PHYSICAL REVIEW C VOLUME 44, NUMBER 6 DECEMBER 1991

Level matrix, ^{16}N β decay, and the ^{12}C$(\alpha,\gamma)^{16}$O reaction

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(Received 1 April 1991)

The level matrix corresponding to the $\mathcal{K}$-matrix parametrization of a resonant nuclear reaction is derived and applied to the spectrum of α particles emitted following ^{16}N β decay. The parametrized spectrum is fitted to data simultaneously with the $E1$ capture cross section of the ^{12}C$(\alpha,\gamma)^{16}$O reaction and the p-wave phase shift of ^{12}C$(\alpha,\alpha)^{12}$C. Our analysis shows that new measurements of the α spectrum from ^{16}N β decay could be used to significantly reduce the uncertainty of the ^{12}C$(\alpha,\gamma)^{16}$O astrophysical S factor at 0.3 MeV. Various constraints on the parameters are analyzed and suggestions are made for further reducing the uncertainty in this crucial reaction rate.

I. INTRODUCTION

As a supplement to a recent paper [1] on the $\mathcal{K}$-matrix parametrization of resonant nuclear reactions, we derive here the level matrix associated with this parametrization. This, in turn, allows us to derive a parametrized form of the spectrum of α particles emitted after the β decay of unstable nuclei. When applied to the α particles emitted following ^{16}N β decay, the parametrization can be used to further constrain the astrophysical S factor for the ^{12}C$(\alpha,\gamma)^{16}$O capture reaction at 0.3 MeV, as compared to a recent analysis [2]. In the latter paper [2], only the cross section for the ^{12}C$(\alpha,\gamma)^{16}$O reaction and the elastic scattering p-wave phase shift for ^{12}C$(\alpha,\alpha)^{12}$C were parametrized in terms of a $\mathcal{K}$ matrix and fitted simultaneously to recent data. Unfortunately, fitting the phase shift proved to be only a weak constraint on the free parameters involved. In contrast, we show here that the simultaneous fit of the α spectrum from ^{16}N β decay is a very stringent constraint on the free parameters. This new $\mathcal{K}$-matrix analysis suggests that a new measurement of the energy spectrum of the α particles following ^{16}N β decay can significantly constrain the S factor for ^{12}C$(\alpha,\gamma)^{16}$O.

In Sec. II below we discuss the level matrix appropriate to the $\mathcal{K}$-matrix description. In Sec. III we give explicit formulas for the $\mathcal{K}$-matrix parametrization of the β-delayed α spectrum from ^{16}N. In Sec. IV we restate the corresponding formulas for the $E1$ ^{12}C$(\alpha,\gamma)^{16}$O cross section and ^{12}C$(\alpha,\alpha)^{12}$C p-wave phase shift, and fit all three types of data simultaneously. Finally, in Sec. V we discuss our results and present our conclusions. Throughout, our notation and definitions are those of Ref. [1].

II. THE LEVEL MATRIX

The particular form of the transition matrix $\mathcal{T}=1-\mathcal{S}$ corresponding to a one-level approximation of the $\mathcal{K}$ matrix has been given in Sec. VI of Ref. [1]. This is the simplest illustration of the fact that in the equation [1]

$$\mathcal{T}=-2ip\mathcal{K}(1-i\mu\mathcal{K})^{-1}p \tag{2.1}$$

the inversion of a channel matrix can be replaced by the inversion of a level matrix. We now obtain $\mathcal{T}$ in terms of a level matrix A (elements $A_{\lambda\mu}$) for any number of levels and channels.

The column vector g_λ has elements $g_{a\lambda}, g_{b\lambda}, \ldots$, and the diagonal matrix μ has diagonal elements $\mu_a, \mu_b, \ldots$, so that $h_\lambda=\mu g_\lambda$ is also a column vector with elements $\mu_a g_{a\lambda}, \mu_b g_{b\lambda}, \ldots$. With this notation we have [1]

$$\mathcal{K}=\sum_\lambda g_\lambda \times g_\lambda/(E_\lambda-E)\ , \tag{2.2}$$

$$1-i\mu\mathcal{K}=1-i\sum_\lambda h_\lambda\times g_\lambda/(E_\lambda-E)\ . \tag{2.3}$$

