Alpha-particle scattering and isoscalar giant resonances in the SU(3) model

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(Received 3 January 1991)

Giant isoscalar monopole and quadrupole resonances are introduced into the SU(3) limit of the interacting boson model by means of S and D bosons simulating collective particle-hole excitations, in addition to the usual s and d bosons, representing either collective particle or hole pairs in the valence shell. The evaluated giant resonance energies, as well as the matrix elements of $E0$ and $E2$ transitions from the giant resonance levels to the 0_1^+-2_1^+-4_1^+ members of the ground-state band, are utilized in coupled-channel calculations of giant resonance excitation through inelastic alpha scattering by the transitional isotopes ^{148}Sm and ^{154}Sm: the calculated differential cross sections are then compared with experimental data.

I. INTRODUCTION

Arima and Iachello's interacting boson model[1] (IBM) has proved to be suitable to describe high-energy collective states in nonmagic even-even nuclei, in addition to the low-energy modes for which the model was originally designed. Calculations of photonuclear reactions involving the excitation of isovector giant dipole resonances (GDR) in various regions of the Periodic Table[2] compare well with experimental data.

Rowe and Iachello[3] have discussed the possibility of treating isoscalar monopole (GMR) and quadrupole (GQR) resonances in an SU(3) coupling scheme, which can be viewed either as a limit of the IBM for deformed nuclei or as a contraction of the algebra of the symplectic model (SM) for collective motion[4] when the number of harmonic-oscillator quanta of the nuclear ground state goes to infinity.

Both the IBM and the SM have their microscopic roots in the shell model. While the original formulation of the IBM, based on an u(6) algebra, contains only collective particle-particle or hole-hole excitations in the valence shell, and $2\hbar\omega$ particle-hole excitations are to be added as a further degree of freedom, the SM incorporates both $0\hbar\omega$ and $2\hbar\omega$ collective excitations into the definition of the sp(3,R) algebra.

Therefore, Sec. II illustrates the IBM Hamiltonian used in the present work and outlines some possible changes in the direction of the SM. Section III is devoted to the prediction of GMR and GQR splitting in the exact SU(3) symmetry and Sec. IV to the discussion of transition operators and energy-weighted sum rules. We remind the reader that schematic calculations of fragmentation patterns of GMR's and GQR's in the transitional isotope chain ^{148}Sm-^{154}Sm have already been carried out[5] by numerical diagonalization of an IBM Hamiltonian without expecting to achieve accurate agreement with experimental data.

Lively interest in the GMR and its connection with the compressibility of nuclei, hence of nuclear matter, leads us to a new analysis of GMR and GQR in the frame of IBM, supplemented by coupled-channel calculations of alpha-scattering cross sections, in order to test our model by comparison with experimental data for the abovementioned isotope chain. In particular, we analyze GMR and GQR excitation due to the scattering of 129 MeV alpha particles by ^{154}Sm,[6] and 115 MeV alphas by ^{148}Sm.[7] We have focused our attention on alpha scattering because alphas are excellent probes for the GMR, whose measurements are not perturbed by the GDR, which is close in energy but weakly excited by alpha scattering. The results of our analysis are discussed in Secs. V and VI.

II. THE HAMILTONIAN

The present work follows the same phenomenologic approach as Ref. 5. GMR and GQR excitations are simulated in version 1 of the IBM by particle-hole bosons S ($L^\pi=0^+$) and D ($L^\pi=2^+$), in addition to the usual particle-particle or hole-hole s and d bosons generating the low-energy collective states of positive parity in even-even nuclei. The basis states are thus of the form

$$|\Psi\rangle=\frac{1}{\sqrt{m!n!p!q!}}(s^\dagger)^m(d^\dagger)^n(S^\dagger)^p(D^\dagger)^q|0\rangle\ , \qquad (1)$$

where $m+n=N$ is the usual s-d boson number and $p,q=0$, or 1, if one adopts the one-boson approximation to the giant resonances. Unlike the s-d case, the S-D boson number need not be conserved. The adopted IBM-1 Hamiltonian reads as follows:

$$\hat{\mathcal{H}}=\hat{\mathcal{H}}(s,d)+\varepsilon_S\hat{n}_S+\varepsilon_D\hat{n}_D+\hat{\mathcal{H}}(s,d,S,D)\ . \qquad (2)$$

