Absolute measurement of the $p+p$ analyzing power at 183 MeV

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The analyzing power A_y for $p+p$ elastic scattering at $\theta_{\rm lab}=8.64°\pm0.07°$ ($\theta_{\rm cms}=18.1°$) and at a bombarding energy of 183.1 ± 0.4 MeV has been determined to be $A_y=0.2122\pm0.0017$. The error includes statistics, systematic uncertainties, and the uncertainty in bombarding energy and angle. This measurement represents a calibration standard for polarized beams in this energy range. The absolute scale for the measurement has been obtained by comparison with $p+C$ elastic scattering at the same energy at an angle where A_y is very nearly unity.

I. INTRODUCTION

The present measurement was motivated by the need for a reaction of accurately known analyzing power, in order to calibrate the beam polarization of 183 MeV protons in the Indiana Cooler Storage Ring. Currently, the polarization of beams stored in the Cooler is determined by detecting the asymmetry of protons scattered from a thick carbon target which partly intercepts the stored beam [1]. This method only yields relative values for the beam polarization since the energy resolution to separate single states, for which the analyzing power would be known, is not available.

One of the reasons to use $p+p$ scattering as a polarization standard is the ease with which scattering events can be identified without requiring high detector resolution. There are no open reaction channels, and in addition it is possible to detect scattered and recoil protons in coincidence for a clean identification of $p+p$ events. Furthermore, the analyzing power in $p+p$ scattering varies only slowly with energy and angle, so that the calibration accuracy is easily maintained from one experimental environment to another. Another reason for $p+p$ scattering as a polarization standard is its use in calibrating internal, polarized hydrogen targets. It has been shown [2] that such targets are technically feasible with a thickness which optimizes the luminosity for 200–300 MeV protons in a storage ring (about 10^{14} atoms/cm^2).

The $p+p$ scattering angle for the calibration was chosen as $\theta_{\rm lab}=8.6°$ where the analyzing power is of the order 0.2 (about two-thirds of its maximum value at this energy). This is within the angular range of $2.5°-10.5°$ studied in a recent $p+p$ experiment at the Indiana Cooler in which analyzing powers were measured using an internal gas jet target [3].

Since there is a lack of accurate proton polarization analyzers for energies around 200 MeV, the present work may be of interest for beam polarization calibrations at other laboratories where either the polarization of an external beam from an accelerator or of a circulating beam in a storage ring is to be measured.

The present calibration was performed on an external beam line of the Indiana University Cyclotron Facility (IUCF), by comparing the $p+p$ analyzing power at $8.6°$ to the analyzing power of $p+C$ scattering at $17.7°$. As discussed in the following section, proton scattering by carbon at this energy and angle has an analyzing power very close to unity, and can serve as a primary standard.

II. THE METHOD

A. Simultaneous observation of two processes

In the measurement described here, $p+p$ scattering is observed concurrently with $p+C$ scattering, for which the analyzing power is well known. The advantage of this method is that the two reactions sample precisely the same beam, so that all uncertainties from possible temporal variation of the beam polarization are removed. In addition, the usual method of suppressing instrumental asymmetries by use of a left-right symmetric detector arrangement is employed in this experiment.

B. Calibration deduced from $p+{}^{12}C$ elastic scattering.

For scattering of spin-$\frac{1}{2}$ particles from nuclei without spin it has been demonstrated [4] that the analyzing power can be *exactly* unity for certain combinations of the bombarding energy T_0, and the scattering angle θ_0. Recently, it has been shown [5] that such a situation [with $A_y(\theta_0,T_0)=+1$] occurs in $p+{}^{12}C$ scattering at $T_0=189\pm2$ MeV, and $\theta_0=17.3°\pm0.3°$ (laboratory angle).