We now assume that the inverse of the latter quantity has the form

$$(1-i\mu\mathcal{K})^{-1}=1+i\sum_{\nu\mu} h_\nu\times g_\mu A_{\nu\mu}\ , \tag{2.4}$$

and justify it by obtaining the corresponding A matrix. The product of the matrices (2.3) and (2.4) being the unit matrix we must have

$$\sum_{\nu\mu} h_\nu\times g_\mu A_{\nu\mu}-\sum_\lambda h_\lambda\times g_\lambda/(E_\lambda-E) -i\sum_{\lambda\nu\mu}A_{\nu\mu}(h_\lambda\times g_\lambda)(h_\nu\times g_\mu)/(E_\lambda-E)=0\ . \tag{2.5}$$

Since the matrix in the third term also reads

$$(h_\lambda\times g_\mu)m_{\lambda\nu}\ ,$$

with

$$m_{\lambda\nu}=\sum_e g_{e\lambda}h_{e\nu}=\sum_e \mu_e g_{e\lambda}g_{e\nu}\ , \tag{2.6}$$

Eq. (2.5) can be given the form

$$\sum_{\lambda\mu}(h_\lambda\times g_\mu)\left[A_{\lambda\mu}-\delta_{\lambda\mu}/(E_\lambda-E) -i\sum_\nu m_{\lambda\nu}A_{\nu\mu}/(E_\lambda-E)\right]=0\ .$$

This equation is satisfied when

$$(E_\lambda - E)A_{\lambda\mu} - i\sum_\nu m_{\lambda\nu}A_{\nu\mu} = \delta_{\lambda\mu}\ , \tag{2.7}$$

or

$$\sum_\nu [(E_\lambda - E)\delta_{\lambda\nu} - im_{\lambda\nu}]A_{\nu\mu} = \delta_{\lambda\mu}\ , \tag{2.8}$$

i.e., when the symmetrical level matrix A, with elements $A_{\lambda\mu}$, is defined by its inverse,

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - i\sum_{e^+} p_e^2 g_{e\lambda}g_{e\mu}\ , \tag{2.9}$$

since in $m_{\lambda\mu}$, according to Sec. VI of Ref. [1], we have $\mu_{e^+} = p_e^2$, $\mu_{e^-} = 0$ in open and closed channels, respectively.

From Eqs. (2.2), (2.4), and (2.7) we obtain

$$\begin{aligned}\mathcal{K}(1 - i\mu\mathcal{K})^{-1} &= \sum_\lambda g_\lambda \times g_\lambda/(E_\lambda - E)\\ &\quad + i\sum_{\lambda\mu\nu} g_\lambda \times g_\mu m_{\lambda\nu}A_{\nu\mu}/(E_\lambda - E)\\ &= \sum_{\lambda\mu} g_\lambda \times g_\mu A_{\lambda\mu}\end{aligned} \tag{2.10}$$

and

$$\mathcal{T} = -2i\sum_{\lambda\mu} p(g_\lambda \times g_\mu)pA_{\lambda\mu}\ . \tag{2.11}$$

For an integrated cross section, and with partial widths defined as in Ref. [1] by $\Gamma_{c\lambda} = 2p_c^2 g_{c\lambda}^2$, we have

$$\sigma_{cd}^J = \frac{4\pi g^J}{k_c^2}\left|\sum_{\lambda\mu} p_c g_{c\lambda}A_{\lambda\mu}g_{d\mu}p_d\right|^2 \tag{2.12}$$

$$= \frac{\pi g^J}{k_c^2}\left|\sum_{\lambda\mu}\Gamma_{c\lambda}^{1/2}A_{\lambda\mu}\Gamma_{d\mu}^{1/2}\right|^2\ . \tag{2.13}$$

The result (2.10) is only formally the same as in R-matrix theory [3], the main difference being in the very definition (2.9) of the level matrix A. Here, $i\mu_e$, which corresponds to $L_e^0 = S_e + iP_e - B_e$ in R-matrix theory, is not only independent of the channel radii, but it also vanishes in all closed channels e^- with two charged fragments, as seen in Sec. VI of Ref. [1].

Nevertheless, from the very form of Eqs. (2.10)–(2.12), we can infer that the applications, which have made use of the level matrix associated with the R matrix, should also be feasible with the level matrix associated with the $\mathcal{K}$ matrix. This should hold in particular for the analysis of the energy spectrum of the α particles following the β^- decay of ^{16}N, whose 2^- ground state is unstable. This and related problems have been analyzed previously by Barker *et al.* [4–6] in papers based on an R-matrix parametrization.

