Semiclassical calculation for $^{16}O+^{16}O$ elastic scattering at high energies

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A semiclassical method has been used to calculate terms of order $\hbar^2$, the $^{16}O+^{16}O$ elastic-scattering cross section for the c.m. energy range 35–90 MeV for various optical model potentials. We find that the calculated results with an energy-dependent absorptive part along with an angular momentum cutoff agree qualitatively with the experimental results.

The availability of extensive experimental data on elastic scattering of the $^{16}O+^{16}O$ system in the low- [1] as well as the high- [2] energy regions has inspired considerable theoretical studies. Some authors use direct numerical calculations with different parametrizations of optical potentials. At higher energies, the potentials assumed have some features different from those found suitable for fitting only low-energy data [1,3]. Chatwin *et al.* [4] and Halbert *et al.* [2] introduced an L-dependent cutoff in the absorptive part to cover both the low- and high- energy data. The agreement with experimental results at higher energies was, however, only qualitative. Considering the Hauser-Feshbach treatment, Pocanic *et al.* [5] have shown the equivalence of the energy dependence of the absorptive potential with its L dependence which, however, is not evident for results at higher energies. Pantis *et al.* [6], on the other hand, have considered an energy dependence of the imaginary radius and diffuseness parameters, which favor possible resonances of near grazing partial waves. Although their parametrization improves the results, the quantitative agreement is still not satisfactory. The problem obviously needs further investigation.

A second class of investigations make use of semiclassical techniques. The method of Miller and Good [8], generalized substantially by a number of authors, [9–18], has been particularly helpful in this connection. A semiclassical method, however, cannot be applied to the heavy-ion scattering problem in a straightforward manner. The optical potential leads to a large number (often infinite) of complex trajectories. Knoll and Schaeffer [19] have given some guidelines for selecting the relevant trajectories, but it is difficult to follow the prescription in a realistic problem. It may be useful, for a preliminary calculation, to follow the conventional perturbative method. This restricts the path integration along a real trajectory, with the turning point determined entirely by the real part of the potential. The imaginary part here only supplies a damping factor to each of the partial waves. The purpose of this paper is to report the results of a calculation done with this approximate semiclassical method for the $^{16}O+^{16}O$ elastic scattering at a higher energy region. A number of potential models have been suggested in the literature in this connection. Gobbi *et al.* [3] considered a four-parameter potential that leads to a comparable fit with experimental results as is obtained with the six-parameter potential (with an L dependence) of Chatwin *et al.* [4] at lower energies. At higher energies, Chatwin potential shows better agreement with the experimental results [2]. Halbert *et al.* [2] also showed that the four-parameter potential of Maher *et al.* [1] gives a poor fit. The potential of Pantis *et al.* [6] gives a better fit, but the agreement is still not satisfactory. As a variation, we have considered also a hybrid potential in which we have introduced an L dependence of the imaginary potential W, while retaining the parameters considered by Maher *et al.* [1]. The calculated results with this potential show better agreement with experimental results. This should, however, be treated only as an indication of the trend of the exact results, because a perturbative treatment of a large absorptive potential is not expected to be accurate.

In the actual calculation, we have made a departure [7] from the usual trend in connection with the choice of the Coulomb part of the potential. The Coulomb potential is usually taken as (a) that between a point charge and a sphere of uniform density or (b) that between two uniformly charged spheres of appropriate radii. The approximation is not good for nuclei like ^{16}O and ^{12}C having modified harmonic-well type of charge distributions. It is well known that the elastic scattering of heavy ions depends more crucially on the real part of the potential around a critical distance $R \sim 1.5(A_1^{1/3}+A_2^{1/3})$ fm. Thus the elastic scattering normally places a weak constraint on the potential. The choice of the correct Coulomb potential will, however, be useful when the entire range of experimental results, for elastic scattering, fusion, and transfer reactions, are sought to be explained with the same set of potential parameters. Keeping this in view, we have considered in this paper two cases: (a) the case of two uniformly charged spheres of radii $(\sqrt{5/3})R_{\rm rms}$ and (b) the case of two diffuse charge distributions of the modified harmonic-well type, as is used in generalized shell model (GSM). We note that the Coulomb potential between two spherical nuclei A and B with charge form factors $f_1(q)$ and $f_2(q)$, $\mathbf{q}$ being the

momentum transferred, is given by

$$V_C(r)=\frac{2Z_1Z_2e^2}{\pi r}\int_0^\infty dq\left[\frac{\sin qr}{q}\right]f_1(q)f_2(q)\ . \quad (1)$$

We consider two special cases: Case I: The colliding nuclei have charge distributions given by the generalized shell model (GSM).

