Projection of the six-quark wave function onto the *NN* channel and the problem of the repulsive core in the *NN* interaction

A. M. Kusainov, V. G. Neudatchin, and I. T. Obukhovsky
Institute of Nuclear Physics, Moscow State University, Moscow 119899, U.S.S.R.
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A modification of the resonating-group method (RGM) is proposed which includes the multiquark shell-model configurations in the nucleon overlap region. The instanton, gluon, and π,σ exchange is taken into account, the interaction constants being consistent with the baryon spectrum. This enables one to cover a wide interval of *NN* scattering energies up to $E_{\rm lab}=2$ GeV. The projection of the six-quark wave function onto the *NN* and other baryon channels is discussed in detail in our approach and in other RGM versions as well, and in this context the problem of repulsive core in the *NN* forces is discussed.

I. INTRODUCTION

The problem of the quark effects in the *NN* interaction is being widely discussed (see the reviews [1–5], and references therein). If, as usual, we restrict ourselves to energies $E_{\rm lab}\le 0.3-0.5$ GeV, we shall fail to reveal any striking manifestations of the quark effects in the *NN* system (even if we consider in addition the deuteron electromagnetic form factors). A good description of this region, given in a number of papers [6–12], can be considered only as indirect evidence for the consistency of the quark notions with the *NN* data. No wonder that much consideration is given to alternative models such as the Skyrme model [13], meson models with the virtually excited baryons [14], the model with the phenomenological repulsive core in the *NN* forces [15], etc.

Meanwhile, an important feature of the quark approach, which has not as yet been appraised at its true worth, is the simplicity in the interpretation of intricate phenomena which in the traditional approach would require the introduction of the multimeson and baryon contributions. For example, the repulsive core in the *NN* interaction and the suppression of the quark effects in the deuteron magnetic form factors were very easily explained through the destructive interference of the quark configurations s^6 and s^4p^2 in the *NN*-system wave function at low energies (including the deuteron) [16]. At higher energies this simple model makes predictions at a qualitative level. As the energy increases, the contribution of the excited configuration s^4p^2 to the *S*-wave *NN* scattering gets larger, leading to changes in the short-range *NN* interaction. At intermediate energy it becomes more appropriate to describe the *NN* interaction using, instead of the commonly used phenomenological repulsive core, the deep attractive potential containing one extra bound state which is regarded as forbidden (the bound 0*S* state would correspond to the configuration s^6 which is suppressed in the present case).

The potential (optical) model of the *NN* interaction [17], which employs the deep attractive potentials with forbidden states (FSP) [18,19], describes well the cross section and the polarization of the *NN* scattering in the hitherto unexplored, in the potential models, energy range from zero to $E_{\rm lab}=5-6$ GeV. At the same time it is here confirmed, in accord with the idea of forbidden states in the *S* and *P* waves, that the *S* and *P* phases are large, start at low energies of π and even 2π and reach the Born region at energies $E_{\rm lab}=4-5$ GeV, and the other phases ($L\ge 2$) are small at any energies. This is likely to be indicative of the decisive role of the quark configurations s^4p^2 and s^3p^3 in the *S* and *P* waves, respectively. It is important to elucidate the degree of consistency of this conclusion with the quark microscopic treatment.

In the present paper these problems are studied on the basis of the Hamiltonian

$$H_q=\sum_i\left[m_i\frac{\nabla_i^2}{2m_i}\right]+\sum_{i<j}V_{ij}(\rho)\ ,\quad \rho=\mathbf{r}_i-\mathbf{r}_j\ , \qquad (1)$$

proceeding from the assumption about the two-particle interaction of quarks. The role of the configuration s^6 and s^4p^2 in the *S*-wave *NN* scattering in a wide energy range $0<E_{\rm lab}\le 1-2$ GeV was considered. We extended the energy range and made a number of improvements in the model with the pair *qq* interactions (the interactions of constituent quarks with the π- and σ-effective fields and the form factors at the πqq and σqq vertices were taken into account, in some variants we introduced the contact four-fermion interactions of different operator form, etc.). This enabled us to describe simultaneously the baryon spectrum with allowance for one- and two-quantum excited states (N, Δ, N^*, N^{**}) and the *NN*-scattering data in a comparable energy interval.

The quark microscopic approach itself is formulated in an unusual way, i.e., through the quark shell configurations [22,23], which contain a very capacious information about the influence of the antisymmetrization upon the *BB* system (*NN*, $\Delta\Delta$, $N\Delta$, etc.), rather than through the resonating-group method (RGM) [20,21]. For example, the question as to the virtual baryons B^*, B^{**} with one or two oscillator quanta of internal excitation in the *NN* interaction is answered with ease in terms of the configurations s^6, s^5p, s^4p^2, s^3p^3, etc. Note that these excited states can be observed as real particles in the quasielastic knock-out of a nucleon from the deuteron, $d(e,e'p)N^*$ or $A(d,N^*)X$ [24], where N^* is the excited nucleon spectator, and can give valuable independent information about the presence of the configuration s^4p^2 in the deuteron.

In connection with all these problems the projection of the six-quark wave function onto the NN channel and other baryon channels is discussed in detail in our approach and in other RGM versions as well [7,25,26].

II. FORMULATION OF THE VARIATIONAL PROBLEM

Just as in the standard RGM approach [20,21], we derive the equations of motion proceeding from the Hulthen-Kohn variational principle. The trial function is written as an expansion in the two-center shell configurations $S_+^3(R)S_-^3(R)$ [21,26] which for the Lth partial wave is written as

$$\frac{1}{r}Y_{LM}(\hat{\mathbf{r}})u_{6q}^{LST}(k,r;\rho_1\xi_1\rho_2\xi_2X)=\sum_f\int_0^\infty \chi_f^L(k,R)[N_f^L(R,R)]^{-1/2}|S_+^3(R)S_-^3(R)\,[f_X]\,[f_{CS}]LST>R\,dR\ . \tag{2}$$

It is implied that the complete six-quark wave function of NN scattering at energy $\hbar^2k^2/m_N$ is expanded as

$$\psi_{6q}^{ST}(\mathbf{r}_1,\ldots,\mathbf{r}_6;\mathbf{k})=\sum_{L=0} i^L(2L+1)e^{i\delta_L}\sin\delta_L P_L(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})\frac{1}{r}u_{6q}^{LST}(k,r;\rho_1\xi_1\rho_2\xi_2X)\ .$$

In the representation (2) the summation is made over all the Young schemes $[f_X]$, $[f_{CS}]$, satisfying the Pauli exclusion principle, in the coordinate (X) and color-spin (CS) space which is briefly designated from here on by a symbol f. $S_\pm(R)$ denotes the quark $0S$ orbitals, centered at points $\pm R/2$,

$$S_\pm(\mathbf{r}_i,\mathbf{R})=(\pi b^2)^{-3/4}\times\exp\left[-\frac{1}{2b^2}\left[r_i\mp\frac{\mathbf{R}}{2}\right]^2\right]\chi(C_i,S_i,T_i) \tag{3}$$

Thus, R is a generator coordinate [21] and the other variables in (2) are Jacobi coordinates,

$$\mathbf{r}=\tfrac{1}{3}\sum_{i=1}^{3}\mathbf{r}_i-\tfrac{1}{3}\sum_{j=4}^{6}\mathbf{r}_j\ ,\quad \mathbf{X}=\tfrac{1}{6}\sum_{i-1}^{6}\mathbf{r}_i\ ,$$

$$\rho_1=\mathbf{r}_1-\mathbf{r}_2\ ,\quad \xi_1=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)-\mathbf{r}_3\ ,$$

etc. In the expression (2) we do not need to use the antisymmetrization operator with respect to permutations of quarks from different nucleons,

$$A=\frac{1}{10}\left[1-\sum_{i=1}^{3}\sum_{j=4}^{6}P_{ij}\right]\ ,\quad A^2=A$$

since the basis two-center functions in (2) are completely antisymmetrized. They are constructed, using the Clebsch-Gordan coefficients of the groups SU(n) from the reduction chain of subgroups

$$\begin{aligned}&\mathrm{SU}(24)_{XCST}\supset\mathrm{SU}(2)_X\times\mathrm{SU}(12)_{CST}\ ,\\&\mathrm{SU}(12)_{CST}\supset\mathrm{SU}(6)_{CS}\times\mathrm{SU}(2)_T\\&\qquad\supset\mathrm{SU}(3)_C\times\mathrm{SU}(2)_S\times\mathrm{SU}(2)_T\end{aligned} \tag{4}$$

and the Young schemes $[f_{XCST}]=[1^6]$, $[f_X]$, $[f_{CST}]=[\tilde{f}_X]$, $[f_{CS}]$, $[f_C]=[2^3]$, $[f_S]$, $[f_T]$ are invariants of these subgroups. The method of construction of the basis and the calculation of the Clebsch-Gordan and fractional parentage coefficients (the method of scalar factors [4,27]) were described by us in Refs. [23,27,28] (see also Refs. [26,29]). In the representation (2) the expansion is performed over the functions, normalized to unit at any fixed value of R,

$$\langle S_+^3(R)S_-^3(R)[f]LST|S_+^3(R')S_-^3(R')[f']L'S'T'\rangle=\delta_{ff'}\delta_{LL'}\delta_{SS'}\delta_{TT'}N_{f_X}^L(R,R')\ , \tag{5}$$

where

$$N_{f_X}^L(R,R')=2(2L+1)e^{-3(R^2+R'^2)/8b^2}\times\left[i_L\left[\frac{3RR'}{4b^2}\right]+\kappa_{f_X}i_L\left[\frac{RR'}{4b^2}\right]\right]$$

$\kappa_{f_X}=9$ at $[f_X]=[6]$ and $\kappa_{f_X}=-1$ at $[f_X]=[42]$. The convenience of the normalized basis consists in that at $R\to0$ the basis vectors $[N_f^L(R,R)]^{-1/2}|S_+^3S_-^3[f]LST\rangle$ change over to usual shell configurations [26] $|s^mp^n[f]LST\rangle$, $m+n=6$. Considering that for even L the Young scheme $[f_X]$ takes on in (2) two values, $[6]_X$ and $[42]_X$, we get in the channel under study $L=0$, $S=1$, $T=0$ (instead of the limit $R\to0$ we record the integral of the δ function)

$$\int_0^\infty\frac{\delta(R)}{R}[N_{f_X}^{L=0}(R,R)]^{-1/2}|S_+^3(R)S_-^3(R)[f]L=0S=1T=0\rangle R\,dR=\begin{cases}|s^6[6]_X[2^3]_{CS}L=0S=1T=0\rangle & \text{if } [f_X]=[6]\ ,\\ |s^4p^2[42]_X[f]_{CS}L=0S=1T=0\rangle & \text{if } [f_X]=[42]\ ,\end{cases} \tag{6}$$

where $[f_{CS}]=[42]$, $[321]$, $[2^3]$, $[31^3]$, $[24^4]$ are all the possible Young scheme from the Clebsch-Gordan series for the inner product $[2^3]_C \cdot [42]_S$.