III. THE SPECTRUM OF α PARTICLES FOLLOWING ^{16}N β DECAY

For the ^{12}C(α,γ)^{16}O reaction, the astrophysical factor

$$S(E) = E\sigma_{\alpha\gamma}(E)\exp(2\pi\eta) \tag{3.1}$$

is of particular interest at the "most effective" ^{12}C+α center-of-mass energy $E = 0.3$ MeV. However, a simultaneous fit to the capture cross section $[\sigma_{\alpha\gamma}(E)]$ and the elastic-scattering ^{12}C(α,α)^{12}C data fixes $S(0.3)$ only within a wide range [2]. It is therefore appropriate to perform a new $\mathcal{K}$-matrix analysis with the further constraint of simultaneously fitting the spectrum of α particles following ^{16}N β decay.

The required fit can be performed by applying Eqs. (2.10)–(2.13) for the $E1$ capture and using a three-level approximation. The first level is at $E_1 = -0.0451$ MeV (the energy of the 1^- bound state of ^{16}O), the second level at E_2 corresponds to the broad 1^- resonance at $E \simeq 2.45$ MeV ($E_x \simeq 9.61$ MeV), while E_3 is associated with background contributions accounting for levels at higher excitation.

The ground state of ^{16}N is 2^-, while ^{12}C and the α particle are both 0^+. Accordingly, if only allowed Gamow-Teller transitions are considered, the corresponding α particles are emitted by ^{16}O* in 1^- and 3^- states. According to Barker and Warburton [6], the parametrized form for the α particle spectrum in the 1^- ^{12}C+α channel has a form corresponding to Eq. (2.12) in which appropriate feeding factors for each energy level are substituted for the factors related to the entrance channel c. With the index α being used to characterize the 1^- ^{12}C+α exit channel, this gives for the number of α particles per unit energy interval and with $l = 1$ angular momentum

$$N_{1\alpha}(E) = f_\beta(E)\left|\sum_{\lambda\mu} B_\lambda A_{\lambda\mu}g_{\alpha\mu}p_{1\alpha}\right|^2 \tag{3.2a}$$

$$= \tfrac{1}{2}f_\beta(E)\left|\sum_{\lambda\mu} B_\lambda A_{\lambda\mu}\Gamma_{\alpha\mu}^{1/2}\right|^2\ , \tag{3.2b}$$

where

$$f_\beta(E) = f(W_0, 8) \tag{3.3}$$

is the integrated Fermi function [7] with $Z = 8$ and $W_0 = (3.768 - E)/m_e$, $E \le E_{\max} = 3.257$ MeV, while the B_λ are feeding factors proportional to the Gamow-Teller matrix elements between the initial and final hadronic states.

In Eqs. (3.2) the elements of the level matrix A are implicitly defined by Eq. (2.9) and here we have

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - i(p_{1\alpha}^2 g_{\alpha\lambda}g_{\alpha\mu} + p_{1\gamma}^2 g_{\gamma\lambda}g_{\gamma\mu})\ . \tag{3.4}$$

We drop the index $l = 1$ when no confusion can arise. In Eq. (3.4) we can neglect the contribution from the γ channel to the last term and write

$$(A^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - ip_{1\alpha}^2 g_{\alpha\lambda}g_{\alpha\mu} \tag{3.5}$$

since, from Table I in Ref. [2], the neglected terms are about 6 orders of magnitude smaller than those associated with the α channel.

The inversion of A^{-1} is easily performed. Defining

$$D = (E_1 - E)(E_2 - E)(E_3 - E)(1 - ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha})\ , \tag{3.6}$$

with

$$\mathcal{K}_{1\alpha\alpha} = \sum_{\lambda=1}^{3} g_{\alpha\lambda}^2/(E_\lambda - E)\ , \tag{3.7}$$

we obtain

$$A_{\lambda\mu}=\frac{\delta_{\lambda\mu}}{E_\lambda-E}\frac{1}{1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha}}+\mathcal{A}_{\lambda\mu}\ , \tag{3.8}$$

