Phase transitions in light nuclei

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(Received 28 March 1991)

The SU(3) Elliott model is used to study the thermal description of ^{20}Ne. This solvable model allows us to work in the canonical ensemble and still be able to define an order parameter, the expectation value of the intrinsic quadrupole moment, to investigate the occurrence of phase transitions.

The finite-temperature mean-field description [1–3] of heavy nuclei has provided the theoretical support for the study of nuclear properties at high excited energy. In a recent series of papers [4], some *sd* shell nuclei, for which exact shell model diagonalizations are available, have been investigated with particular emphasis in their behavior at finite temperature. The specific heat C, previously used in the rare-earth region [5], was identified as a quantity which may signal the appearance of a "phase transition." There are many problems concerning the meaning of phases and phase transitions in nuclei, for the nucleus is a finite system. Moreover, in a finite-temperature mean-field description, quantum as well as statistical fluctuations are present. In regions where quantum fluctuations are less important (heavy nuclei), it has been shown [6] how to correct the mean-field solutions for fluctuations. In light nuclei, where a complete spectrum obtained by a full diagonalization of the Hamiltonian is available, we can perform a statistical description using the canonical ensemble. In this way quantum fluctuations are fully taken into account, but then we lose the possibility of defining an order parameter, usually associated with the intrinsic state, to characterize a nuclear phase. Moreover, a peak in the specific heat due to the finiteness of the space (Schottky effect) [7] is predicted to appear independently of whether a phase transition is taking place or not. A way out of this dilemma is to use, as an effective interaction in the shell-model calculation, the Elliott [8] Hamiltonian. This approach has some unique properties: (a) The energy eigenvalues and their degeneracies can be expressed analytically. (b) There is a one-to-one correspondence between states in the laboratory frame and states in the intrinsic frame. Therefore we define the intrinsic quadrupole moment as an order parameter. (c) The low-lying states, the most important ones for a statistical description, have a large overlap with those of realistic calculations.

The Hamiltonian of the Elliott model is a linear combination of the quadratic Casimir operators of SU(3) and O(3). It is a pure quadrupole-quadrupole interaction, where the quadrupole operator is an SU(3) generator. Its eigenvalues are

$$E(\lambda,\mu,L)=352\kappa-4\kappa[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)]+3\kappa L(L+1)\ , \qquad (1)$$

where λ,μ are the SU(3) quantum numbers, L is the angular momentum, and κ is the strength of the quadrupole-quadrupole interaction. The allowed values of λ,μ for ^{20}Ne are given in Ref. [8]. The other quantum numbers related to λ and μ are

$$\begin{aligned} &K=\min(\lambda,\mu),\ \min(\lambda,\mu)-\ ,\ldots,0 \text{ or } 1\\ &L=K,K+1,\ldots,K+\max(\lambda,\mu),\quad \text{for } K\neq 0 \qquad (2)\\ &L=\max(\lambda,\mu),\ \max(\lambda,\mu)-2,\ldots,0 \text{ or } 1,\quad \text{for} K=0\ . \end{aligned}$$

The value of κ was fixed to $\kappa=0.08$ MeV by fitting the energy of the first 2^+ in ^{20}Ne.

Having now the complete spectrum, we can calculate the partition function Z, the energy E, and the specific heat C in the canonical ensemble as a function of the temperature:

$$Z(\beta)=\sum_i(2I_i+1)(2L_i+1)\exp(-\beta E_i)\ , \qquad (3)$$

$$E(\beta)=-\partial(\ln Z)/\partial\beta\ , \qquad (4)$$

$$C(\beta)=\partial E(\beta)/\partial T\ . \qquad (5)$$

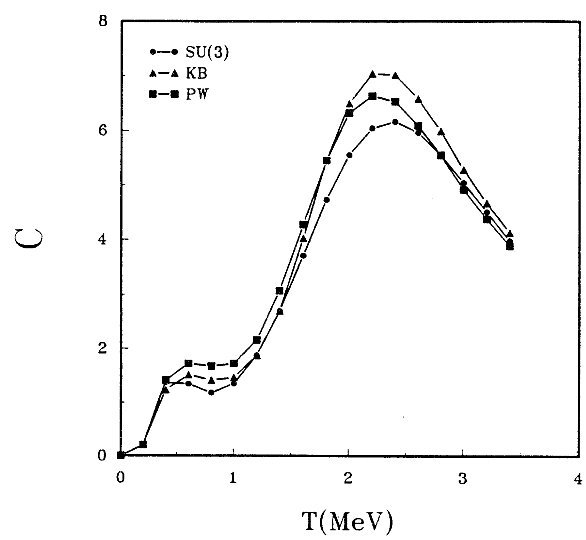

FIG. 1. The specific heat C for ^{20}Ne versus temperature T in MeV for the Kuo and Brown (KB), Preedom and Wildenthal (PW), and Elliott SU(3) interactions.

Here β is the inverse of the temperature T, I is the isospin, and the subscript i labels the irreducible representations of SU(3) and O(3). In Fig. 1 we present results for the specific heat C as a function of the temperature and compare them with the results of other calculations using realistic interactions.

The SU(3) specific heat is similar to the realistic ones, presenting a small peak at 0.5 MeV related to the ground-state rotational band as it was recognized in Ref. [4] and a larger one at $T \approx 2.4$ MeV, which was assumed as a signal of a phase transition from a deformed to a spherical system. In order to study the character of this peak and its relation to deformation we take advantage of the relation between the laboratory frame and the intrinsic frame in the SU(3) model.

The intrinsic quadrupole moment Q_0 is defined in terms of the SU(3) quantum numbers as

$$Q_0(\lambda,\mu)=\begin{cases}(2\lambda+\mu+3), & \text{if } \lambda \geq \mu \ ,\\ (-\lambda-2\mu-3), & \text{if } \lambda < \mu \ .\end{cases} \tag{6}$$

In Fig. 2 we show the expectation value of the intrinsic quadrupole moment as a function of the temperature. The deformation is reduced by about 30% at the temperature associated with the second specific heat peak.

To gain insight into the kind of phenomena that are taking place as a function of temperature, we display in Fig. 3(a) the expectation value of quadrupole moment (as before) and its thermal fluctuations. The 30% reduction in the expectation value of the quadrupole moment, noted above, takes place at a temperature where the thermal fluctuations are exceedingly large.

In order to understand the meaning of these enormous fluctuations, we divide the states of the system into two groups having prolate and oblate shapes, respectively. We then repeat the statistical calculations for each subsystem, getting almost constant expectation values for the quadrupole moment in the two branches and relatively small thermal fluctuations [Fig. 3b]. The reduction in the expectation value of the quadrupole moment thus arises from the mixing of deformed configurations with opposite signs rather than from the appearance of a dominant spherical configuration.

We conclude that there is not a true phase transition to sphericity in this model. Whether the realistic calculations of ^{20}Ne display a true phase transition is still an

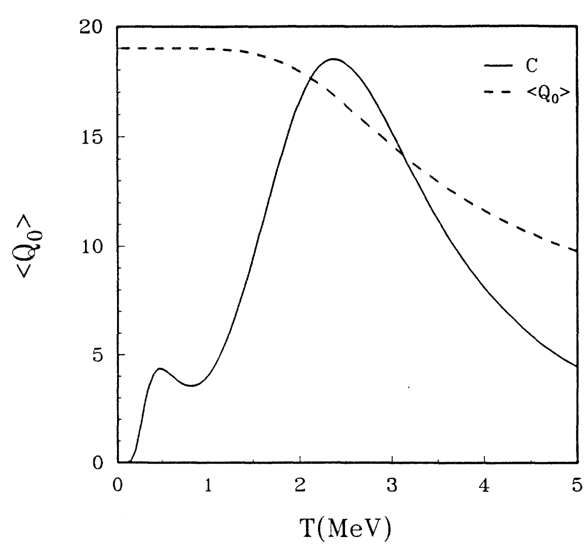

FIG. 2. Expectation value of the intrinsic quadrupole moment and the SU(3) specific heat as a function of temperature.