As it has been noted, the configurations s^6 and s^4p^2 are the most essential components of the NN-system wave function in the internal region and, therefore, in our modified RGM approach we recorded the trial functions $\chi_f^L(k,R)$ in (2) as an expansion

$$\chi_f^L(k,R)=a_f^L(k)\frac{\delta(R)}{R}+U_f^{NN}[C_{f_X}^L(k,R)+\cot\delta_L(k)S_{f_X}^L(k,R)] . \tag{7}$$

The expansion (7) includes only the "basis" internal states $\delta(R)/R$ and the asymptotic states $S_{f_X}^L(k,R)$, $C_{f_X}^L(k,R)$, which in the limit $r\rightarrow\infty$ describe the free motion of $3q$ clusters:

$$\lim_{r\rightarrow\infty}\sum_f U_f^{NN}\int_0^\infty \begin{Bmatrix} S_{f_X}^L(k,R) \\ C_{f_X}^L(k,R) \end{Bmatrix}[N_f^L(R,R)]^{-1/2}|S_+^3(R)S_-^3(R)[f]LST\rangle R\,dR$$
$$=\sqrt{10}\,A\left[\psi_N^{(0)}(\rho_1\xi_1)\phi_N^{(0)}(\rho_2\xi_2)Y_{LM}(\hat{\mathbf{r}})\frac{1}{kr}\begin{Bmatrix}\sin(kr-\pi L/2) \\ \cos(kr-\pi L/2)\end{Bmatrix}\varphi_{00}(\mathbf{X})\right]_{\mathrm{ST}} , \tag{8}$$

where

$$\begin{Bmatrix} S_{f_X}^L(k,R) \\ C_{f_X}^L(k,R) \end{Bmatrix}=\left[\frac{3}{2\pi b^2}\right]^{3/2}\frac{1}{4}e^{k^2b^2/3}[N_f^L(R,R)]^{-1/2}\frac{1}{k}\begin{Bmatrix}\sin(kR-\pi L/2) \\ \cos(kR-\pi L/2)\end{Bmatrix} . \tag{9}$$

In the expressions (7) and (8), U_f^{NN} are elements of unitary matrix $U_f^{B_1B_2}$ which realizes in the CST space the transformation from the baryon quantum numbers $B_1(S_1T_1C_1)B_2(S_2T_2C_2)$ to the quantum numbers of the $6q$ system—the Young schemes $[f_X]$ and $[f_{CS}]$ (see Table I). $\psi_N^{(0)}(\rho_1\xi_1)$ is the nucleon wave function in the form of the ground state of the translationally invariant shell model (TISM) [30] (see, also, [31])

$$\psi_N^{(0)}(\rho_1\xi_1)\equiv|s^3[3]_X L_1=0 S_1=\tfrac{1}{2} T_1=\tfrac{1}{2}\rangle_{\mathrm{TISM}}$$
$$=\left[\frac{3}{2\pi b^2}\right]^{3/2}\exp\left[-\frac{1}{2b^2}\left[\frac{1}{2}\rho_1^2+\frac{2}{3}\xi_1^2\right]\right]|[1^3]_C[21]_{CS}S_1=\tfrac{1}{2}T_1=\tfrac{1}{2}\rangle \tag{10}$$

Here $\varphi_{nl}(x)$ are everywhere the harmonic-oscillator wave functions, for example, $\varphi_{00}(\mathbf{x})=(\pi b^2\mu)^{-3/4}\exp(-\mu^2\mathbf{x}^2/2b^2)$, where $\mathbf{x}$ is the Jacobi coordinate and μ is the corresponding reduced mass; b is the rms ground-state radius. The coefficients a_f^L and $\cot\delta_L$ in (2) and (7) are variational parameters. The trial functional of the Hulthen-Kohn variational principle:

$$\mathcal{J}_E^{LST}(\{a_f^l\},\cot\delta_L)=\frac{\hbar^2k^2}{3m_q}\frac{\cot\delta_L}{k^3}+\int Y_{LM}^*(\hat{\mathbf{r}})\frac{1}{r}u_{6q}^{LST}(k,r;\rho_1\cdots\mathbf{X})(H_q-E)$$
$$\times Y_{LM}(\hat{\mathbf{r}})u_{6q}^{LST}(k,r;\rho_1\cdots\mathbf{X})d^3r\,d^3\rho_1\cdots d^3X \tag{11}$$

is quadratic in a_f^L, $\cot\delta_L$, and we get from the stationary conditions $\partial\mathcal{J}_E^{LST}/\partial a_f^L=0$, $\partial\mathcal{J}_E^{LST}/\partial\cot\delta_L=0$ a set of linear algebraic equations for $\{a_f^L\}$ and $\cot\delta_L$,

TABLE I. Matrix $U_f^{B_1B_2}$.

B_1B_2	NN	$\Delta\Delta$	C_1C_1	C_2C_1	C_2C_2	C_3C_3
$[6]_X[2^3]_{CS}$	$\sqrt{1/9}$	$-\sqrt{4/45}$	$\sqrt{2/9}$	$\sqrt{4/9}$	$\sqrt{1/45}$	$-\sqrt{1/9}$
$[42]_X[42]_{CS}$	$-\sqrt{9/20}$	0	$\sqrt{1/10}$	$\sqrt{1/5}$	$\sqrt{1/4}$	0
$[42]_X[321]_{CS}$	$\sqrt{16/45}$	0	$\sqrt{8/45}$	$-\sqrt{1/45}$	$\sqrt{4/9}$	0
$[42]_X[2^3]_{CS}$	$\sqrt{1/36}$	$\sqrt{16/45}$	$\sqrt{1/18}$	$\sqrt{1/9}$	$\sqrt{1/180}$	$\sqrt{4/9}$
$[42]_X[31^3]_{CS}$	$-\sqrt{1/18}$	0	$\sqrt{4/9}$	$-\sqrt{2/9}$	$\sqrt{5/18}$	0
$[42]_X[21^4]_{CS}$	0	$\sqrt{5/9}$	0	0	0	$-\sqrt{4/9}$

$$\frac{4\pi}{2L+1}(\bar{\bar{v}}^{SS}+\bar{\bar{Q}}^{SS})\cot\delta_L+\sum_f(\bar{v}_{f'}^{s\delta}+\bar{Q}_{f'}^{s\delta})a_{f'}^L=-\frac{1}{k}\left[\frac{\hbar^2}{6m_q}+\frac{4\pi}{2L+1}(\bar{\bar{v}}^{SC}+\bar{\bar{Q}}^{SC})\right],$$

$$(\bar{v}_f^{\delta s}+\bar{Q}_f^{\delta s})\cot\delta_L+\frac{4\pi}{2L+1}\sum_{f'}(v_{ff'}^{\delta\delta}+Q_{ff'}^{\delta\delta})a_{f'}^L=-(\bar{v}_f^{\delta C}+\bar{Q}_f^{\delta C}) \tag{12}$$

The coefficients of these equations, $\bar{\bar{v}}^{SS}$, $\bar{\bar{Q}}^{SS}$, $v_f^{\delta s}$, $\bar{Q}_f^{\delta s}$, $v_{ff'}^{\delta\delta}$, $Q_{ff'}^{\delta\delta}$ are convolutions (in the generator coordinate R and indices f) of the matrix elements of potential (v) and kinetic (Q) energy with the "basis functions" of the representation (7), $\delta(R)/R$, $U_f^{NN}S_{f_X}^L(k,R)$, $U_f^{NN}C_{f_X}^L(k,R)$:

$$\bar{\bar{v}}^{SS}=\sum_{ff'}U_f^{NN}U_{f'}^{NN}v_{ff'}^{SS}\ ,\quad \bar{v}_f^{\delta s}=\sum_{f'}U_{f'}^{NN}v_{ff'}^{s\delta}\ ,$$

$$\bar{\bar{Q}}^{SS}=\sum_{ff'}U_f^{NN}U_{f'}^{NN}Q_{ff'}^{SS}\ ,\quad \bar{Q}_f^{\delta s}=\sum_{f'}U_{f'}^{NN}Q_{ff'}^{s\delta}\ ; \tag{13}$$

$$v_{ff'}^{SS}=\int_0^\infty R\,dR\int_0^\infty R'dR'S_{f_X}^L(k,R)[V_{ff'}^L(R,R')-2\delta_{ff'}\langle\psi_N|V|\psi_N\rangle I_{f_X}^L(R,R')]S_{f'_X}^L(k,R')\ ,$$

$$Q_{ff'}^{SS}=\delta_{ff'}\int_0^\infty R\,dR\int_0^\infty R'dR'S_{f_X}^L(k,R)\left[K_{f_X}^L(R,R')-\left[\frac{15}{4}\frac{\hbar^2}{m_qb^2}+\frac{k^2\hbar^2}{3m_q}\right]I_{f_X}^L(R,R')\right]S_{f'_X}^L(k,R')\ ,$$

$$v_{ff'}^{s\delta}=\int_0^\infty R\,dR\int_0^\infty R'dR'S_{f_X}^L(k,R)[V_{ff'}^L(R,R')-2\delta_{ff'}\langle\psi_N|V|\psi_N\rangle I_{f_X}^L(R,R')]\frac{\delta(R')}{R'}\ , \tag{14}$$

$$v_{ff'}^{\delta\delta}=\int_0^\infty R\,dR\int_0^\infty R'dR'\frac{\delta(R)}{R}[V_{ff'}^L(R,R')-2\delta_{ff'}\langle\psi_N|V|\psi_N\rangle I_{f_X}^L(R,R')]\frac{\delta(R')}{R'}\ .$$

In (11) E is the complete energy of the $6q$ system.

$$E=6m_q+\frac{3}{4}\frac{\hbar^2}{m_qb^2}+\frac{k^2\hbar^2}{3m_q}=2\langle\psi_N^{(0)}|H_q|\psi_N^{(0)}\rangle+\frac{15}{4}\frac{\hbar^2}{m_qb^2}+\frac{k^2\hbar^2}{3m_q} \tag{15}$$

and for the purpose of simplifying the kinematics we use the mass of the constituent quark $m_q=\frac{1}{3}m_N$.

