Barrier region resonance model for heavy ion resonances

B. Sahu, B. M. Jyrwa, P. Susan, and C. S. Shastry
Department of Physics, North Eastern Hill University, Shillong–793 003, Meghalaya, India
(Received 29 May 1991)

Using an explicit analytical expression of the *s*-wave *S* matrix for a typical short-range potential barrier we correlate the classical orbiting and corresponding quantum-mechanical barrier states. These are compared with the results of earlier works on this problem and some additional interesting features, such as dependence of width on height of the barrier and spacing between the levels of the barrier region resonances, are pointed out. On this basis, a four-parameter barrier region resonance model is introduced in the analysis of resonances in heavy ion scattering. The model is then applied to fit satisfactorily the large number of resonances observed in $^{12}C+^{12}C$ and $^{12}C+^{16}O$ systems. The present model directly relates the nucleus-nucleus potential barrier generated by the effective nucleus-nucleus potential to heavy ion resonances.

I. INTRODUCTION

Among the interesting phenomena generated by the advent of heavy ion collision experiments, the resonance structures [1] observed have a very special place. The main emphasis in the theoretical understanding of these structures is centered around the idea of nuclear molecular resonance states. Several microscopic [2] and schematic models [3-6] have been suggested in the analysis of nuclear molecular resonances. Iachello [7] suggested that the phenomenon of nuclear molecular resonances can be understood in a way akin to the rotational vibrational spectra of atomic molecules. Similarly, Cindro and Počanić [5] studied the resonance state as due to interaction of two orbiting nuclei. Khosla *et al.* [8] improved the orbiting cluster model by incorporating the generalized moment of inertia in the two-central-shell model. Satpathy and co-workers [9] have studied the nuclear molecular resonances using larger range Morse potentials to generate the resonance states. Cindro and Greiner [10] have used the potential approach with an anharmonic potential well with a negative quartic term. In this paper we give a description of the nuclear molecular resonances for the $^{12}C+^{12}C$ and $^{12}C+^{16}O$ systems within what we refer to as barrier region resonance model (BRRM). The motivation for the work is as follows.

It is well known that the resonancelike structures are the characteristic features observed in several nucleus-nucleus collisions around the barrier region. The theoretical analysis of the differential cross sections in the nucleus-nucleus collisions is most commonly done within the framework of the nuclear optical model incorporating the special features of the heavy ion collisions, namely, strong absorption and high Coulomb barrier. The approximate expressions for the height V_B and position R_B of the *s*-wave Coulomb barrier is given by the empirical expressions [11]

$$V_B=\frac{Z_1Z_2e^2}{R_B}\left[1-\frac{a}{R_B}\right], \quad (1)$$

$$a=0.63\ \text{fm},\qquad R_B=[1.07(A_1^{1/3}+A_2^{1/3})+2.72]\ \text{fm}, \quad (2)$$

A_i, Z_i, $i=1,2$, denote the mass number and proton number of the colliding nuclei. It is well known that the nuclear effects in heavy ion collisions are generated by the nucleus-nucleus potentials around the barrier region in the vicinity of the strong absorption radius. The nucleus-nucleus potentials in the interior of the barrier are highly absorptive leading to the characteristic variation of the reflection function η_l as a function of l. Hence, it is reasonable to assume that the observed features of the molecular-type resonances should primarily originate from the special features of the total nucleus-nucleus effective potential including the Coulomb and the centrifugal terms in the surface region. If the resonance data can be related in a more direct way to the barrier parameters of the nucleus-nucleus system, a closer connection between the nuclear molecular resonances and the underlying phenomenological optical-model potential becomes clearer.

The effective real nucleus-nucleus potential between two heavy ions is characterized by a Coulomb barrier region and a potential pocket which gets shallower and shallower as l increases and the centrifugal barrier is closer to the origin. Within the framework of such a realistic real nucleus-nucleus effective potential, one may first look for resonances originating from the pocket region. However, the calculations based on the potential models and the fact that, in the pocket region an imaginary part of the phenomenological optical potential is too large, shows that the potential pockets arising from the effective potential within the framework of the optical model may not be able to generate the sequence of well-known resonances in the $^{12}C+^{12}C$ and $^{12}C+^{16}O$ systems. The results of Ref. [9] also lead to these observations. However, the resonance states can originate not only from the potential pocket but also from the barrier top region provided absorption is not large in the barrier region and the barrier is reasonably flat. These have been referred to in literature as barrier top resonances and resonances associated with orbiting and so on [12]. We show in this paper that it is possible to explain the resonances in $^{12}C+^{12}C$ and $^{12}C+^{16}O$ within the framework of resonance model with reasonably realistic parameters for the barrier height. This particular model has the advantage that makes a closer connection to the corresponding optical-model situation. In order to determine the main features of the model it is necessary to examine the special features of barrier region resonances in contrast to potential pocket resonances. We carry this out using an exactly solvable quantum-mechanical model in Sec. II A. Based on this, in Sec. III we describe the BRRM for the analysis of heavy ion resonances. Section IV contains the results of calculations. In Sec. V we summarize the discussions and conclusions.