Here, $\hat{\mathcal{H}}(s,d)$ is the usual s-d boson Hamiltonian, written as a multipole expansion, with the definitions of Ref. 1

$$\hat{\mathcal{H}}(s,d)=\varepsilon_d\hat{n}_d+a_0\hat{P}^\dagger\cdot\hat{P}+a_1\hat{L}\cdot\hat{L}+a_2\hat{Q}\cdot\hat{Q}+a_3\hat{T}_3\cdot\hat{T}_3+a_4\hat{T}_4\cdot\hat{T}_4\ , \tag{3}$$

and ε_S (ε_D) is the S (D) boson energy, $\hat{n}_S$ ($\hat{n}_D$) the S (D) boson number. Finally, $\hat{\mathcal{H}}(s,d,S,D)$ is the interaction Hamiltonian, which couples GMR and GQR to each other and to the low-energy modes. The interaction adopted in the present work is of quadrupole-quadrupole type:

$$\begin{aligned}\hat{\mathcal{H}}(s,d,S,D)&=c_2^{(2)}\hat{Q}(s,d)\cdot\hat{Q}(S,D)\\&=c_2^{(2)}[(s^\dagger\times\tilde{d})^{(2)}+(d^\dagger\times\tilde{s})^{(2)}+\chi(d^\dagger\times\tilde{d})^{(2)}]\cdot[(S^\dagger\times\tilde{D})^{(2)}+(D^\dagger\times\tilde{S})^{(2)}+\chi'(D^\dagger\times\tilde{D})^{(2)}]\ ,\end{aligned} \tag{4}$$

where the quadrupole operator $\hat{Q}(S,D)$ has been chosen of the same form as the s-d quadrupole, $\hat{Q}(s,d)$. Formula (4) is less general than the interaction Hamiltonian adopted in our preliminary work[5] where, for instance, a first-order D quadrupole interaction had been included:

$$\hat{\mathcal{H}}'(s,d,S,D)=c_1^{(2)}(D^\dagger+\tilde{D})\cdot\hat{Q}(s,d)+c_2^{(2)}\hat{Q}(S,D)\cdot\hat{Q}(s,d)\ . \tag{5}$$

The effect of the first term on the right-hand side of Eq. (5) is to transfer D strength to the low-energy modes, thus playing an important role in problems of core polarization, but possibly a minor one in the splitting of GMR and GQR, on which the present work is focused.

An important generalization of the quadrupole-quadrupole interaction is possible if one goes beyond the one-boson approximation and adds to $\hat{\mathcal{H}}'$ in formula (5) two S-D interaction terms:

$$\hat{\mathcal{H}}''(s,d,S,D)=\hat{\mathcal{H}}'(s,d,S,D)+c_0^{(2)}(D^\dagger+\tilde{D})\cdot(D^\dagger+\tilde{D})+c_3^{(2)}\hat{Q}(S,D)\cdot\hat{Q}(S,D)\ . \tag{6}$$

Assuming an interaction Hamiltonian of type (6) and adding to it the s-d quadrupole interaction, $a_2\hat{Q}(s,d)\cdot\hat{Q}(s,d)$, contained in formula (3), amounts to defining a total quadrupole operator of the following form:

$$\hat{Q}_{\text{tot}}(s,d,S,D)=\alpha\hat{Q}(s,d)+\beta(D^\dagger+\tilde{D})+\gamma\hat{Q}(S,D)\ , \tag{7}$$

where α, β, and γ are suitable coefficients and the corresponding quadrupole-quadrupole interaction is

$$\hat{\mathcal{H}}_{\text{int}}(s,d,S,D)=\kappa\hat{Q}_{\text{tot}}\cdot\hat{Q}_{\text{tot}}\ , \tag{8}$$

which has microscopic roots, unlike the simpler interaction adopted in this work. In fact, when $\alpha=\gamma=1$, $\beta=\sqrt{N_0}/2$, where $N_0\simeq 0.9A^{4/3}$ is the number of harmonic-oscillator quanta in the nuclear ground state, and $\chi=\chi'=-\sqrt{7}/2$, the operator of formula (7) is proportional to the total quadrupole operator in U(3)⊗HW(6) limit of the SM.[8,9] The possibility of investigating GMR and GQR by including formulas (7) and (8) in the IBM is already under study.

III. THE SU(3) LIMIT

Coming back to the interaction Hamiltonian (4), it is of some interest to analyze its predictions in the SU(3) limit of the IBM, where energies and transition strengths for GMR and GQR can be estimated by means of algebraic formulas. Here, we follow the procedure outlined by Rowe and Iachello.[3]

When $\varepsilon_d=a_0=a_3=a_4=0$ and $\chi=-\sqrt{7}/2$ in formula (3), $\varepsilon_S=\varepsilon_D$ and $\chi'=-\sqrt{7}/2$ in formula (4), the total IBM Hamiltonian exhibits SU(3) symmetry, since it can be written in terms of quadratic Casimir operators of SU(3) and its subgroup SO(3), $C_2(\text{SU}(3))=\frac{4}{3}\hat{Q}\cdot\hat{Q}+\frac{1}{2}\hat{L}\cdot\hat{L}$ and $C_2(\text{SO}(3))=2\hat{L}\cdot\hat{L}$, respectively, with expectation values $\langle C_2(\text{SU}(3))\rangle=\frac{2}{3}(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)$ in the (λ,μ) irreducible representation (irrep) of SU(3), and $\langle C_2(\text{SO}(3))\rangle=2L(L+1)$ in the L irrep of SO(3). In particular, the ground-state band of an axially symmetric nucleus belongs to the $(2N,0)$ irrep of SU(3), N being the number of s and d bosons, while the S and D boson creators, $S^\dagger,D_\mu^\dagger$ ($\mu=-2,\ldots,+2$), transform according to the (2,0) irrep and quadrupole operator $\hat{Q}_\lambda$ ($\lambda=-2,\ldots,+2$) and the angular momentum $\hat{L}_\nu$ ($\nu=-1,0,+1$) transform according to the (1,1) irrep. Thus, coupling an S,D boson to the ground-state band, in order to build a giant resonance, amounts to the following reducible product of representations:

