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Quadrupole collectivity of summed magnetic dipole orbital strength using schematic interactions

L. Zamick and D. C. Zheng*

Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855

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We perform shell-model calculations in ^{22}Ne for the summed magnetic dipole orbital strength and spin strength with a schematic interaction consisting of single-particle splitting, an isospin-conserving pairing interaction, and a quadrupole-quadrupole interaction. We consider four channels: orbital excitations to $J=1^+$ $T=1$ states, and then to $J=1^+$ $T=2$ states; spin excitations to $J=1^+$ $T=1$ states, and then to $J=1^+$ $T=2$ states. For the orbital $T \rightarrow T$ channel, the summed $M1$ strength varies quadratically with the size of the quadrupole interaction x for small values of this parameter when degenerate single-particle energies are used. Using realistic single-particle splitting does not destroy the quadrupole collectivity. The other three channels respond in different manners. We also consider linear-energy-weighted sum rules for which analytic results are obtained. The relevance of this work to experiments on orbital strengths and their relations to quadrupole excitations is discussed.

I. INTRODUCTION

Interest in magnetic dipole orbital modes became very intense after the discovery of the "scissors-mode" excitations in ^{156}Gd by Bohle *et al.* [1]. There was of course considerable theoretical work previous to this by Lo Iudice and Palumbo [2], Iachello [3], and Dieperink [4], but only when the orbital strength was actually seen experimentally did the field really take off.

In 1981 one of the authors of this work (L.Z.) in collaboration with Halemane and Abbas [5] was also interested in looking separately at the spin and orbital strength distributions in nuclei and addressed the specific problems of the linear-energy-weighted sum rules when a spin-dependent delta interaction was used to induce configuration mixing.

In this work we return to the problem of orbital strength. Instead of considering the detailed spectrum of 1^+ states, we will mainly focus on the *summed* orbital and spin $M1$ strengths. We will try to make our work more relevant to subsequent developments, not only the discovery of the scissors modes, but also their relations to electric quadrupole excitations. In this regard we consider the works of Ziegler *et al.* [6] and Rangacharyulu *et al.* [7], who have claimed that in even-even rare-earth nuclei, there is a strong correlation between the summed *orbital* magnetic dipole strength below 4 MeV, often called scissors-mode excitations, and the $E2$ transition to the first excited 2^+ state. Since the $0_1^+ \rightarrow 2_1^+$ transition rate is known to be proportional to the square of the deformation parameter (δ^2), they feel that this is also true for the summed orbital $M1$ strength. This result is certainly not *a priori* obvious—one does not see it in textbooks.

We feel that the definition of a scissors mode is somewhat in a state of flux, evolving in time and being sharpened up only as experiments became more precise and the number of nuclei considered increases. The simplest definition would be that given by the original picture of Lo Iudice and Palumbo [2] of an out-of-phase rotation of the deformed symmetry axis of neutrons against that of the protons. However, since the original formulation leads to much too strong $B(M1)$, one can go to the next stage of the interacting boson approximation IBA-2 where only the valence nucleons participate [3,4]. This picture gives a more or less correct $B(M1)$ rate at the right energy, e.g., $B(M1) \simeq 2\mu_N^2$ at 3 MeV in ^{156}Gd. However, it is noted in the above recent experimental analysis [6,7] that the IBA-2 predicts a different dependence of $B(M1)_{\text{orbital}}$ and $B(E2)_{0_1^+ \rightarrow 2_1^+}$ on the numbers of valence protons and neutrons in the open sell (N_ν and N_π):

$$B(E2) = C(e_\nu N_\nu + e_\pi N_\pi)^2 \frac{2N+3}{3} , \tag{1.1}$$

$$B(M1)_{\text{orbital}} = \frac{3}{4\pi}(g_\nu - g_\pi)^2 \frac{8N_\nu N_\pi}{2N-1}(\mu_N^2) , \tag{1.2}$$

and therefore the ratio of $B(M1)_{\text{orbital}}$ to $B(E2)$ is not independent of N_ν and N_π, in contradiction with experimental findings [6,7]. Also, the IBA-2 predicts only one state at an energy of ~ 3 MeV for ^{156}Gd. A finer-resolution (γ,γ') experiment by Berg and Kneissl [11] shows that there is considerable intermediate structure in the broad (e,e') peak, which means there are several 1^+ states close by. It is reasonable then to identify the scissors mode, not with one state, but as sum over several states which are localized within a small band of energies.

Does this mean that all the orbital strength will thus be accounted for? The shell model provides a simple answer: No. In a comparison of the Nilsson and shell models by Liu and Zamick [8], it was shown that for ^{22}Ne most orbital strength was located in a few low-lying states. These states have a significant orbital strength so that it can be seen experimentally. They also found that some of the orbital strength was at higher energies, but completely fragmented, so that any one state had very little strength.

Zamick [9] noted some connections between the shell-model calculations and the more collective ones. In the single j shell of protons and neutrons, the $B(M1)$ rate is isovector, e.g., proportional to $(g_{j\pi}-g_{j\nu})^2$. Also, the shell model has "antisymmetric" states in the protons and neutrons, just as IBA-2 [3,4]. This is especially apparent in ^{48}Ti where the wave functions are either symmetric or antisymmetric under the interchange of protons and neutron holes and vice versa. Chaves and Poves [10] noted that the s-d shell offers good examples of scissors-mode excitations, especially in ^{20}Ne and ^{22}Ne.