II. BARRIER REGION RESONANCES IN QUANTAL SCATTERING

The characterization of resonances and their role in the analysis of scattering amplitude is an important aspect of potential scattering. In the S-matrix theory, the bound states and resonances associated with a particle moving under a potential $V(r)$ are represented in terms of poles of the S matrix in the complex momentum (k) plane [20]. The resonance poles are known to occur in the lower half of the k plane and are symmetrically placed with respect to the imaginary axis. The resonance poles represent decaying states with positive energy E^R and nonzero width Γ. One expects that sharp resonances will occur when the potential function has a pocket capable of trapping the particle to generate a long-lived state and its decay is due to the tunneling away from the pocket to infinity. It may be noted that, in classical situations, particles trapped in such potential pockets will generate bound orbits.

In the classical situation, an unstable but bound orbit can arise if the effective potential has a barrier and the total energy becomes equal to the height of the effective barrier. Therefore, the problem of quantum-mechanical resonances generated by the barrier top which can be associated with the orbiting is of considerable interest. There has been considerable work in literature [12–19] on this and related topics. The purpose of this section is to examine several interesting features of barrier region resonances which are important in our heavy ion resonance model.

The formal investigation of barrier resonance states in scattering has been done by Friedman and Goebel [16] and also by Brink [17,18]. In their work Friedman and Goebel [16] give the following expression for the barrier top resonance poles in the complex energy plane for different partial waves, l:

$$E_{n,l}=V_0-i(2n+l+\tfrac{3}{2})w, \quad n=0,1,2,\ldots \qquad (3)$$

in the case of a spherically symmetric parabolic potential barrier

$$V(r)=V_0-mw^2r^2/2, \quad r\geq 0 . \qquad (4)$$

Here, V_0 and w indicate height and oscillator frequency of the barrier, respectively. Brink [18] has used such a barrier to analyze the three turning point Wentzel-Kramers-Brillouin (WKB) approximation for the partial-wave S matrix, S_l, in the complex angular momentum plane. He deduced that one can associate an infinite sequence of Regge poles with barrier top resonances. We refer to this as the semiclassical analog of orbiting phenomena in classical mechanics.

Now we note the following aspects in the above-stated results of Friedman and Goebel [16] and Brink [17,18]. Equation (3) essentially manifests a kind of "degeneracy." That is, at the same real energy, we have an accumulation of infinite levels of steadily increasing widths. A question arises: Can these manifest as an observable sharp resonance? For example, Eq. (3) indicates the situation when a large number of broad resonances ($n=1,2,3,\ldots$) gets superposed on the sharpest resonance corresponding to $n=0$ for particular orbital angular momentum $L=l\hbar$. As a consequence, one has a physical situation in which the overall phenomenon may not be a sharply discernible resonance. Further, it is known that, for a well-behaved short-range superposition of a Yukawa-type real potential, the S matrix will only have a finite number of Regge poles in the right half complex L plane [12]. Thus, an infinite sequence of Regge poles associated with orbiting phenomenon deduced from the WKB formula for the S matrix is not compatible with this [12]. This situation may be due to the mathematical simplifications used in the analysis of the S matrix in the WKB approximation. It is possible to illustrate that a minor change in the expression can lead to a drastic change in the analytical properties as follows: The function e^{-z} is an entire function of z, whereas the function $1/(e^z+\delta)$ has an infinite sequence of poles at $z=\ln\delta+(2n+1)\pi i$ for $\delta>0$, however small, $|n|=0,1,2,3,\ldots$. In view of this observation, we prefer to consider the results of Ref. [17] as a semiclassical analog rather than the quantal analog for the classical orbiting phenomena. In this paper we try to examine the nature of an exact quantum-mechanical analog of classical orbiting or barrier top states in complex k and complex energy planes using an exactly solvable case.