If one desires to make use of this absolute knowledge of the analyzing power at energies and angles which are close but not quite equal to θ_0 and T_0, the shape of $A_y(\theta,T)$ in the neighborhood of the $A_y=1$ point has to be known. Sufficiently close to (θ_0,T_0), A_y can be described by

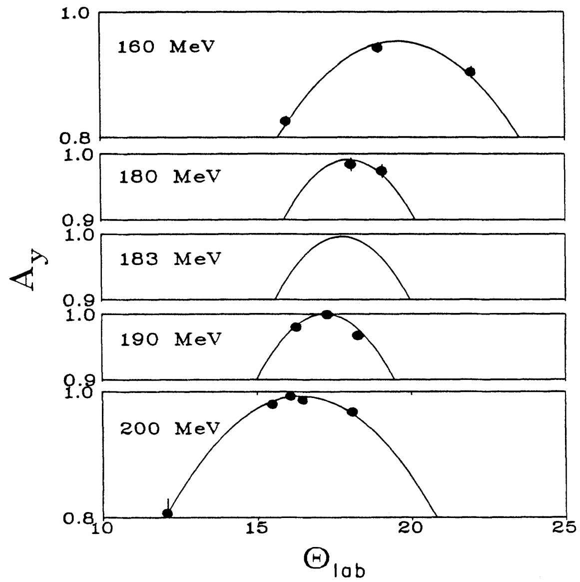

FIG. 1. $p+{}^{12}C$ analyzing powers in the neighborhood of the $A_y=1$ point. Shown are data from Ref. [5,6]. The solid curves are calculated from Eqs. (1) and (2).

$$A(\theta,T)=1-\alpha(T-T_0)^2-\beta(T-T_0)(\theta-\theta_0) -\gamma(\theta-\theta_0)^2 \ . \tag{1}$$

The coefficients are obtained by a usual least-square fit of Eq. (1) to $p+{}^{12}C$ analyzing power measurements [5,6] in the region $160<T<200$ MeV, and $12°<\theta_{lab}<22°$, resulting in

$$\begin{aligned}\alpha&=(1.21\pm0.07)\times10^{-4}\ \mathrm{MeV}^{-2}\ ,\\ \beta&=(1.61\pm0.11)\times10^{-3}\ \mathrm{MeV}^{-1}\,\mathrm{deg}^{-1}\ ,\\ \gamma&=(1.00\pm0.07)\times10^{-2}\ \mathrm{deg}^{-2}\ .\end{aligned} \tag{2}$$

The analyzing power data used in determining the above coefficients are shown in Fig. 1, together with the values calculated from Eq. (1). The fit has a χ^2 per degree of freedom of 1.03. The data which are represented by Eqs. (1) and (2) have been obtained with a spectrometer which allowed the separation of $p+{}^{12}C$ and $p+{}^{13}C$ scattering. In the present experiment this separation was not possible. This point will be addressed in Sec. IV C.

It can now be deduced from Eq. (1) that at $T=183.1$ MeV a maximum analyzing power $A_{y,\max}=0.998\pm0.002$ is reached at a laboratory angle of $\theta_{\max}=17.75°\pm0.16°$. The uncertainties follow from the errors of the coefficients in Eq. (2) as well as from the uncertainty of the location (θ_0, T_0) of the $A_y=1$ point; the latter uncertainty is dominating.

III. THE EXPERIMENT

A. Experimental setup

The experiment was carried out in the 64" scattering chamber with a 183.1 ± 0.4 MeV polarized proton beam from the Indiana University Cyclotron. The beam current was adjusted to be in the range between 5 and 10 nA; it was measured with a current integrator connected to a Faraday cup. The total beam charge accumulated during data taking was about 10^{-3} C. The beam polarization was typically $P=0.8$. During the course of the experiment two different CH_2 targets of 1.1 mg/cm^2 thickness were used, supplemented for comparison by a 1.3 mg/cm^2 thick carbon target. The target was oriented at $45°\pm5°$ with respect to the beam axis, with the normal pointing to either beam left or right.