It seems then to make the most sense in defining operationally the scissors modes to include a few low-lying 1^+ states which exhaust most orbital $M1$ strength from the ground state and can be reached by the $M1$ excitations of predominantly orbital isovector characteristic from the ground state. These do not include the 1^+ states in the higher-energy region where the fragmentation is very strong (although one should always be aware of the presence of this higher-energy region). Of course, our definition would be of no interest if there were no significant chunks of low-lying isovector orbital strength. There are such low-lying states with significant orbital isovector strength supported by experiments and many theories, including the shell model that is here being used.

In order not to lose sight of the collective aspects, we will be using schematic interactions, just as has been done in the past for studying $E2$ excitations. It is important to see if there is a microscopic justification for the picturesque ideas which the collective models yield.

Before going on the calculations, we should mention a surprisingly simple problem that occurs when considering the familiar $B(E2)$'s in deformed nuclei. What does the statement "$B(E2)$ is proportional to δ^2" mean? Does it mean that a $B(E2)$ vanished at zero deformation? In fact, there are strong $E2$ transitions to the first excited 2^+ states even in vibrational nuclei. So somewhere this formula must lose its meaning. In microscopic models, instead of using δ as a deformation parameter, we will use a parameter in our Hamiltonian—the size of the quadrupole-quadrupole interaction as an indication of the collectivity.

We will do the calculations for ^{22}Ne, which is certainly not a rare-earth nucleus. It is one, however, where a reasonably good shell-model calculation can be performed, and as we will show, all the necessary features are present to display the tendencies both for and against the scissors-mode collectivity. A special feature of light nuclei is that there are two orbital modes, one in which the isospin of the 1^+ states is the same as that of the ground state and the other in which the isospin is one unit higher. We shall see that the response of these two modes to the quadrupole collectivity is quite different. For completeness we will consider the spin excitations as well.

II. SUMMED ORBITAL AND SPIN $M1$ STRENGTHS IN ^{22}Ne WITH SCHEMATIC INTERACTIONS

In this section we evaluate the summed orbital and spin $M1$ strengths in ^{22}Ne as a function of certain parameters of a schematic interaction, the two-body part of which is an isospin-conserving pairing (V_p) plus quadrupole-quadrupole (V_q) interaction. The interaction can be written as

$$V=V_{\text{single part}}+V_p+xV_q \ . \tag{2.1}$$

The first term in Eq. (2.1) is simply the single-particle splitting in a major shell. We shall consider two cases. First, we take the single-particle energies to be degenerate. This should enhance the quadrupole collectivity. Then we will use "experimental" single-particle splitting: $\epsilon_{d_{5/2}}=0, \epsilon_{s_{1/2}}=0.87$ MeV, and $\epsilon_{d_{3/2}}=5.08$ MeV. It will be of interest to see whether or not the single-particle term destroys the quadrupole collectivity.

The isospin-conserving pairing interaction V_p is defined by its two-body matrix elements:

$$\langle (jj)^{J,T}|V_p|(j'j')^{J,T}\rangle = -G(-1)^{l-l'}\sqrt{(2j+1)(2j'+1)}\delta_{J,0}\delta_{T,1} \ . \tag{2.2}$$

We here use $G=5/A$(MeV), which is only half the value as given in Ref. [12]. This is justified by noting that the isospin-conserving pairing acts not only between like nucleons, but also between proton-neutron pairs, and for ^{22}Ne, which has 4 valence neutrons and 2 valence protons, the number of neutron-neutron and proton-proton pairs is 7, nearly half of the number of nucleon-nucleon (pp,pn,nn) pairs, which is 15.

The quadrupole-quadrupole (QQ) interaction is

$$V_q=-\chi\sum_{i>j}\sum_{\mu}(-1)^{\mu}(r^2Y_{2,\mu})_i(r^2Y_{2,-\mu})_j \ , \tag{2.3}$$

with the parameter χ given by [12]

$$\chi=-\frac{240}{A^{5/3}b^4} \ , \tag{2.4}$$

where the oscillator length b is defined by the equation

$$\hbar\omega(\text{MeV})=\frac{\hbar^2}{mb^2}=\frac{45}{A^{1/3}}-\frac{25}{A^{2/3}} \ . \tag{2.5}$$

For $A=22$, $\hbar\omega=12.88$ MeV, $b=1.795$ fm, and $\chi=-0.1324$ MeV/fm^4. We may change the strength of the QQ interaction by varying x. For $x=1$ our pairing plus QQ interaction yields the normalized, antisymmetrized two-body matrix elements $(jj|V_q|jj)_{J,T}=-2.912$ MeV for $J=0$, $T=1$ and -1.017 MeV for $J=1$, $T=0$, where $j=d_{5/2}$. The corresponding matrix elements for a universal s-d (USD) phenomenological interaction of Wildenthal [13] are -2.655 and -1.537 MeV.

We shall, with the pairing strength G fixed at $5/A$(MeV), study the summed strength as a function of x from $x=0$ (pure pairing) to $x=\infty$. Presumably, as we increase x, we go toward the rotational limit. At the other limit $x=0$, we should remember that the properties of an isospin-conserving pairing interaction are not the same as those of a pairing interaction which acts only between like particles.

In a light nucleus such as ^{22}Ne, we can have both $J=0^+T\rightarrow J=1^+T$ and $J=0^+T\rightarrow J=1^+(T+1)$ transitions. It will be shown that these two channels respond differently to the change of the schematic interaction parameters. We therefore will consider them separately.

