Parametrization of total and differential cross sections for $\pi d \rightarrow pp$ below 1 GeV

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The energy dependence of the total cross section for $\pi d \rightarrow pp$ below 1 GeV pion laboratory kinetic energy is described using a form similar to a previous parametrization of that energy dependence below 330 MeV. Measured angular distributions for the reaction below 1 GeV were fitted using a Legendre polynomial expansion. The energy dependences of the coefficients of the expansion have been fitted with a semiempirical form, giving a simple parametrization for the energy dependence of the differential cross sections for the reaction below 1 GeV.

A published semiempirical parametrization [1] of the energy dependence of the total cross section $\sigma(T_\pi)$ for $\pi d \rightarrow pp$ below 330 MeV has proven useful for several purposes. For instance, that parametrization has provided a means for normalizing data [2], served as an empirical representation of the energy dependence for the cross section [3], and has been used to approximate the quasideuteron absorption component in pion absorption on heavier nuclei [4]. Since Ref. [1] was published, however, new data have been obtained at many energies below 1 GeV. Also, interest in pion absorption reactions at energies above the delta region has increased. Thus, it appears timely to reexamine and extend the work of Ref. [1]. Of similar utility would be a simple parametrization of the energy dependence of the differential cross sections in the same energy range. The results of such a parametrization are reported here. Of central utility in this process was the availability of the Leningrad database [5], which includes the results of nearly all of the total and differential cross-section measurements.

The initial database for the parametrization of $\sigma(T_\pi)$ consisted of the results tabulated in the Leningrad database, recent results [6] for the reaction below 20 MeV, and results for $np \rightarrow d\pi^0$ above 2 MeV [7], the latter being corrected for Coulomb effects using the work of Reitan [8] and assuming detailed balance. The Leningrad database assumes the validity of detailed balance for the inverse reaction and also provides cross sections for $\pi d \rightarrow pp$ converted from the inverse reaction using that principle. The total cross-section data were fitted using a form similar to Eq. (3) of Ref. [1]:

$$\sigma = a + \frac{b}{\sqrt{T_\pi}} + c_1 W(E_1,\Gamma_1) + c_2 W(E_2,\Gamma_2) + f T_\pi \ , \quad (1)$$

where $W(E_n,\Gamma_n)=\Gamma_n^2/[(E-E_n)^2+\Gamma_n^2]$ and $E=[(m_\pi+m_d)^2+2m_d T_\pi]^{1/2}$. The third term represents contributions principally due to the underlying $\Delta(1232)$ reaction mechanisms, while the fourth term describes an enhancement in the cross section occurring at about 800 MeV. The second term in the equation incorporates semiempirically the fraction of the cross section attributable to *s*-wave strength, while the remaining terms parametrize the nonresonant background.

A trial least-squares fit to the total cross-section database with Eq. (1) revealed several sets of measurements [9–14] which had one or more datapoints very far from the estimated value of the total cross sections. To avoid distortion of the fit due to measurements that may be wrong, datapoints which lay farther than 3.5 times their stated uncertainties from the initial fit estimate were removed from the dataset. As an additional conservative measure, to avoid possible bias, *all* total cross-section datapoints belonging to these sets of measurements with questionable datapoints were removed. This procedure reduced the number of total cross-section data points from 187 to 167, but this final total cross-section database, as seen in Fig. 1, is sufficient to determine $\sigma(T_\pi)$. (In this work, $\eta=p_\pi/m_\pi$, where p_π is the pion center of mass momentum.) Beyond this first, very conservative selection of data, no further pruning of the total cross-section database was performed.

The least-squares fit for Eq. (1) to the total cross-section database yielded parameter values as given in Table I. The agreement between the fit and the data, seen in Fig. 1, is very satisfactory. The uncertainty in the total cross-section estimate obtained using Eq. (1) with the parameters given in Table I varies with energy but is always less than 5%.