$$(2N,0)\otimes(2,0)=(2N+2,0)\oplus(2N,1)\oplus(2N-2,2)\ . \tag{9}$$

The GMR (GQR) excitations are the $L=0$ ($L=2$) members of the SO(3) irreps contained in the SU(3) irreps on the right-hand side of Eq. (9), labeled with a quantum number, K, corresponding to the projection of the angular momentum on the nuclear symmetry axis: $(2N+2,0)$ contains a $K=0$ band with $L=0,2,\ldots,2N+2$; $(2N,1)$ a $K=1$ band with $L=1,2,\ldots,2N+1$; and $(2N-2,2)$ a $K=0$ band with $L=0,2,\ldots,2N-2$ and a $K=2$ band with $L=2,3,\ldots,2N$. Therefore, the SU(3) limit of the model predicts two GMR components and four GQR components, the GMR states being close in energy to the GQR states with $K=0$, one of which has the same energy as the $K=2$ state, since they belong to the same SU(3) irrep, $(2N-2,2)$.

The energies associated with the irreps of Eq. (9) are easily evaluated if one expresses the interaction Hamiltonian (4) by means of the Casimir operators of SU(3) and SO(3) and takes the expectation value in the irreps (9); with the same approximations as in Ref. 3 one obtains

$$E_1 \simeq \varepsilon_{S,D} + \tfrac{3}{8}c_2^{(2)}[\langle C_2(2N+2,0)\rangle - \langle C_2(2N,0)\rangle - \langle C_2(2,0)\rangle] = \varepsilon_{S,D} + 2c_2^{(2)}N \,, \tag{10a}$$

$$E_2 \simeq \varepsilon_{S,D} + \tfrac{3}{8}c_2^{(2)}[\langle C_2(2N-2,2)\rangle - \langle C_2(2N,0)\rangle - \langle C_2(2,0)\rangle] = \varepsilon_{S,D} - c_2^{(2)}\frac{(2N+3)}{2} \,, \tag{10b}$$

$$E_3 \simeq \tfrac{3}{8}c_2^{(2)}[\langle C_2(2N,1)\rangle - \langle C_2(2N,0)\rangle - \langle C_2(2,0)\rangle] = \varepsilon_{S,D} + c_2^{(2)}\frac{(N-3)}{2} \,. \tag{10c}$$

The reduced matrix elements of the $E0$ and $E2$ operators for transitions between the ground state and the GMR and GQR states, respectively, are proportional to the corresponding Wigner coefficients for the SU(3)$\supset$SO(3) reduction chain,[3] given in analytical form in Ref. 10, and the fractions of energy-weighted sum rule (EWSR) exhausted by each state are easily evaluated.

In terms of Wigner coefficients, the $E0$ EWSR is proportional to

$$S(E0) = E_1 |\langle (2N,0),0,(2,0),0 \| (2N+2,0),0\rangle|^2 + E_2 |\langle (2N,0),0,(2,0),0 \| (2N-2,2),0\rangle|^2 \,. \tag{11a}$$

Therefore, the EWSR fractions corresponding to the two GMR states are

$$S_1(E0) = \frac{1}{S(E0)} E_1 |\langle (2N,0),0,(2,0),0 \| (2N+2,0),0\rangle|^2 = \frac{E_1(2N+3)}{E_1(2N+3)+4NE_2} \,, \tag{11b}$$

$$S_2(E0) = \frac{1}{S(E0)} E_2 |\langle (2N,0),0,(2,0),0 \| (2N-2,2),0\rangle|^2 = \frac{4NE_2}{E_1(2N+3)+4NE_2} \,. \tag{11c}$$