The detector arrangement is shown in Fig. 2. Four identical scintillator telescopes at $\theta=\pm8.6°$ and $\theta=\pm17.7°$ were used to detect forward scattered protons. Each telescope consisted of a small plastic scintillator (ΔE) of thickness 3.2 mm, width 6.4 mm, and height 9.5 mm, followed by a cylindrical NaI scintillator (E) 51 mm in diameter and sufficient length (127 mm) to stop 220 MeV protons. In order to shield against unwanted charged particles, 51 mm thick brass collimators with conical holes were placed in front of the telescopes. For the same reason, shielding was inserted between the beam and the innermost NaI detectors. The ΔE detectors were 390 mm from the target and defined the subtended solid angle (4.02×10^{-4} sr). They were smaller than the narrowest part of the conical hole in the brass absorber.

The detector setup was left-right symmetric. The outer pair of telescopes ($C_{L,R}$) was placed at 17.7°, i.e., the angle where the $p+{}^{12}C$ analyzing power is largest (see Sec. II B). The inner pair ($B_{L,R}$) at 8.6° was intended for the desired $p+p$ analyzing power measurement. In order to cleanly define $p+p$ scattering events, a coincidence be-

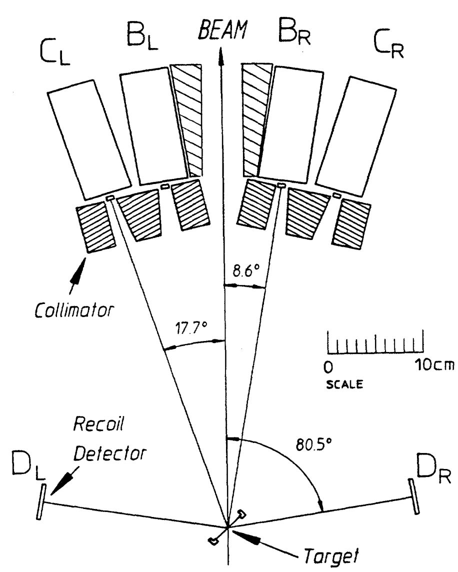

FIG. 2. Top view of the experimental arrangement. Shown are the scintillator telescopes $B_{L,R}$ and $C_{L,R}$ for measuring $p+p$ and $p+{}^{12}C$ elastic scattering, respectively, as well as the microstrip detectors $D_{L,R}$ to detect $p+p$ recoil protons. The shaded parts are the brass collimators with conical holes and the shielding between the beam and the inner detector telescopes.

tween both outgoing protons was required. For the purpose of detecting the recoil protons [7], two silicon (so-called "microstrip") detectors, 40 mm wide and 60 mm high, were placed at symmetric locations to the left and right ($D_{L,R}$), in such a way that they subtended about 1.5 times the recoil angle range associated with forward protons observed in the telescopes $B_{L,R}$.

Energy and time information for all detectors was recorded event by event. In addition, the scaled beam current integrator and discriminator counts for all detectors were written to tape every 10 sec. When replaying the data, it was thus possible to determine the time dependence of the recorded rates. It was discovered that the gain of a given NaI detector was related to the rate at which the respective discriminator fired. For example, increasing the rate of one of the inner NaI detectors from 750 to 1750 Hz caused an increase of 1.5% in the amplitude of the output signal (the rates quoted were those typically observed for spin down and up, the difference being due to the $p+\mathrm{C}$ analyzing power). Over the range of rates observed in a given run, the gain was found to be linearly dependent on rate. This information was used to apply a rate-dependent correction to the recorded energy signals before any further processing. Examples of measured spectra, after correction, are shown in Fig. 3. The FWHM width of the $p+\mathrm{C}$ ground-state peak is 1.5 MeV, corresponding to a resolution of 0.8%.

B. Measurements

Data were accumulated with the direction of the polarization vector either up ($\uparrow$) or down ($\downarrow$) with respect to the scattering plane. The spin orientation was reversed every 10 sec. Runs with two different CH_2 targets, with a carbon target, with different target orientations, with different average beam intensities, and with the beam position displaced from the center position were carried out to provide the database needed to assess the systematic uncertainties of the measurement.

