# Shell-model interpretation of the $\beta^-$ decay of $^{212}Bi^g$, $^{212}Bi^{m1}$, and $^{212}Bi^{m2}$

E. K. Warburton
*Brookhaven National Laboratory, Upton, New York 11973*
(Received 01 February 1991)

The $\beta^-$ decay of the $^{212}$Bi ground state ($T_{1/2} = 61$ min) and the isomers $^{212}Bi^{m1}$ ($T_{1/2} = 27$ min), and $^{212}Bi^{m2}$ ($T_{1/2} = 8$ min) are considered in the spherical shell model. The wave functions and energy spectra of all even and odd states in the Kuo-Herling model space were calculated for both $^{212}$Bi and $^{212}$Po. Possible candidates for the two $^{212}$Bi isomers were located from these results and the allowed and first-forbidden $\beta^-$ decays rates were formed using a recently developed parametrization of first-forbidden decays in the lead region. It was concluded that $^{212}Bi^{m1}$ is the yrast $8^-$ state and $^{212}Bi^{m2}$ the yrast $18^-$ state of $^{212}$Bi. The decay rates calculated for these two $\beta^-$ emitters are in excellent agreement with experiment. The predictions for the decay of the ground state of $^{212}$Bi are not in good agreement with experiment, illustrating the difficulty of calculating weak first-forbidden decay modes reliably.

## I. INTRODUCTION

$^{212}$Bi has three isomers which have been observed to decay at least in part by $\beta^-$ emission leading to $^{212}$Po. The ground state with $T_{1/2} = 61$ min, $Q(\beta^-) = 2252 \pm 4$ keV, and $J^\pi = 1^-$ (Ref. [1]) is designated $^{212}Bi^g$. An isomer, $^{212}Bi^{m1}$, with $T_{1/2} = 26.5 \pm 1.0$ min (Refs. [2,3]) and at an excitation energy $E_x$ of 250 keV [2] has been speculated [2] to have $J^\pi = 9^-$. An $8 \pm 1$-min activity, $^{212}Bi^{m2}$, with $E_x > 670$ keV decays by $\beta^-$ (and possibly subsequent $\gamma$) emission to a $^{212}$Po level at $2921 \pm 15$ keV which itself is isomeric with $T_{1/2} = 45.1 \pm 0.6$ s (Refs. [1–3]) and early on was speculated to have $J^\pi = 16^+$ (Ref. [4]) and $18^+$ (Ref. [5]).

The existence of these three $^{212}$Bi isomers and the $^{212}$Po isomer has been known for some time yet the answers to the following questions have not been answered: (1) What is the situation which causes the isomerism of the 27- and 8-min activities? (2) Specifically, what are the spin-parity values of these two activities? (3) Can the $\beta^-$ decay rates of the three $^{212}$Bi activities be understood quantitatively? These three questions pose an interesting and challenging spectroscopic problem which is the subject of this study.

To address these questions shell-model calculations for both even- and odd-parity states were performed for both $^{212}$Bi and $^{212}$Po in the full Kuo-Herling particle space [6] which consists of the six proton orbits and seven neutron orbits above an inert (assumed) $^{208}$Pb core. Beta-decay rates were then calculated using a recently developed parametrization of first-forbidden decays which is applicable to $A \sim 208$ nuclei [7].

## II. RESULTS

### A. Energy spectra

The calculations were performed with the shell-model code OXBASH [8]. The interaction used is a slightly modified version of the Kuo-Herling "bare + 1p1h" (KHP) interaction [6,9]. The modifications leading to the interaction KHP$_e$ consisted mainly of changes to the proton-neutron interaction with the aim of reproducing the low-lying spectrum of $^{210}$Bi. The changes and an appraisal of the interaction are described elsewhere [10].

A rather complete knowledge of the predicted energy spectra of $^{212}$Bi and $^{212}$Po is necessary for a thorough exploration of the questions posed in the Introduction. The necessary predictions are listed in Tables I and II. These results are used to construct the theoretical decays schemes shown in Fig. 1. The only experimental data included in the left two-thirds of the figure are the $T_{1/2}$ and $\beta^-$ branching ratio for the $^{212}$Bi isomers and the experimental $\log f_0 t$ values for $^{212g}$Bi decay [1]. The experimental data on the right is that of Poletti *et al.* [11] which was obtained in a $^{208}$Pb($^9$Be,$\alpha n\gamma$)$^{212}$Po fusion-evaporation study. A similar $^{210}$Pb($\alpha, 2n\gamma$)$^{212}$Po study by Sugawara *et al.*[12] gave similar and consistent results. The unadorned $J^\pi$ assignments in the experimental decays scheme are assumed here to be firm, although Poletti *et al.* and Sugawara *et al.* do not claim definite assignments for those level above 1500 keV. The $J^\pi$ values in square brackets are speculations for $J$ of Poletti *et al.* In four cases the shell-model calculation has been invoked to add an even-parity choice to these speculations. Rather obvious identifications with shell-model states can be inferred for all five of the speculated assignments by reference to Table II. The fusion-evaporation studies ended at the $14^+$ state at 2885 MeV.