As noted above, the Leningrad database also contains nearly all the results of differential cross-section measurements for the reaction $\pi d \rightarrow pp$. The original sources for these data used a number of different parametrizations to describe the angular distributions, and used several criteria to establish the need for different terms. In order to parametrize the experimental differential cross sections with a consistent set of assumptions concerning statistical significance, each of the measured center-of-mass differential cross-section datasets in the database and in Ref. [6] were fitted using the rapidly converging series

$$\frac{d\sigma}{d\Omega} = \sum_{n \text{ even}} \alpha_n P_n(\cos\Theta_{\text{c.m.}}) \ ,$$

where the P_n's are the Legendre polynomials. Only datasets with at least four points were used. In determining the number of terms to be kept in this series expansion, the series was truncated when the chi-squared difference per degree of freedom (χ^2/ν_D) failed to improve by at least 1. Statistical errors for the coefficients

TABLE I. Total cross-section parameters determined for Eq. (1).

a(mb)	−0.577(39)
b(mb MeV$^{1/2}$)	3.09(20)
c_1(mb)	12.50(6)
Γ_1(MeV)	69.0(7)
E_1(MeV)	2133.9(4)
c_2(mb)	0.075(16)
Γ_2(MeV)	54(19)
E_2(MeV)	2644(6)
f(μb MeV^{-1})	0.38(4)
χ^2/ν_D	2.4

were determined simultaneously with the fit to the differential cross sections. None of the differential cross-section datasets required an α_8 term. The resulting values for α_2, α_4, and α_6 are shown in Figs. 2 and 3.

The α_n values obtained in this procedure were generally found to be in reasonable agreement with published values. In one notable exception, the published results of Akemoto *et al.* [15] for the α_n differed substantially from the values obtained from fitting the Legendre expansion to the points recorded in the Leningrad database. This is probably explained by the fact that the published α_n's were obtained by incorporating results from several other measurements which are separately included in the Leningrad database [16–18] with their measurements. In order to be conservative and to avoid any possible double counting of the measurements of Refs. [16–18], the measurements of Akemoto *et al.* were excluded from the database and are not shown in Figs. 2 and 3.

Using this Legendre series expansion, $2\pi\alpha_0=\sigma$, the total cross section. Since all terms in the expansion for

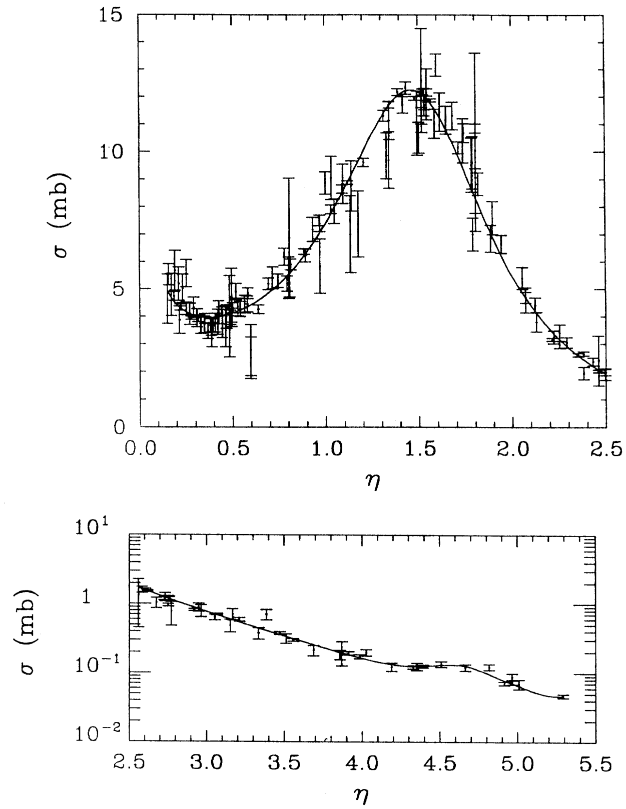

FIG. 1. Measured total cross sections for the reaction $\pi d \rightarrow pp$ from the database described in the text. Also shown are the results of the parametrization described in the text (solid line).