A. Barrier region resonances—an exactly solvable example

Let us consider the potential

$$V(r)=U_0/\cosh^2\alpha(r-R), \quad 0\leq r\leq\infty, \quad U_0>0 . \qquad (5)$$

This generates a barrier of height U_0 peaked at $r=R$ with range parameter α. This barrier represents a more realistic short-range potential to analyze the barrier region resonance scattering states than the infinite-range inverted harmonic-oscillator potential given by Eq. (4) which tends to $-\infty$ as $r\to\infty$. The modified s-wave Schrödinger equation for a particle of mass m and energy E for the potential given by Eq. (5) can be written as

$$\frac{d^2\Psi}{dy^2}+(k^2-V_0/\cosh^2\alpha y)\Psi=0 , \qquad (6)$$

where $y=r-R$, $k^2=2mE/\hbar^2$, and $V_0=2mU_0/\hbar^2$. One can readily obtain [19] the two linearly independent solutions of Eq. (6) as

$$\Psi_1(\zeta)=2^{ik/\alpha}(1-\zeta^2)^{-ik/2\alpha}F\left[-\frac{ik}{\alpha}-s,\frac{-ik}{\alpha}+s+1,\frac{-ik}{\alpha}+1;\frac{1-\zeta}{2}\right] \tag{7}$$

$$\xrightarrow[y\to\infty]{} e^{iky} ,$$

$$\Psi_2(\zeta)=2^{ik/\alpha}(1-\zeta^2)^{-ik/2\alpha}\left[\frac{1-\zeta}{2}\right]^{ik/\alpha}F\left[s+1,-s,\frac{ik}{\alpha}+1;\frac{1-\zeta}{2}\right] \tag{8}$$

$$\xrightarrow[y\to\infty]{} e^{-iky} ,$$

where

$$\zeta=\tanh\alpha y , \tag{9}$$

$$s=\tfrac{1}{2}[-1+(1-4V_0/\alpha^2)^{1/2}] . \tag{10}$$

We seek a general solution of the form

$$\Psi(\zeta)=A\Psi_1(\zeta)-B\Psi_2(\zeta) \tag{11}$$

such that Ψ is zero at $r=0$. Then the expression for the s-wave scattering matrix $S(k)$ for real k is given by

$$S(k)=e^{-2ikR}(A/B)$$

$$=e^{-2ikR}\left[\frac{\operatorname{sech}^2\alpha R}{4}\right]^{ik/\alpha}\frac{F((ik/\alpha)-s,(ik/\alpha)+1+s,(ik/\alpha)+1;(1+\tanh\alpha R)/2)}{F(-(ik/\alpha)-s,-(ik/\alpha)+1+s,-(ik/\alpha)+1;(1+\tanh\alpha R)/2)} , \tag{12}$$

where $F(a,b,c;z)$ denotes the hypergeometric function and s is assumed to be real. When $4V_0>\alpha^2$, in the denominator one should use, instead of s, its complex conjugate s^*. When $\alpha R \gg 1$, Eq. (12) gives

$$S(k)=(1-e^{-2\alpha R})^{ik/\alpha}\frac{\Gamma[(ik/\alpha)+1]}{\Gamma[(-ik/\alpha)+1]}\left[\frac{a+be^{-2ikR}}{a^*+b^*e^{2ikR}}\right] , \tag{13}$$

where

$$a=\frac{\Gamma(ik/\alpha)}{\Gamma[(ik/\alpha)-s]\Gamma[ik/\alpha)+s+1]} , \tag{14}$$

$$b=\frac{\Gamma(-ik/\alpha)}{\Gamma(s+1)\Gamma(-s)} , \tag{15}$$

a^* and b^* are complex conjugates of a and b, respectively. Clearly, the unitarity condition $S(k)S^*(k)=1$ is satisfied. If we set $R=0$, we get the potential barrier peak at the origin having no potential pocket. Incidently it may be pointed out that this situation is akin to studying the resonances for the potential given by Eq. (5) with $R\neq 0$ but adding a highly absorptive imaginary potential in the region $0<r<R$, as discussed in Ref. [16]. Hence, the resonance poles of the S matrix corresponding to this case ($R=0$) can be identified with the barrier region resonances. We make use of these concepts in formulating the BRRM in Sec. III. Making use of the properties of $F(a,b,c;z)$ in this case ($R=0$), the S matrix given by Eq. (12) can be reduced to a comparitively simpler form and is given by

$$S(k)=(2)^{-2ik/\alpha}\frac{\Gamma[(ik/\alpha)+1]\Gamma[-(ik/2\alpha)-(s/2)+\frac{1}{2}]\Gamma[(-ik/2\alpha)+(s/2)+1]}{\Gamma[(ik/2\alpha)-(s/2)+\frac{1}{2}]\Gamma[(ik/2\alpha)+(s/2)+1]\Gamma[(-ik/\alpha)+1]} . \tag{16}$$