### B. Calculation of the $\beta^-$ decay rates

The $Q(\beta^-)$ value for a specific branch is taken as

$$Q(\beta^-) = 2252 + E_x(^{212}\mathrm{Bi}) - E_x(^{212}\mathrm{Po})\ \mathrm{keV} \qquad (1)$$

with the excitation energies taken from experiment when possible and from the KHP$_e$ predictions otherwise. With the $Q(\beta^-)$ values pertaining for the three $^{212}$Bi isomers, the only conceivable $\beta$-decay branches are either Gamow-Teller ($\Delta J \leq 1$, $\pi_i\pi_f = +$) or first forbidden ($\Delta J \leq 2$, $\pi_i\pi_f = -$); and if first forbidden, $\Delta J \leq 1$. The latter restriction follows from the restriction on the strength of unique ($\Delta J = 2$) first-forbidden decays which is stated in the form [13]: *First-forbidden unique decays have* $\log f_1 t \geq 8.5$.

The Gamow-Teller (GT) beta moment is defined by [14]

$$B(\mathrm{GT}) = 6166/f_0 t \quad [\mathrm{GT(allowed)\ decay}] , \qquad (2)$$

where $f_0$ is the Fermi integral and t the half-life of the decay branch in question [15,7]. For first-forbidden decays, comparison to experiment is made here via the averaged shape factor [16]

$$\overline{C(W)} = 9195 \times 10^5/f_0 t = B_1^{(0)} + B_1^{(1)}\ \mathrm{fm}^2 , \qquad (3)$$

where advantage is taken of the fact that the decay is incoherent in the rank $R$ of the contributing operators and, as stated above, the rank-two contribution is negligible. The rank-zero ($R0$) and rank-one ($R1$) components of first-forbidden decays can be formulated in terms of two $R0$ matrix elements $M_0^S$ and $M_0^T$ — the spacelike and timelike components of the axial current — and two $R1$ matrix elements $M_1^x$ and $M_1^u$ — the $E1$-like and $R1$ spin-dipole operators. Although the $R2$ contribution was calculated for all decay branches of interest, it was always negligible and need not be considered further. The $B_1^{(R)}$ were calculated using the full rigour and accuracy of the Behrens-Bühring formalism [16], however, the $\zeta$ approximation is useful for displaying the physics involved in the $R0$ and $R1$ decays [15,7]:

$$\begin{aligned} B_1^{(0)} &= [M_1^{(0)}]^2 = [\epsilon_{\mathrm{mec}} M_0^T + a_S M_0^S]^2 , \\ B_1^{(1)} &= [M_1^{(1)}]^2 = [a_u M_1^u - a_x M_1^x]^2 , \end{aligned}$$

$$(\zeta \text{ approximation}) \qquad (4)$$

where $\epsilon_{\mathrm{mec}}$ is the meson-exchange-current (mec) enhancement factor evaluated for the lead region [7] as $\epsilon_{\mathrm{mec}} = 2.01 \pm 0.05$. In the present calculations a value of 2.0 is used. The $a_\alpha$ are positive, largely kinematical and insensitive to nuclear structure. The $\zeta$ approximation has errors of $\leq$4% for $R0$ and $\leq$10% for $R1$.

The $\beta$ matrix elements are calculated via

$$\begin{aligned} M_R^\alpha &= \sum_{j_i j_f} \mathcal{M}_R^\alpha(j_i j_f) = \sum_{j_i j_f} D_R(j_i j_f) M_R^\alpha(j_i j_f, \mathrm{eff}) \\ &= \sum_{j_i j_f} D_R(j_i j_f) q_\alpha(j_i j_f) M_R^\alpha(j_i j_f) \end{aligned} \qquad (5)$$

TABLE I. KHP$_e$ predictions for the energy spectrum of $^{212}$Bi. The index $k$ orders states of a given $J^\pi$ in energy. All levels are shown for $E_x < 1$ MeV while only yrast and yrare ($k$ = 1–2) levels are shown for $E_x > 1$ MeV.

