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Role of the pygmy resonance in the synthesis of heavy elements with radioactive beams

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It is suggested that the inclusion of the virtual excitation of the soft giant dipole (pygmy) resonance in the calculation of the cross section for very neutron-rich radioactive beam-induced fusion reactions may enhance the formation probability of the heavy compound nucleus produced at low excitation energy.

I. INTRODUCTION

Recently, it has been suggested [1] that the beams of neutron-rich radioactive nuclei offer a rather unique possibility for synthesizing both the superheavy nuclei lying around the magic neutron and proton numbers $N=184$ and $Z=114$ and the heavy isotopes with $N\geq 160$ of new elements. Owing to the larger N/Z ratio of these exotic nuclei the effective Coulomb barrier is basically lowered, permitting the appreciable formation of not so excited compound nuclei at low energies. These cold compound nuclei have lower fission probability, thus increasing the possibility of observing them.

The theoretical calculation of the survival probability of heavy elements using radioactive neutron-rich beams has been done using the macroscopic model of extra-extra push of Swiatecki [2]. Substantial lowering of the effective fissility and, consequently a lower effective fusion barrier is obtained. The degree of lowering of these physical parameters has, however, been recently questioned [3].

In the present paper, we address ourselves to another, dynamical effect involving neutron-rich nuclei. It has been theoretically established that nuclei in the neutron drip region exhibit appreciable collective behavior at quite low excitation energies. In particular, the soft giant dipole resonance, in nuclei such as ^{11}Li, is predicted to be situated in the 1–2-MeV energy region, exhausting about 12% of the classical dipole sum rule and thus accounting for about 90%, of the observed fragmentation cross section [4–7]. We shall demonstrate here that these "pygmy resonances" (PR's) could enhance the fusion probability of neutron-rich nuclei by as much as a factor of 50 or more. We base our discussion on known facts about ^{11}Li and make reasonable extrapolations to the Fe isotopes induced fusion considered in Ref. [1].

II. THE PYGMY RESONANCE

In nuclei such as ^{11}Li it has been suggested that the two neutrons in the $p_{1/2}$ level form a "halo," and as such are very distanced from the ^{9}Li core. When discussing the collective dipole excitation of such a loosely bound system, one is bound to consider two types of vibrations: the usual ($E^*\approx 20$ MeV) isovector proton versus neutron vibration in the core, with the halo neutrons taken as mere spectators, and the oscillation of the whole core nucleus against the halo neutrons (the pygmy resonance). In this latter case the rather extended distribution of the halo results in a weak restoring force, and consequently a low excitation energy of the pygmy resonance (also known as the soft giant dipole resonance).

Recent microscopic calculation [4–6] of the structure of neutron-rich nuclei clearly confirmed the above qualitative picture. For the purpose of the present paper, however, we shall use microscopic, Steinwedel-Jensen (SJ) modeling guided with appropriate sum-rule arguments to discuss the pygmy resonance in the Fe isotopes.

In a recent Letter, Suzuki, Ikeda, and Sato [7] predicted the following excitation energy of the pygmy resonance, using the *SJ* model:

$$E^*_{\rm PR}=\left[\frac{Z(N-N_c)}{N(Z+N_c)}\right]^{1/2}E^*_{\rm GDR}\ , \tag{1}$$

where $\hbar\omega_{\rm GDR}$ is the excitation energy of the usual giant dipole resonance ($\approx 80/A^{1/9}$ MeV) and N_c refers to the neutron number of the core. N and Z are the neutron and proton numbers of the whole nucleus. Thus for the ^{A}Fe isotopes with $A=56,\ldots,70$, we have, e.g., $E^*_{\rm PR}(70)=0.38E^*_{\rm GDR}$. This shows that in ^{70}Fe the pygmy resonance occurs at ≈ 5 MeV. This value could well be lower if the separation energy of the excess neutron is small as, e.g., the case in ^{11}Li. The pygmy resonance in this latter nucleus is found to occur at ≈ 2 MeV [5,6].

A pure cluster model supplies slightly different results from those of Ref. 7. Within this model, the dipole strength is distributed according to [8]

$$\frac{dB(E1)}{dE^*}=\frac{3\hbar^2e^2}{\pi^2}\frac{Z^2\Delta N}{A\,A_c}\frac{\sqrt{\epsilon}(E^*-\epsilon)^{3/2}}{E^{*4}}\ , \tag{2}$$

where ΔN refers to the excess neutrons treated as a cluster and ϵ is the binding energy (separation energy) of the excess neutron clusters. The position of the maximum of $dB(E1)/dE^*$ is just the energy of the pygmy resonance and is easily calculated to be

$$E^*_{\rm PR}=\tfrac{8}{5}\epsilon\ ; \tag{3}$$

thus, the smaller ϵ is, the lower $E_{\rm PR}$ will be. In a nucleus such as ^{70}Fe ϵ could very well be in the few-keV region.