For nuclei with an incomplete $1p$ shell, the charge distribution is given by [20]

$$\rho(r)=\frac{2}{\pi^{3/2}}\frac{1}{b_0^3(2+3\alpha)}\left[1+\alpha\frac{r^2}{b_0^2}\right]\exp\left[\frac{-r^2}{b_0^2}\right], \quad (2)$$

where $b_0=(\hbar^2/M\epsilon)^{1/2}$, and ϵ is the energy interval between two consecutive levels of the harmonic oscillator. The rms radius of the distribution is given by $a=b_0[3(2+5\alpha)/2(2+3\alpha)]^{1/2}$ with $\alpha=(Z-2)/3$. The form factor for the distribution (2) is given by

$$F(q)=\left[1-\left[\frac{\alpha q^2b_0^2}{2(2+3\alpha)}\right]\right]\exp\left[-\frac{q^2b_0^2}{4}\right], \quad (3)$$

which gives for the Coulomb potential

$$V_C(r)=Z_1Z_2e^2\left[\frac{1}{r}\mathrm{erf}\left[\frac{r}{2B}\right]+(\beta+\gamma r^2)e^{-r^2/4B^2}\right], \quad (4)$$

where

$$\beta=[3A_1A_2-2(A_1+A_2)B^2]/4B^5\sqrt{\pi}\ , \quad (5)$$

$$\gamma=-A_1A_2/8B^7\sqrt{\pi},\quad B^2=B_1^2+B_2^2\ , \quad (6)$$

$$A_i=\alpha_i b_{0i}^2/[2(2+3\alpha_i)],\quad B_i^2=b_{0i}^2/4\ , \quad (7)$$

the subscript $i(i=1,2)$ referring to the ith nuclei.

Case II: When both colliding nuclei have uniform charge distributions of density ρ_0 and radius R, the potential is given by

$$V_C(r)=\begin{cases}\dfrac{Z^2e^2}{r}\left[1+\dfrac{1}{160R^6}(30r^4R^2-r^6-80R^3r^3+192R^5r-160R^6)\right], & r<2R,\\ \dfrac{Z^2e^2}{r}, & r>2R\ .\end{cases} \quad (8)$$

We consider the following nuclear potentials for the $^{16}O+^{16}O$ system, all of which have the general form

$$V_N(r)=\frac{-(V_0+iW_0)}{1+\exp[(r-R_0)/a_0]}\ . \quad (9)$$

Case A: We consider here the optical model potentials [2,4] with

$$W_0=\overline{W}\{1+\exp[(L-L_c)/0.4]\}^{-1}\ , \quad (10)$$

and $\overline{W}=-7.0+0.5E_{\mathrm{c.m.}}$. Here L_c is the cutoff angular momentum given [2] by $L_c=\overline{R}[2\mu(E+\overline{Q})]^{1/2}\hbar$ with $\overline{R}=9.8$ fm and $\overline{Q}=-20$ MeV.

Case B: We consider next a potential obtained by combining forms of Chatwin *et al.* [4] and Maher *et al.* [1]. Thus W_0 is given by Eq. (10) but $\overline{W}=0.4+0.1E_{\mathrm{c.m.}}$. Our motivation for this choice is to check if any improvement of the results at high energies follows without disturbing the results at lower energy. This may be possible provided $L_c>L_i$, where L_i are the angular momenta of the partial waves dominating the scattering processes at low energies.