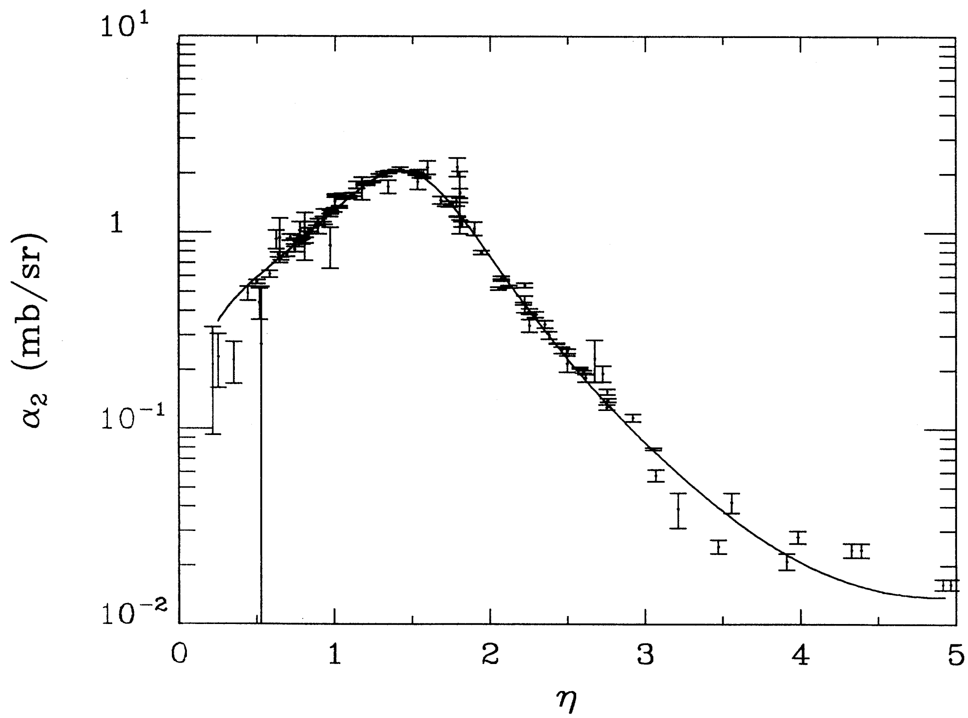

FIG. 2. α_2 values determined from the differential cross-section database described in the text. Also shown are the parametrization results (solid line).

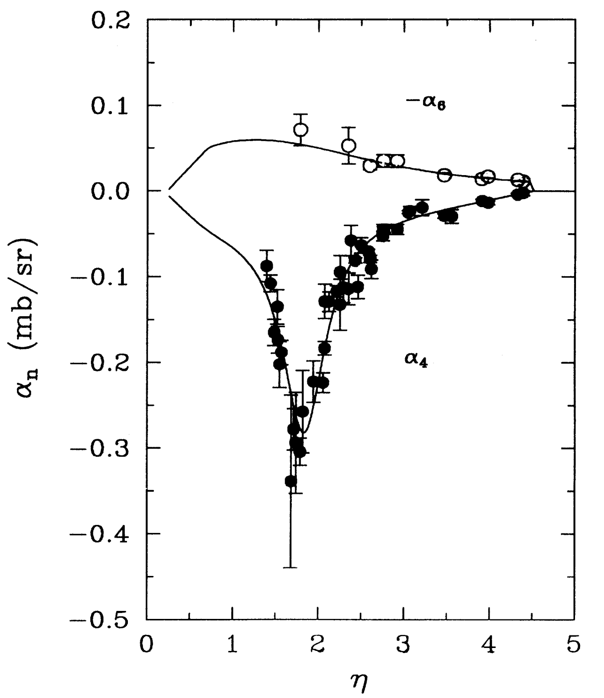

FIG. 3. α_4 (solid circles) and α_6 (open circle) values determined from the differential cross-section database. The results obtained in this work for the parametrization of these coefficients are indicated by solid lines.