Of course, an ambiguity remains as to what should be the core. However, equations (1) and (2) should serve our purposes of supplying estimates.

From the above discussion one may safely assume that the pygmy resonance may occur in the 0.2–2-MeV excitation energy region in the Fe isotopes.

In discussing the energy-weighted sum rule for neutron-rich nuclei one may consider the usual classical sum rule which reads

$$S(E1)=14.8\frac{NZ}{A}\ \text{MeV fm}^2 e^2 \tag{4}$$

and the dipole cluster sum rule for the core plus excess neutron system [9]:

$$\begin{aligned} S_c(E1)&=S(E1)-S_{\text{cluster}}(E1)-S_{\text{excess}}(E1) \\ &=14.8\left[\frac{NZ}{A}-\frac{N_c Z_c}{A_c}\right]\ \text{MeV fm}^2 e^2 \ . \end{aligned} \tag{5}$$

It is usually found that the pygmy resonance exhausts about 10% of the classical sum rule and 80% of the cluster sum rule.

III. THE FUSION OF ^{A}Fe$+^{208}$Pb

Aside from the static, barrier penetration effects considered in reference (1), there are dynamic effects arising from the virtual excitation of giant resonances. Here we consider the effects of the pygmy resonances. It has been shown in the last few years that the fusion cross section at low energies is appreciably increased over the static value, when channel-coupling effects are taken into consideration [10]. The enhancement is largest when the Q value of the nonelastic channel is lowest. We show below how the excitation of the pygmy resonance may help increase the fusion cross section of the system ^{A}Fe$+^{208}$Pb. Although the theoretical description of coupled-channels effect in the fusion of heavy ions is well developed, we opt here for a simple two-channel model that can be solved exactly. Calling H_0+V_0 the entrance channel Hamiltonian, $H_{\text{PR}}+V_{\text{PR}}$ the pygmy resonance channel Hamiltonian, V_c the coupling Hamiltonian, and Q_{PR} the Q value of the pygmy resonance channel, the two-channel Schrödinger equation then reads

$$\begin{bmatrix} H_0+V_0 & V_c \\ V_c & H_{\text{PR}}+V_{\text{PR}}+Q_{\text{PR}} \end{bmatrix}\psi=E\psi, \quad Q_{\text{PR}}<0 \ . \tag{6}$$

From the previous discussion we know that $|Q_{\text{PR}}|$ is small and we neglect it in what follows (the c.m. energy is much larger than Q_{PR}). Further $H_{\text{PR}}+V_{\text{PR}}$ describes the relative motion of the excited ^{A}Fe nucleus with respect to ^{208}Pb. We are safe in taking this Hamiltonian to be equal to H_0+V_0. We thus have

$$(H_0+V_0+V_c\sigma_x)\psi=E\psi \ , \tag{7}$$

where σ_x is a Pauli spin matrix which is introduced here for notational convenience. Fusion with no coupling is accounted for by the complex bare optical potential V_0. The corresponding cross section is

$$\begin{aligned} \dot{\sigma}_F&=\frac{k}{E}\langle\dot{\psi}^{(+)}|-\text{Im}V_0|\dot{\psi}^{(+)}\rangle \\ &=\frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\dot{T}_l \ . \end{aligned} \tag{8}$$

Taking into account the coupling interaction to all orders amounts to replacing $\dot{\sigma}_F$ above by

$$\sigma_F=\frac{k}{E}\langle\psi^{(+)}|-\text{Im}V_0|\psi^{(+)}\rangle-\sigma_{\text{PR}} \ , \tag{9}$$

where $\psi^{(+)}$ is the spinor

$$\begin{bmatrix}\psi_0^{(+)} \\ \psi_{\text{PR}}^{(+)}\end{bmatrix}$$

and σ_{PR} is the angle-integrated inelastic cross-section for the direct transition $0\rightarrow\text{PR}$. The fusion cross section σ_F can be written in closed form after performing a convenient transformation that diagonalizes σ_x. The result of σ_F is

$$\sigma_F=\tfrac{1}{2}[\sigma_R(V_c)+\sigma_R(-V_c)] \ , \tag{10}$$

where $\sigma_R(V_c)$ is the total reaction cross section obtained from the Hamiltonian $H_0+V_0+V_c$ and $\sigma_R(-V_c)$ from $H_0+V_0-V_c$. We should stress that in all our discussion above we have disregarded the angular momentum (1) of the pygmy resonance, which is quite valid considering the large values of the orbital angular momentum involved. Equation (10) has been previously derived in a slightly different manner by Lindsay and Rowley [11].