In the same way one obtains the EWSR fractions of the GQR states:

$$\begin{aligned} S(E2) &= E_1 |\langle (2N,0),0,(2,0),2 \| (2N+2,0),2\rangle|^2 + E_3 |\langle (2N,0),0,(2,0),2 \| (2N,1),2\rangle|^2 \\ &\quad + E_2 [\,|\langle (2N,0),0,(2,0),2 \| (2N-2,2),2\rangle_{K=0}|^2 + |\langle (2N,0),0,(2,0),2 \| (2N-2,2),2\rangle_{K=2}|^2] \\ &= E_1 \frac{2}{15}\frac{(2N+3)(2N+5)}{(2N+1)(2N+2)} + E_2 \frac{56N^3-36N^2-42N+27}{15(2N+1)(4N^2-3)} + E_3 \frac{2}{5}\frac{2N+3}{2N+2} \,. \end{aligned} \tag{12a}$$

$$S_1(E2) = \frac{1}{S(E2)} E_1 \frac{2}{15}\frac{(2N+3)(2N+5)}{(2N+1)(2N+2)} \,, \tag{12b}$$

$$S_2(E2) = \frac{1}{S(E2)} E_2 \frac{56N^3-36N^2-42N+27}{15(2N+1)(4N^2-3)} \,, \tag{12c}$$

$$S_3(E2) = \frac{1}{S(E2)} \frac{2}{5} E_3 \frac{2N+3}{2N+2} \,. \tag{12d}$$

As an example, let us consider ^{232}Th, whose low-energy spectrum and GDR states are described with good accuracy in the SU(3) limit of the IBM.[2] Inelastic alpha-particle scattering[11] has revealed two GMR states at 9.6 and 13.8 MeV and a GQR state at 10.9 MeV. By imposing $E_1 = 9.6$ MeV and $E_2 = 13.8$ MeV, with $N=12$, one obtains $\varepsilon_{S,D} = 12.29$ MeV and $c_2^{(2)} = -0.112$ MeV from formulas (10a) and (10b) and $E_3 = 11.78$ MeV from formula (10c), about 1 MeV higher than the experimental energy of the main GQR component. The monopole EWSR fractions from formulas (11a)–(11c) are $S_1 = 28\%$, in comparison with experimental values ranging from 21% to 28%, and $S_2 = 72\%$, against an experimental range of 63–66%. The quadrupole EWSR fractions exhausted by the states at 9.60, 11.78, and 13.80 MeV are12.6%, 39.8%, and 47.6%, while the experiments assign a strength of 62–63% to the intermediate state, located at 10.9 MeV.

Although the pure SU(3) symmetry fails to reproduce all the experimental values, the predicted GMR splitting for a deformed nucleus is good enough to encourage a numerical analysis based on the interaction Hamiltonian (4), provided the SU(3) symmetry is broken by assuming either $\varepsilon_S \neq \varepsilon_D$ or $\chi' \neq -\sqrt{7}/2$. The amount of SU(3) breaking is expected to increase along a transitional isotope chain, going from deformed to spherical nuclei.

IV. TRANSITION OPERATORS AND ENERGY-WEIGHTED SUM RULES

The simplified interaction (4) makes a clear distinction between the wave functions of the low-energy excitations, consisting of s and d components only, and the GR wave functions, containing also S and D components. Therefore, the transitions between members of the ground-state band and those involving the GR states are described by different operators.

According to Ref. 1, the electric-multipole transition densities between the low-lying states of interest in the present work are describable in terms of the following operators:

$$\hat{T}(E0)=\varepsilon_{dd}^{(0)}(r)(d^{\dagger}\times\tilde{d})^{(0)} , \tag{13a}$$

$$\hat{T}(E2)=\varepsilon_{sd}^{(2)}(r)[(s^{\dagger}\times\tilde{d})^{(2)}+(d^{\dagger}\times\tilde{s})^{(2)}] + \varepsilon_{dd}^{(2)}(r)(d^{\dagger}\times\tilde{d})^{(2)} , \tag{13b}$$

$$\hat{T}(E4)=\varepsilon_{dd}^{(4)}(r)(d^{\dagger}\times\tilde{d})^{(4)} . \tag{13c}$$

For instance, the $E2$ transition density from an initial state of spin I_i to a final state of spin I_f is given by the reduced matrix element of (13b) between the abovementioned states:

$$\rho_{I_iI_f}^{(2)}=\varepsilon_{sd}^{(2)}(r)\langle I_f\|(s^{\dagger}\times\tilde{d})^{(2)}+(d^{\dagger}\times\tilde{s})^{(2)}\|I_i\rangle + \varepsilon_{dd}^{(2)}(r)\langle I_f\|(d^{\dagger}\times\tilde{d})^{(2)}\|I_i\rangle . \tag{14}$$

The matrix elements on the right-hand side of formula (14) contain full information about the nuclear structure.

The constants in front of the usual IBM-1 transition operators are obtained by integrating the radial functions $\varepsilon_{ij}^{(\lambda)}(r)$ in formulas (13a)–(13c), for $\lambda=2$ and 4:

$$e_{ij}^{(\lambda)}=\int_0^{\infty}\varepsilon_{ij}^{(\lambda)}(r)r^{\lambda+2}dr \tag{15a}$$

and for $\lambda=0$:

$$e_{dd}^{(0)}=4\pi\int_0^{\infty}\varepsilon_{dd}^{(0)}(r)r^4dr . \tag{15b}$$

For the samarium isotopes studied in this work the constants on the left-hand side of Eqs. (15a) and (15b) have been obtained from the effective boson charges given in Ref. 12, multiplied by a factor of A/Z, and are used for normalizing the transition densities.