The poles of this matrix arise from the poles of gamma functions in the numerator. However, the poles arising out of $\Gamma[(ik/\alpha)+1]$ are along the imaginary k axis and they do not correspond to resonances but to the so-called virtual states. The resonance pole positions evaluated by setting

$$\frac{-ik}{2\alpha}-\frac{s}{2}+\frac{1}{2}=-n, \quad n=0,1,2,3,\ldots , \tag{17}$$

$$\frac{-ik}{2\alpha}+\frac{s}{2}+1=-n, \quad n=0,1,2,3,\ldots \tag{18}$$

can be identified with the resonance poles. The positions of these resonance poles in complex k plane are thus found to be

$$k_n=\pm V_0^{1/2}(1-\alpha^2/4V_0)^{1/2}-i2\alpha(n+\tfrac{3}{4}) . \tag{19}$$

Clearly, the resonance poles are symmetrically placed with respect to the imaginary k axis in the lower half of the complex k plane. The expression for positions of resonance poles in the lower half of the complex energy plane becomes

$$E_n=[V_0(1-\alpha^2/4V_0)-4\alpha^2(n+\tfrac{3}{4})^2] -i4\alpha V_0^{1/2}(1-\alpha^2/4V_0)^{1/2}(n+\tfrac{3}{4}) =E_n^R-iE_n^I=E_n^R-i\frac{\Gamma_n}{2},\quad n=0,1,2,3,\ldots, \tag{20}$$

where Γ_n indicates the width of the resonance states. An inspection of Eq. (20) reveals that, unlike Eq. (3), the real part E_n^R of E_n in Eq. (20) is no longer equal to the barrier height V_0 but has different values for different n and these are smaller than V_0. This implies that identifying all barrier region resonances with energy $E_n^R=V_0$ is not found to be valid in this exactly solvable example. In other words, the so-called barrier top resonance "degeneracy" pointed out earlier is shown not to be valid in this example. We believe this statement is likely to be valid in the exact quantum-mechanical calculations for the other well-behaved finite-range potential barrier also. Further, we observe that, in the above example, [see Eq. (20)] there are no resonances having a real energy larger than V_0, and expectedly the sharpest resonance occurs when E_n^R is closest to V_0.

Equation (20) also indicates that, unlike Eq. (3), the number of poles which can be identified as resonances have $E_n^R>0$ is finite. Denoting by $[x]$ the largest integer less than or equal to x, we find that, for a given V_0 and α, the number of resonance poles N with $E_n^R\geq 0$ is given by

$$N=[(4V_0-\alpha^2)^{1/2}/4\alpha-3/4]\ . \tag{21}$$

Clearly, we should have $V_0\geq 5\alpha^2/2$ for N to be positive or zero. In this sense, expression (20) is quite different from Eq. (3), which predicts infinite resonances with different widths and having the same energy. Hence, we expect that, in the case of realistic examples of barrier-type potentials, the number of actual barrier region resonances states will be finite and have different energies and widths.

Another interesting feature which is evident from the comparison of Eqs. (3) and (20) is the following. In Eq. (20), pertaining to the present analysis, the imaginary part, E_n^I, of resonance energy E_n is given by

$$E_n^I=\frac{\Gamma_n}{2}=4\alpha V_0^{1/2}(1-\alpha^2/4V_0)^{1/2}(n+3/4)\ . \tag{22}$$

This explicitly indicates the relation between the height of the barrier, V_0, and width, Γ_n, of the resonance state, implying that the barrier region resonances are likely to be much broader in the case of a higher barrier and sharper in the case of a lower barrier; that is, for example, in the case of lighter ion-ion systems. We believe that this point is of some significance in the study of heavy ion resonances. The proportionality $\Gamma_n\propto V_0^{1/2}$ obtained from Eq. (22) implies that the lifetime of the resonance state is proportional to $V_0^{-1/2}$; that is, the barrier resonance corresponding to a higher barrier with the same range parameter has a smaller lifetime as compared to the corresponding state with a lower barrier having the same range. If one analyzes the classical situation, this can be understood as follows. In the classical case, the time τ_c required for the particle with total energy $E=V_0$ to traverse across the barrier region generated by the potential $V(r)=V_0/\cosh^2\alpha r$ in the interval $0<r_0\leq r<1/2\alpha$ is given by

$$\tau_c=\left[\frac{m}{2}\right]^{1/2}\int_{r_0>0}^{1/2\alpha}\frac{dr}{[E-V(r)]^{1/2}} =m^{1/2}(2V_0)^{-1/2}I\ , \tag{23}$$

where

$$I=(1/\alpha)\ln\left[\frac{\sinh\frac{1}{2}}{\sinh(\alpha r_0)}\right]>0\ .$$

Equation (23) indicates that the classical barrier states can get out of the barrier region in time $\tau_c\propto V_0^{-1/2}$. Thus, our result in Eq. (22) is consistent with the physical situation one can visualize from the classical arguments.