The kernels of kinetic $\delta_{ff'}K_{f_X}^L$ and potential $V_{ff'}^L$ energy and, also, the overlap kernel $\delta_{ff'}I_{f_X}^L$ are defined by the standard relations of the generator coordinate method

$$\begin{Bmatrix} V_{ff'}^L(R,R') \\ \delta_{ff'}K_{f_X}^L(R,R') \\ \delta_{ff'}I_{f_X}^l(R,R') \end{Bmatrix}=[N_{f_X}^L(R,R)N_{f'_X}^L(R',R')]^{-1/2}$$

$$\times\langle S_+^3(R)S_-^3(R)[f]LST|\begin{Bmatrix} \sum_{i>j}V_{ij} \\ -\sum_i\frac{\nabla_i^2}{2m_q} \\ 1 \end{Bmatrix}|S_+^3(R')S_-^3(R')[f']LST\rangle\ . \tag{16}$$

Note that the first two integrals in (14) contain singular terms proportional to the δ functions. However, it is easy to see that in the combination $(K_{f_X}^L-\tilde{E}I_{f_X}^L)$, where $\tilde{E}=\frac{15}{4}\hbar^2/m_qb^2+k^2\hbar^2/3m_q$ the singularities are canceled in pairs. A more fine moment is the cancellation of singularities in the combination $(V_{ff'}^L-2\delta_{ff'}\langle\psi_N^{(0)}|V|\psi_N^{(0)}\rangle I_{f_X}^L)$. This can be demonstrated if we express the interaction matrix elements through the Casimir invariants of group SU(n), considering that $V=\sum_{i<j}V_{ij}$,

$$m_N=\langle\psi_N^{(0)}|H_q|\psi_N^{(0)}\rangle=\langle\psi_N^{(0)}|V|\psi_N^{(0)}\rangle+\frac{3}{2}\frac{\hbar^2}{m_qb^2}+3m_q\ ;$$

V_{ij} contains the operators $\lambda_i\lambda_j\sigma_i\sigma_j$, $\tau_i\tau_j\sigma_i\sigma_j$, $\sigma_i\sigma_j$, $\tau_i\tau_j$, etc., invariant with respect to the group $SU(6)_{CS}$, $SU(4)_{ST}$, $SU(2)_S$, $SU(2)_T$, respectively.

III. THE CONCRETE DEFINITION OF THE INTERACTION AND THE SOLUTION OF THE VARIATIONAL PROBLEM

In the quark-cluster approach to the nucleon-nucleon interaction, it is usually assumed that the Hamiltonian (1) describes not only the hadronic system but also the interaction of hadrons as quark clusters [7]. This makes possible a comparison between different quark interactions and a wide range of experimental data.

It is known that the splitting in the spectrum of light hadrons are well described by the color-exchange potentials carried over from the charmonium spectroscopy [32] (the spin-orbital forces are omitted)

$$V_{ij}^{OGE}(\rho)=\alpha_S\frac{\lambda_i\lambda_j}{4}\left[\frac{1}{\rho}-\frac{\pi}{m_q^2}(1+\tfrac{2}{3}\sigma_i\sigma_j)\delta(\rho)-\frac{1}{m_q^2\rho^3}S_{ij}(\hat{\rho})\right]; \quad (17)$$

$$S_{ij}(\hat{\rho})=3(\sigma_i\hat{\rho})(\sigma_j\hat{\rho})-\sigma_i\sigma_j ,$$
$$V_{ij}^{\rm conf}=-\frac{\lambda_i\lambda_j}{4}(a_c\rho-V_0) , \quad (18)$$

but in this case the constants α_S, a_c, m_q should be considered as phenomenological parameters. This "renormalization" of the QCD constants merely imitates the nonperturbative contributions and is inconsistent even on the *NN* system where the interaction (17), (18) leads to the short-range repulsion and fails to account for the mutual attraction of nucleons in a medium range $0.5\le r\le 1.5$ fm. Though it is not *a priori* clear that the universal potentials, describing both the $3q$ and $6q$ systems, are existent, the construction of the phenomenological potentials V_{ij}, which take into account the nonperturbative dynamics, is quite justified and led to certain positive results [6–12]. In the framework of the "naive nonperturbative" models [8,11], allowing for the π and σ exchanges at the quark level, the success was achieved due to introduction of new parameters (such as m_σ, $g_{\sigma qq}$, etc.) which were fitted directly to the *NN* data. However, the fitting of the parameters cannot lead to a better understanding of the dynamics. In the present work we did not fit parameters of the chiral interaction $H_{\rm ch}$ especially to the *NN* data or the baryon spectrum but we used the constrains upon the πqq and σqq interactions which follow from known models of the spontaneous breaking of chiral symmetry (SBCS) [33–35] in QCD vacuum. According to known Nambu–Jona-Lasinio results [33] the effective fields interaction is of the chiral-invariant form, $H_{\rm ch}\sim g_{\rm ch}\bar{\psi}(\sigma+i\gamma_5\tau\pi)\psi$; the constrains upon the masses of the effective fields—pseudoscalar (π), scalar (σ), and fermion (in our case it is the constituent quarks with the mass $m_q\approx m_N/3$) are $m_\pi\approx 0$, $m_\sigma\approx 2m_q$. Besides, we followed Refs. [34,35] and kept in mind that the description of the interaction in terms of the effective fields (π, σ, q) makes sense only at distances greater than the characteristic size ρ_c of the instanton fluctuations responsible for SBCS (according to Refs. [34–36], $\rho_c\approx 0.3$ fm). In the presence of the instanton fluctuations of the gluon field, the light QCD quarks ($m_{u,d}\approx 0$) acquire the dynamic mass $m_q(Q^2)$ which depends on the momentum transfer Q [34,35]. At large momenta, when $Q\ge Q_c\approx\rho_c^{-1}$, this mass $m_q(Q^2)\to m_{u,d}\approx 0$ and only at small momenta $Q\to 0$ it equals the constituent mass $m_q(0)=m_q\approx m_N/3$. The interaction with the effective chiral field $\phi(x)$ can be recorded in the form, suggested in Refs. [34,35]:

$$H_{\rm ch}=m_q(Q^2)\bar{\psi}\exp(i\gamma_5\tau\phi/f_\pi)\psi . \quad (19)$$

At $Q\ge Q_c\approx 0.6$ GeV/c the interaction is automatically switched off. We used the expression (19) in the linear approximation in $\pi=\hat{\phi}f_\pi\sin(\phi/f_\pi)$ and $\sigma=f_\pi[\cos(\phi/f_\pi)-1]$

$$H_{\rm ch}\approx m_q\bar{\psi}\psi+g_{\rm ch}F(Q^2)\bar{\psi}(\sigma+i\gamma_5\tau\pi)\psi , \quad (20)$$

where $m_q=m_q(0)$, $g_{\rm ch}=m_q/f_\pi$, $F(Q^2)=m_q(Q^2)/m_q(0)$. The expression (20) differs from the Hamiltonian of the linear σ model [37] only in that the vertex constant $g_{\rm ch}$ is reduced by the form factor $g_{\rm ch}\to g_{\rm ch}F(Q^2)$.

The form factor $F(Q^2)$ from [34] was approximated here by the analytic expression

$$F(Q^2)=\left[1+\sum_{k=1}^{n}C_k\frac{Q^2+m_\pi^2}{Q^2+\Lambda_k^2}\right]^{1/2} . \quad (21)$$

The convenience of (21) is in that it enables us to record immediately the π- and σ-exchange potentials, generated by the Hamiltonian (20)

$$\begin{aligned}V_{ij}^{\rm ch}(\rho)=\frac{1}{3}\alpha_{\rm ch}m_\pi\Bigg\{&\tau_i\tau_j\sigma_i\sigma_j\left[Y_0(m_\pi\rho)+\sum_{k=1}^{n}C_k\frac{\Lambda_k^3}{m_\pi^3}Y_0(\Lambda_k\rho)+\left[1+\sum_{k=1}^{n}C_k\right]\frac{4\pi}{m_k^3}\delta(\rho)\right]\\&+\tau_i\tau_jS_{ij}(\hat{\rho})\left[Y_0(m_\pi\rho)+\sum_{k=1}^{n}C_k\frac{\Lambda_k^3}{m_\pi^3}Y_0(\Lambda_k\rho)\right]\\&-3\left[\frac{4m_q^2}{m_\pi^2}\right]\left[\frac{m_\sigma}{m_\pi}\left[1+\sum_{k=1}^{n}C_k\frac{m_\pi^2-m_\sigma^2}{\Lambda_k^2-m_\sigma^2}\right]Y_0(m_\sigma\rho)+\sum_{k=1}^{n}C_k\left[\frac{\Lambda_k^2-m_\pi^2}{\Lambda_k^2-m_\sigma^2}\right]Y_0(\Lambda_k\rho)\right]\Bigg\} ,\end{aligned} \quad (22)$$

$$Y_0(x)=e^{-x}/x , \quad Y_2(x)=(3/x^2+3/x+1)Y_0(x)$$

The fact is that after the form factor (21) has been introduced, the pion propagator takes the form typical of the Pauli-Villars regularization

$$\frac{1}{m_\pi^2+Q^2}\to\frac{F(Q^2)}{m_\pi^2+Q^2}=\frac{1}{m_\pi^2+Q^2}+\sum_{k=1}^{n}C_k\frac{1}{\Lambda_k^2+Q^2} , \quad (23)$$

where Λ_k ($k=1,2,\ldots,n$) are masses of the subtracted fields ($m_\pi^2\ll\Lambda_1^2<\Lambda_2^2<\cdots<\Lambda_n^2$). As a rule, the authors of Refs. [7,8,12] use the regularization (23) with one subtraction ($n=1$) and the constant Λ_1 is fitted to the data. In our case $2n$ parameters C_k, Λ_k are used merely to provide the best description of the function $m_q(Q^2)/m_q(0)$ from Ref. [34]. We understand that the authors of Ref.

[34] discussed a kind of idealization in which the nonperturbative effects were reduced only to the contributions of the instanton fluctuations. In fact, there exist in addition some other contributions, for example, the constituent mass includes the relativistic energy of current quarks in it, etc. Besides, the confinement forces, which were not studied in [34,35], are essential. Therefore, proceeding from the natural physical requirements (see below), we imposed in our work additional restrictions upon the constants C_k, Λ_k. First of all, we studied the problem of the stability of the observables (baryon spectrum, NN-scattering phase shifts) with respect to variations of the parameters Λ_k, C_k. If we require that starting with the distances of the order of the confinement radius, $\rho \sim r_{\text{conf}} \approx 1$ fm the potential (22) should go over to the usual π and σ exchange potential in the region $\rho \geq r_{\text{conf}}$, then all the observables become stable and depend critically only on one parameter—the minimum mass Λ_1 in the expression (23). Choosing $\Lambda_1 \sim 1/\rho_c$, we set the boundary $\rho \sim \rho_c$, at which the potential begins to die out rapidly at $\rho \to 0$. In accord with the fact that $m_q(Q^2) \to 0$ at $Q^2 \geq 1/\rho_c^2$ and the interaction (19) becomes zero at $\rho \to 0$, it is necessary to require that the potential (22) should satisfy the conditions $V_{ij}^{\text{ch}}(0)=0$, $dV_{ij}^{\text{ch}}/d\rho=0$ at $\rho=0$. These conditions are equivalent to the relations

$$\sum_{k=1}^{n} C_k = -1 \ , \quad \sum_{k=1}^{n} C_k \frac{\Lambda_k^2}{m_\pi^2} = -1 \ , \quad \sum_{k=1}^{n} C_k \frac{\Lambda_k^3}{m_\pi^3} = -1 \ , \quad \sum_{k=1}^{n} C_k \frac{\Lambda_k^4}{m_\pi^4} = -1 \tag{24}$$

but the most stringent requirement is the one upon V_{ij}^{ch} in the region $\rho \geq r_{\text{conf}}$. Note that the scale $r_{\text{conf}} \approx 1$ fm is not used in Refs. [34,35] and the introduction of this scale leads to the form factor $F(Q^2)$ which is somewhat different from the function $m_q(Q^2)/m_q(0)$ of Ref. [34] in the region $1/r_{\text{conf}} \leq Q \leq 1/\rho_c$. All these conditions are satisfied only at $n=8-10$. Table II lists the values of the parameters C_k Λ_k at $n=10$.