The result from a given run consists of nine numbers, namely the accumulated number of $p+p$ scattering events B_{ik} with spin up or down ($i=\uparrow,\downarrow$) detected on the left or right ($k=L,R$), the accumulated number of $p+\mathrm{C}$ elastic-scattering events C_{ik}, and the ratio ξ of the accumulated beam charge for spin down divided by the same number for spin up. If one assumes that the target thickness during the spin-up part of the run was the same as during the spin-down part, it is straightforward to prove that the four parameters

$$b_k=\xi\frac{B_{\uparrow k}}{B_{\downarrow k}}\ ,\quad C_k=\xi\frac{C_{\uparrow k}}{C_{\downarrow k}}\quad (k=L,R) \tag{3}$$

are independent of the integrated luminosities, the subtended solid angles, and the detector efficiencies. Elastic $p+\mathrm{C}$ scattering can now be used to determine the two beam polarizations $P_\uparrow, P_\downarrow$, averaged over the duration of the run, using the relations

$$C_k=\frac{1\pm P_\uparrow A_y(\mathrm{C})}{1\mp P_\downarrow A_y(\mathrm{C})}\ , \tag{4}$$

where the upper sign applies for $k=L$ and the lower for $k=R$. Note that $P_\uparrow$ and $P_\downarrow$ are not assumed to be equal. At this stage, the analyzing power for $p+p$ scattering, $A_{y,k}(^1\mathrm{H})$, can be determined either from the left or the right telescope alone ($k=L$ or R) by solving

$$b_k=\frac{1\pm P_\uparrow A_{y,k}(^1\mathrm{H})}{1\mp P_\downarrow A_{y,k}(^1\mathrm{H})}\ , \tag{5}$$

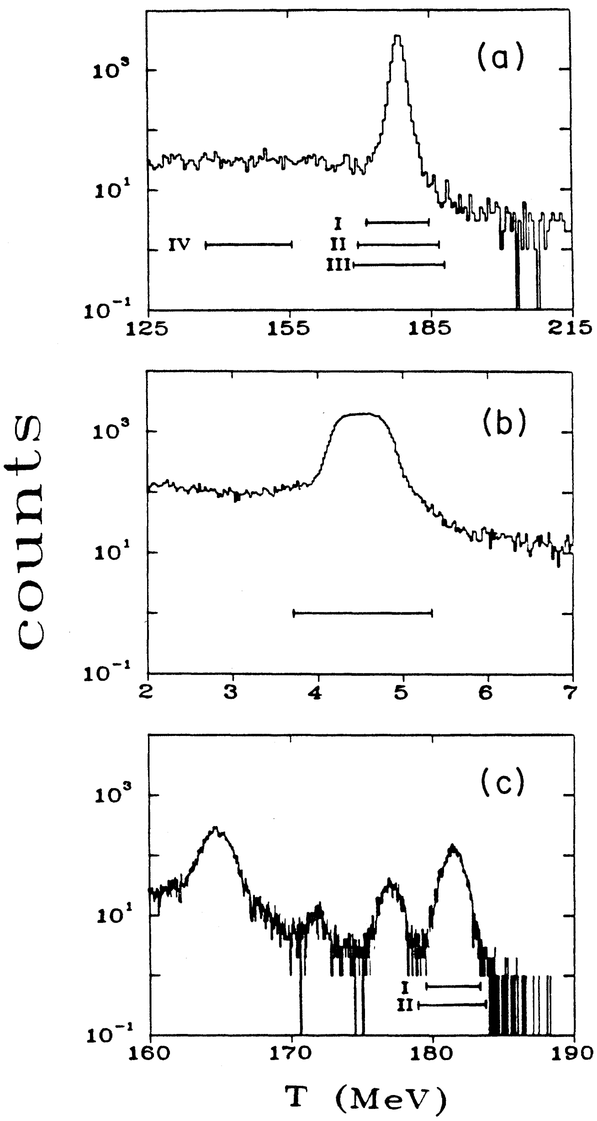

FIG. 3. Representative energy spectra. The spectrum of forward protons from $p+p$ scattering in the NaI scintillators at 8.6° ($B_{L,R}$ in Fig. 2) is shown in (a). For these protons a coincidence was required with the recoil protons detected with a microstrip detector ($D_{L,R}$ in Fig. 2); the microstrip spectrum is shown in (b). A spectrum obtained with a pure carbon target, scaled to the same integrated luminosity, has been subtracted to arrive at the spectrum shown in (a). The singles spectrum obtained with the NaI scintillators in the outer telescopes ($C_{L,R}$ in Fig. 2) is shown in (c). A rate-dependent gain correction has been applied to spectra (a) and (c). Note that the ground-state peak is cleanly separated from the first excited state at 4.4 MeV.