| $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $1^-$ | 1 | 911 | $6^-$ | 2 | 1586 | $6^+$ | 1 | 2519 | $16^+$ | 1 |
| 186 | $2^-$ | 1 | 936 | $4^-$ | 3 | 1599 | $7^+$ | 1 | 2543 | $18^-$ | 2 |
| 220 | $0^-$ | 1 | 968 | $4^-$ | 4 | 1615 | $6^+$ | 2 | 2561 | $18^+$ | 1 |
| 263 | $3^-$ | 1 | 991 | $6^-$ | 3 | 1620 | $5^+$ | 2 | 2568 | $21^+$ | 1 |
| 281 | $9^-$ | 1 | 1074 | $11^-$ | 1 | 1629 | $15^-$ | 2 | 2581 | $16^+$ | 2 |
| 303 | $8^-$ | 1 | 1080 | $9^-$ | 2 | 1633 | $10^+$ | 2 | 2586 | $0^+$ | 2 |
| 319 | $4^-$ | 1 | 1103 | $10^-$ | 2 | 1636 | $2^+$ | 2 | 2647 | $17^+$ | 2 |
| 345 | $6^-$ | 1 | 1188 | $11^-$ | 2 | 1637 | $7^+$ | 2 | 2669 | $19^+$ | 1 |
| 346 | $5^-$ | 1 | 1250 | $12^-$ | 1 | 1661 | $9^+$ | 1 | 2689 | $19^+$ | 2 |
| 347 | $1^-$ | 2 | 1261 | $0^-$ | 2 | 1664 | $8^+$ | 2 | 2712 | $20^+$ | 2 |
| 363 | $7^-$ | 1 | 1292 | $12^+$ | 1 | 1668 | $17^-$ | 1 | 2735 | $18^+$ | 2 |
| 436 | $10^-$ | 1 | 1293 | $10^+$ | 1 | 1680 | $12^+$ | 2 | 3079 | $21^+$ | 2 |
| 511 | $1^-$ | 3 | 1323 | $4^+$ | 1 | 1760 | $11^+$ | 2 | 3138 | $22^+$ | 2 |
| 654 | $2^-$ | 2 | 1325 | $11^+$ | 1 | 1812 | $1^+$ | 2 | 3285 | $24^-$ | 1 |
| 703 | $2^-$ | 3 | 1358 | $1^+$ | 1 | 1825 | $9^+$ | 2 | 3408 | $22^-$ | 1 |
| 708 | $3^-$ | 2 | 1367 | $13^-$ | 1 | 1905 | $13^+$ | 1 | 3595 | $23^-$ | 1 |
| 709 | $1^-$ | 4 | 1413 | $12^-$ | 2 | 2015 | $16^-$ | 2 | 3610 | $20^-$ | 1 |
| 731 | $7^-$ | 2 | 1429 | $2^+$ | 1 | 2143 | $13^+$ | 2 | 3683 | $24^-$ | 2 |
| 782 | $8^-$ | 2 | 1434 | $3^+$ | 2 | 2196 | $19^-$ | 1 | 3697 | $19^-$ | 2 |
| 784 | $1^-$ | 5 | 1461 | $15^-$ | 1 | 2204 | $17^-$ | 2 | 3698 | $21^-$ | 1 |
| 826 | $3^-$ | 3 | 1496 | $18^-$ | 1 | 2240 | $14^+$ | 1 | 3701 | $20^-$ | 2 |
| 835 | $5^-$ | 2 | 1500 | $5^+$ | 1 | 2276 | $0^+$ | 1 | 3877 | $22^-$ | 2 |
| 863 | $7^-$ | 3 | 1521 | $4^+$ | 2 | 2360 | $17^+$ | 1 | 3924 | $21^-$ | 2 |
| 865 | $3^+$ | 1 | 1522 | $13^-$ | 2 | 2368 | $15^+$ | 1 | 4022 | $23^-$ | 2 |
| 870 | $4^-$ | 2 | 1532 | $14^-$ | 1 | 2392 | $20^+$ | 1 | 5073 | $24^+$ | 1 |
| 879 | $2^-$ | 4 | 1547 | $14^-$ | 2 | 2406 | $14^+$ | 2 | 5096 | $23^+$ | 1 |
| 899 | $5^-$ | 3 | 1557 | $8^+$ | 1 | 2439 | $22^+$ | 1 | 5192 | $23^+$ | 2 |
| 904 | $8^-$ | 3 | 1581 | $16^-$ | 1 | 2510 | $15^+$ | 2 | 5329 | $24^+$ | 2 |

using Woods-Saxon (WS) wave functions. In Eq. (5) $\alpha$ labels a matrix element of rank $R$, $M_R^\alpha(j_i j_f)$ is a single-particle matrix element for the transition $j_i \rightarrow j_f$ in the impulse approximation, and the quenching factor $q_\alpha(j_i j_f)$ corrects $M_R^\alpha(j_i j_f)$ for the finite size of the model space and some effects of the nuclear medium. The $D_R(j_i j_f)$ are the one-body-transition densities which are the result of the shell-model calculations performed with the code OXBASH [8]. The quenching factor $q_{\rm GT}(j_i j_f)$ for allowed decays was taken to be state independent and equal to 0.66 which is a value suggested by results obtained by Towner [17]. For first-forbidden decays recent results [18] were used which are associated with the parametrization for $A$ = 205–212 nuclei [7] already alluded too.

With Fig. 1 as a reference we are now ready to consider the three $^{212}$Bi activities. The information given in Fig. 1 and not as yet touched on will become clear as the discussions develop. We start with the 27-min $^{212}$Bi$^{m2}$ activity.

### C. $^{212}$Bi$^{m2}$ decay

As pointed out by Poletti *et al.* [11] an isomer in either $^{212}$Bi or $^{212}$Po with a half-life in the 10-min region must in all probability be one which has no available states to decay to by electromagnetic radiation with $L < 4$. From Fig. 1 it is clear that there are no predicted $^{212}$Bi states with $E_x > 670$ keV which so qualify. In order to explain the $^{212}$Bi$^{m2}$ activity (with $E_x > 670$ keV) it is necessary to assume that either the ordering of the levels differs from the KHP$_e$ predictions or that a highly unlikely retardation of one or more $L < 4$ electromagnetic transitions is occurring. The former alternative is considered much more likely on general grounds and calculations of electromagnetic transition strengths with the KHP$_e$ wave functions supports this view. There are three obvious possibilities for isomers. The yrast $15^-$ and $18^-$ states are predicted to be separated by only 35 keV. This is well within the rms deviation of 62 keV for the ten states of $^{212}$Po for which a correspondence between theory and experiment is indicated in Fig. 1. With the $18_1^-$ state below that of $15_1^-$, it would have only $L \geq 5$ transitions available to lower $^{212}$Bi states. A second possibility is that the $22_1^+$ state lies below the $19_1^-$ and $20_1^+$ states in which case all decays of the $22_1^+$ state to lower states would have $L \geq 4$. This is considered less likely because the energy separation of the $19_1^-$ and $22_1^+$ states is predicted to be 243 keV. Likewise, if the $15_1^-$ state were lowered by 211 keV relative to the $J^\pi = 12^+$, $12^-$, and $13^-$ states shown in Fig. 1, it would have only $L \geq 4$ electromagnetic transitions available. These two pos-