Combining the potential V_{ij}^{ch} with the color-exchange interactions V_{ij}^{OGE} and V_{ij}^{conf}, we get a potential model,

$$V_{ij} = V_{ij}^{OGE} + V_{ij}^{\text{conf}} + V_{ij}^{\text{ch}} \ , \tag{25}$$

which uses in fact different types of interaction in three different regions $0 < \rho \leq \rho_c$, $\rho_c \leq \rho \leq r_{\text{conf}}$ and $r_{\text{conf}} \leq \rho < \infty$. Hence, this model has two characteristic scales: ρ_c and r_{conf} and gives stable observables as other model parameters are varied.

It has already been noted that the masses of the effective fields (q, π, σ) in our model are fixed

$$m_q = \tfrac{1}{3} m_N \ , \quad m_\pi = 140 \text{ MeV} \ , \quad m_\sigma = 2 m_q \tag{26}$$

and are not adjustable parameters, just as is not the constant $\alpha_{\text{ch}} = (g_{\text{ch}}^2/4\pi)(m_\pi^2/4m_q^2)$, which is normalized to known constant of the pseudoscalar πNN coupling $g_{\pi NN}^2/4\pi = 14.2$ (see, for example, [7,8])

$$\alpha_{\text{ch}} = \left[\frac{3}{5}\right]^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_\pi^2}{4m_N^2} = 0.0284 \ . \tag{27}$$

The adjustable parameters of the interaction (25) were, as usual, only the constants α_s, α_{ch}, and V_0, which were

TABLE II. Parameters of approximation of the form factor (21).

k	$\frac{\Lambda_k}{m_\pi}$	C_k
1	6.4	−0.961 036 32
2	7.3	4.264 053 03
3	8.9	−12.485 067 2
4	11.0	26.493 810 4
5	13.6	−30.817 173 9
6	21.5	29.724 046 9
7	28.58	−29.720 098 3
8	36.5	20.299 925 3
9	42.5	−9.394 768 24
10	48.5	1.596 308 15

TABLE III. Parameters of the qq interaction.

No.	b (fm)	α_s	a_c (MeV/fm)	V_0 (MeV)	α_{ch}	x	α_{eff}	$k=\frac{\alpha_G}{\alpha_{\text{eff}}}$
I	0.55	1.33	411.35	300.6	0	0	0	0
I[a]	0.475	0.97	246.4		0	0	0	0
II	0.525	0.78	161.78	94.44	0.0284	0	0	0
III	0.50	0.57	253.74	231.46	0.0284	0	0	0
IV	0.525	1.03	506.36	445.31	0	1	0	0
V	0.50	0	293.75	404.97	0	8.08	0	0
VI	0.49	0.82	316.1	237.08	0.0284	0	0.05	3
VII	0.50	0.48	195.25	186.31	0.0284	1	0	0
VIII	0.475	0.22	222.07	265.25	0.0284	1.778	0	0
IX	0.45	0	254.06	343.24	0.0284	2.25	0	0
Exp.								

[a]From Ref. [7].

TABLE IV. The nucleon excitation spectra for variants I–IX of parameters from Table III.

No.	m_N (MeV)	m_Δ (MeV)	$m_N^*(J^P=\frac{1}{2}^+)$ (MeV)		$m_N^*(J^P=\frac{1}{2}^-)$ (MeV)			m_Δ^* (MeV)
I	939	1234	1454	2037	1345	1501	2005	2188
I*a*	939	1232						
II	939	1235	1412	1848	1426	1628	1951	2157
III	939	1232	1497	1884	1483	1682	2077	2181
IV	939	1234.3	1500	2089	1379	1533	2092	2227
V	939	1225	1362	2010	1159	1202	1832	2078
VI	939	1234.7	1495	2028	1499	1701	2204	2210
VII	939	1235.8	1458	1874	1398	1590	2014	2172
VIII	939	1237.1	1512	1907	1412	1596	2098	2185
IX	939	1236.9	1584	1953	1443	1612	2213	2192
Exp.	939	1232±2	1440±40	1710±30	1540±20	1650±30	2100(?)	(?)

chosen to provide the best description of the nucleon mass, the Δ isobar, and the masses of the nucleon excited states of positive and negative parity $N^*_{1/2^-}$ (1535) $N^{**}_{1/2^+}$ (1440), etc. (Tables III and IV). In the calculation of the baryon spectrum, the TISM basis was used. It included the excitations up to three oscillator quanta—the configurations $s^3[3]_X$, $sp^2[3]_X$, $sp^2[21]_X$, $sp^2[1^3]_X$, $s^2p[21]_X$, $p^3[3]_X$, $p^3[21]_X$, $L=0$, 1, 2; $S=\frac{1}{2},\frac{3}{2}$; $T=\frac{1}{2},\frac{3}{2}$ [see the expression (10)]:

$$\psi_0=|s^3[3]_X[21]_{CS}L=0\ S=\tfrac{1}{2}\ T=\tfrac{1}{2}\rangle_{\rm TISM},$$

$$\begin{aligned}\psi_1&=|sp^2[3]_X[21]_{CS}L=0\ S=\tfrac{1}{2}\ T=\tfrac{1}{2}\rangle_{\rm TISM}\\ &=\left[\frac{3}{\pi b^2}\right]^{3/2}\sqrt{3/4}\left[\left[1-\frac{\rho_1^2}{3b^2}\right]+\left[1-\frac{4}{9}\frac{\xi_1^2}{b^2}\right]\right]\\ &\quad\times\exp\left[-\frac{1}{2b^2}(\tfrac{1}{2}\rho_1^2+\tfrac{2}{3}\xi_1^2)\right]\\ &\quad\times|[1^3]_C[21]_{CS}S=\tfrac{1}{2}\ T=\tfrac{1}{2}\rangle\ ,\end{aligned}\tag{28}$$

etc. (see Ref. [31]).

The oscillator radius b was chosen such that the lowest configurations, $s^3[3]_X$ and $sp^2[3]_X$, be minimally mixed in the nucleon wave function. To this end we imposed an additional condition,

$$\langle\psi_0|H_q|\psi_1\rangle=0\tag{29}$$

which plays here the same role as does the known minimum condition of the nucleon mass, calculated in the zeroth order approximation

$$dm_N^{(0)}/db=0\ ,\quad m_N^{(0)}=\langle\psi_0|H_q|\psi_0\rangle\ .\tag{30}$$

Table IV lists the results of the calculation of the baryon spectrum with the neglect of the tensor forces (in what follows in the *NN*-scattering calculation we neglected the tensor forces). For the splitting N-Δ, N-N^{**}, N-N^*, which were of particular importance to us, the tensor forces do not play the significant role. The inclusion of the tensor forces leads only to a small renormalization of the parameters α_s, a_c, V_0, etc., and to small (0.5–1.5 %) admixtures of the D waves in the lowest states of the spectrum.

For the sake of comparison, we present the results of the variant in which the π- and σ-exchange interactions (22) (variant I, $\alpha_{\rm ch}=0$) were neglected. We see that in the nonrelativistic approach at $V_{ij}^{\rm ch}=0$ it is impossible to obtain even an approximate degeneracy of the levels corresponding to the lowest resonances of positive and negative paraity, $N^{**}_{1/2^+}$ (1440) and $N^*_{1/2^-}$ (1535).

For the sake of completeness we also considered, along with the principle, in our model, interaction (25), some variants of the effective four-fermion interactions, discussed in the literature on the hadron spectroscopy [38,39]. In the context of our work they could be considered as residual interaction. Therefore, the constants of four-fermion interactions of different operator form (see below), x, α_G, $\alpha_{\rm eff}$ are considered here as phenomenological parameters, which are fitted to the baryon spectrum, rather than as the fundamental constants.

(1) The interaction, induced by the instantons [40], which in the sector of u and d quarks leads to the contact term [38,41]

$$V_{ij}^{\rm inst}(\rho)=-x(\rho_c m_q)^2\frac{1-\tau_i\tau_j}{4}\times\left[1+\frac{3}{8}\frac{\lambda_i\lambda_j}{4}(1+3\sigma_i\sigma_j)\right]\frac{\pi}{3m_q^2}\delta(\rho)\ .\tag{31}$$

(2) The interaction connected with the exchange of the effective heavy "gluon bunch" [39] which is approximated by the contact term

$$V_{ij}^G=\alpha_G\frac{\lambda_i\lambda_j}{4}\left[(1+\tfrac{2}{3}\sigma_i\sigma_j)\frac{1}{m_q^2b^2}\right]\frac{\pi}{m_q^2}\delta(\rho)\ .\tag{32}$$

(3) The chiral-invariant combination of the scalar and pseudoscalar four-fermion interactions of Ref. [33], which corresponds to the contact term

$$V_{ij}^{\rm eff}(\rho)=\alpha_{\rm eff}\left[1+\tfrac{1}{3}\sigma_i\sigma_j\tau_i\tau_j\frac{1}{m_q^2b^2}\right]\frac{\pi}{m_q^2}\delta(\rho)\ .\tag{33}$$

If we vary the phenomenological parameters x, α_G, $\alpha_{\rm eff}$, we can verify the stability of the results to changes in the interaction at small distances $\rho \leq \rho_c$, which is essential for the NN scattering at intermediate energies.

The inclusion of the interactions (31)–(33) does not change qualitatively the baryon spectrum (Table IV). An optimal set of the parameters corresponds to variant VI which has but a slight advantage over variant III, obtained with no account of the interactions (31)–(33). But in the $^3S_1$-wave phase shifts of the NN scattering (with no account of the tensor forces) the difference between variants III and VI is not felt. Hence, the obtained results are stable to changes of the interaction (31)–(33).

The $^3S_1$-waves phase shifts of the NN scattering were calculated in the energy interval $E_{\rm lab}=0-2$ GeV for all the interaction variants from Table III. The results obtained above the $\Delta\Delta$ threshold, $E_{\rm lab} \geq 1.2$ GeV, lie in the energy above the range of applicability of the model and are presented here to demonstrate qualitatively the transition of the $^3S_1$ phases into the Born region when the absolute value of the phase shifts starts to decrease.

It will be noted first of all that the coincidence of our results for variant Ia with the results of Ref. [7] for the same variant (in the standard RGM calculation with the channel coupling $NN+\Delta\Delta+CC$) indicates that the chosen approximation (7) is adequate. Figure 1 shows the phase shifts for variants I, Iα, II, III, and VI. From the comparison between the results for variants I, II, III one can see that the π- and σ-exchange interaction $V_{ij}^{\rm ch}$ leads to the required nucleon attraction ($^3S_1$ phases become negative each time when the interaction $V_{ij}^{\rm ch}$ is neglected). Despite the fact that the model parameters were not especially fitted to the NN data, we obtained a not bad description of the $^3S_1$-phase shifts in the whole energy interval $0<E_{\rm lab} \leq 1$ GeV which supplied the reliable phase-shift data [42]. (The underrated values of the $^3S_1$ phases at $E_{\rm lab} \leq 0.2$ GeV can be accounted for by the fact that the calculation did not include the tensor forces and the $^3S_1$-$^3D_1$ mixing.) The best description of the phase shifts in the region $0<E_{\rm lab} \leq 1$ GeV is obtained for variants III and VI, i.e., for the cases when the baryon spectrum is described most accurately.