We show our results in Table I and in Figs. 1 and 2 for the summed orbital and spin strengths from the ground state of ^{22}Ne. The strengths for the $T=1 \rightarrow T=1$ channel and the $T=1 \rightarrow T=2$ channel are separated. The $B(E2)$ for the transition $0_1^+ \rightarrow 2_1^+$ is also listed in Table I and Fig. 3. For the fixed pairing strength, we give the results for various choices of the QQ strength parameter x. Just to gauge where we are, we also give results for the Wildenthal interaction (denoted by "Wldnthl" in Table I) [13], which fits most of the empirical data in the *s-d* shell very well.

We now discuss four different cases in the following.

A. Orbital strength, $T=1 \rightarrow T=1$

We consider first the calculations for the $T=1 \rightarrow T=1$ channel with degenerate (deg.) in Table I single-particle energies. For a pure isospin-conserving pairing interaction ($x=0$), the summed orbital strength, as seen in column A of Table I, is *zero*. Of course, since every term in the sum is non-negative, each individual orbital transition rate must be zero for this case. This corresponds to zero deformation. We now turn on the quadrupole interaction xV_q. As we increase x, we see that for small x the summed orbital strength is indeed *quadratic* in x, as we would expect if $B(M1)_{\text{orbital}}$ should be quadratic in the deformation parameter δ. However, when x is increased further, a *saturation* sets in and the summed strength flattens out to reach an asymptotic value of $0.981\mu_N^2$ for a pure QQ interaction. Note that with the Wildenthal interaction [13], we obtain a summed strength of $0.531\mu_N^2$, not very different from the value of $0.610\mu_N^2$ for the schematic interaction with $x=1.0$.

We then come to column B of Table I where the summed orbital strength for experimental single-particle

TABLE I. Summed orbital and spin strengths (in μ_N^2) for the $M1$ transitions from the ground state of ^{22}Ne and the $B(E2)$ (in $e^2\,\text{fm}^4$) for the transition $0_1^+ \rightarrow 2_1^+$ using the schematic interaction ($V_p + xV_q$) with different choices of x. Degenerate (deg.) or experimental (expt.) single-particle energies are used.

QQ	$T=1 \rightarrow T=1$						$T=1 \rightarrow T=2$			
size	M1 Orbital		M1 Spin		$B(E2)$		M1 Orbital		M1 Spin	
x	Deg.	Exp.	Deg.	Exp.	Deg.	Exp.	Deg.	Exp.	Deg.	Exp.
0.0	0	0.094	0	5.032	233.7	89.02	0.283	0.218	3.937	4.942
0.1	0.021	0.101	0.006	4.803	272.2	167.5	0.273	0.201	3.010	4.444
0.2	0.085	0.133	0.020	4.615	308.8	215.6	0.260	0.192	2.145	3.858
0.3	0.177	0.187	0.034	4.435	338.8	249.1	0.253	0.188	1.463	3.246
0.4	0.275	0.254	0.041	4.235	360.2	275.5	0.251	0.189	0.994	2.675
0.5	0.361	0.325	0.042	4.008	374.3	297.0	0.250	0.192	0.692	2.181
0.6	0.434	0.393	0.039	3.761	383.5	314.2	0.251	0.197	0.498	1.778
0.7	0.492	0.454	0.035	3.507	389.5	327.9	0.253	0.203	0.371	1.457
0.8	0.539	0.506	0.031	3.258	393.5	338.6	0.255	0.209	0.284	1.205
0.9	0.578	0.550	0.028	3.021	396.4	347.1	0.256	0.215	0.224	1.006
1.0	0.610	0.587	0.024	2.800	398.4	353.8	0.258	0.220	0.180	0.849
1.1	0.637	0.618	0.022	2.596	399.9	359.2	0.259	0.225	0.148	0.723
1.2	0.659	0.644	0.019	2.409	401.0	363.7	0.260	0.230	0.124	0.621
1.3	0.678	0.667	0.017	2.239	401.9	367.3	0.262	0.234	0.105	0.539
1.4	0.695	0.686	0.015	2.084	402.6	370.4	0.263	0.237	0.090	0.470
1.5	0.710	0.703	0.014	1.942	403.2	373.0	0.263	0.240	0.078	0.414
∞	0.981	0.981	0	0	374.6	374.6	0.278	0.278	0	0
Wldnthl	0.531	0.562	0.940	3.877	337.3	307.5	0.388	0.292	0.164	0.709
Column	A	B	C	D	E	F	G	H	I	J

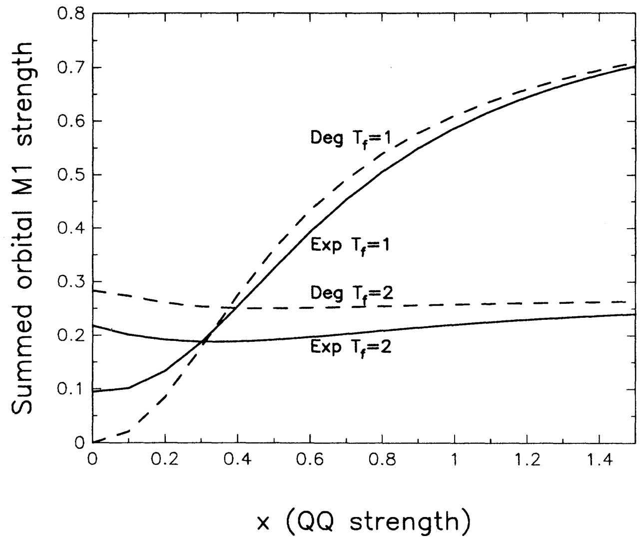

FIG. 1. Summed orbital $M1$ strength (in μ_N^2) in ^{22}Ne as a function of QQ strength x in the schematic interaction (V_p+xV_q). Degenerate (dashed lines) and experimental (solid lines) single-particle energies are used.