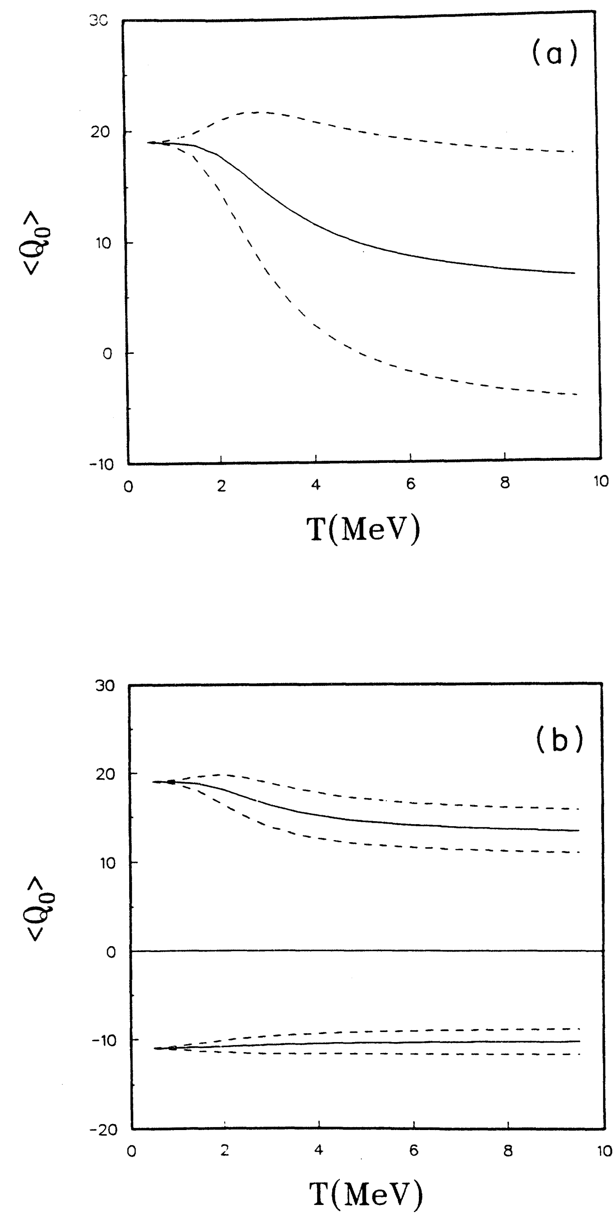

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FIG. 3. (a) Expectation value of the intrinsic quadrupole moment (solid line) and its thermal fluctuation (dashed line). (b) The same as (a) but for prolate and oblate separately.

open question, but in light of the present calculations we can state that a peak in the specific heat does not necessarily signal one. Furthermore, as commented above, this peak may be to the finiteness of the space.

The present work does not contradict previous calculations in larger spaces (heavy nuclei), but calls attention to the proper definition of a phase in nuclear physics, especially in light nuclei where large quantum fluctuations are expected. We suggest that a study of the fluctuations in the order parameter (intrinsic quadrupole moment, pairing gap, etc.) around its thermal mean field value is important in order to be able to define a nuclear phase. We also show how exact solvable models containing the relevant dynamics and defined in the appropriate Hilbert space may be powerful tools to study thermal properties of nuclei in the canonical ensemble.

One of the authors (J.D.) acknowledges support of the Dirección General de Investigacion Científica y Técnica of Spain.

*On leave from the Comisión Nacional de Energía Atómica of Argentina.

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