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Infrared structure of quantum hadrodynamics

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It is shown that quantum hadrodynamics as a formal field theory has a much richer infrared structure at high energies than previously realized. This arises from large vertex corrections due to vector meson exchange. By selectively summing all leading and nonleading logarithms, it is demonstrated that the infrared structure of the theory may in fact overwhelm the corresponding ultraviolet divergences arising from the Landau poles. Since competing infinites can thus be found, caution is urged in the use of any nonperturbative summation, especially one that emphasizes merely one sector of the theory.

Quantum hadrodynamics (QHD) [1] is an attempt at a relativistic formulation of nuclear physics using a renormalizable Lagrangian of effective hadronic fields. Within the context of mean-field theory, it has achieved considerable success in describing nuclear matter. Quantum corrections to these mean-field results are, however, more difficult to assess. Although apparently resulting in improvements at low energies, they seemingly inevitably lead to catastrophes at high energies at which propagator poles (in the random-phase approximation [2]) and/or tachyons (in the loop expansion [3]) apparently appear. These are generally interpreted as a manifestation of the fact that the field theory is not asymptotically free [2–6] and that the Landau pole in the running coupling enters at invariant masses of only a few GeV^2. Such calculations and/or arguments, however, are based in one form or another upon the ultraviolet structure of the theory. QHD, though, as a formal field theory, contains a rich infrared structure at high energies due to vector-meson exchange. This structure is exactly analogous to the structure found in gauge theories and is related to the so-called infrared problem of QED. It arises, to put it succinctly, because matter fields (electrons, nucleons, etc.) strongly wish to radiate vectors when accelerated, and the more one restricts such radiation, the more the process is suppressed. This phenomenon is solely the property of vector theories. A theory with only spinless bosons, although qualitatively having the same ultraviolet behavior, i.e., Landau poles, does not contain this infrared structure. In this Brief Report an estimate for this infrared suppression in QHD is given using techniques developed in gauge theories and generically known as the Sudakov form factor [7]. Its application to QHD, a strongly coupled theory, is questionable, as is any other approach based upon perturbation theory, such as the renormalization-group arguments, leading to the existence of Landau poles. The purpose of this Brief Report is to show that within the same approximations that lead to the Landau pole, strong infrared suppressions can be found that may in fact overwhelm the divergences found from the ultraviolet sphere, thus suggesting that QHD might be much more stable at high energies than previously thought. Conversely, since competing infinites can be found, one might be suspicious of any nonperturbative summation. Ultimately, the stability of the theory to quantum corrections is still an open question.

For simplicity, consider a theory with only fermions and a vector-meson exchange:

$$\mathcal{L}=\bar{\psi}\gamma_\mu(i\partial^\mu-g_v V^\mu-M)\psi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_v^2 V_\mu V^\mu \ . \qquad (1)$$

Since the vector-meson exchange is designed in the full form of QHD to dominate at short distances, i.e., high energies, by providing the repulsive core in the nuclear force and since it also generates the infrared structure, we should not be losing too much generality by using this particularly simple Lagrangian. The above theory is effectively a massive version of QED. The asymptotic fermion form factor in QED is obtained by summing the leading-log contributions of all vertex corrections and yields an exponentially suppressed form. Since this result is infrared divergent, a regulator must be introduced. In his original work, Sudakov kept the external fermions off shell, their invariant mass then providing the infrared regulator. The form of use to us is that first derived by Jackiw [8] and Fishbane and Sullivan [9], who considered on-shell electron scattering and introduced a fictitious photon mass. They then found that the form factor $\Gamma(q^2)$ exponentiates in the form

$$\Gamma(q^2)=\exp\left[-\frac{g^2}{16\pi^2}\ln^2\frac{|q^2|}{\mu^2}\right] , \qquad (2)$$

where q^2 is the invariant mass of the external photon, and μ^2 is the photon's fictitious mass. In the application to (1), this fictitious mass becomes simply the vector meson's mass m_v^2. In the calculation of cross sections in QED, it should be noted that the sum of all bremstrahlung graphs also exponentiate [10, 11], which, added to (2), thereby replaces μ by the detector width δ. We thereby recover the well-known result of Bloch and Nordseick that the lowest-order correction is proportional to $-g^2\ln(q^2/\delta^2)$. With only this first-order result, one might have wondered what happens as one improves the detectors accuracy—it would appear the cross section could become negative, signifying the breakdown of perturbation theory. The all-order analysis shows this does not happen, but that instead the cross section is greatly suppressed as one restricts the total radiation allowed to escape.