The results for variants II, III, VI at low energies $E_{\rm lab} \leq 0.3$ GeV are rather close to the RGM calculations [8,11,12] where the σ exchange was included at the nucleon or quark level but the coupling constant $g_{\sigma qq}$ ($g_{\sigma NN}$) and the mass m_σ were fitted to the NN data. In our model the constant at the σqq vertex coincides with the πqq constant and no σ-exchange parameters whatever were fitted especially. If we go over to the nucleon level and calculate the effective σNN constant, averaging the σNN vertex in the quark wave functions, we shall get

$$(g_{\sigma NN}^{\rm eff})^2/4\pi = 9\alpha_{ch}\frac{4m_q^2}{m_\pi^2}=5.1$$

which is close to 5–8 used in the OBEP models [9,14,15]. However, the form factor at the σNN vertex (the same as at the πNN vertex), connected with the quark structure of the nucleon, automatically appears in our approach. This enables us to make progress in the NN interaction description, without introducing new parameters, from low energies, $E_{\rm lab} \leq 0.3-0.5$ GeV to intermediate energies, $E_{\rm lab} \approx 1$ GeV, where the form factors at the πNN and σNN vertices play the significant role.

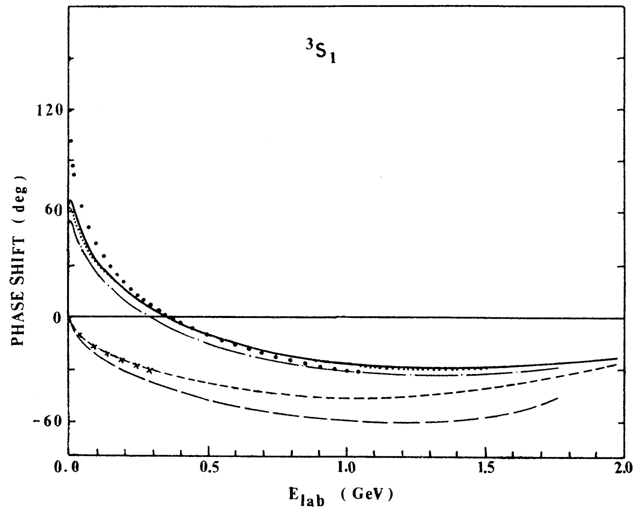

FIG. 1. $^3S_1$-wave phase shifts. The calculated results for variants I–III, VI from Table III: I (long dashed line). Ia (dashed line), II (dot-dashed line), III (solid line), VI (dotted line); the results of the phase-shift analyses [42] (circles). The results from Ref. [7] are shown by crosses.

In a wide energy range $0.1-0.2 \leq E_{\rm lab} \leq 1-1.5$ GeV the $^3S_1$ phases in each variant we considered, I–X, have approximately the same negative slope. In the present model this behavior is undoubtedly determined by the color-exchange interactions (17), (18) which are most essential at $r \leq r_{\rm conf}$. In Sec. V we analyze this phenomenon using the rich information about the six-quark wave functions obtained in the calculation. It will be recalled that for imitating the negative slope of the S- and P-phase shifts in "realistic" NN potentials [43] it is a common practice to use the phenomenology of the repulsive core. To understand just to which type of nucleon interaction there corresponds to the results of the quark calculations, it is necessary to have, first of all, the unambiguous procedure of projection of the six-quark wave function onto the NN channel. This problem is discussed in Sec. IV.

IV. PROJECTION OF THE SIX-QUARK FUNCTION ONTO THE NN CHANNEL

Compare the approach which employs the quark shell configurations (see Sec. II) with the multichannel RGM variant [6–8] where the trial wave function is written as

$$\psi_{6q}^{ST}(r_1,\ldots,r_6;k)=A\{N(123)N(456)\chi_{NN}(r)\}_{ST}+A\{\Delta(123)\Delta(456)\chi_{\Delta\Delta}(r)\}_{ST} + \sum_{S_1T_1}\sum_{S_2T_2} A\{C_{S_1T_1}(123)C_{S_2T_2}(456)\chi_{C_1C_2}(r)\}_{ST}\ . \quad (34)$$

In the RGM the relative motion function of baryons $\chi_\alpha(r)$ in the channels $\alpha=NN$, $\Delta\Delta$, CC satisfy the set of coupled equations

$$\sum_\beta \int [\mathcal{H}_{\alpha\beta}(\mathbf{r},\mathbf{r}')-E\mathcal{N}_{\alpha\beta}(\mathbf{r},\mathbf{r}')]\chi_\beta(\mathbf{r}')d^3r'=0 , \tag{35}$$

where

$$\begin{Bmatrix}\mathcal{H}_{\alpha\beta}(\mathbf{r}',\mathbf{r}'')\\ \mathcal{N}_{\alpha\beta}(\mathbf{r}',\mathbf{r}'')\end{Bmatrix}=\int d^3\rho_1 d^3\xi_1 d^3\rho_2 d^3\xi_2 d^3r A\{\delta(\mathbf{r}-\mathbf{r}')B_{\alpha_1}(\rho_1\xi_1)B_{\alpha_2}(\rho_2\xi_2)\}\begin{Bmatrix}H_q\\ 1\end{Bmatrix}A\{\delta(\mathbf{r}-\mathbf{r}'')B_{\beta_1}(\rho_1\xi_1)B_{\beta_2}(\rho_2\xi_2)\} \tag{36}$$

Here the functions χ_α are normalized by the relation

$$\sum_{\alpha\beta}\int\int \chi^*_\alpha(\mathbf{r})\mathcal{N}_{\alpha\beta}(\mathbf{r},\mathbf{r}')\chi_\beta(\mathbf{r}')d^3r\, d^3\mathbf{r}'=\begin{cases}\delta(E-E'), & E>0 ,\\ 1, & E<0 ,\end{cases} \tag{37}$$

The RGM representation (34) has a number of disadvantages when it is used in the overlap region of nucleons. The fact that the terms, entering in (34), are nonorthogonal to each other, is not, as a matter of fact, very essential—we know examples when the nonorthogonal basis proves to be versatile and effective [44]. What is worse is that in the very important overlap region of nucleons these terms merely repeat each other. We can illustrate it based on a more universal and powerful concept of quark configurations. For example, the configuration s^6 can be represented as the $0S$ state in any RGM channel.

$$\begin{aligned}|s^6[6]_X[2^3]_{CS}L=0\ S=1\ T=0\rangle_{\mathrm{TISM}}&=\sqrt{9}A\{\psi_N^{(0)}(\rho_1\xi_1)\psi_N^{(0)}(\rho_2\xi_2)\varphi_{00}(\mathbf{r})\}_{ST}\\ &=-\sqrt{45/4}A\{\psi_N^{(0)}(\rho_1\xi_1)\psi_N^{(0)}(\rho_2\xi_2)\varphi_{00}(\mathbf{r})\}_{ST}\\ &=\sqrt{9/2}A\{\psi_{C_1}^{(0)}(\rho_1\xi_1)\psi_{C_1}^{(0)}(\rho_2\xi_2)\varphi_{00}(\mathbf{r})\}_{ST}=\cdots .\end{aligned} \tag{38}$$

The second, more pithy example is the configuration s^4p^2 with a whole set of the states with different Young schemes $[f]_{CS}$, each of which can be represented as a superposition of RGM channel, for example,

$$\begin{aligned}|s^4p^2[42]_X[42]_{CS}L=0\ S=1\ T=0\rangle_{\mathrm{TISM}}=&-\sqrt{81/16}A\{\psi_N^{(0)}\psi_N^{(0)}\phi_{20}(r)\}_{ST}-\sqrt{9/8}A\{\psi_\Delta^{(0)}\psi_\Delta^{(0)}\varphi_{20}(r)\}_{ST}\\ &+\sqrt{9/4}A\{\psi_{C_1}^{(0)}\psi_{C_1}^{(0)}\varphi_{20}(r)\}_{ST}+\sqrt{45/16}A\{\psi_{C_2}^{(0)}\psi_{C_2}^{(0)}\varphi_{20}(r)\}_{ST} .\end{aligned} \tag{39}$$

Here

$$C_1=\{[21]_C S=\tfrac{1}{2}\ T=\tfrac{1}{2}:[1^3]_{CST}\} ,$$

$$C_2=\{[21]_C S=\tfrac{3}{2}\ T=\tfrac{1}{2}:[1^3]_{CST}\} .$$

The terms on the right-hand side of (39) are not orthogonal to each other and, therefore, the coefficients are not unambiguous. Besides, on the right-hand side of (39), one could use also the channel with orbitally excited baryons, N^*, N^{**}, Δ^*, etc., which have never been taken into account in the RGM. Thus, by multiplying the number of bound channels in the RGM, we merely repeat one and the same shell component rather than improve the quality of the many-particle wave function, when the question concerns the overlap region of nucleons. The attempts to make different baryon channels [different terms in (34)] orthogonal to each other [25] are completely ineffective in the indicated region since this reduces, to a considerable extent, to orthogonalization of the many-particle wave function to itself. These attempts lead only to an arbitrary redistribution of the weights of different quark configurations in the RGM channels regardless of the cluster dynamics of the system. For example, after this "orthogonalization " the node in the NN channel can appear or not in a completely arbitrary manner [25].

Of course, in the outer region, where the antisymmetrization effects are small, the difference between the baryon channels has the definite meaning and we use this fact in our combined approach [see Eqs. (6)–(8)].