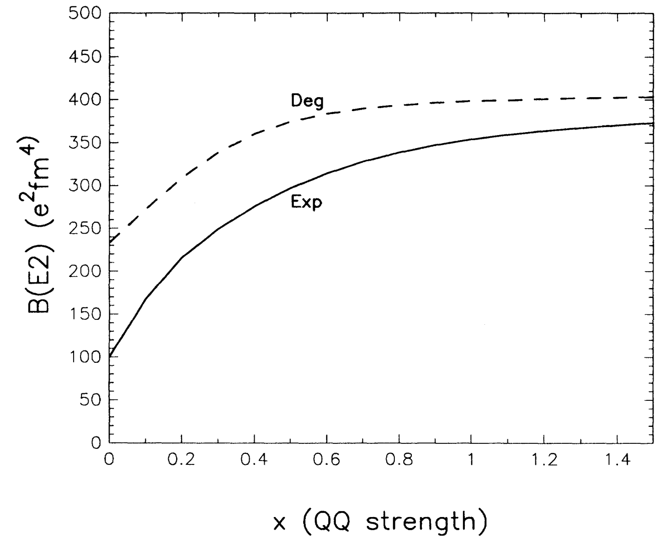

FIG. 3. $B(E2)_{0_1^+\to 2_1^+}$ in $e^2\,\text{fm}^4$ in ^{22}Ne as a function of QQ strength x in the schematic interaction (V_p+xV_q). Degenerate (dashed line) and experimental (solid line) single-particle energies are used.

energies is given. Here a very important question to consider is whether or not the simple picture of quadrupole collectivity for orbital $M1$ transitions gets destroyed when we remove the unrealistic constraint of degenerate single-particle energies. We note of course that there are differences in columns A and B. As we approach the pairing limit ($x\to 0$), the summed strength in column B does not approach zero, although the result is small:

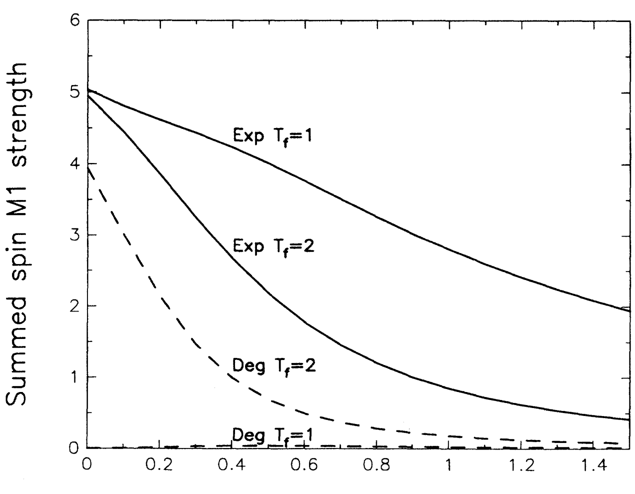

FIG. 2. Summed spin $M1$ strength (in μ_N^2) in ^{22}Ne as a function of QQ strength x in the schematic interaction (V_p+xV_q). Degenerate (dashed lines) and experimental (solid lines) single-particle energies are used.

$\sum B(M1)_{\text{orbital}}=0.094\mu_N^2$. However—and this is one of the most important points in this work—when we increase x toward realistic values, the results in columns A and B come close to each other. For example, for $x=0.5$, the corresponding values are $0.361\mu_N^2$ and $0.325\mu_N^2$; for $x=1.0$, they are $0.610\mu_N^2$ and $0.587\mu_N^2$; for $x=1.5$, they are $0.710\mu_N^2$ and $0.703\mu_N^2$. This means that for realistic values of x ($x\sim 1$) the quadrupole collectivity is sufficiently strong so that the single-particle energies do not play a significant role as far as the summed orbital strength is concerned, at least in the $J=0^+T\to J=1^+T$ channel.

However, as clearly seen from Fig. 3, the $B(E2)$ for the transition from the ground state to the first 2^+ state is *not quadratic* in the QQ strength x even the degenerate single-particle energies are used. In this case the value of $B(E2)_{0_1^+\to 2_1^+}$ for $x=0$ is not zero; it is rather 233.7 $e^2\,\text{fm}^4$. With some thought the nonzero value is not surprising. When the deformation becomes small, we approach the vibrational limit, and it is well known that the $B(E2)_{0_1\to s_1}$ is quite large in this limit. As we increase x, the value of $B(E2)$ increases, but not quadratically. Indeed, the curve $B(E2)$ vs x is concave down, whereas a quadratic dependence would give a concave-up curve.

As is seen in Fig. 1, whereas the curve for $\sum B(M1)_{\text{orbital}}T=1\to T=1$ starts out quadratically (concave up), beyond $x\sim 0.4$ it becomes concave down just as the one for $B(E2)$ vs x.