In calculating the enhancement of σ_F, we use the Wong formula [12] which reads

$$\dot{\sigma}_F\equiv\sigma_F(V_c=0)=\frac{\hbar\omega R_B^2}{2E}\ln\left[1+\exp\left[\frac{E-V_B}{\hbar\omega/2\pi}\right]\right] \ , \tag{11}$$

where $\hbar\omega$ measures the curvature of the Coulomb barrier and V_B is its height. Here the Coulomb barrier is obtained from $V_0(r)+\hbar^2 l(l+1)/2\mu r^2$, and the Coulomb interaction is contained in V_0. We define the enhancement factor $\mathcal{E}(V_c)$ as

$$\mathcal{E}(V_c)=\frac{\sigma_F(V_c)}{\sigma_F(V_c=0)}=\frac{\ln\{1+\exp[(E-V_B-V_c)/\hbar\omega/2\pi]\}+\ln\{1+\exp[(E-V_B+V_c)/\hbar\omega-2\pi]\}}{2\ln\{1+\exp[(E-V_B)/\hbar\omega/2\pi]\}} \ . \tag{12}$$

At the barrier, $E=V_B$, one has

$$\mathcal{E}(V_c)=\frac{\ln\{1+\exp[-V_c/(\hbar\omega/2\pi)]\}+\ln\{1+\exp[V_c/(\hbar\omega/2\pi)]\}}{2\ln 2} \ . \tag{13}$$

We now write fully the structure of V_c guided with the results for stable nuclei supplied by the collective model

$$V_c = C_1[B_{\mathrm{PR}}(E1)]^{1/2}F(r) + V_c^{\mathrm{Coul}} , \tag{14}$$

where C_1 is a strength which may be calculated within the Tassie model, $F(r)$ is the radial form factor given by

$$\int \frac{d}{dr'}\rho_{\mathrm{Fe}}(r')\rho_{\mathrm{Pb}}(r'-r)dr' ,$$

and V_c^{Coul} is the Coulomb piece of V_c which is also proportional to $[B_{\mathrm{PR}}(E1)]^{1/2}$. Thus

$$V_c = \mathcal{F}(r)[B_{\mathrm{PR}}(E1)]^{1/2} . \tag{15}$$

In Eqs. (14) and (15) $B_{\mathrm{PR}}(E1)$ is the $B(E1)$ value of the pygmy resonance, which in a cluster model (core and excess neutrons) can be written as [8] [by integrating Eq. (2) over E^*]

$$B_{\mathrm{PR}}(E1) = \frac{3\hbar^2 e^2}{16\pi}\left[\frac{Z^2\Delta N}{A A_c}\right]\frac{1}{\epsilon} , \tag{16}$$

where ϵ is the binding energy of the excess neutron cluster with respect of the core. It is obvious that ϵ is the determining factor in the degree of enhancement of σ_{fusion}. Thus, we obtain the final explicit form of the enhancement factor $\mathcal{E}$ (at $E_{\mathrm{c.m.}} = V_B$) showing its dependence on the relevant physical parameters that characterize the exotic neutron-rich nucleus ${}^{A}\mathrm{Fe}$, with $A = A_c + \Delta N$, and using Eq. (3)

$$\mathcal{E} = \frac{1}{2\ln 2}\ln\left\{2\left[1+\cosh\left[\frac{\mathcal{F}(R_B)}{\hbar\omega/2\pi}[B_{\mathrm{PR}}(E1)]^{1/2}\right]\right]\right\} , \tag{17}$$

where $\mathcal{F}(r)$ of Eq. (15) is evaluated at the barrier $r = R_B$. Since Z^2/A_c, in Eq. (16), is fixed once the core is decided upon, the quantity that varies as more neutrons are added is $\Delta N/A\epsilon$. The argument of cosh could become very large for very neutron-rich isotopes such as ${}^{70}\mathrm{Fe}$, where $B_{\mathrm{PR}}(E1)$ is expected to be large, rendering $\mathcal{E}$ to attain large values.