The transition operators between GMR, GQR, and low-lying states are, to the lowest order,

$$\hat{T}'(E0)=\alpha^{(0)}(r)(S^{\dagger}+\tilde{S}) , \tag{16a}$$

$$\hat{T}'(E2)=\alpha^{(2)}(r)(D^{\dagger}+\tilde{D}) . \tag{16b}$$

In analogy with Eqs. (15a) and (15b), one obtains the constants

$$a^{(2)}=\int_0^{\infty}\alpha^{(2)}(r)r^4dr , \tag{17a}$$

$$a^{(0)}=4\pi\int_0^{\infty}\alpha^{(0)}(r)r^4dr . \tag{17b}$$

$a^{(0)}$ and $a^{(2)}$ can be directly calculated if one assumes that the GMR and GQR states exhaust the corresponding EWSR's:[13]

$$\sum_m E(0_m^+)|\langle 0_m^+\|a^{(0)}(S^{\dagger}+\tilde{S})\|0_1^+\rangle|^2=2\frac{\hbar^2}{m_N}A\langle r^2\rangle , \tag{18}$$

$$\sum_n E(2_n^+)|\langle 2_n^+\|a^{(2)}(D^{\dagger}+\tilde{D})\|0_1^+\rangle|^2=\frac{25}{4\pi}\frac{\hbar^2}{m_N}A\langle r^2\rangle . \tag{19}$$

Here, index m (n) runs over the GMR (GQR) states, m_N is the atomic mass unit, and the mean square radius $\langle r^2\rangle$ is evaluated over the mass distribution in the ground state. We assume a uniform distribution of characteristic radius $R_0=1.2A^{1/3}$ fm, so that $\langle r^2\rangle=\frac{3}{5}R_0^2$.

The radial functions $\varepsilon_{ij}^{(\lambda)}(r)$ appearing in the transition densities for the low-lying states, according to formulas (13a)–(13c) are taken from the geometrical model[14] and fitting the form factors in electron scattering.[15,16] More precisely, $\varepsilon_{sd}^{(2)}(r)$ and $\varepsilon_{dd}^{(4)}(r)$ are taken to be proportional to the first derivative of the ground-state density, $\varepsilon_{dd}^{(2)}(r)$ is proportional to the second derivative, and $\varepsilon_{dd}^{(0)}(r)$ is a linear combination of the ground-state density and its first derivative, with the coefficients chosen so that the volume integral, $\int_0^{\infty}\varepsilon_{dd}^{(0)}(r)r^2dr$, vanishes, in accordance with version 2 of the monopole transition density of Ref. 17. The radial functions are normalized according to formulas (15a) and (15b).

The functions $\alpha^{(2)}(r)$ and $\alpha^{(0)}(r)$ of formulas (16a) and (16b), defining the transition densities for GQR and GMR states, have the same form as $\varepsilon_{sd}^{(2)}(r)$ and $\varepsilon_{dd}^{(0)}(r)$, respectively, and are normalized according to formulas (17a) and (17b).

As for the Hamiltonian parameters, those of $\hat{\mathcal{H}}(s,d)$ in formula (3) are taken from Ref. 12 for the samarium isotopes studied in this work. The parameters of the S-D terms, namely, ε_S, ε_D, $c_2^{(2)}$, and χ', are chosen so as to reproduce experimental energies and scattering cross sections for the GMR and GQR states. The values adopted for ^{148}Sm and ^{154}Sm are listed in Table I, the resulting GR energies and EWSR fractions in Table II. The energies and the transition matrix elements, obtained by means of the GR-GRT codes,[18] are to be used, with an appropriate choice of optical model potentials, in coupled-channel calculations of angular distributions of alpha

TABLE I. IBM parameters.

Isotope	^{148}Sm	^{154}Sm
N	8	11
ε_d (MeV)	0.7703	0.3710
a_0 (MeV)	0.0523	0.0080
a_1 (MeV)	0.0040	0.0005
a_2 (MeV)	−0.0126	−0.019 55
a_3 (MeV)	0.0296	0.0084
a_4 (MeV)	0.0197	0.005 65
χ	−0.650	−1.200
ε_S (MeV)	14.30	12.50
ε_D (MeV)	12.80	12.50
$c_2^{(2)}$ (MeV)	−0.130	−0.130
χ'	−0.300	−0.300

TABLE II. GR energies and strengths.

$E(0^+)$ (MeV)	S (% EWSR)	$E(2^+)$ (MeV)	S (% EWSR)
	^{148}Sm		
12.58	23	12.59	75
13.90	4	13.22	18
14.47	1	13.48	4
14.95	71	15.61	3
15.36	1		
	^{154}Sm		
10.38	38	10.46	10
11.78	1	12.47	50
13.86	1	12.88	30
13.98	7	14.08	2
14.30	1	14.48	2
14.42	49	14.49	3
15.17	1	14.51	3

particles scattered through GMR and GQR excitation, as discussed in the next section.