with

$$\begin{aligned} D\mathcal{A}_{11}&=-ip_{1\alpha}^2[g_{\alpha2}^2(E_3-E)+g_{\alpha3}^2(E_2-E)]\ ,\\ D\mathcal{A}_{12}&=-ip_{1\alpha}^2g_{\alpha1}g_{\alpha2}(E_3-E)\ , \end{aligned} \tag{3.9}$$

and all the other $\mathcal{A}_{\lambda\mu}$ being obtained by circular permutation. Hence

$$\sum_\mu \mathcal{A}_{\lambda\mu}g_{\alpha\mu}=0 \tag{3.10}$$

and, instead of Eq. (3.2a), with the approximation (3.5), we simply have

$$N_{1\alpha}(E)=f_\beta(E)p_{1\alpha}^2(E)\left|\frac{B_1g_{\alpha1}/(E_1-E)+B_2g_{\alpha2}/(E_2-E)+B_3g_{\alpha3}/(E_3-E)}{1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha}}\right|^2\ , \tag{3.11}$$

a formula similar to the R-matrix expression used by Barker in his early paper [5].

In Eq. (3.11), B_2 and B_3 are free parameters obtained from fitting the energy spectrum of emitted α particles. However, B_1 can be obtained from the β-delayed γ-ray intensity from the E_1 bound state, since when the total lepton energy is larger than 3.257 MeV, E is negative, the α channel is closed for the decay of $^{16}O^*(1^-)$, and only the γ channel is open. With $Q=7.1616$ MeV and

$$p_{1\gamma}^2=[(Q+E)/\hbar c]^3,\quad \Gamma_{\gamma\mu}=2p_{1\gamma}^2g_{\gamma\mu}^2\ , \tag{3.12a}$$

$$(A^{-1})_{\lambda\mu}=(E_\lambda-E)\delta_{\lambda\mu}-ip_{1\gamma}^2g_{\gamma\lambda}g_{\gamma\mu}\ , \tag{3.12b}$$

the γ spectrum is given by

$$N_{1\gamma}(E)=\tfrac{1}{2}f_\beta(E)\left|\sum_{\lambda\mu}B_\lambda A_{\lambda\mu}\Gamma_{\gamma\mu}^{1/2}\right|^2 . \tag{3.13}$$

Since the γ widths are very small, so is $N_{1\gamma}(E)$, except when $E\simeq E_1$. Hence, we are justified in using a one-level approximation to Eq. (3.13). With $A_{11}=1/(E_1-E-ip_{1\gamma}^2g_{\gamma1}^2)$ we have

$$N_{1\gamma}(E)=\tfrac{1}{2}f_\beta(E)B_1^2\frac{\Gamma_{\gamma1}}{(E-E_1)^2+\Gamma_{\gamma1}^2/4} . \tag{3.14}$$

To a very good approximation, because $\Gamma_{\gamma1}(E_1)$ is only 55 meV, this gives for the total number of γ's emitted

$$N_{1\gamma}=\int_{-Q}^{0}N_{1\gamma}(E)dE=\pi B_1^2f_{\beta1}\ , \tag{3.15}$$

with $f_{\beta1}=f_\beta(E_1)=f(7.462,\ 8)$, and hence

$$B_1^2=\frac{N_{1\gamma}}{\pi f_{\beta1}} . \tag{3.16}$$

Because E_2 is a broad resonance, a similar evaluation does not apply to B_2. We note, however, that $N_{1\alpha}/\pi f(2.581,\ 8)$ obtained from the data has the correct order of magnitude and hence is a good starting value for B_2 in the search for the best fit.

IV. FITTING σ_{E1}, δ_1, AND $N_{1\alpha}$ TO THE DATA

The parametrized expressions for the $E1$ capture cross section and the δ_1 phase shift to be fitted to data are the same as those in Ref. [2]. With [8]

$$\mathcal{K}_{1\alpha\alpha}=\frac{g_{\alpha1}^2}{E_1-E}+\frac{g_{\alpha2}^2}{E_2-E}+\frac{g_{\alpha3}^2}{E_3-E}+b_{\alpha\alpha}\ , \tag{4.1}$$

$$\mathcal{K}_{1\gamma\alpha}=\frac{g_{\gamma1}g_{\alpha1}}{E_1-E}+\frac{g_{\gamma2}g_{\alpha2}}{E_2-E}+\frac{g_{\gamma3}g_{\alpha3}}{E_3-E}+b_{\gamma\alpha}\ , \tag{4.2}$$