TABLE II. KHP$_e$ predictions for the energy spectrum of $^{212}$Po. The index $k$ orders states of a given $J^\pi$ in energy. Only yrast and yrare ($k$ = 1–2) levels are shown with the exception of some levels with $k > 2$ which are of present interest.

| $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ | $E_x$ (keV) | $J^\pi$ | $k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $0^+$ | 1 | 2422 | $7^+$ | 2 | 2942 | $7^-$ | 2 | 3822 | $16^-$ | 2 |
| 788 | $2^+$ | 1 | 2443 | $8^+$ | 5 | 2955 | $12^+$ | 2 | 3841 | $18^-$ | 2 |
| 1172 | $4^+$ | 1 | 2465 | $7^+$ | 3 | 2969 | $8^-$ | 2 | 3842 | $20^-$ | 1 |
| 1356 | $6^+$ | 1 | 2474 | $10^+$ | 2 | 3032 | $16^+$ | 2 | 3849 | $15^-$ | 2 |
| 1405 | $8^+$ | 1 | 2477 | $8^+$ | 6 | 3039 | $13^+$ | 1 | 3877 | $17^-$ | 2 |
| 1429 | $2^+$ | 2 | 2494 | $7^+$ | 4 | 3056 | $1^-$ | 1 | 3958 | $21^-$ | 2 |
| 1515 | $1^+$ | 1 | 2515 | $3^-$ | 2 | 3083 | $14^+$ | 2 | 3964 | $20^-$ | 2 |
| 1543 | $0^+$ | 2 | 2527 | $9^+$ | 2 | 3098 | $21^-$ | 1 | 4103 | $23^+$ | 1 |
| 1669 | $4^+$ | 2 | 2549 | $9^+$ | 3 | 3102 | $13^+$ | 2 | 4135 | $24^+$ | 1 |
| 1727 | $6^+$ | 2 | 2575 | $2^-$ | 2 | 3124 | $18^+$ | 2 | 4421 | $22^+$ | 1 |
| 1729 | $8^+$ | 2 | 2586 | $7^-$ | 1 | 3138 | $12^-$ | 2 | 4718 | $21^+$ | 1 |
| 1732 | $10^+$ | 1 | 2607 | $12^-$ | 1 | 3149 | $15^+$ | 1 | 4751 | $20^+$ | 1 |
| 1858 | $3^+$ | 1 | 2625 | $6^-$ | 1 | 3187 | $15^+$ | 2 | 4847 | $20^+$ | 2 |
| 1904 | $8^+$ | 3 | 2650 | $9^-$ | 1 | 3204 | $1^-$ | 2 | 4883 | $21^+$ | 2 |
| 1952 | $3^-$ | 1 | 2659 | $13^-$ | 1 | 3245 | $17^+$ | 1 | 4887 | $22^+$ | 2 |
| 2003 | $4^+$ | 3 | 2665 | $11^-$ | 2 | 3263 | $19^-$ | 1 | 4900 | $19^+$ | 1 |
| 2084 | $6^+$ | 3 | 2714 | $4^-$ | 2 | 3303 | $13^-$ | 2 | 4994 | $19^+$ | 2 |
| 2089 | $5^+$ | 1 | 2753 | $12^+$ | 1 | 3375 | $17^+$ | 2 | 5049 | $23^+$ | 2 |
| 2111 | $1^+$ | 2 | 2780 | $8^-$ | 1 | 3492 | $17^-$ | 1 | 5687 | $25^-$ | 1 |
| 2164 | $8^+$ | 4 | 2796 | $11^+$ | 1 | 3509 | $15^-$ | 1 | 6044 | $24^-$ | 1 |
| 2172 | $7^+$ | 1 | 2815 | $10^-$ | 1 | 3540 | $14^-$ | 1 | 6146 | $23^-$ | 1 |
| 2190 | $9^+$ | 1 | 2850 | $5^-$ | 2 | 3599 | $18^-$ | 1 | 6251 | $23^-$ | 2 |
| 2223 | $4^+$ | 4 | 2855 | $14^+$ | 1 | 3632 | $16^-$ | 1 | 6287 | $24^-$ | 2 |
| 2282 | $11^-$ | 1 | 2873 | $16^+$ | 1 | 3653 | $18^+$ | 3 | 6376 | $22^-$ | 1 |
| 2287 | $2^-$ | 1 | 2877 | $9^-$ | 2 | 3714 | $0^-$ | 1 | 6490 | $22^-$ | 2 |
| 2289 | $3^+$ | 2 | 2912 | $11^+$ | 2 | 3719 | $19^-$ | 2 | 6666 | $25^-$ | 2 |
| 2307 | $4^-$ | 1 | 2913 | $18^+$ | 1 | 3773 | $17^+$ | 3 | 7745 | $26^+$ | 1 |
| 2403 | $5^-$ | 1 | 2919 | $6^-$ | 2 | 3784 | $0^-$ | 2 | 8020 | $24^+$ | 2 |
| 2411 | $5^+$ | 2 | 2931 | $10^-$ | 2 | 3802 | $14^-$ | 2 | 8417 | $25^+$ | 1 |

sibilities are not considered further because, as it turns out, the $18_1^-$ possibility provides a natural explanation for the activity [19]. The $J = 17, 18$, and 19 states which are predicted to be energetically available for the $\beta^-$ decay of the $^{212}$Bi $18_1^-$ level are shown in Fig. 1.