Note that the value of $B(E2)$ reaches a maximum somewhere beyond $x=1.5$, but it must then decrease slowly because, for $x\to\infty$, the $B(E2)$ is slightly smaller than that for $x=1.5$. The decrease may be an artifact of our model in the sense that we do not allow $\Delta N=2$ admixture.

We now demonstrate that the summed orbital $M1$ strength for the $T \rightarrow T$ channel is dominantly isovector. We give the summed orbital $M1$ strengths for the Wildenthal interaction with its self-consistent single-particle energies and for the schematic pairing plus quadrupole interaction ($x=1$) with experimental single-particle energies as follows:

Interaction			Wildenthal	Pairing + quadrupole
	$g_l(p)$	$g_l(n)$	$\sum B(M1)_{\text{orbital}, T\rightarrow T}$	$\sum B(M1)_{\text{orbital}, T\rightarrow T}$
Normal	1	0	0.562	0.587
Isovector	0.5	−0.5	0.539	0.575
Isoscalar	0.5	0.5	0.055	0.045

We see that despite the fact that the bare orbital $M1$ operator $\sum_i g_l(i) l(i)$ is half isoscalar and half isovector, the orbital strength is about 10 times larger than the corresponding isoscalar strength.

The reason for this is that configuration mixing brings the ground-state wave function of ^{22}Ne toward the LS limit with $L=0$ [14]. The isoscalar orbital operator is just $0.5L$, i.e., half the total orbital angular momentum operator. When acting on a state with orbital angular momentum $L=0$, the above operator yields zero.

We also note that the summed orbital $M1$ strength shown in Table I is exhausted by a few low-lying 1^+ states in all cases. In the top portion of Fig. 4, we show the orbital $M1$ strength distribution for the $T=1 \rightarrow T=1$ transitions using the Wildenthal interaction [13] with its self-consistent single-particle energies. In this case over 65% ($0.366\mu_N^2$) of the total ($0.562\mu_N^2$) orbital $M1$ strength goes to the lowest three 1^+, $T=1$ states (their excitation energies are 5.431, 6.660, and 8.584 MeV, respectively) and the rest strength is strongly fragmented over the other 348 states. Thus, in practice, only the lowest few states can be seen in the orbital $M1$ excitations.

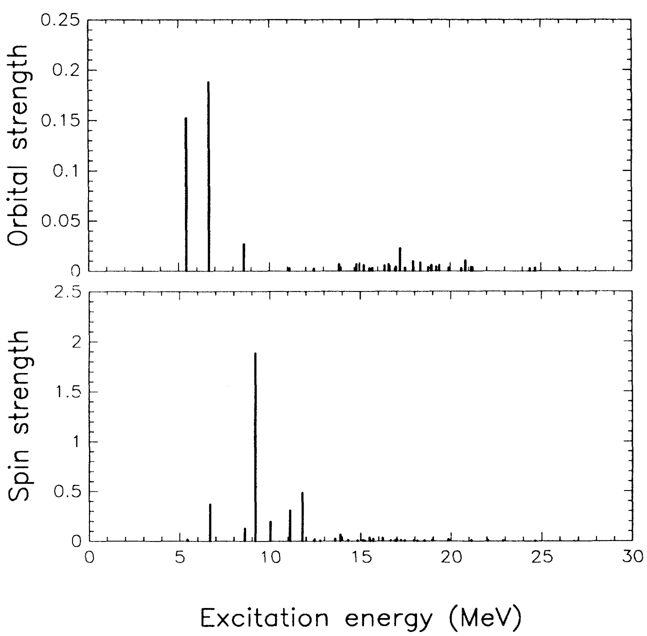

FIG. 4. Orbital (top) and spin (bottom) $M1$ strength distributions (in μ_N^2) for the transitions 0^+, $T=1 \rightarrow 1^+$, and $T=1$ in ^{22}Ne using the Wildenthal interaction with its self-consistent single-particle energies.

B. Orbital strength, $T=1 \rightarrow T=2$

As seen in column G of Table I (also Fig. 1), the $T=1 \rightarrow T=2$ orbital strength for the case of degenerate single-particle energies behaves in a completely different manner than that of the $T=1 \rightarrow T=1$ channel (column A). The summed strength is *not* zero in the pairing limit ($x \rightarrow 0$). From $x=0$ to 0.5 the strength decreases very slowly from $0.273\mu_N^2$ to $0.250\mu_N^2$. It increases after $x=0.6$ very slowly and approaches the value $0.277\mu_N^2$ for $x=\infty$. The summed orbital strength when the experimental single-particle energies are used is slightly smaller than in the degenerate case, but the results as a function of x are qualitatively the same.

Thus we see that the association of summed orbital $M1$ strength with deformation only holds for the lower isospin branch $T=1 \rightarrow T=1$. The upper branch $T=1 \rightarrow T=2$ scarcely responds to the onset of deformation. This is seen clearly in Fig. 1 where the curves for $T=1 \rightarrow T=2$ are quite flat as a function of x.

C. Spin strength, $T=1 \rightarrow T=1$

From column C of Table I, we see that the summed spin strength in the case of degenerate single-particle energies is zero for the case of a pure pairing interaction, and as does its orbital counterpart (column A), it increases quadratically in x for small x. But then it turns around and approaches zero as x becomes very large, i.e.; for a nearly pure QQ interaction. More generally, the summed spin strength would vanish for any spin-independent central interaction in the case of degenerate single-particle energies because of SU(4) symmetry. We see in Fig. 2 that for all values of x considered here, the summed spin strength is always *very small* when degenerate single-particle energies are used.