On the basis of the above analysis, we can expect that, even in the case of a cutoff parabolic barrier

$$V(r)=\begin{cases}V_0-mw^2r^2/2, & r<R\ ,\\ 0, & r\geq R\ ,\end{cases} \tag{24}$$

the lifetime of the barrier state will be proportional to $V_0^{-1/2}$. This can be seen in the corresponding expression for classical time τ_p in this case for the particle to traverse the interval $0<r_0\leq r\leq R$, which is given by

$$\tau_p=m^{1/2}R\ \ln(R/r_0)/(2V_0)^{1/2}>0 \tag{25}$$

for a fixed range $R=(2V_0/m)^{1/2}/w$. Thus, the proportionality $\Gamma_n\propto V_0^{1/2}$ illustrated in the above examples may have a more general validity.

It may be pointed out that, if the number N of resonances as per Eq. (21) is small and if the spacing between the adjacent levels

$$E_n^R-E_{n+1}^R=8\alpha^2(n+5/4)<\Gamma_n\ , \tag{26}$$

then one may expect this physical situation to manifest as a broad resonance having average energy $\bar{E}^R=\sum_{n=0}^{N}(E_n^R/N)$ and width

$$\Gamma_n<8\alpha V_0^{1/2}(1-\alpha^2/4V_0)^{1/2}(N+3/4)$$

and, as a consequence, individual resonances may not be discernible.

Some more comments regarding the resonances corresponding to the outgoing states of the barrier given by Eq. (4) are in order. As mentioned earlier, such a potential does not correspond to a scattering situation occurring in a real system since $V(r)\rightarrow-\infty$ as $r\rightarrow\infty$. On the other hand, the potentials describing scattering, in general, vanish as $r\rightarrow\infty$, so that an amplitude with respect to the outgoing spherical wave (in the case of the long-range Coulomb potential it is the distorted outgoing spherical wave) can be defined. In the case of such potentials, as our explicit example has shown, the resonance energy E_n^R corresponding to barrier region resonances is unlikely to remain fixed at $E_n^R=V_0$. Various observations on the detailed aspects of barrier states stated earlier in this section are relevant in the study of barrier resonances in physical situations like heavy ion collisions.

III. BARRIER REGION RESONANCE MODEL FOR HEAVY ION RESONANCES

In typical effective heavy ion potentials in the region $r < R_B$, the imaginary part of the potential, and particularly for higher partial waves, the highly repulsive interior centrifugal potential dominates the real part. The former attenuates the wave function drastically and the latter also makes the wave functions in the regions $r < R_B$ very small. Hence, for the purpose of the present discussion, we may consider the heavy ion collision for a given l to be an equivalent s-wave problem dominated by the effective barrier and having negligible contribution to the wave functions from the region's interior to the barrier. In such a situation we also assume that the imaginary part of the potential is small inside the barrier region and outside, since the occurrence of sharp resonances naturally implies less absorption. With these assumptions, and based on the implication of Eq. (20), we write the following semiempirical expression for the resonance energy $E(n,l)$ associated with the barrier:

$$E(n,l)=V_0(l)-c(n+3/4)^2 , \tag{27}$$

where $V_0(l)$ and c are the parameters to be adjusted so as to fit the experimental resonances. If the underlying assumptions of the model have empirical validity, $V_0(l)$ should be reasonably close to the realistic barrier height of effective potential $V_B^{(l)}$. Since the above expression is based on a particular type of barrier potential, some deviations of $V_0(l)$ from the effective barrier height $V_B^{(l)}$ can be expected. In view of this, we parametrize $V_0(l)$ as $V_B^{(l)}+a+bl+dl^2$.

Clearly, $a+bl+dl^2$ gives the deviation of $V_0(l)$ from $V_B^{(l)}$. Thus, we obtain a four-parameter (a,b,c,d) semiempirical formula to analyze the barrier region resonances:

$$E(n,l)=V_B^{(l)}+a+bl+dl^2-c(n+3/4)^2 . \tag{28}$$

Evidently we have implied, in the other expression, the slope parameter c to be practically the same for different l's. The formula used by Erb and Bromley [21] introduced on the basis of group-theoretical results of Iachello is also a four-parameter formula of the type

$$E(n,L)=-D+a(n+1/2)-b(n+1/2)^2+cL(L+1) . \tag{29}$$

In Eq. (28), $V_B^{(l)}$ denotes the height of the effective barrier given by

$$V_B^{(l)}=V_B+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{R_B^2(l)} . \tag{30}$$

Unlike Eq. (29), the theoretical basis for expression (28) is based on the barrier region resonances discussed in the last section. It is interesting to note that the expression can be readily seen to contain a linear and a quadratic term in $(n+1/2)$ since

$$(n+3/4)^2=(n+1/2)^2+1/2(n+1/2)+1/16 .$$

The interesting aspect of expression (28) is that it has parabolic behavior as a function of n which is seen to be observed in the resonance data of well-known heavy ion systems like $^{12}C+{}^{12}C$ and $^{12}C+{}^{16}O$. Further, the formula directly connects the barrier height to the resonance data. In the next section we analyze the resonance data using the semiempirical relations [Eq. (28)].