where again the upper and lower signs are for $k=L$ and $k=R$, respectively. The final result is then the average $A_y(^1\mathrm{H})$ between $A_{y,L}(^1\mathrm{H})$ and $A_{y,R}(^1\mathrm{H})$, which is insensitive to left-right positioning errors, depending only on the angle *between* the two inner telescopes, which has been determined with special care (see Sec. IV B).

IV. SYSTEMATIC UNCERTAINTIES

A. Background and peak sums

The spectrum from the inner telescopes at 8.6°, for which a coincidence with the microstrip detector was required, contained unwanted events arising from $p+\mathrm{C}$ scattering accompanied by an accidental pulse in the associated microstrip detector. Most of these events were removed by applying a gate to the microstrip spectrum as indicated in Fig. 3(b). Remaining background events (for which the accidental pulse fell within this gate) were removed by acquiring data with a pure carbon target under otherwise identical conditions (including the gate in the microstrip spectrum), and subtracting the result, properly scaled with target thickness and accumulated beam current, from the 8.6° spectrum obtained with a CH_2 target. The spectrum resulting from this subtraction is shown in Fig. 3(a). The remaining smooth component outside the $p+p$ peak is due to $p+p$ events misplaced in energy by reaction losses and outscattering. This explanation is supported by the fact that A_y below [gate IV in Fig. 3(a)] and above the $p+p$ peak (0.207 ± 0.003 and 0.24 ± 0.03 respectively) is very close to A_y in the peak (see Sec. V).

Carrying out the correction for the accidental background in the way described above lowered the analyzing power by 0.005. The method used for background subtraction was tested by repeating it without a gate on the microstrip spectrum; the background correction thus relies entirely on the subtraction of a spectrum obtained with a C target. The resulting A_y was within 0.0002 of the value obtained with the microstrip gate applied. Thus, it was felt that, in the uncertainty of the final result, the contribution from background subtraction was negligible.

The integration limits used to determine the numbers $B_{L,R}$ of $p+p$ protons are shown as gate II in Fig. 3(a). The $p+p$ analyzing powers obtained when using either gate I or III in Fig. 3(a) differed by 0.0003, demonstrating that the result is insensitive to the exact choice of the integration limits. The limits used to obtain the number of $p+\mathrm{C}$ events from the outer detectors were tested in a similar way, with a change of 0.0002 in the $p+p$ analyzing power when using either gate I or II in Fig. 3(c).

Using the procedure outlined in Sec. III B, the $p+p$ analyzing power $A_y(^1\mathrm{H})$ was determined individually for every run. The results are shown in Fig. 4. Different symbols correspond to runs taken under different conditions; they will be discussed in detail in the following sections.

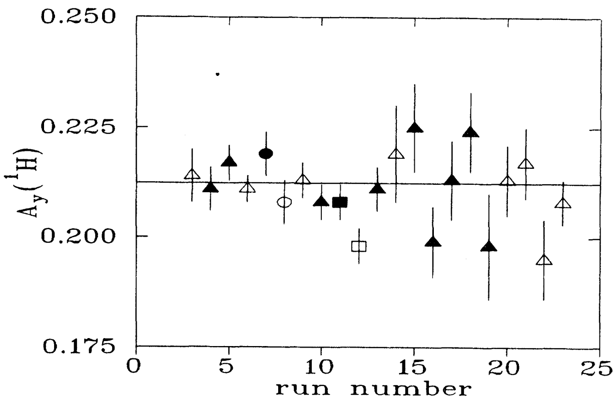

FIG. 4. The left-right averaged $p+p$ analyzing power $A_y(^1\mathrm{H})$ for individual runs. Open and solid symbols correspond to the target orientations +45° and −45°, respectively. Circles indicate runs with displaced beam spot and squares those with high beam current. Triangles are "normal" runs used to obtain the final result.