each set of differential cross-section data incorporate the overall normalization error for each measurement, ratios of the coefficients to α_0 were determined, which are independent of the experimental normalization. Renormalized α_n values were then determined by multiplying these ratios by the value of $\sigma(T_\pi)/2\pi$ for each energy. The renormalized values were then used in parametrizing the energy dependence of the differential cross-section parameters. This procedure has the advantage of removing the scatter of the data attributable to normalization errors in the various experiments. This renormalization procedure entails the uncertainty in the total cross-section estimate described above. This uncertainty was taken to be 5% for all points, and was added in quadrature to the statistical uncertainty found in the fit to the differential cross section.

Upon inspection, it was also found that this procedure brought the angular distributions for Refs. [9–12] in close agreement with measurements at the same or nearby energies; their renormalized coefficients were used despite the aforementioned disagreement of their measured total cross-section data from these datasets with $\sigma(T_\pi)$, since those datasets seem incorrect only in their overall absolute normalization. The final database included 109 angular distributions with 1361 differential cross-section measurements.

There are many possible forms for parametrizing the energy dependence of the Legendre polynomial coefficients. If the parametrization is to be used near threshold, it is important that whatever form is chosen it gives the appropriate threshold behavior of the coefficients. For this work, the semiempirical formula assumed for fitting the energy dependence of the α_n for $n \geq 2$ was similar to Eq. (1):

$$\alpha_n = \tau_n [a_n + b_n \eta^2 + c_n W(\epsilon_n, \gamma_n)] , \tag{2}$$

where the factor τ_n

$$\tau_n = \eta^{n-1}/(\eta^{n-1} + \eta_t^{n-1})$$

yields the physically required threshold behavior [3,19] for each α_n for $\eta \ll \eta_t$. The value of η_t chosen here is 0.5, or about 20 MeV, which is suggested by the behavior of the ratio α_2/α_0 shown in Fig. 3 of Ref. [6]. For each α_n, the Breit-Wigner term represents a resonance dom-

TABLE II. Parameters for the energy dependence of the Legendre polynomial expansion coefficients below 1 GeV using Eq. (2) as discussed in the text. Note that an energy-dependent width is required for $n=2$. As indicated in the text, both α_4 and α_6 are set to zero above 750 MeV.

	$n=2$	$n=4$	$n=6$
No. of points	109	42	10
a_n (μb/sr)	$-2(2)$	$-40(8)$	
b_n (μb/sr)	0.6(4)	2.1(5)	
c_n (mb/sr)	2.80(6)	$-0.255(23)$	$-0.06(1)$
γ_n (MeV)	[a]	44(5)	245(33)
ϵ_n (GeV)	2.126(1)	2.184(2)	2.1[b]
χ^2/ν_D	3.0	3.0	0.6

[a] $\gamma_2 = [(-14\pm2)(\eta-\eta_0)+(67\pm2)]$ MeV, where $\eta_0=\eta(\epsilon_2)=1.398$.

[b] See text.