IV. ANALYSIS OF RESONANCES OF $^{12}C+{}^{12}C$ AND $^{12}C+{}^{16}O$

We have analyzed the resonance data of $^{12}C+{}^{12}C$ using the BRRM. In Table I we list the parameters a, b, d, and c and, also, the height of the effective potential $V_B^{(l)}$ for different l's has been calculated using the global nucleus-nucleus potential [17].

$$V(r)=-50(R_1R_2)/(R_1+R_2)\exp[(R_1+R_2-r)/a_0] , \tag{31}$$

where

$$R_i=1.233A_i^{1/3}-0.978A_i^{-1/3}\text{ fm } (i=1,2) ,$$

$$a_0=0.63\text{ fm}$$

along with the Coulomb and centrifugal potentials. It is seen that, in the case of the $^{12}C+{}^{12}C$ system, the difference between $V_0(l)$ and $V_B^{(l)}$ is in the range of 0.6–2.1 MeV where $V_B^{(l)}$ varies from 6.19 to 17.2 MeV.

TABLE I. The quantities $R_B(l)$, $V_B^{(l)}$, $V_0(l)$, and typical set of $E(n,l)$ corresponding to the $^{12}C+{}^{12}C$ system. The parameters used to compute $E(n,l)$ are $a=0.5985$ MeV, $b=0.2119285$ MeV, $c=0.060085$ MeV, and $d=-0.008393$ MeV.

l	$R_B(l)$ (fm)	$V_B^{(l)}$ (MeV)	$V_0(l)$ (MeV)	$E(n,l)=V_B^{(l)}+a+bl+dl^2-c(n+3/4)^2$ (MeV)				
				$n=0$	1	2	3	4
0	7.70	6.19	6.99	6.96	6.81	6.54	6.15	5.64
2	7.62	6.54	7.70	7.67	7.52	7.24	6.85	6.34
4	7.46	7.40	8.85	8.82	8.67	8.39	8.01	7.49
6	7.26	8.82	10.5	10.5	10.3	10.0	9.65	9.14
8	7.04	10.9	12.7	12.7	12.5	12.3	11.9	11.4
10	6.82	13.6	15.5	15.5	15.3	15.1	14.7	14.2
12	6.60	17.2	19.1	19.1	18.9	18.7	18.3	17.8

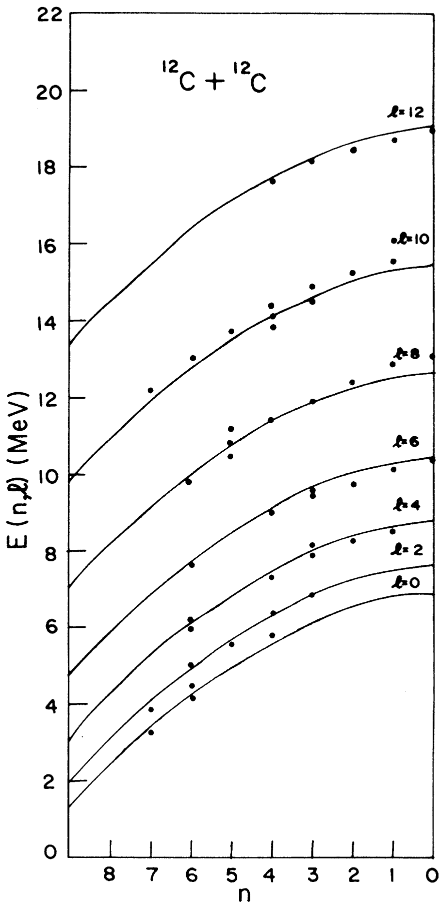

FIG. 1. Plots of $E(n,l)$ against the assumed values of n for the resonance data of the $^{12}C+^{12}C$ system [Eq. (28)]. The parameters are listed in Table I.