We solve the problem of the projection of the six-quark wave function onto the NN channel (and other baryon channels) by means of the fractional parentage technique (or Racah's method) [4,45] which is employed as a rule to calculate the cluster spectroscopic factors S_x, widely used in nuclear physics for the analysis of the direct reactions on light nuclei [46]. Unfortunately, this technique is not well known among the theorists who are indirectly concerned with the problems of cluster spectroscopy. The paraodoxical, at first sight, detail of this method, which was long ago mentioned by Racah [45], is that the completely antisymmetrized wave function (for example, a superposition of the configurations s^6, s^4p^2, etc.) is expanded as a finite series in the nonantisymmetrized, but orthogonal basis consisting of the product of clusters with fixed quark numbers:

$$\Psi^d_{6q}(1,2,\ldots,6)=\sum_{B_iB_j}\Gamma^{CST}_{B_iB_j}\phi_{B_iB_j}(r)B_i(123)B_j(456) . \tag{40}$$

For example, the configuration $s^4p^2[42]_X[42]_{CS}$, which was first discussed in Refs. [22,23] is expanded in the orthogonal basis

$$
\begin{aligned}
&|s^4p^2[42]_X[42]_{CS}L=0\ S=1\ T=0\rangle_{\text{TISM}}\\
&=\sqrt{1/25}\{N(123)N(456)\}_{ST}\varphi_{20}(r)\\
&\quad+\sqrt{2/225}\{C_1(123)C_1(456)\}_{ST}\varphi_{20}(r)-\sqrt{4/225}\{C_2(123)C_2(456)\}_{ST}\varphi_{20}(r)\\
&\quad-\sqrt{1/45}\{C_2(123)C_2(456)\}_{ST}\varphi_{20}(r)-\sqrt{1/100}\{N^{**}(123)N(456)\}_{ST}\varphi_{00}(r)\\
&\quad-\sqrt{1/450}\{C_1^{**}(123)C_1(456)\}_{ST}\varphi_{00}(r)+\sqrt{1/225}\{C_1^{**}(123)C_2(456)\}_{ST}\varphi_{00}(r)\\
&\quad-\sqrt{1/180}\{C_2^{**}(123)C_2(456)\}_{ST}\varphi_{00}(r)+\sqrt{1/50}\{N(123)\tilde{N}^{**}(456)\}_{ST}\varphi_{00}(r)\\
&\quad+\sqrt{1/225}\{C_1(123)\tilde{C}_1^{**}(456)\}_{ST}\varphi_{00}(r)-\sqrt{2/225}\{C_1(123)\tilde{C}_2^{**}(456)\}_{ST}\varphi_{00}(r)\\
&\quad-\sqrt{1/90}\{C_2(123)\tilde{C}_2^{**}(456)\}_{ST}\varphi_{00}(r)-\sqrt{8/225}\{C_1'(123)\tilde{C}_1^{**}(456)\}_{ST}\varphi_{00}(r)\\
&\quad-\sqrt{32/225}\{C_1'(123)C_2^{**}(456)\}_{ST}\varphi_{00}(r)-\sqrt{1/50}\{\{N^*(123)N(456)\}_{ST}\varphi_{11}(r)\}_L\\
&\quad-\sqrt{1/225}\{\{C_1^*(123)C_1(456)\}_{ST}\varphi_{11}(r)\}_L+\sqrt{2/225}\{\{C_2^*(123)C_1(456)\}_{ST}\varphi_{11}(r)\}_L\\
&\quad+\sqrt{1/90}\{\{C_2^*(123)C_2(456)\}_{ST}\varphi_{11}(r)\}_L+\sqrt{8/225}\{\{C_1^*(123)C_1'(456)\}_{ST}\varphi_{11}(r)\}_L\\
&\quad+\sqrt{32/225}\{\{C_2^*(123)C_1'(456)\}_{ST}\varphi_{11}(r)\}_L+\sqrt{2/225}\{N^*(123)N^*(456)\}_{LST}\varphi_{00}(r)\\
&\quad+\sqrt{1/225}\{C_1^*(123)C_1^*(456)\}_{LST}\varphi_{00}(r)-\sqrt{2/225}\{C_1^*(123)C_2^*(456)\}_{LST}\varphi_{00}(r)\\
&\quad-\sqrt{1/90}\{C_2^*(123)C_2^*(456)\}_{LST}\varphi_{00}(r)+\sqrt{4/225}\{C_1^*(123)C_1'(456)\}_{LST}\varphi_{00}(r)\\
&\quad+\sqrt{16/225}\{C_2^*(123)C_1'^*(456)\}_{LST}\varphi_{00}(r)+\sqrt{1/9}\{C_2'^*(456)\}_{LST}\varphi_{00}(r)\\
&\quad+\sqrt{9/100}\{\Delta^*(123)\Delta^*(456)\}_{LST}\varphi_{00}(r)-\sqrt{1/50}\{C_3^*(123)C_3^*(456)\}_{LST}\varphi_{00}(r)\\
&\quad-\sqrt{2/25}\{C_3^*(123)C_4^*(456)\}_{LST}\varphi_{00}(r)-\sqrt{1/20}\{C_4^*(123)C_4^*(456)\}_{LST}\varphi_{00}(r)\ .
\end{aligned}
\tag{41}
$$

Here N, N^*, $\tilde{N}^{**}$, Δ^*, C_i, C_i^*, C_i^{**}, $C_i'^*$ are baryons possessing different spin-isospin and color structure (the number of asterisks corresponds to the number of the oscillator quanta of excitation; the complete enumeration of quantum numbers would occupy much space). In (41) the generalized fractional parentage coefficient is the function of relative motion of baryons ($\varphi_{00}(r),\varphi_{20}(r),\ldots$), multiplied by the fractional parentage coefficient (fpc) in the $XCST$ space. In the general case, it is a certain function $\Phi_{B_iB_j}(r)$ which describes the relative motion of baryons $B_i(123)$ and $B_j(456)$. In accord with the above mentioned it seems that it should be determined by the relation

$$
\Phi_{B_iB_j}(r)=\sqrt{6!/3!3!2}\{\langle B_i(123|\langle B_j(456)|\}_{ST}\times|\psi_{6q}^{LST}(123456)\rangle\ . \tag{42}
$$

The combinatorial factor on the right-hand side of (42) takes into effective account the identity of quarks of different baryons and leads to correct normalization of the function $\Phi_{B_iB_j}$ to the "effective number" of clusters B_i, B_j in the $6q$ system. It is just this normalization that is usually used in the cluster nuclear physics [46].

However, formally the expression (42) is inadequate since a probability density can only be calculated by projecting on states which are normalized to unity (δ function). But it needs a minimal modification for us to interpret this overlap as the wave function for relative motion. To calculate the normalization corresponding to a bra in Eq. (42) we can restore the antisymmetrization operator

$$
\mathcal{A}=\begin{bmatrix}6\\3\end{bmatrix}^{-1}\left[1-\sum_{i=1}^{3}\sum_{j=4}^{6}P_{ij}\right][1-P_{14}P_{25}P_{36}]\ ,\quad \mathcal{A}^2=\mathcal{A} \tag{43}
$$

in Eq. (42) and rewrite Eq. (42) in the form

$$
\begin{aligned}
\Phi_{B_iB_j}(\mathbf{r})=\begin{bmatrix}6\\3\end{bmatrix}^{1/2}&\int d^3\rho_1d^3\xi_1d^3\rho_2d^3\xi_2d^3r'\\
&\times\mathcal{A}\{\delta(\mathbf{r}-\mathbf{r}')B_i(\boldsymbol{\rho}_1\xi_1)B_j(\boldsymbol{\rho}_2\xi_2)\}^*\\
&\times\Psi_{6q}^{LST}(\mathbf{k},\mathbf{r}';\rho_1\xi_1\rho_2\xi_2)
\end{aligned}
\tag{42$'$}
$$

The normalization of the bra in Eq. (42′) is

$$
\left\langle\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\mathcal{A}\{\delta(\mathbf{r}-\mathbf{r}')B_iB_j\}\middle|\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\mathcal{A}\{\delta(\mathbf{r}-\mathbf{r}'')B_kB_j\}\right\rangle=\delta(\mathbf{r}'-\mathbf{r}'')-k(\mathbf{r}',\mathbf{r}'')\ ,
$$

where

$$
k(\mathbf{r}',\mathbf{r}'')=\begin{bmatrix}6\\3\end{bmatrix}\langle\delta(\mathbf{r}-\mathbf{r}')B_iB_j|P_{14}|\delta(\mathbf{r}-\mathbf{r}'')B_iB_j\rangle \tag{44}
$$

is the exchange kernel [47].

The modification of Eq. (42) which we look for consists in the inclusion of the renormalization factor $(\hat{1}-\hat{k})^{-1/2}$

in Eq. (42′) (see, Ref. [48]). We omit the antisymmetrization operator $\mathcal{A}$ since $\mathcal{A}\Psi_{6q}^{LST}=\Psi_{6q}^{LST}$ and rewrite Eq. (42′) in the form

$$\Phi_{B_iB_j}(\mathbf{r})=\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\int d^3\rho_1 d^3\xi_1 d^3\rho_2 d^3\xi_2 d^3r' \times B_i(\rho_1\xi_1)B_j(\rho_2\xi_2)(1-\hat{k})^{-1/2}_{r,r'} \times \Psi_{6q}^{LST}(\mathbf{k},\mathbf{r}';\rho_1\xi_1\rho_2\xi_2)\ . \qquad (42'')$$

Further we mean that the kernel $(1-\hat{k})^{-1/2}_{r,r'}$ of the operator $(1-\hat{k})^{-1/2}$ is included in the overlap (42). In our case (quark configurations s^6, s^4p^2, etc., as wave functions Ψ_{6q}^{LST}) the inclusion of the kernel $(\hat{1}-\hat{k})^{-1/2}_{r,r'}$ into the overlap (42) leads to a nonessential modification of both Eqs. (42) and (42′). To see it we express the kernel $(\hat{1}-\hat{k})^{-1/2}_{r,r'}$ in terms of harmonic-oscillator wave functions φ_{nl} (see, e.g., Ref. [49]) and obtain for $L=0$

$$\{(\hat{1}-\hat{k})^{-1/2}_{r,r'}\}^{L=0}=\sqrt{9/10}\varphi_{00}(r)\varphi_{00}(r')+\sqrt{81/82}\varphi_{20}(r)\varphi_{20}(r')+\cdots\ . \qquad (45)$$

This expression almost coincides with the analogous expansion of the unit operator

$$\{\hat{1}_{r,r'}\}^{L=0}=\frac{\delta(r-r')}{rr'}=\varphi_{00}(r)\varphi_{00}(r')+\varphi_{20}(r)\varphi_{20}(r')+\cdots \qquad (46)$$

which is the consequence of a small value of permutation matrix elements (44) in the CST space. If we substitute in Eq. (42″) fpc expansions of the type (41) for Ψ_{6q}^{LST} and the expansion (45) for $(\hat{1}-\hat{k})^{-1/2}$ we obtain that in the case of quark configurations s^6, s^4p^2, etc., the Fliessbach's formula (42″) differs from Eq. (42′) only in the factors $\sqrt{9/10}$ or $\sqrt{81/82}$ instead of the unit.

The functions $\Phi_{B_iB_j}(r)$ are quite analogous to the function of relative motion of cluster and residual nucleus, for example, in the quasielastic α-particle knock-out $A(p,p'\alpha)(A-4)$ [46]. Namely, we can write Eq. (41) for the six-quark deuteron by the general form, commonly used in the low-energy nuclear physics [4,28,46]

$$\Phi_{B_iB_j}(r)=\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\Gamma^{CST}_{B_iB_j}\phi_{B_iB_j}(r)\ . \qquad (47)$$

Here, $\Gamma^{CST}_{B_iB_j}$ are the fractional parentage coefficients in the CST space, calculated using the scalar factors of the Clebsch-Gordan coefficients of unitary groups (4) (see Refs. [4,26–29]). The quasielastic knock-out experiments $d(e,e'p)N^*$ and the deuteron fragmentation reactions $A(d,N^*)X$ at the intermediate energies (see, for example, Refs. [24]) offer an excellent possibility of the immediate experimental observation of individual terms in the expansion (41), as far as the high value of the final-state relative momentum $k\gtrsim 1$ GeV/c makes the final-state quark antisymmetrization not essential ($k\gg b^{-1}$, where $b\approx 0.5$ fm is the characteristic size of the three-quark system).