Case C: We consider the optical potential by Pantis *et al.* [6] with

$$W_0=C_1E^{-5/2}\exp(2\sqrt{2C_2E}\ )\ , \quad (11)$$

which follows from a statistical model of the nucleus [21]. The imaginary radius and the diffuseness parameters considered here have explicit energy dependence, viz.,

$$r_0=r_0'\{1+\beta_1\exp[(E-E_0)/\beta_2]\}^{-1},\quad a_0=\gamma E\ , \quad (12)$$

where $\beta_1,\beta_2,E_0,r_0',\gamma$ are constants. We have taken the values of all these constants from Ref. [6].

To calculate the cross section, we follow the generalized Miller-Good method [9–18] in the conventional perturbative approach. The contribution of the imaginary part in this approximation is contained in a damping factor $e^{-2S(L)}$ for each partial wave of angular momentum L, where

$$S(L)=\frac{2\mu}{\hbar^2}\int_{r_0}^\infty\frac{V_I dr}{2[K^2-(2\mu/\hbar^2)V_R-L(L+1)/r^2]^{1/2}}\ . \quad (13)$$

Apart from this damping factor, the problem is now similar to that of a real potential. The phase shifts in zeroth order can be written as $\sigma_L^0=\sigma_L^C+\sigma_L^{\mathrm{diff}}$, where σ_L^C is the Coulomb phase shift and

$$\sigma_L^{\mathrm{diff}}=(L+\tfrac{1}{2})\left[\frac{\pi}{2}-\sin^{-1}\left[\frac{n\bar{y}+(L+\frac{1}{2})^2}{\bar{y}\sqrt{n^2+(L+\frac{1}{2})^2}}\right]\right]+n\ln\left[\frac{\bar{y}-n+\sqrt{\bar{y}^2-2n\bar{y}-(L+\frac{1}{2})^2}}{\sqrt{n^2+(L+\frac{1}{2})^2}}\right]-\sqrt{\bar{y}^2-2n\bar{y}-(L+\tfrac{1}{2})^2}+\int_{y_t}^{\bar{y}}\sqrt{t_1^L(y)}dy\ , \quad (14)$$

where

$$t_1^L(y)=1-\frac{(L+\frac{1}{2})^2}{y^2}-\frac{2\mu}{\hbar^2K^2}(V_N'+V_C)\ , \quad (15)$$

TABLE I. Phase shifts (radians) for $E_{c.m.}=60$ MeV and $\theta_{c.m.}=90°$.

	Zeroth order		With first-order correction	
L	GSM	Uniform	GSM	Uniform
0	13.1553	13.2434	13.1422	13.2279
2	13.2079	13.2912	13.0497	13.1435
10	13.8466	13.8714	13.8586	13.7005
30	13.6127	13.6106	13.5732	13.5711
36	13.3247	13.3247	13.3319	13.3319

with $V_N'=\mathrm{Re}(V_N)$, V_C the Coulomb part of the potential, n the Sommerfeld parameter, and y_t the classical turning point, given by $t_1^L(y_t)=0$. We have taken the outermost turning point, real and positive, for each L for evaluating the integrals. The quantity $\bar{y}$ is a chosen large value of y.

The correction term of order $\hbar^2$ is given by

$$\Delta_L=\tfrac{1}{12}\int_{y_t}^{\infty}\mathcal{D}[\tau_1(y)]\sqrt{t_1^L(y)}\,dy - \tfrac{1}{12}\int_{z_t}^{\infty}\mathcal{D}[\tau_2(z)]\sqrt{t_2^L}\,(z)dz\ , \tag{16}$$

where

$$t_2^L(z)=1-\frac{(L+\frac{1}{2})^2}{z^2}-\frac{2n}{z}\ , \tag{17}$$

$$\tau_1(y)=y^2t_1^L(y),\quad \tau_2(z)=z^2t_2^L(z)\ , \tag{18}$$

and

$$\mathcal{D}(\tau_i)=\frac{\tau_i''''}{\tau_i'^2}-4\frac{\tau_i'''\tau_i''}{\tau_i'^3}+3\frac{\tau_i''^3}{\tau_i'^4}\ , \tag{19}$$

where primes indicate differentiation with respect to respective arguments. In calculating terms of order $\hbar^2$, there is a well-known problem of divergence at the extrema of τ. These have been tackled following the method of Wald and Lu [12]. This essentially consists in dividing the range of integration into parts and making use of partial integrations to remove τ' from the denominator of the integrand in the segment where it vanishes.