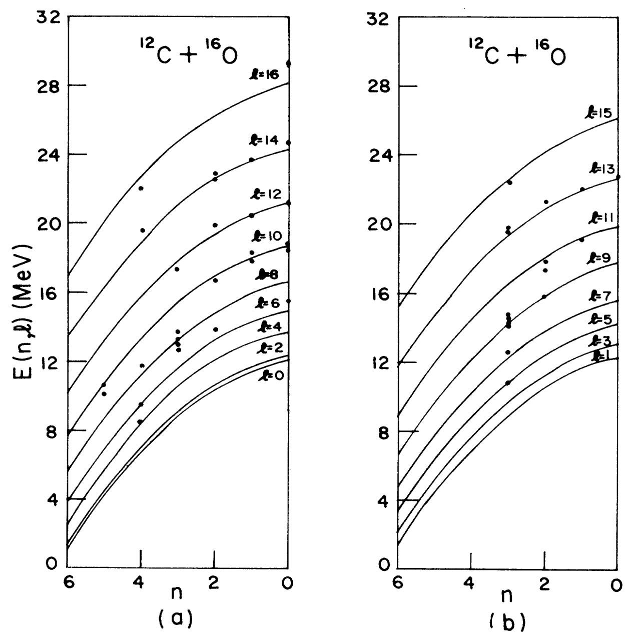

FIG. 2. Plots of $E(n,l)$ against the assumed values of n for the resonance data of the $^{12}C+^{16}O$ system [Eq. (28)]. The parameters are listed in Table II.

This means the height of the barrier $V_0(l)$ implied in Eq. (27) is about 10% higher than the corresponding Coulomb barrier height. A possible explanation for this will be given later. In Fig. 1 we depict the results obtained for even l along with the experimental data. It is clear that the present approach gives quite a good fit to the experimental data.

TABLE II. The quantities $R_B(l)$, $V_B^{(l)}$, $V_0(l)$, and typical set of $E(n,l)$ corresponding to the $^{12}C+^{16}O$ system. The parameters used to compute $E(n,l)$ are $a=4.15049$ MeV, $b=0.1942653$ MeV, $c=0.2453$ MeV, $d=-0.012391$ MeV.

l	$R_B(l)$ (fm)	$V_B^{(l)}$ (MeV)	$V_0(l)$ (MeV)	$E(n,l)=V_B^{(l)}+a+bl+dl^2-c(n+3/4)^2$ (MeV) $n=0$	1	2	3	4
0	7.91	8.05	12.2	12.1	11.5	10.3	8.75	6.66
1	7.89	8.15	12.5	12.3	11.7	10.6	9.03	6.94
2	7.86	8.34	12.8	12.7	12.1	11.0	9.38	7.30
3	7.81	8.64	13.3	13.1	12.5	11.4	9.81	7.73
4	7.76	9.04	13.8	13.6	13.0	11.9	10.3	8.24
5	7.69	9.55	14.4	14.2	13.6	12.5	10.9	8.83
6	7.61	10.2	15.1	14.9	14.3	13.2	11.6	9.52
7	7.53	10.9	15.8	15.7	15.1	13.9	12.4	10.3
8	7.44	11.8	16.7	16.6	16.0	14.9	13.3	11.2
9	7.36	12.8	17.7	17.6	16.9	15.8	14.3	12.2
10	7.26	13.9	18.8	18.7	18.1	16.9	15.4	13.3
11	7.18	15.2	20.0	19.9	19.3	18.2	16.6	14.5
12	7.09	16.7	21.4	21.2	20.6	19.5	17.9	15.8
13	7.01	18.3	22.9	22.7	22.1	21.0	19.4	17.3
14	6.92	20.0	24.5	24.3	23.7	22.6	21.0	18.9
15	6.84	22.0	26.3	26.1	25.5	24.4	22.8	20.7
16	6.76	24.1	28.2	28.0	27.4	26.3	24.7	22.6

In Table II we list the quantities used in a similar analysis for the $^{12}C+^{16}O$ data. Here $V_0(l)$ is also found to be larger than $V_B^{(l)}$, implying that the barrier at the resonance implied in Eq. (27) is somewhat higher than $V_B^{(l)}$. In the present case, the fit to the experimental data obtained is shown in Fig. 2. It is seen that our semiempirical four-parameter formulation is successful in analyzing these resonances also.

V. DISCUSSIONS AND CONCLUSIONS

From the semiempirical four-parameter expression for barrier region resonances constructed on the basis of the properties of barrier region resonances, derived in an exactly solvable quantum-mechanical model, we have been able to fit a large number of resonances of $^{12}C+^{12}C$ and $^{12}C+^{16}O$. Since our model did not incorporate a more realistic potential in the region $r > R_B$, some deviation of $V_O(l)$ from $V_B^{(l)}$ is not very surprising. However, we feel that this difference may also, to some extent, be related to the so-called threshold anomaly observed in heavy ion collisions. During recent years it has been found that the elastic optical potential has strong local energy dependence at energies close to the Coulomb barrier [22,23]. This phenomenon is normally referred to as the threshold anomaly. This, together with the dispersion relations connecting the real and imaginary parts of the optical potential, indicate a sharp rise in the real potential at energies close to the Coulomb barrier and the sharp fall in the magnitude of the imaginary potential in the barrier region at energies around the barrier. A decrease in the imaginary potential implies a decrease in the number of nonelastic open channels. This, in turn, should facilitate sharper resonances. The very fact that there exist a large number of reasonably sharp heavy ion resonances around the barrier region should imply a smaller imaginary part of the potential for resonance energies in the region of the barrier and, hence, a corresponding increase in real part of the potential. Even though in the global nucleus-nucleus potentials such subtle aspects are not incorporated, the fact that, in our analysis of heavy ion resonances, we need to use a somewhat higher effective barrier $V_0(l)$ as compared to the empirical global $V_B^{(l)}$ is indicative of the fact that threshold anomaly is likely to have an implicit role in the generation of the resonances in the heavy ion system. The orbiting cluster model, the potential model, and the present model are consistent with the general interpretation that resonances in $^{12}C+^{12}C$ collisions could be interpreted as highly deformed shape isomeric rotational states of ^{24}Mg [24,25].