Important subsequent progress was made by Mueller [12] and Collins [13], who showed that all nonleading logarithms of the form $g^{2n}[\ln(q^2/\mu^2)]^m$, where $2n>m$, also exponentiate to also give a decreasing exponential. Since (2) provides such a large suppression, it was important to show that the nonleading terms did not ruin the result. Sen [14] then made the highly nontrivial extension to non-Abelian gauge theories, showing that for the asymptotic electromagnetic form factor of quarks, all the leading and nonleading logarithms from gluon exchange also exponentiate, with a leading-log result of

$$\Gamma(q^2)=\exp\left[-4\frac{C_F}{\beta}\ln\frac{(q^2)^{1/2}}{\mu}\ln\ln\frac{(q^2)^{1/2}}{\mu}\right], \tag{3}$$

where $C_F=(N^2-1)/2N$, $\beta=11N/3-2f/3$, f is the number of quark flavors, and $N=3$ for QCD. Comparing Eqs. (2) and (3), the lnln in (3) is a direct result of including the running coupling. These exponentiating logarithms are known to be quite important in QCD. For example, the fact that exclusive events are strongly suppressed has found recent application in the work of Botts and Sterman [15], in which an analogous exponentiation leads in the case of high-energy, exclusive meson-meson scattering to nearly a full extra order power suppression to the naive counting result from independent quark-scattering diagrams [16].

We now wish to apply these results to the theory of Eq. (1). Again, we will be summing all leading logarithms, which in a strongly coupled theory is not strictly justifiable as the terms we are ignoring could be of comparable size. Nevertheless, let us proceed. Since we are concerned with the interplay between the ultraviolet and infrared behavior of the theory, we use a running coupling constant. (Otherwise the form factor would, to this approximation, be given simply by Eq. (2)—see Refs. [17] and [18] for early work along these lines.) From the ultraviolet structure of the theory, one would then associate at high energies a factor $\alpha_v^{1/2}(q^2)$ with each vertex as is done in QCD, where, though, in this case

$$\alpha_v(q^2)=\frac{\alpha_v}{1-\beta_v\ln(|q^2|/M^2)}, \tag{4}$$

in which $\beta_v=\alpha_v/3\pi$ and we have used on-shell renormalization as is commonly done in QHD. This association alone, of course, leads to all the problems concerning the Landau pole. The infrared vertex corrections should also be included. From Sen, the leading-log result appearing in the exponential for a theory with a running coupling is given by [19]

$$\ln\Gamma(q^2)=\frac{-2}{4\pi^2}\int_{m_v}^{Q}\frac{dx}{x}\int_{m_v}^{x}\frac{dy}{y}g^2(y). \tag{5}$$

Inserting (4) for the running coupling and ignoring the differences between M and m_v, which leads to only small errors since they are in practice nearly identical, one gets

$$\ln\Gamma(q^2)=-\frac{g_v^2}{8\pi^2\beta_v^2}\left[\left[1-\beta_v\ln\frac{|q^2|}{M^2}\right]\times\ln\left[1-\beta_v\ln\frac{|q^2|}{M^2}\right]+\beta_v\ln\frac{|q^2|}{M^2}\right]. \tag{6}$$

Taking the limit $\beta_v\to 0$ and expanding the logarithms, one recovers the standard QED result of Eq. (2). As we approach the Landau pole, i.e.,

$$1-\beta_v\ln\frac{|q^2|}{M^2}\to 0, \tag{7}$$

we find that (6) is now essentially behaving as a single logarithm which cannot be played off of the divergence arising from (4). To find the results in this limit, therefore, we must consider the contributions in the exponential of the "nonleading"-log results.

To estimate these terms we focus on the leading infrared contributions to the lowest-order vertex correction (Fig. 1) and use the knowledge that it exponentiates. We will first rederive the result of Eq. (6) by following the development of Fishbane and Sullivan [9], although now modified to the case of a running coupling. We then estimate the nonleading corrections to this expression to obtain the true result in the limit of Eq. (7).

We choose to use light-cone coordinates and to work in the Breit frame with q^2 spacelike. The external momenta to Fig. 1 are then given by

$$q=(Q,-Q,0),\quad p_1=(M^2/Q,Q,0),\quad p_2=(Q,M^2/Q,0), \tag{8}$$

where terms power suppressed, i.e., $O(M/Q)$, have been ignored. Let us first concentrate on the numerator contractions for Fig. 1. Since we are interested in the infrared behavior of the graph, we ignore all terms of $O(k)$

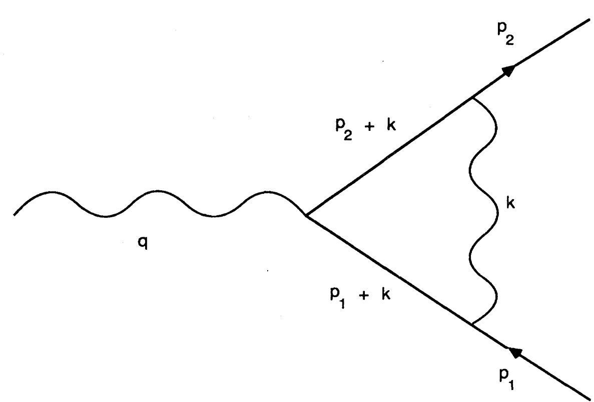

FIG. 1. Lowest-order vertex correction. Wiggly lines are the vector mesons, while straight lines are nucleons. The external meson is taken as incoming and way off shell. The external fermions are on shell.