When we use experimental single-particle energies, the summed spin strength becomes very large, e.g., $5.032\mu_N^2$ for $x=0$. As we increase x, the strength decreases. It is $1.942\mu_N^2$ for $x=1.5$ and it approaches to zero as $x \rightarrow \infty$. Unfortunately, Fig. 2 does not extend enough in x to show this.

In contrast to the case of the orbital strength, the single-particle energies are *always* a major factor as far as the spin strength is concerned. In fact, in evaluating the linear-energy-weighted sum rule for $M1$ strength, Kurath [17] took only the one-body spin-orbit interaction into account—this gives the single-particle splitting between $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$; he did not take any two-body interaction into account.

In the bottom portion of Fig. 4, we show the spin $M1$ strength distribution for the $T=1\rightarrow T=1$ transitions using the Wildenthal interaction [13] with its self-consistent single-particle energies. This should be compared with top portion of Fig. 4, where we show the orbital $M1$ strength distribution. It is very interesting to see that the spin strengths to the lowest three 1^+, $T=1$ states (especially the first and third states) are relatively small, in contrast to the orbital case where the lowest three states exhaust over 65% of the total orbital $M1$ strength. In particular, the calculated orbital strength to the first 1^+ state is $0.152\mu_N^2$, much larger than the corresponding spin strength of $0.017\mu_N^2$.

D. Spin strength, $T=1\rightarrow T=2$

The $T=1\rightarrow T=2$ spin strength, in contrast to the $T=1\rightarrow T=1$ case, does not vanish for a pure pairing interaction. It is quite large even in the case of degenerate single-particle energies ($3.937\mu_N^2$ when $x=0$). That the $T\rightarrow T$ spin transition vanishes but the $T\rightarrow T+1$ does not for an isospin-conserving pairing interaction was also shown by Halse [15]. As we turn on the QQ interaction, this strength goes steadily toward zero and indeed vanishes when $x=\infty$. Again, this is a consequence of the SU(4) symmetry.

When the experimental single-particle energies are included, the summed spin strength increases somewhat, but the qualitative behavior (decrease of the summed spin strength with increasing x) is the same as in the degenerate case. We see that, in general, for the spin excitations there is a fairly complex interplay between the effects of the single-particle energies and two-body residual interactions.

Thus far, we have discussed mostly the summed $M1$ strength. Under what conditions, however, is the strength sufficiently concentrated so that the concept of a scissors mode is interesting? An answer is given in the works of Retamosa *et al.* [16] and Zamick, Zheng, and Moya de Guerra [14]. Retamosa *et al.* showed that in the SU(3) limit there is only one scissors state in ^{20}Ne, and three in ^{24}Mg. Zamick, Zheng, and Moya de Guerra also got the same result for ^{22}Ne by considering the one-body approach in the LS limit. In that case the deformed potential is triaxial and the only way to get $M1$ excitations is to have a transition proportional to $\omega_x-\omega_z$ or $\omega_y-\omega_z$. It should be remarked that there are no spin excitations in this limit.

III. LINEAR-ENERGY-WEIGHTED SUM RULE FOR $M1$ EXCITATIONS

It is easier to obtain analytic results by considering energy-weighted sums. But it should be kept in mind that for $M1$ transitions these sum rules are highly model dependent and should be used for illustrative purpose only. The linear-energy-weighted sum rule (LEWSR) for $M1$ transitions is defined as

$$S_{M1}^{\rm EW}=\frac{3}{4\pi}\sum_f (E_f-E_g)|\langle f|\boldsymbol{\mu}|0^+\rangle|^2 = \frac{3}{8\pi}\langle 0^+|[\boldsymbol{\mu},[H,\boldsymbol{\mu}]]0^+\rangle\ , \tag{3.1}$$

where $|0^+\rangle$ is the initial ground state and $\boldsymbol{\mu}$ is the magnetic dipole operator:

$$\boldsymbol{\mu}=\sum_i [g_l(i)\boldsymbol{l}_i+g_s(i)\mathbf{s}_i]\ . \tag{3.2}$$

The LEWSR for $M1$ transitions can be divided into a one-body part and a two-body part as

$$S_{M1}^{\rm EW}=\frac{3}{8\pi}\langle 0^+|\left[\boldsymbol{\mu},\left[\left[\sum_i -a\boldsymbol{l}_i\cdot\boldsymbol{\sigma}_i+\sum_{i<j}V_{ij}\right],\boldsymbol{\mu}\right]\right]|0^+\rangle = (S_{M1}^{\rm EW})_{\text{1-body}}+(S_{M1}^{\rm EW})_{\text{2-body}}\ . \tag{3.3}$$

The one-body part arises from the single-particle spin-orbit splitting and can be easily evaluated to yield the Kurath sum rule [17]

$$(S_{M1}^{\rm EW})_{\text{1-body}}=\frac{3}{16\pi}[(g_s^S-g_l^S)^2+(g_s^V-g_l^V)^2] \times\langle 0^+|\sum_i a\boldsymbol{l}_i\cdot\boldsymbol{\sigma}_i|0^+\rangle\ , \tag{3.4}$$

where $g_s^S=g_s(p)+g_s(n)=1.760$, $g_s^V=g_s(p)-g_s(n)=9.412$, and $g_l^S=g_l^V=g_l(p)=1.000$. Note that

$$\frac{(g_s^S-g_l^S)^2}{(g_s^V-g_l^V)^2}=0.008\,163\ , \tag{3.5}$$

and so the one-body part is dominantly isovector. The expectation value of the one-body spin-orbit interaction $\langle\sum_i a\boldsymbol{l}_i\cdot\boldsymbol{\sigma}_i\rangle$ for a given l is equal to $a[n_{j=l+1/2}l-n_{j=l-1/2}(l+1)]$, where n_j is the number of nucleons in the orbit j. The parameter a is 1 MeV if the spin-orbit splitting between $d_{3/2}$ and $d_{5/2}$ is 5 MeV.