As a final remark on transition operators, it is to be pointed out that adoption of the quadrupole-quadrupole interaction (8) would require different definitions for the transition operators, because, in that case, both low-lying and high-lying states are made of s, d, S, and D bosons; thus, for instance, all the $E2$ transitions should be described by an operator proportional to the quadrupole of formula (7).

V. ANALYSIS OF ALPHA-PARTICLE SCATTERING

The differential cross sections for the scattering of alphas leaving the target nucleus either in one of the first three excitations of the ground-state band, with spin and parity $J^\pi=0^+, 2^+, 4^+$, or in a GMR or GQR excitation have been evaluated in the coupled-channel formalism by means of the ECIS88 code.[19]

Since the experimental groups[6,7] who measured the angular distributions of alphas scattered by samarium isotopes in the energetic and angular ranges revealing the GMR and GQR excitations do not yield the corresponding angular distributions for elastic scattering, we cannot determine the optical model parameters needed in our calculations from these measurements. Therefore, we resort to the optical model adopted in the analysis of differential cross sections for 120 MeV alphas elastically and inelastically scattered by $^{144-154}$Sm.[20] The optical model has the following form:

$$V(r)=V_C(r)-V_R f^2(r,a_R,R_R) -iW_V f(r,a_V,R_V)+4iW_S\frac{df}{dr}(r,a_S,R_S)\ , \quad (20)$$

where $V_C(r)$ is the Coulomb potential and

$$f(r,a,R_0)=\frac{1}{1+\exp[(r-R_0A^{1/3})/a]} \quad (21)$$

is a Woods-Saxon form. The adopted optical model parameters are listed in Table III.

The incident alpha energies in our calculations are 115 MeV,[7] and 129 MeV:[6] In the former case we adopted the optical model parameters labeled with "c" in Table I of Ref. 20; in the latter we modified the above-mentioned parameters by decreasing the depth of the real potential well, V_R, and increasing the depths, W_V and W_S, of the imaginary volume and surface potentials, in order to take into account the fact that the alpha energy is higher than that of Ref. 20.

The transition potentials appearing in the coupled-channel equations have been determined by means of an implicit folding procedure,[21,22] which allows us to derive the transition potentials from the optical potential through the same functional relations existing between the transition densities and the ground-state density. Moreover, the transition potentials have been normalized so as to satisfy the Satchler theorem,[23] valid for density-independent nuclear forces; in this case the transition potentials have the same normalized multipole moments as the transition densities:

$$\frac{\int_0^\infty V^{(\lambda)}(r)r^{\lambda+2}dr}{\int_0^\infty V(r)r^2dr}=\frac{\int_0^\infty \rho^{(\lambda)}(r)r^{\lambda+2}dr}{\int_0^\infty \rho(r)r^2dr}\ , \quad (22)$$

TABLE III. Optical model parameters.

$^{148-154}$Sm	$E_\alpha=115$ MeV	$E_\alpha=129$ MeV
V_R (MeV)	164.17	150.17
R_R (fm)	1.329	1.329
a_R (fm)	1.265	1.265
W_V (MeV)	20.31	27.31
R_V (fm)	1.481	1.481
a_V (fm)	0.641	0.641
W_S (MeV)	3.557	6.557
R_S (fm)	1.266	1.266
a_S (fm)	0.711	0.711
R_C (fm)	1.300	1.300

where $V^{(\lambda)}$ is the transition potential of multipolarity λ, V the optical potential, $\rho^{(\lambda)}$ the transition density of multipolarity λ, and ρ the ground-state density.

In transitions of multipolarity 2 and 4 the Coulomb excitation has been taken into account by means of standard Coulomb form factors.[14] Its influence is great at small scattering angles, where it interferes destructively with the nuclear potential, thus reducing the differential cross section with respect to pure nuclear scattering.

For the sake of comparison between calculations and experiments, we have chosen two members of the transitional samarium chain having different structure, ^{154}Sm, an axially symmetric rotor, and ^{148}Sm, an anharmonic vibrator. According to Sec. III, the SU(3) symmetry breaking is expected to increase from ^{154}Sm to ^{148}Sm; that is why we have kept $\varepsilon_S=\varepsilon_D$ for ^{154}Sm, the symmetry of the interaction Hamiltonian (4) being broken only by $\chi'\neq-\sqrt{7}/2$, while for ^{148}Sm we have assumed in addition that $\varepsilon_S\neq\varepsilon_D$. Figure 1 shows the differential cross sections for GQR and GMR excitation by 129 MeV alphas scattered by ^{154}Sm: The calculated cross sections appear to be in good agreement with the experimental data[6] and are certainly not worse than the results of the geometrical model, thus suggesting that the simple interaction (4) with moderate SU(3) breaking makes it possible to analyze GQR and GMR in deformed nuclei.