B. Geometrical alignment and symmetry

Prior to the experiment an optical reference line was established through the centers of two quadrupole lenses 3 m upstream of the scattering chamber. A beam alignment scintillator at the target location was centered on this line. During the run, the beam was positioned on target by the use of this scintillator. Individual telescope angles were determined by triangulation with a precision of ±0.2°. The geometry was fixed during the experiment and the position of all detectors was checked again afterward. The angle *between* the inner telescopes could be measured more accurately since it is given by the distance between the ΔE detectors (known to ±0.5 mm), yielding a left-right average for the $p+p$ scattering angle of $\theta=8.64°\pm0.07°$. From the known angle dependence $dA_y(^1\mathrm{H})/d\theta_{\mathrm{lab}}=0.011\ \mathrm{deg}^{-1}$ at the angle and energy of interest, one finds that this angular uncertainty contributes $\delta A_\theta=0.0008$ to the final error.

An overall test of the symmetry of the setup is obtained from a study of the difference of the $p+p$ analyzing powers $A_{y,L}(^1\mathrm{H})$ and $A_{y,R}(^1\mathrm{H})$ determined from B_L and B_R, respectively. Such a difference could arise from a displacement of the beam position on target (±1.5 mm is the positioning accuracy of the beam on the target), from a nonzero angle of the beam with respect to the optical reference line, or from the breaking of the symmetry of the setup by the target, inclined by 45° to the left or the right. In principle, a spin-dependent beam shift is also conceivable but such an effect has been shown to be small [8] for an atomic beam source, as was used for the present experiment, where RF transitions are switched for spin reversal. The difference between $A_{y,L}(^1\mathrm{H})$ and $A_{y,R}(^1\mathrm{H})$ is shown in Fig. 5; the average difference for all data runs (triangles) is 0.004 ± 0.003. A possible small left-right difference cancels to first order in the average $A_y(^1\mathrm{H})$, shown in Fig. 4. In Figs. 4 and 5, the open (closed) symbols denote runs with the target oriented at +45° (−45°).

Runs with the beam spot on the target deliberately displaced to one side by 3 mm are indicated by circles.

C. Other systematic effects

The accumulated beam current during a run was measured by integrating the charge in a Faraday cup serving as the beam stop. The digitized integrator output was gated with the busy signal from the acquisition computer. The same busy signal was used to determine the ratio of the dead time during spin-up and spin-down runs. The dead time (typically 3%) was found to be equal to within ± 0.001 for spin-up and spin-down runs, which translates into a contribution of $\delta A_I = 0.0004$ to the error of the final result. Results from runs carried out with a beam current of 10 nA (instead of the usual 5 nA) are indicated by squares in Figs. 4 and 5. At this current the dependence of the pulse heights in the NaI detectors on the count rate was nonlinear, making a correction less reliable.

The method used in this analysis makes the assumption that the target thickness is the same for spin-up and spin-down beams. Only an inhomogeneous target in conjunction with a spin-dependent beam shift could violate this assumption. For this reason two different targets were used during the experiment; no effect was found.

Possible contributions of beam halo scattered from material near the beam were ruled out by taking data with an empty target frame. No events in the regions of interest were found when accumulating a similar beam charge as for a run with target.

The experiment was carried out with natural carbon targets which contain 1.1% ^{13}C. The NaI detectors were unable to resolve the 100-keV energy difference between protons scattered from ^{12}C and ^{13}C. At 200 MeV it is known [9] that scattering from ^{12}C and ^{13}C at a given laboratory angle yields the same analyzing power within 0.05. It is reasonable to assume that the situation is similar at 183 MeV; thus, the ^{13}C admixture has a negligible effect on the final result.