in the numerator when compared to terms of $O(Q)$. We then obtain that [20]

$$\bar{u}(p_2)\gamma^{\nu}(\not{p}_2+\not{k}+M)\gamma^{\mu}(\not{p}_1+\not{k}+M)\gamma_{\nu}u(p_1) \rightarrow 2Q^2\bar{u}(p_2)\gamma^{\mu}u(p_1) . \quad (9)$$

The fact that this numerator is proportional to Q^2 is crucial for the leading-log results of Eqs. (2) and (3). The corresponding contractions from spinless boson theories are power suppressed with respect to (9) and thus do not lead to the infrared structure contained in vector-boson exchange theories. Using (9), the vertex correction $\Gamma_g(Q^2)$ becomes [21]

$$\Gamma_g(Q^2)=-2iQ^2\int\frac{d^4k}{(2\pi)^4}g_v^2(k^2)\left[\frac{1}{(p_1+k)^2-M^2+i\varepsilon}\right]\times\left[\frac{1}{(p_2+k)^2-M^2+i\varepsilon}\right]\times\left[\frac{1}{k^2-m_v^2+i\varepsilon}\right] . \quad (10)$$

Note that we have associated a running coupling with the exchanged meson. Analyzing the pole structure in the k^+ plane, one finds that, unless $-Q<k^-<0$, all the poles lie on the same side of the k^+ axis and the integral (10) is zero. Restricting k^- accordingly and closing on the lower half-plane [i.e., $k^+=k_\perp|(Q+k^-)$], we then get that

$$\Gamma_g(Q^2)=-\frac{Q^2}{(2\pi)^3}\int d^2k\int_{-Q}^{0}\frac{dk^-}{Q+k^-}g_v^2(k^2)\left[\frac{1}{k^-[Q+k_\perp^2/(Q+k^-)]-k_\perp^2}\right]\left[\frac{1}{k^-k_\perp^2/(Q+k^-)-k_\perp^2-m_v^2}\right] . \quad (11)$$

The double-log result comes from the double region $m_v^2 \ll k_\perp^2 \ll Q^2$ and $-Q \ll k^-$. With the approximations $Q+k^- \sim Q$, $k^2 \sim -k_\perp^2$, and that $k^- < -k_\perp^2/Q$, (11) becomes

$$\Gamma_g(Q^2)=\frac{1}{8\pi^2}\int_{m_v^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}g_v^2(k_\perp^2)\int_{-Q}^{-k_\perp^2/Q}\frac{dk^-}{k^-} = \frac{-g_v^2}{8\pi^2}\int_{m_v^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\ln(Q^2/k_\perp^2)}{1-\beta_v\ln(k_\perp^2/M^2)} . \quad (12)$$

Performing the last integration, we obtain Eq. (6).

The corrections of interest are obtained by relaxing the constraint that $k^- < -k_\perp^2/Q$ [22]. The k^- integral from (11) then decomposes as

$$\int_{-Q}^{0}\frac{dk^-}{k^- - k_\perp^2/Q} \rightarrow \int_{-k_\perp^2/Q}^{0}\frac{dk^-}{-k_\perp^2/Q}+\int_{-Q}^{-k_\perp^2/Q}\frac{dk^-}{k^-} = -1-\ln(Q^2/k_\perp^2) . \quad (13)$$

The correction to (12) is therefore

$$\Gamma_g^c(Q^2)=\frac{-g_v^2}{8\pi^2}\int_{m_v^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{1}{1-\beta_v\ln(k_\perp^2/M^2)} = \frac{g_v^2}{8\pi^2\beta_v}\ln[1-\beta_v\ln(Q^2/M^2)] . \quad (14)$$

Exponentiating (14) and using that $\beta_v=g_v^2/12\pi^2$, one obtains that the leading contribution to the form factor as we approach the Landau pole (7) behaves as

$$\Gamma(q^2)=[1-\beta_v\ln(Q^2/M^2)]^{3/2} , \quad (15)$$

which is apparently sufficient to compensate the divergence arising from the running coupling of the vertex $\alpha_v^{1/2}(q^2)$.

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[19] See Eq. (6.35) of Ref. [14].
[20] By current conservation, we can take the massive vector-

meson propagator as just proportional to $g^{\mu\rho}$.

[21] The fact that we have a renormalizable field theory is crucial both for making sense out of $\Gamma(Q^2)$ and for being able to ignore the ultraviolet region of the integrals for the present discussion.

[22] There is actually another region of importance, the so-called colinear region, when $k^- \sim -Q$. To obtain a logarithmic result such as (14), one must though maintain that $|k^2| < Q^2$; otherwise, it is an ultraviolet contribution. This then strongly restrains the $k_\perp$ integration. Altogether, one obtains another correction comparable to (and with the same sign as) (14).