Here, the spectator N^* momentum distribution (i.e., the initial-state momentum distribution of the B_1B_2 mutual motion) is described, ideally, by the square of the Fourier transform of the function (42)

$$\bar{\Phi}_{B_1B_2}(\mathbf{k})=(2\pi)^{-3/2}\int\Phi_{B_1B_2}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d^3r \qquad (48)$$

(see the Appendix). Its normalization integral provides us with the corresponding "effective number" of baryons, mentioned above.

On the contrary, the RGM channel functions $\chi_{NN}(r)$, $\chi_{\Delta\Delta}(r)$, etc., do not have such a simple relation with the observed cross section (likewise the renormalized functions $\tilde{\chi}_\alpha=N^{1/2}_{\alpha\beta}\chi_\beta$, $\beta=NN$, $\Delta\Delta$, etc. [7,25]). The expressions (40) and (41) show that the configurations s^6, s^4p^2, etc., carry a lot of information about the quark degrees of freedom in the system. For example, if the inclusive [such as $d(A,N^*)X$] or exclusive [such as $d(e,e'p)N^*$] experiments detect as particle spectators the excited nucleons N^*, N^{**}, etc., with the characteristic momentum distributions, this will be a direct indication to the presence of the excited quark configuration s^4p^2 in the ground state of the system.

Of course, the number of shell-model configurations used in calculations cannot be very large and at the periphery of the system it is more convenient to represent the wave function in terms of RGM baryon channels—just as is done in Eq. (7). After these general comments we proceed to analyze our calculation in its different versions.

V. THE *NN*-SYSTEM WAVE FUNCTION ACCORDING TO THE QUARK CALCULATIONS

Using the technique of projection onto the NN and other BB channels (42), we show in Figs. 2(a)–2(e) the NN-scattering wave functions for different energies $E_{\text{lab}}=5,200$, 1000, 1500, and 2000 MeV for variant III which brings the phase shifts close to the experimental. (The results for the other optimal variant VI do not differ from variant III). Figures 2(a)–2(e) show in the form of projections onto the NN channel the complete wave function $u_{6q}^{L=0ST}$ and its components: (1) the shell component $s^6[6]_X[2^3]_{CS}$ which is symmetric in the x space; (2) the additional component $\tilde{u}_{6q}^{LST}$ which belongs to mixed symmetry in the x space

$$\tilde{u}_{6q}^{LST}=u_{6q}^{LST}-a_{[6]_X}|s^6[6]_X[2^3]_{CS}LST\rangle\ . \qquad (49)$$

By definition, these components are orthogonal to each other $\langle\tilde{u}_{6q}^{LST}|s^6[6]_X[2^3]_{CS}LST\rangle=0$ and the component $\tilde{u}_{6q}^{LST}$ consists of the asymptotic part of wave function (8) which will be written in what follows as $A\{\psi_N\psi_N\chi_{as}(r)\}$, without specifying the form of $\chi_{as}(r)$ and the superpositions of the remaining five states of the shell configuration $s^4p^2[42]_X$

$$\tilde{u}_{6q}^{LST}=\sum_{f_{cS}}\tilde{a}_f|s^4p^2[42]_X[f_{CS}]LST\rangle+A\{\psi_N\psi_N\chi_{as}(4)\}\ . \qquad (50)$$

Here the coefficients $\tilde{a}_f$ differ from the calculated a_f. The coefficients $\tilde{a}_f$ include the correction for the overlap of the configurations s^6 and $s^4p^2[42]_X$ with the asymptotic part of the wave function [hence, the first and second

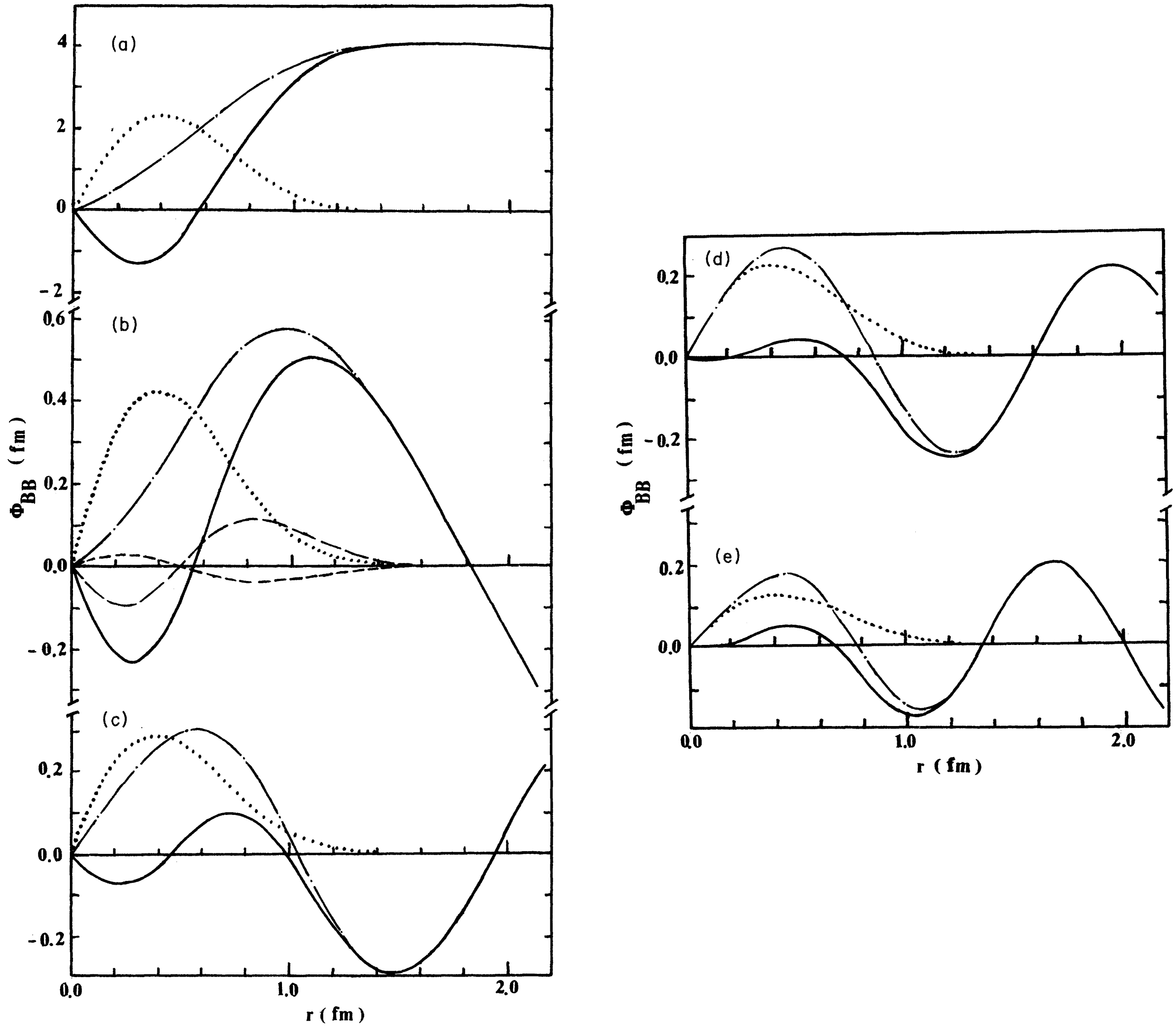

FIG. 2. Projections of the six-quark wave functions onto the NN channel. The projections of the complete wave function u_{6q}^{LST} (dot-dashed line) and its components, $\tilde{u}_{6q}^{LST}$ (solid line) and s^6 (dotted line), are shown at energies $E_{\text{lab}}=5$ MeV (a), 200 MeV (b), 1000 MeV (c), 1500 MeV (d), and 2000 MeV (e). The projections of $\tilde{u}_{6q}^{LST}$ onto $\Delta\Delta$ (dashed line) and CC (long dashed line) are shown at energy $E_{\text{lab}}=200$ MeV.

term (49) are orthogonal to each other].

Figure 3 shows the projections of the component $\tilde{a}_{[6]_X}|s^6[6]_X[2^3]_{CS}LST\rangle$ onto the NN, $\Delta\Delta$, and CC channels [for the case of $E_{\text{lab}}=200$ MeV, shown in Fig. 2(b)] and Fig. 2(b)—the analogous projections for the component $\tilde{u}_{6q}^{LST}$ at an energy $E_{\text{lab}}=200$ MeV. (At other energies the relation between the projections NN, $\Delta\Delta$, CC does not change qualitatively.) We see that the components $s^6[6]_X$ and $\tilde{u}_{6q}^{LST}$ differ qualitatively in relative value of the amplitudes of the functions $\Phi_{NN}(r)$ and $\Phi_{BB}(r)$, where $BB=\Delta\Delta$, C_iC_j. The symmetric component $s^6[6]_X$ is projected onto any of the baryon channels NN, $\Delta\Delta$, CC with the weights close to unity (i.e., none of the baryon channels is predominant) whereas the component $\tilde{u}_{6q}^{LST}$ is projected mainly onto the NN channel ($\tilde{\Phi}_{NN} \gg \tilde{\Phi}_{\Delta\Delta}, \tilde{\Phi}_{CC}$) not only in the asymptotic region but in the overlap region, $r \le r_{\text{conf}}$ as well. This means that in the configuration $s^4p^2[42]_X$ there forms (dynamically) a such superposition of states $[f_{CS}]$, which corresponds to the predominantly clustered wave function in the form of $N+N$ with the other nonexcited baryon cluster, $\Delta+\Delta$ and C_i+C_j, being suppressed. This is not connected with some kind of special status of the configuration s^4p^2, since each basis state in this configuration belonging to a definite Young scheme $[f_{CS}]$, does not possess the predominant NN-clusterized NN states and its projec-

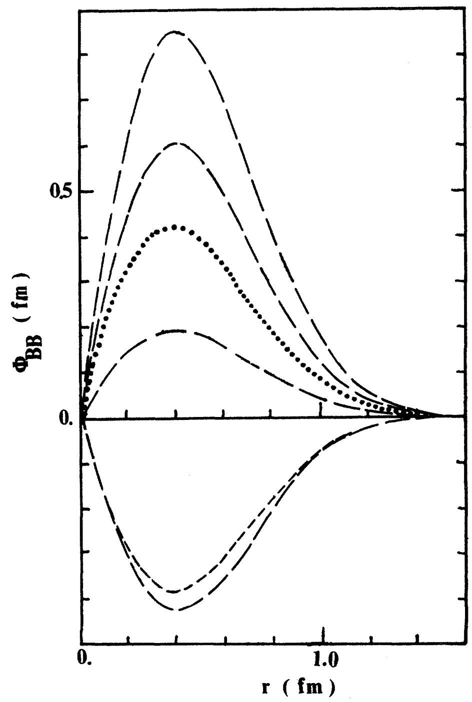

FIG. 3. The projections of the s^6 component at $E_{\rm lab}=200$ MeV. The notations are the same as in Fig. 2.

tions onto baryon-baryon channels are uniformly distributed [see the series expansion (41)].