they read

$$\sigma_{E1}=\frac{12\pi}{k^2}p_{1\alpha}^2p_{1\gamma}^2|\mathcal{K}_{1\gamma\alpha}/(1-ip_{1\alpha}^2\mathcal{K}_{1\alpha\alpha})|^2\ , \tag{4.3}$$

$$\delta_1=\arctan(p_{1\alpha}^2\mathcal{K}_{1\alpha\alpha}) . \tag{4.4}$$

Here, as in Ref. [2], a single background pole term in $\mathcal{K}_{1\alpha\alpha}$ does not lead to a sufficiently good fit of δ_1. Rather than introducing as in Ref. [9] a second background pole term with a very large pole energy, we simply add a constant term $b_{\alpha\alpha}$ as we did in Ref. [2]. Similarly a constant $b_{\gamma\alpha}$ is added to $\mathcal{K}_{1\gamma\alpha}$. These background terms are introduced as a low-energy approximation of the contributions from distant levels E_λ ($\lambda>2$). In order to constrain as much as possible the background parameters, we have, as in Ref. [2], used the presently available data over the widest possible energy range, namely, up to $E=4.9$ MeV for the phase shift δ_1, $E=2.9$ MeV for the capture cross section σ_{E1}, and $E=2.7$ MeV ($E_\alpha=3.6$) for the α spectrum.

As in Ref. [2], δ_1 is fitted simultaneously to three sets of data, [9] while here, for illustration, σ_{E1} is fitted only to the Kremer *et al.* [10] data. For the experimental α spectrum, we use as in Ref. [12], the data of Hättig *et al.* [11] as obtained by Barker [4,13], with counts corresponding to the $l=3$ α channel subtracted from the total α spectrum. This reduces the total number of counts from $N_\alpha=3.24\times10^7$ to $N_{1\alpha}=3.15\times10^7$, where the latter number is to be used in the evaluation of the feeding factor B_1 as given by Eq. (3.16). This is accomplished using

$$N_{1\gamma}=N_{1\alpha}Y_{1\gamma}(E_1)/Y_{1\alpha}(E_2)\ , \tag{4.5}$$

where the branching ratios are [14] $Y_{1\gamma}(E_1)=0.048\pm0.004$, $Y_{1\alpha}(E_2)=(1.20\pm0.05)\times10^{-5}$, and thus we obtain

TABLE I. Parameter values for the best fits with $g_{\gamma 3}, b_{\gamma\alpha}$ as free parameters (second column) and with $g_{\gamma 3}=b_{\gamma\alpha}=0$ (third column). The numbers in parentheses are fixed parameters and have been obtained from earlier work (see Ref. [2]) and Eq. (4.6). To give the reduced with amplitudes their usual dimensions, they have been multiplied by $a^{-3/2}$, with $a=5.46$ fm as in Ref. [2].

E_1(MeV)	(-0.0451)	(-0.0451)
$g_{\alpha 1}a^{-3/2}$(MeV$^{1/2}$)	-5.34	-5.89
$g_{\gamma 1}a^{-3/2}$(MeV$^{1/2}$)	(1.897×10^{-3})	(1.897×10^{-3})
B_1	(6804)	(6804)
E_2(MeV)	2.452	2.453
$g_{\alpha 2}a^{-3/2}$(MeV$^{1/2}$)	7.02	6.97
$g_{\gamma 2}a^{-3/2}$(MeV$^{1/2}$)	0.659×10^{-3}	0.632×10^{-3}
B_2	-2385	-2365
E_3(MeV)	(7.000)	(7.000)
$g_{\alpha 3}a^{-3/2}$(MeV$^{1/2}$)	$12.00i$	$12.04i$
$g_{\gamma 3}a^{-3/2}$(MeV$^{1/2}$)	$-2.66\times 10^{-3}i$	—
B_3	$4028i$	5116
$b_{\alpha\alpha}a^{-3}$	70.83	72.12
$b_{\gamma\alpha}a^{-3}$	-5.71×10^{-3}	—
$\Gamma_{\gamma 1}$(MeV)	(55×10^{-9})	(55×10^{-9})
$\Gamma_{\alpha 2}$(MeV)	0.467	0.461
$\Gamma_{\gamma 2}$(MeV)	16.4×10^{-9}	15.0×10^{-9}
$S_{E1}(0.3)$(MeV b) at $\chi^2_{\min}$	0.043	0.055
$S_{E1}(0.3)$(MeV b) range	0.027–0.063	0.038–0.074

$$B_1=6804\pm 635\ . \tag{4.6}$$

Other fixed parameters are E_1, E_3, $g_{\gamma 1}$, as in Ref. [2].