Since we are interested in the orbital strength, the two-body part is of greater interest to us. For a spin-independent central interaction, the two-body part is of orbital isovector characteristic and can be written as

$$(S_{M1}^{\rm EW})_{\text{2-body}}=-\frac{3}{64\pi}\langle 0^+|\sum_{i,j}(t_i^z-t_j^z)^2\sum_{\alpha=x,y,z}[(l_i^\alpha-l_j^\alpha),[(l_i^\alpha-l_j^\alpha),V_{ij}]]|0^+\rangle\ . \tag{3.6}$$

For the QQ interaction (xV_q), the above double commutator can be greatly simplified. In fact, we have

$$\sum_{\alpha=x,y,z}[(l_i^\alpha-l_j^\alpha),[(l_i^\alpha-l_j^\alpha),Y_m^2(i)Y_{-m}^2(j)]]$$
$$=(6+2m^2)Y_m^2(i)Y_{-m}^2(j)$$
$$-(6-m^2+m)Y_{m-1}^2(i)Y_{-m+1}^2(j)$$
$$-(6-m^2-m)Y_{m+1}^2(i)Y_{-m-1}^2(j)\ . \qquad (3.7)$$

Therefore,

$$\sum_{\alpha=x,y,z}[(l_i^\alpha-l_j^\alpha),[(l_i^\alpha-l_j^\alpha),[Y^2(i)Y^2(j)]^0]]$$
$$=18\sqrt{5}[Y^2(i)Y^2(j)]^0\ , \qquad (3.8)$$

and the two-body part of the LEWSR for $M1$ transitions is

$$(S_{M1}^{\rm EW})_{\text{2-body}}$$
$$=\frac{135x\chi}{32\pi}\langle 0^+|\sum_{i,j}(t_i^z-t_j^z)^2r_i^2r_j^2[Y^2(i)Y^2(j)]^0|0^+\rangle\ . \qquad (3.9)$$

Except for an isospin factor $(t_i^z-t_j^z)^2$, which is nonzero for neutron-proton pairs only, the above quantity is closely related to the expectation value of the QQ interaction (xV_q) in the ground state:

$$\langle xV_q\rangle=\langle 0^+|\sum_{i<j}(xV_q)_{ij}|0^+\rangle$$
$$=\sqrt{5}x\chi\langle 0^+|\sum_{i<j}r_i^2r_j^2[Y^2(i)Y^2(j)]^0|0^+\rangle\ . \qquad (3.10)$$

This in turn can be related to the (non-energy-weighted) summed $E2$ strength, which can be obtained by inserting a complete set of 2^+ states:

$$S_{E2}=\sum_f B_f(E2)$$
$$=\sqrt{5}\langle 0^+|\sum_{i,j}e_ie_jr_i^2r_j^2[Y^2(i)Y^2(j)]^0|0^+\rangle\ , \qquad (3.11)$$

where the $E2$ operator is defined as

$$\Omega_M^2(E)=\sum_{i=1}^{A}(er^2Y_M^2)_i\ . \qquad (3.12)$$

The relations (3.9) and (3.11) are the closest we can come to establishing a relation between the LEWSR for $M1$ transitions and the summed $B(E2)$ strength. The basic idea is that the expectation value of the commutator for orbital $M1$ transitions using a QQ interaction is proportional to the expectation value of the neutron-proton part of the QQ interaction.

IV. CLOSING REMARKS

Rather than establishing a direct link between the summed orbital strength and deformation parameter δ, we tried to correlate the former quantity with the size of the QQ interaction x by studying the dependence of this strength on the parameter x. For the orbital strength of $J=0^+$, $T=1\to J=1^+$, and $T=1$ transitions, we find for a fixed pairing strength G, a quadratic dependence on x for small x. We find that for a sufficiently large but nevertheless realistic value of x, the results with experimental single-particle energies are almost the same as for the results with degenerate single-particles energies. This indicates that the quadrupole collectivity is not destroyed by the single-particle splitting.

Ironically, although $\sum B(M1)_{\rm orbital}T\to T$ starts out varying quadratically with x, this is not the case for $B(E2)_{0_1^+\to 2_1^+}$. Indeed, the latter quantity is nonzero as $x\to 0$. We explained this by noting that in the vibrational limit the value of $B(E2)_{0_1^+\to 2_1^+}$ is substantial. In this regard it might be better to establish a connection between $B(M1)$ and the quadrupole moment of the 2_1^+ state, as suggested by Chaves and Poves [10]. Beyond a certain value of x ($x>0.4$), the curves for $B(E2)_{0_1^+\to 2_1^+}$ and $\sum B(M1)_{\rm orbital}T\to T$ as a function of x look quantitatively similar and both curves are concave down.