The $J^\pi$ = $^{212}$Bi $18_1^-$ state is found to be 98% $\pi 0h_{9/2}\nu 1g_{9/2}^2 0i_{11/2}$. First consider the first-forbidden decays of this state. There are two $17^+$ and two $18^+$ $^{212}$Po states energetically available. The rank-one ($R1$) decays to all four are weak due to cancellation between the contributions of $M_1^u$ and $M_1^x$ to the $R1$ beta moment — see Eq. (4). This destructive interference is characteristic of $R1$ decays for $A = 209$–212 nuclei [7]. In the present instance it is due to the fact that the relative phases of the $\nu 1g_{9/2} \to \pi h_{9/2}$ and $\nu 1g_{9/2} \to \pi 1f_{7/2}$ single-particle $R1$ transitions are both even. The two KHP$_e$ $18^+$ states are largely an orthogonal mixture of $\pi 0h_{9/2}^2\nu 1g_{9/2}0i_{11/2}$ and $\pi 0h_{9/2}1f_{7/2}\nu 1g_{9/2}0i_{11/2}$, respectively. The $R1$ contributions to the two decays is predicted to be only 2.3% and 0.8% of the $R0$ decays. There are two contributions to the $R0$ matrix elements $\nu 1g_{9/2} \to \pi h_{9/2}$ and $\nu 1g_{7/2} \to \pi 1f_{7/2}$. These are in phase for $18_1^+$ and out of phase for $18_2^+$ with the result that the log$f_0t$ values for the decay to these two states are 5.68 and 6.38, respectively, and — using the KHP$_e$ predictions for the relative energies — $\beta^-$ branching ratios to these two states of 7.8% and 0.6% are calculated for $T_{1/2}$ = 8 min.

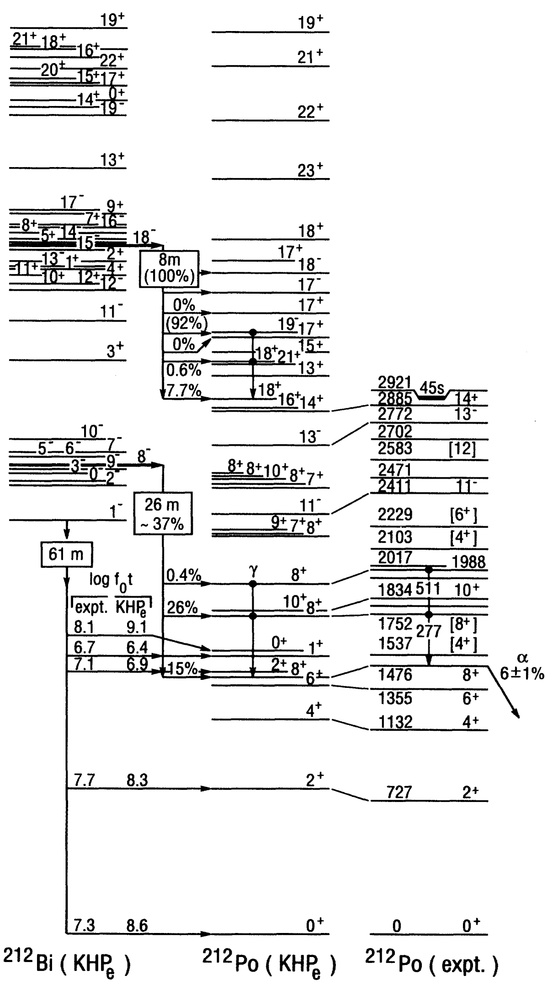


FIG. 1. Selected energy levels of $^{212}$Bi and $^{212}$Po and schematic of the $\beta^-$ decays of $^{212}$Bi. See the text for details.

Now consider the allowed decays. The $J^\pi$ = $17^-$, $18^-$, and $19^-$ yrast states are predicted to be 80%, 86%, and 86% $\pi 0h_{9/2}0i_{13/2}\nu 1g_{9/2}^2$, respectively. The Gamow-Teller beta moment $B$(GT) is calculated to be 0.0009, 0.0039, and 1.15 for these three final states so that the decays to the $17^-$ and $18^-$ states are predicted to be negligible compared to that to the $19^-$ state. The much larger value of $B$(GT) for the $19^-$ state is due to its being a "stretched" state as is the $^{212}$Bi $18_1^-$ state. Thus for the transition between these two states the coupling coefficients associated with the only contributing single-particle transition, $\nu 0i_{11/2} \to \pi 0i_{13/2}$, are maximal. The $f_0$ value for the transition to the $19_1^-$ state is calculated to be 12.3 giving a predicted partial half-life of 440 s. This is in excellent agreement with the value of 520 s expected for a 92% branch (with 8% going via first-forbidden decays). Note that it would only take a 30-keV decrease in the relative excitation energies of the initial and final states to lower $f_0$ enough so as to exactly reproduce the $T_{1/2}$ = 520-s expectation.