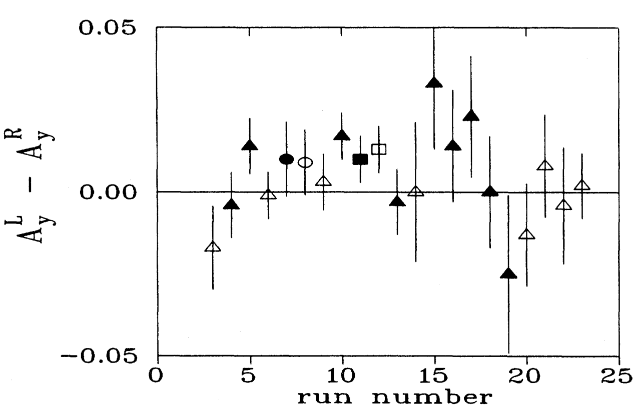

FIG. 5. The difference $A_{y,L}(^1\mathrm{H}) - A_{y,R}(^1\mathrm{H})$ between $p+p$ analyzing powers for individual runs determined separately with the left and right detectors. The symbols are explained in the caption for Fig. 4.

D. Uncertainty in $p + {}^{12}$C reference point

The shape of the angular distribution of the $p+$ C analyzing power at 183 MeV is shown in Fig. 1. Averaging over the angular acceptance of 0.94° of the detector telescope lowers the effective analyzing power by 0.001 from the maximum value. Thus, from Sec. II B one obtains the value $A_y(\mathrm{C}) = 0.997 \pm 0.002$ for the $p+$C analyzing power that enters the analysis of the present experiment. From the shape of the angular distribution, it is easy to see that the angular positioning error of the outer detectors of $\pm 0.2°$, as well as the uncertainty of $\pm 0.2°$ in the angle at which the extremum occurs, do not contribute significantly to the error of $A_y(\mathrm{C})$.

V. Result and Discussion

The final result is obtained from averaging 17 individual runs, shown as triangles in Fig. 4. The set of runs is internally consistent: the χ^2 for the distribution of the individual measurements with respect to the mean was 0.96 per degree of freedom. Thus, the analyzing power for $p+p$ elastic scattering at a bombarding energy of 183.1 ± 0.4 MeV and a laboratory scattering angle of $\theta_{\mathrm{lab}} = 8.64° \pm 0.07°$ ($\theta_{\mathrm{cms}} = 18.1°$) has been determined to be

$$A_y(^1\mathrm{H}) = 0.2122 \pm 0.0017 . \quad (6)$$

The quoted error has been obtained by quadratically combining the uncertainties due to statistics ($\delta A_S = 0.0013$), the uncertainty in the angle of the inner detectors ($\delta A_\theta = 0.0008$), the uncertianty in the current integration ($\delta A_I = 0.0004$), and the error in the $p+$C analyzing power [$\delta A_C = 0.21 \delta A_y(\mathrm{C}) = 0.0004$]. Also included was an error $\delta A_T = 0.0004$ from the uncertainty of the bombarding energy, deduced from the known $dA_y(^1\mathrm{H})/dT = 0.0011$.

When comparing the present result to previously available experimental information, we note that there is a shortage of analyzing power data for $p+p$ scattering at medium energies, especially at small angles. Between 150 and 300 MeV only four experiments have reported analyzing power data forward of $\theta_{\mathrm{cms}} = 30°$. [10–13] Three of these are more than 25 years old. The two measurements that are closest in angle to the present result, and bracket it in energy are $A_y(^1\mathrm{H}) = 0.241 \pm 0.036$ at 174 MeV and $\theta_{\mathrm{cms}} = 20.8°$ (Ref. [10]), and $A_y(^1\mathrm{H}) = 0.264 \pm 0.025$ at 209 MeV and $\theta_{\mathrm{cms}} = 18.9°$ (Ref. [13]). Clearly, the present result constitutes a significant addition to the database for $p+p$ scattering. The consequences with respect to the NN interaction that follow from it will be discussed in conjunction with a measurement of the $A_y(^1\mathrm{H})$ angular distribution at angles in the Coulomb-nuclear interference region, carried out with the Indiana Cooler; this will be the subject of a forthcoming paper.

ACKNOWLEDGMENTS

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