is the quadrupole Legendre polynomial, while

$$B = \chi\beta_2 \left[0.397\,89 \frac{kR}{\chi} \left[1 - \frac{kR}{\chi} \right] \right]^{1/2} \left[0.8 - 0.4 \frac{kR}{\chi} \right] .$$

The contribution of the other values of β (β_λ with $\lambda \neq 2$) is given by the matrix $a_{l\Omega}$ in Eq. (5). It is

$$a_{l\Omega}(R) = \int_0^{2\pi} d\varphi \int_0^\pi \sin\vartheta \, d\vartheta \, Y_{l\Omega}^*(\vartheta\varphi) \Psi_1(R\vartheta\varphi) \quad (7a)$$

with

$$\Psi_1(R\vartheta\varphi) = F_0(R\vartheta\varphi) \exp\left[\chi \sum_{\substack{\lambda>0 \\ \lambda\neq 2}} \beta_\lambda \left\{ \left[\frac{R}{r_0} \left[1 - \frac{R}{r_0} \right] \right]^{1/2} - f_\lambda \right\} Y_{\lambda 0}(\vartheta) \right] , \quad (7b)$$

where $r_0 = 2.88(Z-2)/E$ and

$$f_\lambda = \frac{1.5}{2\lambda+1} \left[\frac{R}{r_0} \right]^\lambda \times \sum_{m=0}^{\lambda-1} \frac{(\lambda-1)!}{m!(\lambda-1-m)!} \frac{(r_0/R-1)^{m+1/2}}{(m+1/2)} . \quad (7c)$$

The expression for the partial decay width $\Gamma(R)$ thus obtained may be strongly dependent upon the distance R. This actually provides a test of the reliability of the formalism. If Γ is indeed strongly dependent upon R on the nuclear surface (where we assumed the validity of the shell model as well as of the semiclassical description), then the theory is incorrect.

The angular distribution of alpha emission is also given by (5) but without integrating on the angle ϑ in Eq. (7a).

We applied this formalism to the alpha decay of ^{222}Ra. The deformed Woods-Saxon potential was diagonalized with the so-called "universal choice" of parameters.[9] For the deformation parameters we used the values $\beta_2 = 0.119$ and $\beta_3 = 0.095$. We expanded the corresponding single-particle wave functions in terms of a h.o. basis with size parameter b as the one corresponding to the α particle. This allows one to perform all integrals analytically. With such a value of b, one may think that the convergence of the expansion of the ^{222}Ra wave function would be very poor. This is indeed the case for $r > 12$ fm. But, in the region around the nuclear surface the number of h.o. basis states needed is manageable. This is illustrated in Table I, where we present the alpha-particle formation probability calculated using up to $N = 18$ shells. The important feature of Table I is the strong increase of the formation probability u as a function of the number of configurations included in the calculation. The very large interference of levels, especially those with different parity, is responsible for this enhancement.[10] Just outside the touching point of the nuclear surfaces of the alpha particle and the daughter nucleus, i.e., around 9.2 fm, the value of u for $N = 18$ is more than 4 orders of magnitude larger than the corresponding value for $N = 8$. This is a manifestation of clustering. That is, the more

TABLE I. Formation probability $u(R)=\int d\hat{R}\,|F_0(\mathbf{R})|^2$ (in units of 10^{-4} fm^{-3}) as a function of the number N of shells.

R (fm) \ N	8	14	16	18
7	5.7	7.5	7.9	7.9
8	9.0×10^{-2}	1.1	1.2	1.2
9	3.3×10^{-5}	0.11	0.16	0.17
10	4.1×10^{-12}	5.9×10^{-3}	2.3×10^{-2}	3.6×10^{-2}
11	2.4×10^{-15}	1.0×10^{-5}	9.0×10^{-4}	7.6×10^{-3}

the clustering features are pronounced in the mother nucleus wave function, the larger the value of F_0, will be as can be seen from Eq. (1), and the better the assumptions of our formulation will be fulfilled. This requirement is satisfied by our calculation, but it is not enough. As mentioned above (and as it can be suspected from the values of the formation probability of Table I), the calculated alpha-decay width $\Gamma(R)$ can be strongly dependent on the distance R. But we found that, using $N=18$ h.o. major shells in the single-particle representation, dependence is weak on a wide region surrounding the touching point distance. Moreover, the calculated value of Γ in that region agrees with the corresponding experimental data within a factor of 3, as seen in Fig. 1. This is a rather good agreement if one considers that, at its maximum value, the ratio $\mathcal{R}=\Gamma^{\text{exp}}/\Gamma^{\text{th}}$ is $\mathcal{R}=33\,461$ for $N=8$, $\mathcal{R}=357$ for $N=14$, and it converges to the value $\mathcal{R}=3$ for $N=18$.

We also analyzed the influence of deformations on alpha decay by performing the calculations with and without the octupole deformation. We thus found that the effect of the octupole deformation is to increase the formation probability (and the corresponding absolute alpha-decay width) by 30%.

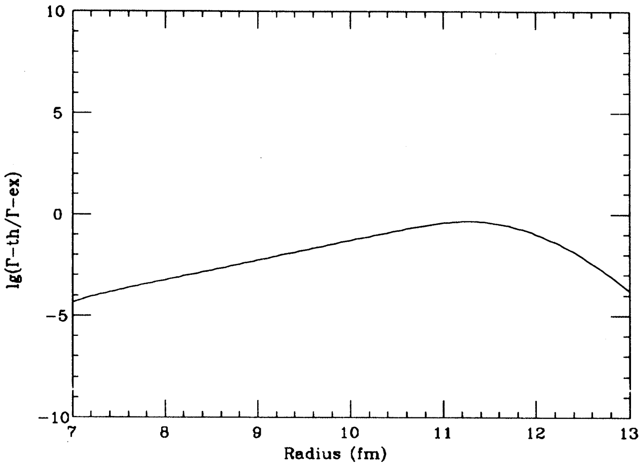

FIG. 1. The ratio $\Gamma^{\text{th}}/\Gamma^{\text{exp}}$ as a function of R.

In conclusion, we have presented in this paper a microscopic formalism to calculate the absolute alpha-decay width of deformed nuclei. The formalism was applied to the decay of ^{222}Ra. The calculation describes well the clustering process if a large number of shells, reflecting the influence of the continuum on alpha decay, is used in the deformed (Woods-Saxon) single-particle representation. A reasonable agreement with the corresponding experimental data was obtained.

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