### D. $^{212}$Bi$^{m1}$ decay

From the relative intensities observed for the $\alpha$ decay of the $^{212}$Bi 27-min isomer and the $\beta^-$-delayed $\alpha$ emission of the $^{212}$Po 1476-keV $8_1^+$ state [2], and with the aid of the $\alpha$ branching ratio for the $8_1^+$ state of 6 ± 1% (Ref. [20]), a $\beta^-$ branching ratio of ~ 37% can be deduced for the 27-min isomer [21]. In their study of this isomer, Lemmertz *et al.* [3] observed the $\gamma$-ray cascades corresponding to $8_1^+ \to 6_1^+ \to 4_1^+ \to 2_1^+ \to 0_1^+$ in coincidence with the $\alpha$ decay of the $^{212}$Po ground state (see Fig. 1). They also observed a 275-keV $\gamma$ transition which is quite probably the same as the 277-keV 1752 → 1476 transition observed by Poletti *et al.* [11]. Since the 1834-keV level is identified as the yrast $10^+$ state, the only reasonable identification of the 1752-keV level with the KHP$_e$ states of Fig. 1 is with the $8_2^+$ state.

There are two obvious possibilities for a 27-min isomer at $E_x$ = 250 keV in $^{212}$Bi; the yrast $9^-$ or $8^-$ state, depending on which lies lower. The $9^-$ possibility was the only one suggested previously. The reason for this preference was that analogy was made to $^{210}$Bi and in that nucleus the yrast $9^-$ and $8^-$ states lie at 271 and 583 keV, respectively. The compression of the spectrum of low-lying states of the even Bi isotopes as $A$ increases from 210 to 214 has been remarked on previously [22]. This is another illustration. The odds as to which of the two states lies lowest seems about even. The KHP$_e$ prediction is that the $9^-$ state is lowest by 22 keV. However, there are actually four $8^-$ states predicted below the second $9^-$ state and it would take only trivial changes in the interactions between them to lower the $8_1^-$ state by 22 keV. The decays of both the $9^-$ and $8^-$ possibilites were considered. Note that both states have only first-forbidden decay channels available.

The predicted $\beta^-$ decay of the $9^-$ state was found to be in severe disagreement with the experimental facts. First of all, the major $\beta^-$ branch (4.9%) is predicted to be that to the $10_1^+$ state and this decay mode would be accompanied by a 358-keV $\gamma$ ray which was not observed experimentally [3]. The remaining $\beta^-$ branching ratios add up to 0.24% and so the disagreement with experiment (~37% $\beta^-$ branching) is considerable for any branch which could be associated with a 275-keV $\gamma$-ray transition. The weakness of the $R1$ contributions to the decays is due to the same sort of destructive interference as seen for the 8-min isomer. The weakness of the $R0$ contribution to the $9_1^- \to 9_1^+$ transition is due to very small values for the relevant $D_0(j_i j_f)$.

The predicted $\beta^-$ decay of the $^{212}$Bi $8_1^-$ state is a quite different story. The $R1$ decays to the available $7^+$ and $9^+$ states exhibit the same destructive interference between $M_1^x$ and $M_1^u$ as the other decays considered so far. The decay branches to these states and to the $8_4^+$ and $8_5^+$ states are predicted to be negligible (< 0.2% in total). However the decays to the three lowest $8^+$ states are predicted to sum to a $\beta^-$ branching ratio of 41% in nice agreement with the experimental value of ~37%. The predicted log$f_0t$ values for these three decays are 6.23, 5.51, and 6.76. They are, respectively, 99%, 99%, and 94% $R0$. For all three branches the $R0$ contribution from $\nu 1g_{9/2} \to \pi 0h_{9/2}$ is dominant. For $8_1^+$ and $8_2^+$ all five possible single-particle transitions contribute in phase. Thus, these two states share the role of the "pygmy" resonance of the initial state as recently discussed [7] (also see Sec. III). As shown in Fig. 1, the predicted branching ratio of 26% into the $8_2^+$ state provides an explanation for the 275-keV $\gamma$-ray transition observed by Lemmertz *et al.* [3].

### E. $^{212}$Bi$^g$ decay

Beta decay of the $^{212}$Bi ground state to the first two $0^+$ and $2^+$ states and the first $1^+$ state was considered. The resulting log$f_0t$ values are compared to experiment in Fig. 1. The agreement is seen to be very poor. This is not surprising. All five transitions are predominantly $R1$ and with one exception the overlap of the initial and final states is very poor, i.e., the $D_0(j_i j_f)$ are small. In addition, the contributions of $M_1^x$ and $M_1^u$ are destructive. The one exception is the decay to $2_2^+$ which has a strong $\nu 1g_{9/2} \to \pi 0h_{9/2}$ component and is dominated by $M_1^u$. It is gratifying that the agreement for the latter decay is reasonably good. There are two general reasons why $R1$ decays with log$f_0t \gtrsim 7.0$ can often not be predicted well. First, for such small rates the results are usually very sensitive to the wave functions — often to a point beyond which the wave functions can be expected to give meaningful results. Second, at this level the next order of terms in the Behrens-Bühring expansion begin to become non-negligible [16]. Note that all the first-forbidden decays considered for the decay of $^{212}$Bi$^{m1}$ and $^{212}$Bi$^{m2}$ have log$f_0t$ < 6.8. In summary we should be satisfied that the three weakest decays are predicted as such and the agreement is not too bad for the two strongest branches.