We fitted simultaneously σ_{E1}, δ_1, and $N_{1\alpha}$ as given by Eqs. (4.3), (4.4), and (3.11) to the three sets of data. We define an effective χ^2 by [2]

$$\chi^2_{\rm eff}=\tfrac{1}{3}(\chi^2_\gamma+\chi^2_\delta+\chi^2_\beta)\ , \tag{4.7}$$

where $\chi^2_\gamma, \chi^2_\delta, \chi^2_\beta$ are χ^2 per data point for the $E1$ capture cross section, the $l=1$ phase shift and the $l=1$ α spectrum, respectively. Our best fit corresponds to $\chi^2_{\min}$, the minimum of $\chi^2_{\rm eff}$. The numerical results for the best fit are given in Table I, while Fig. 1 gives $\chi^2_{\rm eff}$ versus $S_{E1}(0.3)$ when this quantity is used as a free parameter instead of $g_{\alpha 1}$, as discussed in Ref. [2]. The $\chi^2_{\rm eff}$ is minimized at $S_{E1}(0.3)=0.043$ MeV b. From Fig. 1 we also see that the range of acceptable values for $S_{E1}(0.3)$, defined as those whose $\chi^2_{\rm eff}$ does not exceed $\chi^2_{\min}$ by more than 30% is 0.027–0.063 MeV b. Ten free parameters and 106 data points are involved in this fit.

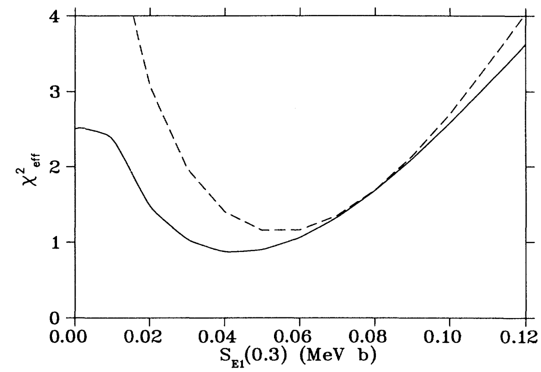

FIG. 1. The effective χ^2 vs $S_{E1}(0.3)$. The two curves correspond to fits without constraint on the free parameters (solid line), and with $g_{\gamma 3}=b_{\gamma\alpha}=0$ (dashed line).

Fits with fewer free parameters have also been obtained by introducing constraints corresponding to those used by Barker [5,15] in his R-matrix fits. They are $g_{\gamma 3}=b_{\gamma\alpha}=0$, and $B_3=0$. Complete results with $g_{\gamma 3}=b_{\gamma\alpha}=0$ are also given in Table I, while for $B_3=0$, only the main results are reported in Table II, together with those obtained in Ref. [2]. As seen from Fig. 1, the constraints $g_{\gamma 3}=b_{\gamma\alpha}=0$ shift the range of acceptable $S_{E1}(0.3)$ values slightly higher, while the allowed range (0.038–0.074 MeV b) is not reduced. These constraints on the γ channel parameters mainly increase χ^2_γ, from 0.88 to 1.48. This suggests that the parametrization of σ_{E1} is then less reliable, and hence, so should be the astrophysical factor $S_{E1}(0.3)$.

The constraint $B_3=0$ increases χ^2_β by more than a factor of 10 as seen in Table II, an unacceptably large increase. With both constraints, $\chi^2_\gamma=4.92$ is also unacceptable.

V. DISCUSSION OF THE RESULTS AND CONCLUSIONS

The overall situation could be much improved by better data for σ_{E1} in the 1–3-MeV energy range along with data points below and above this range, but this does not appear likely in the near future. However, a new measurement of the α spectrum extending below and above the energy range of the Hättig *et al.* [11] data, $E=1.5$–2.7 MeV, appears more feasible and has recent-

TABLE II. Values of χ^2 for simultaneous fits to the three sets of data, with different constraints, and the corresponding range of allowed values for $S_{E1}(0.3)$. For the sake of comparison, also given in the first line are the results obtained in Ref. [2], where the α spectrum from ^{16}N β decay was not fitted.