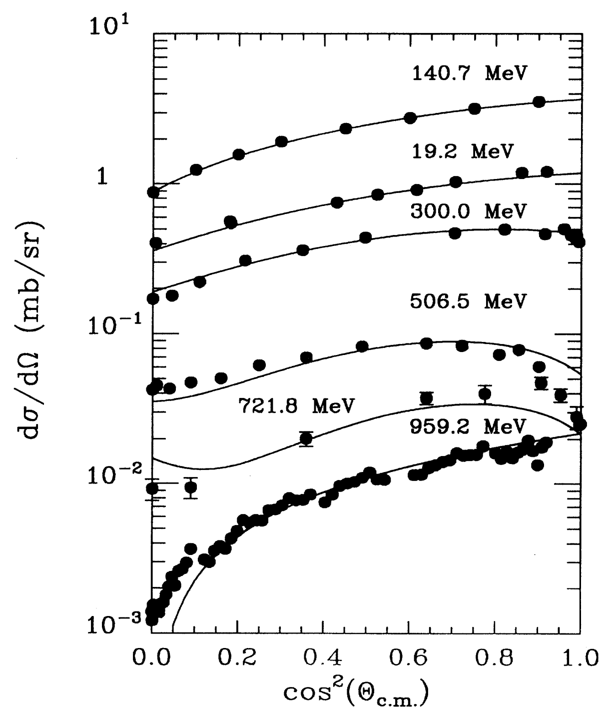

FIG. 4. Measured $\pi d \to pp$ differential cross sections (Refs. [16–18,23–25]) compared with the results of the parametrization (solid line) described in the text. The pion energies of 19.2, 140.7, 300.0, 506.5, 721.8, and 959.2 MeV correspond to η values of approximately 0.5, 1.5, 2.5, 3.5, 4.4, and 5.3, respectively.

inating that particular term, while the remaining terms parametrize the nonresonant strength.

The parameters determined for Eq. (2) for the various α_n are given in Table II; the resulting fits are shown in Figs. 2 and 3. The α_2 parameter requires an energy-dependent width as indicated in the table, whereas neither fit to the other parameters requires any such variation. This is perhaps due to the limited data on α_4 and α_6, since it is probable that the interference of different amplitudes would distort the α_n resonant terms from simple Breit-Wigner shape.

It is expected [20] that the Breit-Wigner centroids of Eq. (2) for $n=2$, 4, and 6 should lie near the resonances in the ${}^1D_2$, ${}^3F_3$, and ${}^1G_4$ partial waves. The values found in the fitting process for ϵ_n for α_2 and α_4 are near the values tabulated for the ${}^1D_2$ and ${}^3F_3$ partial-wave resonances by Locher, Sainio, and Svarc [21], 2.14–2.18 and 2.20–2.26 GeV, respectively. These are also relatively close to the same quantities found in the partial-wave analysis of Strakovsky, Kravtsov, and Ryskin [22], who found values of 2.18 and 2.17 GeV, respectively. Given the very simple form of Eq. (2), the agreement is interesting, but clearly the more detailed analyses should yield values with more credibility.

The data for α_6 are too sparse to determine a value for the location of ϵ_6 with any certainty. The value obtained in Ref. [20], 2.39 GeV, results in a very poor fit to the α_6 data; this may be due to the sparseness of the data or error. For the purposes of parametrization only, then, ϵ_6 was set to 2.1 GeV. Further, in the case of α_6, the available data are also too scarce to determine the first two terms of Eq. (2) for that coefficient, and the remaining terms have large uncertainties.

Finally, for α_4 and α_6, the available data are too sparse to determine the energy dependence above 750 MeV. For the purposes of this parametrization, then, both α_4 and α_6 are set to zero above 750 MeV.

Results of a comparison of this parametrization of the differential cross sections to measurements [16–18, 23–25] made at six energies from 19.2 to 959.2 MeV, spanning the range of η from 0.5 to 5.3, are shown in Fig. 4. These were chosen since the experimental values for the total cross section agree with $\sigma(T_\pi)$ and hence the differential cross sections require no renormalization. The predicted values are seen to be generally in good agreement with the data. The agreement with the 959.2 MeV dataset indicates that setting the α_4 and α_6 terms to zero above 750 MeV still provides a satisfactory fit to the data. Other datasets may require overall renormalization to $\sigma(T_\pi)$ at the energies of those datasets.

In summary, parametrizations for the energy dependences of the total and differential cross sections below 1 GeV pion laboratory energy have been developed. The semiempirical parametrizations, which include appropriate threshold behavior, describe the data well, though the uncertainty and sparseness of data for α_4 and α_6 prevent as accurate a parametrization for them as is available for the total cross section and α_2.

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