## III. SUMMARY

It was expected that the Kuo-Herling model space for $Z \geq 82, A > 208$ will give a good description of the yrast and yrare states of $^{212}$Bi and $^{212}$Po and that an examination of the spectra of these states would reveal the possible candidates for $^{212}$Bi$^{m1}$ and $^{212}$Bi$^{m2}$. Accordingly the spectra were calculated and indeed several possibilities were obvious for each isomer. Calculations of the $\beta^-$ decays rates were then performed to test the proposed candidates and obvious choices for both isomers were found. The success of this approach is due to two factors: (1) The availability of shell-model results for all $^{212}$Bi and $^{212}$Po states of the Kuo-Herling model space — heretofore only the even-parity spectrum of $^{212}$Po was known [23]. (2) The ability to calculate first-forbidden decay rates with good accuracy [7].

The three isomers studied exhibit quite different decay properties which can be characterized by the proposed principal decay mode. The 8-min high-spin isomer $^{212}$Bi$^{m2}$ is predicted to decay by an allowed transition to the yrast $19^-$ state of $^{212}$Po. The transition is quite fast due to the stretched nature of the $^{212}$Bi $18_1^-$ and $^{212}$Po $19_1^-$ states. It is proposed that the $19_1^-$ state then decays by an $E1$ transition to the $18_1^+$ state of $^{212}$Po as indicated in Fig. 1. Arguments and experimental evidence advanced by Poletti *et al.* [11] and Kudo *et al.* [24] strongly support the identification of the $18_1^+$ state as the 45-s isomer of $^{212}$Po. (However, it would be reassuring to see evidence for formation of the $^{212}$Po $16_1^+$ and $18_1^+$ states in a fusion-evaporation study such as that of Poletti *et al.* [11].) If the $^{212}$Po isomer is actually the $16_1^+$ state, then the proposed decay of the $19_1^-$ state would be $19_1^- \xrightarrow{E1} 18_1^+ \xrightarrow{E2} 16_1^+$. In Table III are listed KHP$_e$ predictions for some $E2$ and $E4$ transitions. These are given because of their potential use in obtaining an understanding of the $^{212}$Po states involved in the decay of the $18_1^+$ state. They also display adequate agreement with experiment with reasonable values of the effective charges.

The 27-min $^{212}$Bi$^{m1}$ isomer at an excitation energy of 250 keV is predicted to be the $8_1^-$ state and to decay mainly by $R0$ first-forbidden components to the first two $8^+$ states of $^{212}$Po. As discussed in detail previously [7], the initial (final) state in first-forbidden beta decay has $R0$ and $R1$ particle-hole giant resonances at high excitation (~ $1\hbar\omega$) in the final (initial) nucleus. This is well known and follows from the repulsive nature of the particle-hole interaction. What is less well known is that the attractive particle-particle interaction gives rise to another (albeit weaker) resonance (termed "pygmy" [7]) situated at low excitation energy. First-forbidden decay rates are fastest when the initial and final states are each others "pygmy" resonance. In the present instance the

TABLE III. $\mathrm{KHP}_e$ predictions for $E\lambda$ ($\lambda = J_f - J_i$) transitions between even-parity states of $^{212}$Po. $B(E\lambda)$ is given by $(e_p M_p + e_n M_n)^2/(2J_i+1)\ e^2\ \mathrm{fm}^{2\lambda}$ where $e_p$ and $e_n$ are the effective proton and neutron charges in units of $e$. The values of $B(E\lambda)$ given in the next-to-last column are for $e_p = 2$, $e_n = 1$ and are in Weisskopf units (W.u.). For A = 212, the $B(E2)$ and $B(E4)$ values corresponding to 1 W.u. are 75.09 $e^2$ fm$^4$ and $1.00\times10^5$ $e^2$ fm$^8$, respectively. The transitions were calculated with WS radial wave functions. The experimental (expt) $B(E2)$ values are from Ref. [11] and the $B(E4)$ value is from Ref. [24].

| $J_i$ | $J_f$ | $M_p$ ($e$ fm$^\lambda$) | $M_n$ ($e$ fm$^\lambda$) | $B(E\lambda)$ ($\mathrm{KHP}_e$) | $B(E\lambda)$ (expt) |
|---|---|---|---|---|---|
| 2 | 0 | 15.29 | 27.39 | 8.95 | |
| 4 | 2 | 21.27 | 42.70 | 10.75 | |
| 6 | 4 | 19.23 | 45.30 | 7.19 | 13.5±3.7 |
| 8 | 6 | 14.33 | 36.95 | 3.37 | 3.95±0.05 |
| 10 | 8 | 7.02 | 17.17 | 0.62 | 2.2±0.6 |
| 12 | 10 | 19.19 | 11.93 | 1.35 | |
| 14 | 12 | 36.94 | 61.30 | 8.39 | |
| 16 | 14 | 29.10 | 48.83 | 4.62 | |
| 18 | 16 | 12.11 | 26.15 | 0.91 | |
| 18 | 14 | 421.80 | 1168.00 | 1.09 | $2.3^{+4.6}_{-1.4}$ |

$8_1^+$ and $8_2^+$ states of $^{212}$Po share the role of the "pygmy" resonance of the $^{212}$Bi $8_1^-$ state. Hence the fast decays.