Constraints	χ^2_γ	χ^2_δ	χ^2_β	$\chi^2_{\min}$	$S_{E1}(0.3)$ range
None (from Ref. [2])	0.92	1.34	—	1.13	0.00–0.16
None	0.88	1.51	0.20	0.86	0.027–0.063
$g_{\gamma 3}=b_{\gamma\alpha}=0$	1.48	1.57	0.35	1.13	0.038–0.074
$B_3=0$	0.99	1.91	2.17	1.69	
$g_{\gamma 3}=b_{\gamma\alpha}=B_3=0$	4.92	2.25	1.99	3.05	

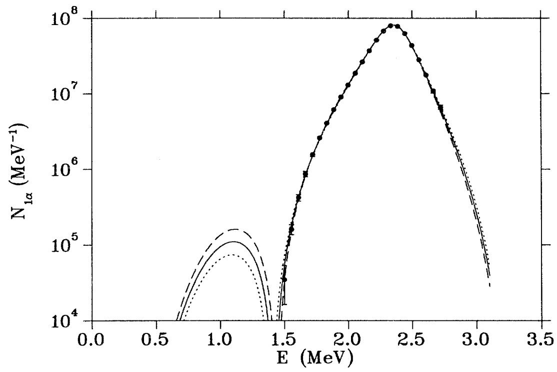

FIG. 2. The parametrized α spectrum vs center-of-mass energy. The solid line gives the α spectrum when σ_{E1}, δ_1, and $N_{1\alpha}$ are fitted simultaneously without constraint. The others give the α spectrum when $S_{E1}(0.3)$ is given the extreme values of its allowed range, i.e., $S_{E1}(0.3)=0.027$ MeV b for the dotted line, and $S_{E1}(0.3)=0.063$ MeV b for the dashed line. All three spectra vanish at $E \approx 1.4$ MeV.

ly been proposed at TRIUMF [16]. In this context, the parametrized spectra in Fig. 2 are shown well above and below the range of the present data. One spectrum corresponds to the best fit with $S_{E1}=0.043$ MeV b, while the others correspond to $S_{E1}(0.3)=0.027$ and 0.063 MeV b, respectively; i.e., the end points of the range of acceptable values of $S_{E1}(0.3)$ deduced from Fig. 1. The computed spectra have two maxima because the numerator in Eq. (3.11) vanishes at $E \simeq 1.4$ MeV. The main parameters involved in these fits are reported in Table III. The parameters $g_{\alpha 2}$ and B_2 are nearly the same for the three fits. In contrast, from the fit with $S_{E1}(0.3)=0.027$ MeV b to that with $S_{E1}(0.3)=0.063$ MeV b, $g_{\alpha 1}^2$ increases by as much as a factor of 2. But this large variation is partially compensated by an important increase in $|B_3|$. Nevertheless, with $S_{E1}(0.3)=0.063$ MeV b, the number of counts at the energy of the first maximum ($\simeq 1.1$ MeV) increases by 45% relative to that when $S_{E1}(0.3)=0.043$ MeV b. With $S_{E1}(0.3)=0.027$ MeV b, it is lowered by 33%. Even a 10% error in the number of counts at energies near 1.1 MeV could result in an important reduction in the range of acceptable values for $g_{\alpha 1}$ and hence $S_{E1}(0.3)$. This analysis of our results at low energy agrees with Baye and Descouvemont [17]. They emphasized the strong correlation between the R-matrix reduced width $\gamma_{\alpha 1}^2$ of the E_1 bound state and the α spectrum in the 0.8–1.2-MeV energy range.

In all of our fits we have kept B_1 fixed, although we have estimated that an uncertainty close to 10% must be attached to the numerical value $B_1=6804$. Since B_1 and $g_{\alpha 1}$ are strongly correlated in the energy spectrum (3.11), it is also desirable to improve the precision of the numerical value of B_1. According to Eqs. (3.16) and (4.5), this requires a better determination of the branching ratios $Y_{1\gamma}(E_1)$ and $Y_{1\alpha}(E_2)$, if no direct measurement is made of the β-delayed γ-ray intensity from the E_1 bound state.