The situation is more controversial for ^{148}Sm, as shown in Fig. 2, where the cross sections for 115 MeV alphas scattered by ^{148}Sm are compared with the experimental data of Ref. 7. It is to be pointed out, however, that these older measurements had originally been interpreted without taking the GMR into account: For instance, the data at excitation energy $E^x=14.9$ MeV had been fitted by $L=2$ distorted-wave Born-approximation calculations, with $S_2(E2)=25\%$ EWSR.

More recent data on ^{148}Sm pose further problems, since they appear to be in partial disagreement. In fact, the ratios of differential cross sections at $E^x=12.1$ MeV and 14.6 MeV for 129 MeV alphas scattered in the angular range[24] $\theta=2°-6°$ turn out to be close to 1 and suggest a large mixture of GMR and GQR at both excitation energies; these results cannot be explained by the present model.

On the other hand, the authors of Ref. 25 measure the ratios of the intensity of the GMR and the GQR for 120 MeV alphas scattered into two narrow angular bins (0°–1.5°) and (1.5°–3.0°) and conclude that the excitation at the higher energy, $E^x=14.95\pm0.14$ MeV, has a rather pure monopole character, exhausting $117\pm27\%$ EWSR. We have evaluated the ratios of angle-averaged cross sections according to the formula

$$R=\frac{\int_0^{1.5}\sigma(\theta)\sin\theta\,d\theta}{\int_0^{1.5}\sin\theta\,d\theta}\,\frac{\int_{1.5}^{3.0}\sin\theta\,d\theta}{\int_{1.5}^{3.0}\sigma(\theta)\sin\theta\,d\theta} \tag{23}$$

at the energy centroids of both GMR and GQR and obtained the values $R_{\rm calc}({\rm GMR})=2.85$, which compares well with the experimental datum, $R_{\rm exp}({\rm GMR})=2.65\pm0.37$ in Ref. 25, and $R_{\rm calc}({\rm GQR})=1.61$, slightly higher

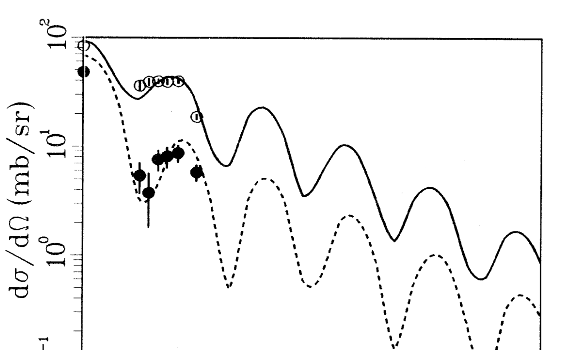

FIG. 1. Differential cross sections for GMR and GQR excitation by 129 MeV alpha particles scattered by ^{154}Sm. Open dots, experimental data at excitation energy $E^x=11.8\pm0.3$ MeV (Ref. 6); solid line, calculated cross section, including contributions from $L=0$ and $L=2$ states in the energy range $10.38\leq E^x\leq 12.88$ MeV (see Table II). Solid dots, experimental data at $E^x=14.9\pm0.3$ MeV (Ref. 6); dashed line, calculated cross section, including contributions from $L=0$ and $L=2$ states in the range $13.98\leq E^x\leq 14.51$ MeV (see Table II).

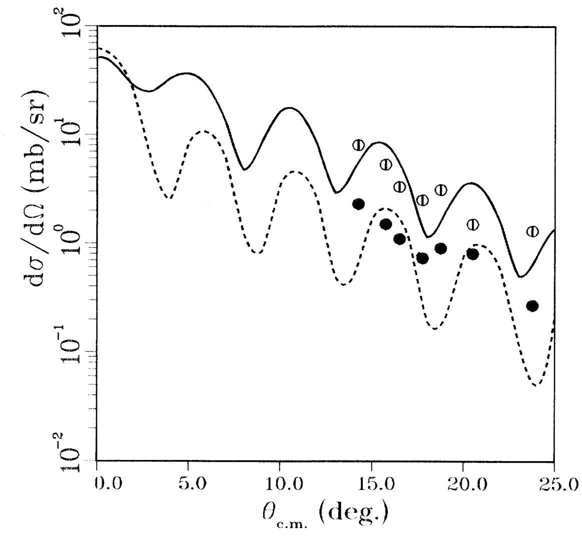

FIG. 2. GMR and GQR excitation by 115 MeV alpha particles scattered by ^{148}Sm. Open dots, experimental data at $E^x\simeq 12.0$ MeV (Ref. 7); solid line, calculated cross section, including contributions from $L=0$ and $L=2$ states in the range $12.58\leq E^x\leq 13.22$ MeV. Solid dots, experimental data at $E^x\simeq 14.9$ MeV (Ref. 7); dashed line, calculated cross section, with contributions from $L=0$ and $L=2$ states in the range $13.48\leq E^x\leq 14.95$ MeV (see Table II).

than the experimental value, $R_{\mathrm{exp}}(\mathrm{GQR})=1.26\pm0.27$, due to the strong monopole component at $E^x \simeq 12$ MeV in our calculations. New measurements might throw light on the problem and provide a more reliable test of the model for transitional nuclei.