It is known that the best set of nuclear molecular resonance data in nucleus-nucleus collision occur when the colliding nuclei are comparatively lighter, like $^{12}C+^{12}C$, $^{12}C+^{16}O$, $^{16}O+^{16}O$, etc. Our analysis in Sec. III shows that an interesting feature of the barrier region resonances lies in the fact that the width of the resonance is proportional to $V_0^{1/2}$. This means that the barrier region resonances in the case of two heavy colliding nuclei are likely to have significantly larger widths, making their observation rather difficult.

On the basis of our results, we conclude that our formulation provides a way to relate the orbiting cluster model and the BRRM and makes a connection to the role of the effective barrier of the optical-model potentials and the threshold anomaly. The physical picture in the present model is fairly close to the macroscopic situation that is likely to occur when the two nuclei collide, leading to rotational states in the barrier region.

We thank Professor C. S. Warke and Dr. R. Singh for useful discussions.

[1] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. Lett. **4**, 365 (1960).

[2] W. Greiner and W. Scheid, J. Phys. (Paris) Colloq. **32**, C6-91 (1971); D. Hahn, G. Terlecki, and W. Scheid, Nucl. Phys. A **325**, 283 (1979).

[3] N. Kondo, Y. Abe, and T. Matsuse, Phys. Rev. C **19**, 1356 (1979).

[4] D. Baye, Phys. Lett. **97B**, 17 (1980).

[5] N. Cindro and D. Počanić, J. Phys. G **6**, 359 (1980).

[6] F. Haas and Y. Abe, Phys. Rev. Lett. **46**, 1667 (1981).

[7] F. Iachello, Phys. Rev. C **23**, 2778 (1981).

[8] H. S. Khosla, S. S. Malik, and R. K. Gupta, Nucl. Phys. **A513**, 115 (1990).

[9] L. Sathpathy, P. Sarangi, and A. Faessler, J. Phys. G **12**, 201 (1986); L. Sathpathy and P. Sarangi, *ibid.* **16**, 469 (1990).

[10] N. Cindro and W. Greiner, J. Phys. G **9**, L175 (1983).

[11] R. A. Broglia and A. Winther, *Heavy Ion Reactions Lecture Notes* (Benjamin/Cummings, Massachusetts, 1981) Vol. 1, Chap. 3, p. 116.

[12] C. S. Shastry and I. Parija, Phys. Rev. C. **27**, 2042 (1983).

[13] K. W. Ford and J. A. Wheeler, Ann. Phys. **7**, 259 (1959).

[14] K. W. Ford, D. L. Hill, M. Wakano, and J. A. Wheeler, Ann. Phys. **7**, 239 (1959).

[15] M. S. Child, *Molecular Collision Theory* (Academic, New York, 1974).

[16] W. A. Friedman and C. J. Goebel, Ann. Phys. **104**, 145 (1977).

[17] D. M. Brink, *Semi-Classical Methods for Nucleus-Nucleus Scattering* (Cambridge University, Cmabridge, 1985).

[18] D. M. Brink and N. Takigawa, Nucl. Phys. **A279**, 159 (1977).

[19] L. D. Landau and E. M. Lifshitz, *Quantum Mechanic, Non-relastivistic Theory,* 2nd ed. (Pergamon, New York, 1965) Vol. 3, p. 79.

[20] R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill, New York, 1966), Chap. 12.

[21] K. A. Erb and D. A. Bromley, Phys. Rev. C **23**, 2781 (1981).

[22] J. S. Lilley, B. R. Fulton, M. A. Nagarajan, I. J. Thomson, and D. W. Banes, Phys. Lett. **151B**, 181 (1985).

[23] M. A. Nagarajan, C. Mahaux and G. R. Satchler, Phys. Rev. Lett. **54**, 1136 (1985).

[24] K. A. Erb and D. A. Bromley, in *Treatise on Heavy-ion Science,* edited by D. A. Bromley (Plenum, New York, 1985), Vol. 3, p. 216.

[25] H. Chandra and U. Mosel, Nucl. Phys. A **298**, 151 (1978).