In fact if, as a reference, we take for the factor

$$\mathcal{X} \equiv \frac{\mathcal{F}(R_B)[B_{\mathrm{PR}}(E1)]^{1/2}}{\hbar\omega/2\pi}$$

the value 0.1 for ${}^{56}\mathrm{Fe}$ then $\mathcal{E}({}^{56}\mathrm{Fe}) \cong 1$. Using Eqs. (16) and (3) to obtain a rough estimate of $B_{\mathrm{PR}}(E1)$ for ${}^{70}\mathrm{Fe}$ and assuming $E^*_{\mathrm{PR}}({}^{70}\mathrm{Fe}) \approx 1$ MeV and $E^*_{\mathrm{PR}}({}^{56}\mathrm{Fe}) \approx E_{\mathrm{GDR}}({}^{56}\mathrm{Fe}) \approx 20$ MeV, we have $\mathcal{X}({}^{70}\mathrm{Fe}) \simeq 10$ and thus we get

$$\mathcal{E}({}^{70}\mathrm{Fe}) \simeq \frac{10}{2\ln 2} = 7.2 .$$

We should stress that the above value of $\mathcal{E}({}^{70}\mathrm{Fe})$ was obtained at $E = V_B$.

At lower c.m. energies, one may obtain much larger enhancement. At $E \ll V_B$, the fusion cross section, Eq. (10), can be approximated by taking only the term with the lowest effective barrier (assuming $V_c > 0$):

$$\sigma_F \approx \frac{\hbar\omega R_B^2}{4E}\ln\left\{1+\exp\left[2\pi\left[\frac{E - V_B + V_c}{\hbar\omega}\right]\right]\right\}$$
$$\approx \frac{\hbar\omega R_B^2}{4E}\exp\left[\frac{2\pi}{\hbar\omega}(E - V_B + V_c)\right] . \tag{18}$$

Thus compared to the no-coupling fusion, the effect of the pygmy resonance at low center-of-mass energies can be represented by an effective increase in the center-of-mass energy $E \to E + V_c$.

With Eq. (18), the enhancement factor $\mathcal{E}$ attains the very simple energy-independent form

$$\mathcal{E} = \tfrac{1}{2}\exp(2\pi V_c/\hbar\omega) , \tag{19}$$

which, with the estimate given earlier, namely, $\mathcal{X} = 2\pi V_c/\hbar\omega = 10$ for ${}^{70}\mathrm{Fe}$, yields $\mathcal{E}({}^{70}\mathrm{Fe}) \approx 10^4$; at higher energies this factor is of course reduced.

Before presenting our concluding remarks, we warn the reader that our formula for $\mathcal{E}$, Eq. (17), was derived in the sudden limit ($Q_{\mathrm{PR}} = 0$) and using the cluster model for the pygmy resonance. The validity of the sudden approximation becomes suspect for small ΔN and one has to consider Eq. (17) as a great overestimation. In fact, in such cases, namely, large values of E^*_{PR}, a more valid approximation is to simulate the excitation of the PR through an attractive local energy-independent polarization potential [13]. Further, the nuclear part of the form-factor function $\mathcal{F}(r)$ is proportional to the derivative of the density and thus is sensitive to the binding energy of the excess neutrons. The low binding energy of these neutrons imply a large diffuseness and accordingly a smaller form factor [since $\mathcal{F}_{\mathrm{nuclear}}(r)$ is inversely proportional to the diffuseness]. This would reduce the value of V_c and correspondingly $\mathcal{E}$. As far as the cluster model of the PR is concerned it has recently been demonstrated that this model overestimates the Coulomb fragmentation cross-section of ${}^{11}\mathrm{Li} + {}^{208}\mathrm{Pb}$ at $E_{\mathrm{lab}} = 800$ MeV/nuclear [8,14] and greatly overestimates the cross section at lower energies [15]. However, the analytic simplicity of the cluster model justifies its use here for the obtention of the simple estimate for $\mathcal{E}$.

In conclusion, we have considered in this paper the influence of the excitation of the soft giant dipole resonance on the fusion of the neutron-rich nuclei with heavy targets. The enhancement over the static fusion calculation, exemplified by Eq. (17), shows clearly that the determining factor is the smallness of the excitation energy E^*_{PR} or, more precisely the large value of $B(E1)$ as the number N/Z is increased. Any static fusion calculation of the type discussed in Ref. [1] must be amended by the multiplication with $\mathcal{E}$ of Eq. (17) or, even better, by a detailed coupled-channel calculation.

Of course, several questions have to be answered before a definite conclusion concerning the value of $\mathcal{E}$ can be reached. The most important of these questions is the precise value of $B_{\mathrm{PR}}(E1)$ and $\mathcal{F}(r)$, which can be only settled through detailed measurement and analysis of the elastic scattering and break-up of these exotic neutron-rich nuclei. Another question that has to be addressed here is the fact that, owing to the loosely bound nature of the excess neutrons, the projectile may simply dissociate before fusion with the target takes place. This will certainly reduce the fusion cross section by an amount which is related to the width of the pygmy resonance. We have not attempted in this work to discuss the width of these low-lying modes, and will leave this as well as the influence on σ_F for a future investigation.

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