Thus, upon solving the set of equations (12), we get that the NN-scattering wave function in a wide energy range has a characteristic structure—it consists of two qualitatively different components. The component $\tilde{u}_{6q}^{LST}$, defined in (50), is the clustered NN state which in the region of small distances is described by the superposition of states in the configuration s^4p^2 [see (50)] and has (in the projection onto the NN channel) a node at distances $r\approx b$ (see Fig. 2). The second component is the symmetric shell states $s^6[6]_X$ which is more like a $6q$ bag (none of the baryon channels is predominant in it). Both the components coexist in the region of distances $r\leq 2b$ and for the understanding of the NN dynamics at small distances this effect of mixing of the configurations s^6 and s^4p^2 is very essential (this was first noted in the paper by Harvey [26]).

Here we note that the configuration s^4p^2 plays the significant role in the NN scattering. The nodal position of the wave function

$$\tilde{\Phi}_{NN}(r)=\sqrt{10}\langle N(123)|N(456)|\tilde{u}_{6q}^{LST}\rangle \qquad (51)$$

at small distances $r\approx b$ is stable in a wide energy interval $0<E_{\rm lab}\leq 1$ GeV (Fig. 2) which accounts for the negative slope of the phase shifts in the $^3S_1$ waves up to energies $E_{\rm lab}\leq 1.2$ GeV. Only at the energies $E_{\rm lab}\approx 1.2-1.5$ GeV, when the relative weight of the component $s^6[6]_X$ starts to decrease [Figs. 2(d) and 2(e)] and the relative sign of the components $s^6[6]_X$ and $\tilde{u}_{6q}^{LST}$ changes, the $^3S_1$-phases start to decrease slowly in absolute value (reach the Born region).

These nodal wave functions $\tilde{\Phi}_{NN}(r)$ could play a key role in the optical model description of NN scattering [17–19] mentioned in the Introduction.

In the P waves the wave function seems to have the analogous structure (indicative of the constant negative slope of the experimental phase shifts at $E_{\rm lab}\geq 0.3-0.5$ GeV). To make sure, it is necessary to perform microscopic calculations including the contribution of the configurations $s^5p[51]_X$ and $s^3p^3[3^2]_X$. Thus, the calculations should be continued to include $L=3$ and it is necessary to take into consideration the tensor forces.

In the present work we obtained not only the absolute values of the amplitudes of the configurations s^6 and s^4p^2, entering into the wave function of the NN system, but also their relative sign (negative) which remains unchanged up to energies $E_{\rm lab}\approx 1-1.2$ GeV. There is no doubt that in the deuteron the relative sign will also be negative (and the preliminary calculations [50] confirm it). The latter is especially important since it permits one to understand that in the deuteron electromagnetic form factors the contributions of the s^6 and s^4p^2 configurations will interfere destructively [51,52]. Just because of the destructive interference of the quark contributions the deuteron magnetic form factor takes on a zero value in the region $Q^2\approx 2$ GeV$^2/c^2$ and, accordingly, rapidly decreases in absolute value at $Q^2\leq 0.8-1$ GeV$^2/c^2$. This behavior of the magnetic form factor is quite different from the previous predictions made in the hybrid model [53,54] in which the $6q$ bag (configuration s^6) and the NN component are spaced apart and their contributions interfere weakly in the electromagnetic processes.

According to our calculations, the node of the NN component of the wave function is stable only up to energies $E_{\rm lab}\approx 1$ GeV, and at higher energies it starts to shift into the region of small r [Figs. 2(d) and 2(e)]. As a result, the $^3S_1$ phase fails to pass through an additional interval π prior to its transition into the Born region, as it is predicted by the phenomenological FSP models [17,18]. However, it should be kept in mind that in our model we use the effective qq potentials, which well describe the baryon spectrum only in the region of lowest excitations (the mass region of 0.94–1.6 GeV), and the use of these potentials at higher energies is not guaranteed at all. Besides, we use the nonrelativistic approach and in the region $E_{\rm lab}>1$ GeV this is not justified at all. Further, the contact interactions (31)–(33), whose contribution should increase with increasing energy and momentum transfer, is used in our calculations on a limited basis (configurations s^6 and s^4p^2) which strongly deforms the contributions of the δ-shaped functions. At the same time, if the basis is extended, the interactions, proportional to the δ functions, will lead to the collapse of not only the $6q$ but also $3q$ system. Thus, to describe adequately the region of intermediate energies $E_{\rm lab}\geq 1$ GeV it is necessary to have a more detailed description of the interaction at $r\rightarrow 0$ and to carry out calculations by means of the adequate procedure of regularization of the matrix elements of the interactions (17) and (31)–(33). [Here we have carried it out only for the effective interaction (20) which at $F(Q^2)\equiv 1$ would generate the singular terms $\sim\delta(\rho)$, $\sim 1/\rho^3$.]

Finally, there exists the intriguing problem of the colored van der Waals interaction between nucleons [55]. It gives a rather strong attraction and is based just on the excited quark configurations of each nucleon $s^2p[21]_X[21]_C$ which is in line with the discussed phenomenology [17]. However, the present status of this problem is not free of some difficulties with the gauge invariance.

In view of these reasons our results at $E_{\rm lab}\geq 1$ GeV should not be considered as contradicting the FSP model [17] in which the nodal position of the NN-wave function in the S and P waves is stable at least in the energy range $E_{\rm lab}\approx 0-4$ GeV and the Born region starts at $E_{\rm lab}\geq 4-5$ GeV [17]. It seems that experiment should play here the decisive role. If S and P phases do pass through an additional interval π, this would be a vivid quantitative manifestation of the quark degrees of freedom in the NN interaction and, also, this would be indicative of the significant contribution of the contact terms of the attractive qq interaction. Note that the phenomenological FSP model [17], which agrees very well with the differential cross sections and the polarizations in the NN scattering in a wide energy interval $0\leq E_{\rm lab}\leq 5$ GeV, proceeds exactly from this behavior of the S- and P-phase shifts and a more sluggish behavior of the phase shifts in the highest partial waves $L\geq 2$.

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APPENDIX

We shall demonstrate that in the impulse approximation the amplitude of quasielastic knock-out of a baryon in the reaction $d(e,e'B_1)B_2$ is proportional to the Fourier transform of the wave function (42). If the momentum k is greater than the characteristic quark scale b^{-1} ($b\approx 0.5$ fm, $k\gg b^{-1}$) the final-state wave function can be written as the antisymmetrized plane wave,

$$\Psi_{6q}^{B_1B_2}(\mathbf{k},\mathbf{q};\mathbf{r}_1,\ldots,\mathbf{r}_6)=\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\mathcal{A}\{B_1(123)B_2(456)(2\pi)^{-3}e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{X}}\}$$

normalized, as usual, to the δ function. Here $X=\frac{1}{6}\sum_{i=1}^{6}r_i$ is a coordinate of the center of mass, q is the photon momentum received by the baryon B_1 in the process of quasielastic knock-out, $\mathcal{A}$ is antisymmetrization operator (43) normalized like a projector $\mathcal{A}^2=\mathcal{A}$. In the impulse approximation the amplitude of quasielastic knock-out of the baryon B_1 is of the form

$$T=\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\langle\mathcal{A}\{B_1(123)B_2(456)(2\pi)^{-3}e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{X}}\}|\times\sum_{j=1}^{6}t_je^{i\mathbf{q}\cdot\mathbf{r}_j}|\Psi_{6q}^{d}(1,\ldots,6)\rangle\ , \qquad (A1)$$

where $t_je^{i\mathbf{q}\cdot\mathbf{r}_j}$ is the interaction operator of an incident electron with the jth quark. We shall write the quark wave function of the deuteron $\Psi_{6q}^d(1,\ldots,6)$ as the fractional parentage cluster expansion (40) and make use of the interaction symmetry with respect to quark permutation which allows the antisymmetrization operator $\mathcal{A}$ to be transposed to the right-hand side of the matrix element (A1) and next exclude it, considering that $\mathcal{A}^2=\mathcal{A}$ and $\mathcal{A}\Psi_{6q}^d=\Psi_{6q}^d$. The elementary calculations give

$$T=2^{-1}\begin{bmatrix}6\\3\end{bmatrix}^{1/2}\left[\sum_i\Gamma_{B_iB_2}^{CST}\langle B_1(123)|3t_3e^{-(2i/3)\mathbf{q}\cdot\boldsymbol{\xi}_1}|B_i(123)\rangle\bar{\phi}_{B_iB_2}(\mathbf{k}-\mathbf{q}/2)+(-1)^{1+S+T}\sum_j\Gamma_{B_2B_j}^{CST}\langle B_1(456)|3t_6e^{-(2i/3)\mathbf{q}\cdot\boldsymbol{\xi}_2}|B_j(456)\rangle\bar{\phi}_{B_2B_j}(-(\mathbf{k}-\mathbf{q}/2))\right].$$

We made use of the fact that, owing to the symmetry of the expression (A1) with respect of the quark numbers, it is possible to make a replacement

$$\sum_{j=1}^{6}t_je^{i\mathbf{q}\cdot\mathbf{r}_j}=3t_3e^{i\mathbf{q}\cdot\mathbf{r}_3}+3t_6e^{i\mathbf{q}\cdot\mathbf{r}_6}\ .$$

in which case $\mathbf{r}_3=\mathbf{X}+\mathbf{r}/2-2\boldsymbol{\xi}_1/3$ and $\mathbf{r}_6=X-\mathbf{r}/2-2\boldsymbol{\xi}_2/3$. The matrix elements

$$\langle B_1(123)|3t_3e^{-(2i/3)\mathbf{q}\cdot\boldsymbol{\xi}_1}|B_i(123)\rangle$$

and

$$\langle B_1(456)3t_6e^{-(2i/3)\mathbf{q}\cdot\boldsymbol{\xi}_2}|B_j(456)\rangle$$

are amplitudes of the electroproduction of the baryon B_1 from the clusters $B_i(123)$ or $B_j(456)$, entering into the right-hand side of Eq. (40). If we restrict ourselves to the diagonal transition $B_1\to B_1$, we shall get a very simple expression

$$T=F_{B_1}(q^2)\bar{\Phi}_{B_1B_2}(k-q/2)\ , \qquad (A2)$$

where $\mathbf{k}-\mathbf{q}/2$ is the recoil momentum of the baryon spectator B_2, $F_{B_1}(q^2)=\langle B_1|\sum_{j=1}^{3}t_je^{i\mathbf{q}\cdot\mathbf{r}_j}|B_1\rangle$ is the form factor of the baryon quasielastic knock-out. The terms of the expression (A1), neglected in (A2), are the corrections for the electroproduction of the baryon B_1 from the initial-state virtual baryons B_i, $i\neq 1$ or B_j, $j\neq 1$. They can be, in principle, taken into account too.

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