The other three channels respond differently to the increase of the QQ strength x. The $T=1\to T=2$ $M1$ orbital strength is nonzero for $x=0$ (pure isospin-conserving pairing interaction) and does not vary much with increasing x.

The case of $T=1\to T=1$ $M1$ spin excitations with degenerate single-particle energies is interesting. The summed strength vanishes for both $x=0$ (pure pairing) and $x\to\infty$. But the above strength is completely overshadowed by the enormous contributions arising from the nondegeneracy of the single-particle energies. The $T=1\to T=2$ spin excitation strengths are large for $x=0$, but they steadily decrease as x is increased.

The closest we come to an analytic relation between magnetic dipole orbital and electric quadrupole strengths is the relation between the linear-energy-weighted sum rule for $M1$ orbital transitions and summed $B(E2)$ strength, where the relevant double commutator is evaluated for the quadrupole interaction.

In this work, by limiting ourselves to summed strengths, we have shown some necessary but not sufficient conditions for the scissors-mode interpretation of the orbital magnetic dipole excitations ($T\to T$) in deformed nuclei to have been met. The summed orbital strength vanishes for this channel in the isospin-conserving pairing limit, it starts to vary quadratically with the strength of the QQ interaction, and for realistic values of the QQ interaction, the inclusion of single-particle energies does not change this sum very much. Despite the fact that the orbital operator is half isoscalar and half isovector, we find that the isovector orbital summed strength is much larger than the corresponding isoscalar sum, and this is explained as being due to the fact that configuration mixing tends to produce an $L=0$ ground state. The above conditions are of course not sufficient for the scissors-mode concept to be of interest. As we have noted in this and previous work [8], about 35% of the strength is hopelessly fragmented for ^{22}Ne. The fact that 65% is not fragmented, but resides in the three lowest-lying states, and thus amenable to experimental observation is what makes the scissors-mode idea interesting. The summed strength that we have calculated obviously provides an upper limit to the scissors-mode strength.

The shell-model approach does not yield the simple picture of a single purely orbital state. The orbital strength can be in several states. There can be spin admixtures. There are states for which the spin and orbit terms nearly cancel [8], and so one does not see them in electron scattering. But the orbital strength can be obtained, as indeed experimental groups have shown [6,7,1], by a combined analysis of electron and proton scattering. And the final picture is one in which there is a concentration of low-lying orbital strength which acts like a collective scissors mode.

We hope that these considerations have shed some light on the relation between scissors-mode excitations and electric quadrupole transitions.

Note added in proof. Since the time of receipt of this manuscript two works have appeared which relate $B(M1)$ (scissors) to $BE2$ $(0_1 \to 2_1)$. Using somewhat different approaches, Hamamoto and Magnusson [18] and Garrido *et al.* [19] conclude that in order to get the orbital $B(M1)$ strength to be proportional to δ^2 rather than to δ one must include pairing.

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*Present address: McMaster University, Hamilton, Ontario, Canada.

[1] D. Bohle, A. Richter, W. Steffen, A. E. L. Dieperink, N. Lo Iudice, F. Palumbo, and G. Scolter, Phys. Lett. **137B**, 27 (1984); D. Bohle, G. Kuchler, A. Richter, and W. Steffen, *ibid.* **148B**, 260 (1984); D. Bohle *et al.*, Nucl. Phys. **A458**, 205 (1986).

[2] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).

[3] F. Iachello, Nucl. Phys. **A358**, 89c (1981); Phys. Rev. Lett. **53**, 1427 (1984).

[4] A. E. L. Dieperink, Prog. Part. Nucl. Phys. **9**, 121 (1983).

[5] T. R. Halemane, A. Abbas, and L. Zamick, J. Phys. G **7**, 1639 (1981).

[6] W. Ziegler, C. Rangacharyulu, A. Richter, and C. Spieler, Phys. Rev. Lett. **65**, 2515 (1990).

[7] C. Rangacharyulu, A. Richter, H. J. Wörtche, W. Ziegler, and R. F. Casten, Phys. Rev. C **43**, R949 (1991).

[8] H. Liu and L. Zamick, Phys. Rev. C **36**, 2057 (1987).

[9] L. Zamick, Phys. Rev. C **31**, 1955 (1985).

[10] L. Chaves and A. Poves, Phys. Rev. C **34**, 1137 (1986).

[11] U. E. P. Berg and U. Kneissl, Annu. Rev. Nucl. Part. Sci. **37**, 33 (1987).

[12] Daniel R. Bes and Raymond A. Sorensen, in *Advances in Nuclear Physics,* edited by Michel Baranger and Erich Vogt (Plenum, New York, 1969), Vol. 2.

[13] B. H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1984).

[14] L. Zamick, D. C. Zheng, and E. Moya de Guerra, Phys. Rev. C **39**, 2370 (1989).

[15] P. Halse (unpublished).

[16] J. Retamosa, J. M. Udias, A. Poves, and E. Moya de Guerra, Nucl. Phys. **A511**, 221 (1990).

[17] D. Kurath, Phys. Rev. **130**, 1525 (1963).

[18] I. Hamamoto and C. Magnusson, Phys. Lett. B **260**, 6 (1991).

[19] E. Garrido, E. Moya de Guerra, P. Sarriguren, and J. M. Udías, Phys. Rev. C **44**, 1250 (1991).