The $^{212}$Bi ground state decays 64% by $\beta^-$ emission and 55% of the $\beta^-$ decay is to the $^{212}$Po ground state [1]. The predicted decay for this branch displays pathological cancellations between contributing terms. For this branch Eq. (4) gives

$$B_1^{(1)} = [9.15 M_1^u - 26.88 M_1^x]^2 = [4.44 - 4.45]^2\ \mathrm{fm}^2\ \ (\zeta\ \text{approximation})\ , \tag{6}$$

i.e., almost complete cancellation. Recall that for first-forbidden transitions the $\beta^-$ shape factor is [16]

$$C(W_e) = k(1 + aW_e + b/W_e + cW_e^2)\ , \tag{7}$$

where Eq. (4) is a good approximation for k. In the present case what little of $k$ survives the cancellation shown in Eq. (6) is itself almost exactly cancelled by the $ka$ term so that the rate is given by the $kc$ term (for $R1$ decays $kb = 0$). Hence the very large $\log f_0 t$ prediction for this transition. As might be imagined, reasonable changes in the wave functions and in the $q_\alpha(j_i j_f)$ and contributions from higher-order terms in the Behrens-Bühring expansion can bring the predicted rate into agreement with experiment.

## ACKNOWLEDGMENTS

I thank B. A. Brown for helpful communications. Research was supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

[1] M. J. Martin, Nucl. Data Sheets **27**, 637 (1979).
[2] P. A. Baisden, R. E. Leber, M. Nurmia, J. M. Nitschke, M. Michel, and A. Ghiorso, Phys. Rev. Letts. **41**, 738 (1978).
[3] P. Lemmertz, L. J. Alquist, R. Fass, H. Wollnik, D. Hirdes, H. Jungclas, R. Brandt, D. Schardt, and J. Żylicz, Z. Phys A **298**, 311 (1980).
[4] N. Auerbach and I. Talmi, Phys. Lett. **10**, 297 (1964).
[5] N. K. Glendenning and K. Harada, Nucl. Phys. **72**, 481 (1965).
[6] T. T. S. Kuo and G. H. Herling, US Naval Research Laboratory Report No. 2258, 1971(unpublished).
[7] E. K. Warburton, Phys. Rev. Lett. **66**, 1823 (1991); E. K. Warburton, Phys. Rev. C **44**, 233 (1991).
[8] B. A. Brown, A. Etchegoyen, W. D. M. Rae, and N. S. Godwin, OXBASH, 1984 (unpublished).
[9] J. B. McGrory and T. T. S. Kuo, Nucl. Phys. **A247**, 283 (1975).
[10] E. K. Warburton and B. A. Brown, Phys. Rev. C **43**, 602 (1991).
[11] A. R. Poletti, G. D. Dracoulis, A. P. Byrne, and A. E. Struchbery, Nucl. Phys. **A473**, 595 (1987).
[12] M. Sugawara, Y. Gono, Y. Ishikawa, and M. Fukuda, Nucl. Phys. **A443**, 461 (1985).
[13] S. Raman and N. B. Gove, Phys. Rev. C **7**, 1995 (1973).
[14] B. R. Brown and B. H. Wildenthal, At. Data Nucl. Data Tables **33**, 347 (1985).
[15] E. K. Warburton, J. A. Becker, B. A. Brown, and D. J. Millener, Ann. Phys. (N.Y.) **187**, 471 (1988).
[16] H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, 1982).
[17] I. S. Towner, Phys. Rep. **155**, 263 (1987).
[18] E. K. Warburton, Phys. Rev. C **42**, 2479 (1990).

[19] The hypothetical $\beta^-$ decay of the $^{212}$Bi $15_1^-$ state was considered. The total decay rate was found to be a factor of $\geq$43 too low. Furthermore 99.8% of the decays was predicted to proceed to $14^+$ and $15^+$ states which would be expected to $\gamma$ decay predominantly to $J \leq 14$ states and thereby bypass the 45-s isomer of $^{212}$Po.

[20] R. M. Lieder, J. P. Didelez, H. Beuscher, D. R. Haenni, M. Müller-Veggian, A. Neskakis, and C. Mayer-Böricke, Phys. Rev. Lett. **41**, 742 (1978).

[21] Unfortunately Baisden *et al.* [2] did not give the information necessary to render this branching ratio more quantitative.

[22] Z. Berant, R. B. Schuhmann, D. E. Alburger, W. T. Chou, R. L. Gill, E. K. Warburton, and C. Wesselborg, Phys. Rev. C **43**, 1639 (1991).

[23] D. Strottman, Phys. Rev. C **20**, 1150 (1979).

[24] H. Kudo, T. Nomura, K. Sueki, M. Magara, and N. Yoshida, Nucl. Phys. **A494**, 203 (1989).