VI. CONCLUSIONS AND PERSPECTIVES

The results presented in the preceding section seem to indicate that the one-boson approximation plus a simple interaction Hamiltonian, reminiscent of SU(3) symmetry, allow the isoscalar giant resonances to be treated in the frame of the IBM, at least in the case of deformed nuclei; the results are less conclusive for transitional nuclei where, in addition, the experimental situation is not completely clear.

In any case, they encourage us to study possible improvements of the model, such as removing the one-boson approximation and adopting a more general quadrupole-quadrupole interaction, reminiscent of the symplectic model, in an attempt to move toward microscopic determination of the IBM parameters.

We wish to thank Professor F. Iachello and Professor M. Pignanelli for useful discussions, Dr. J. Raynal for providing us with the ECIS88 code, and Dr. U. Garg for the experimental data of samarium isotopes.

[1] F. Iachello and A. Arima, *The Interacting Boston Model* (Cambridge University Press, Cambridge, England, 1987).
[2] See R. F. Casten and D. D. Warner, Rev. Mod. Phys. **60**, 389 (1988) for a review of papers prior to 1988; recent works are G. Maino, A. Ventura, and L. Zuffi, Phys. Rev. C **37**, 1379 (1988); S. Turrini, G. Maino, and A. Ventura, *ibid.* **39**, 824 (1989); E. Bortolani and G. Maino, *ibid.* **43**, 343 (1991).
[3] D. J. Rowe and F. Iachello, Phys. Lett. **130B**, 231 (1983).
[4] G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. **38**, 10 (1977); Ann. Phys. (N.Y.) **126**, 343 (1980).
[5] G. Maino, A. Ventura, P. Van Isacker, and L. Zuffi, Europhys. Lett. **2**, 345 (1986).
[6] D. H. Youngblood, P. Bogucki, J. D. Bronson, U. Garg, Y. W. Lui, and C. M. Rozsa, Phys. Rev. C **23**, 1997 (1981); U. Garg, private communication.
[7] D. H. Youngblood, J. M. Moss, C. M. Rozsa, J. D. Bronson, A. D. Bacher, and D. R. Brown, Phys. Rev. C **13**, 994 (1976).
[8] G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. **47**, 223 (1981).
[9] P. Rochford and D. J. Rowe, Nucl. Phys. **A492**, 253 (1989).
[10] J. D. Vergados, Nucl. Phys. **A111**, 681 (1968).
[11] H. P. Morsch, M. Rogge, P. Turek, C. Mayer-Böricke, and P. Decowski, Phys. Rev. C **25**, 2939 (1982).
[12] O. Scholten, Ph.D. thesis, University of Groningen, 1980 (unpublished).
[13] A. M. Lane, *Nuclear Theory* (Benjamin, New York, 1964); A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.
[14] G. R. Satchler, *Direct Nuclear Reactions* (Oxford University Press, New York, 1983), p. 581.
[15] A. E. L. Dieperink, F. Iachello, A. Rinat, and C. Creswell, Phys. Lett. **76B**, 135 (1978).
[16] M. A. Moinester, J. Alster, G. Azuelos, J. B. Bellicard, B. Frois, M. Huet, P. Leconte, and P. X. Ho, Phys. Rev. C **24**, 80 (1980); M. A. Moinester, J. Alster, G. Azuelos, and A. E. L. Dieperink, Nucl. Phys. **A383**, 264 (1982).
[17] G. R. Satchler, Part. Nucl. **5**, 105 (1973).
[18] P. Van Isacker, computer programs GR and GRT (unpublished).
[19] J. Raynal, in *Applied Nuclear Theory and Nuclear Model Calculations for Nuclear Technology,* edited by M. K. Mehta and J. J. Schmidt (World Scientific, Singapore, 1989).
[20] T. Ichihara, N. Sakaguchi, M. Nakamura, T. Noro, H. Sakamoto, H. Ogawa, M. Yosoi, M. Ieiri, N. Isshiki, Y. Takeuchi, and S. Kobayashi, Phys. Rev. C **35**, 931 (1987).
[21] G. J. Wagner, P. Grabmayr, and H. R. Schmidt, Phys. Lett. **113B**, 447 (1982).
[22] W. Bauhoff, Z. Phys. A **318**, 219 (1984).
[23] G. R. Satchler, J. Math. Phys. **13**, 1118 (1972).
[24] U. Garg, P. Bogucki, J. D. Bronson, Y. W. Lui, and D. H. Youngblood, Phys. Rev. C **29**, 93 (1984).
[25] M. M. Sharma, W. T. A. Borghols, S. Brandenburg, S. Crona, A. Van der Woude, and M. M. Harakeh, Phys. Rev. C **38**, 2562 (1988).