Turning to the high-energy part of the α spectrum, new data extending to energies higher than $E=2.7$ MeV should better constrain the B_3 feeding factor. As seen earlier, this in turn could better constrain $g_{\alpha 1}$, since the term in B_3 is not negligible at 1.1 MeV. At $E=3$ MeV, with $S_{E1}(0.3)=0.063$ MeV b, the parametrized spectrum is reduced by 22% relative to that with $S_{E1}(0.3)=0.043$ MeV b. With $S_{E1}=0.027$ MeV b, it increases by 24%. These variations are significant, but as seen in Fig. 2, at $E=3$ MeV, the spectrum varies rapidly with E. Hence, extending the range of the data up to 3 MeV might not be as useful in constraining the parametrization as data taken near 1 MeV.

When our results are compared with those of Barker, [4,15,18] it appears from Table II that the constraints we have applied ($g_{\gamma 3}=b_{\gamma\alpha}=0$ and $B_3=0$) are more stringent than his. This may be related to the fact that when, e.g., we introduce the $B_3=0$ constraint in the α spectrum, the corresponding term disappears completely from the parametrization. This is not the case in Barker's parametrization. An R-matrix many-level fit is complicated by the fact that any physical constraint associated with a level (energy, reduced width amplitude, or feeding factor) must be applied only when the boundary condition constant is chosen to have a vanishing energy shift at that same level. In this context, it is obvious that the absence of boundary condition constants in the $\mathcal{K}$-matrix parametrization greatly simplifies the fitting procedure. A single fit contains all the physical parameters, resonance energies, reduced width amplitudes, and feeding factors.

We can conclude that the parametrized spectrum (3.11) of the α particles from ^{16}N β decay, with its three feeding factors, allows a very good fit to the present data. This

TABLE III. Variation of several key parameters for the fits with $S_{E1}(0.3)$ within its allowed range. Note the strong correlation between $S_{E1}(0.3)$ and $g_{\alpha 1}$, B_3, and the α spectrum at 1.1 and 3.0 MeV.

$S_{E1}(0.3)$ (MeV b)	0.063	0.043	0.027
$g_{\alpha 1}a^{-3/2}$(MeV$^{1/2}$)	-6.33	-5.34	-4.43
$g_{\alpha 2}a^{-3/2}$(MeV$^{1/2}$)	6.96	7.02	7.09
B_2	-2370	-2385	-2399
$g_{\alpha 3}a^{-3/2}$(MeV$^{1/2}$)	$12.39i$	$12.00i$	$11.69i$
B_3	$5724i$	$4028i$	$2324i$
$N_{1\alpha}$ at 1.1 MeV	+45%	Reference	−33%
$N_{1\alpha}$ at 3 MeV	−22%	Reference	+24%

introduces a very stringent constraint on the parameters of the $\alpha+{}^{12}$C channel involved in the $E1$ capture cross section. The value we have obtained for the $E1$ part of the astrophysical factor is

$$S_{E1}(0.3)=0.043^{+0.020}_{-0.016}\,\text{MeV b} . \tag{5.1}$$

If we take for the $E2$ part of the S factor the result obtained in Ref. [2],

$$S_{E2}(0.3)=0.007^{+0.024}_{-0.005}\,\text{MeV b} , \tag{5.2}$$

we obtain for the total S factor

$$S(0.3)=0.05^{+0.03}_{-0.02}\,\text{MeV b} . \tag{5.3}$$

However, the degree of confidence one can have in the results (5.1) and (5.3) is limited by the fact that we have not included in our fit the f-wave part of the α spectrum. In addition, since the original experiment [11] did not attempt to accurately extract that α spectrum, there are potential uncertainties associated with the detector response and resolution [19].

We have given arguments justifying the importance of remeasuring the α spectrum of ^{16}N β decay over a wider energy range, and also of obtaining better branching ratios for the β decay to the 7.12-MeV bound state and the 9.61-MeV resonance of ^{16}O.

ACKNOWLEDGMENTS

The authors are grateful to R.E. Azuma, F.C. Barker, and C.A. Barnes for useful discussions. B.W.F. acknowledges support from the A.P. Sloan Research Foundation. This work was supported in part by National Science Foundation Grants No. PHY88-17296 and No. PHY90-13248.

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