We have calculated the phase shifts and the symmetrized cross sections for the $^{16}O+^{16}O$ elastic scattering in the energy range $E_{c.m.}=35\sim90$ MeV. For the GSM charge distribution, we choose $\alpha=2$, $a=2.625$ fm and for the uniform charge distribution, we choose $R=3.39$ fm. Our results are as follows:

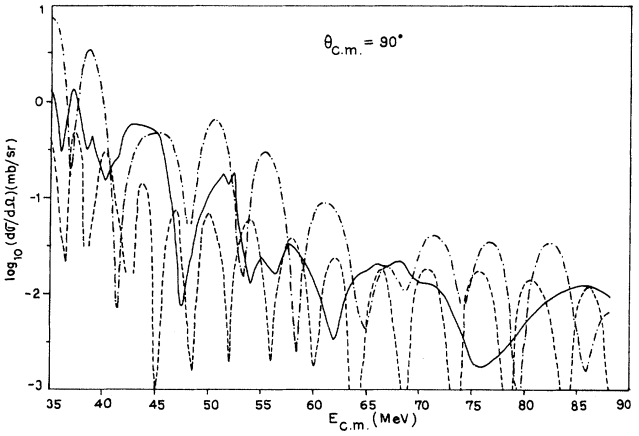

FIG. 1. $O^{16}+O^{16}$ elastic-scattering excitation functions. The solid line shows the experimental results. The dashed curve shows the theoretical results of Ref. [2] and the dash-dotted curve the semiclassical results, both using potential A.

(a) Effect of the charge distribution: The phase shifts for the GSM and uniform charge distributions show small differences as indicated in Table I. This small difference of phase-shifts may not be sharply reflected in the excitation function or through angular distribution for elastic scattering. However, the inaccuracy of the potential parameters may affect the cross sections for other processes, viz., fusion or transfer processes. It is, therefore, useful to consider the realistic GSM charge distribution for nuclei like ^{16}O and ^{12}C.

(b) Energy dependence of the cross section: The experimental results of Ref. [2] have been used for comparison with our calculated results, obtained with the GSM Coulomb charge distribution and the nuclear potentials shown in Table II. We note the strength of the imaginary potentials in the cases considered. In case A, at $E_{c.m.}=90$ MeV, $38\geq W_0\geq0$ MeV for $L=0$–58 and at $E_{c.m.}=35$ MeV, $10.5\geq W_0\geq0$ MeV for $L=0$–30. In case B, at $E_{c.m.}=90$ MeV, $9.4\geq W_0\geq0$ MeV for $L=0$–58 and at $E_{c.m.}=35$ MeV, $3.9\geq W_0\geq0$ MeV for $L=0$–30. In case C, W_0 is L independent, and at $E_{c.m.}=90$ MeV, $W_0\sim115$ MeV and at $E_{c.m.}=35$ MeV, $W_0\sim15$ MeV. Thus the magnitude of W_0 is lowest in case B, and rather large in case C. The accuracy of the

TABLE II. Parameters of optical potential for $^{16}O+^{16}O$ elastic scattering. $R_0=r_0(16^{1/3}+16^{1/3})$.

V_N	V_0 (MeV)	W_0 (MeV)	r_0(Re) (fm)	a_0(Re) (fm)	r_0(Im) (fm)	a_0(Im) (fm)
A	16.0	$\dfrac{-7.0+0.5E}{1+\exp[(L-L_c)/0.4]}$	1.349	0.49	1.349	0.49
B	17.0	$\dfrac{0.4+0.1E}{1+\exp[(L-L_c)/0.4]}$	1.349	0.49	1.349	0.49
C	17.0	Eq. (11)	1.349	0.49	Eq. (12)	Eq. (12)

first-order perturbative results in case C is, therefore, not expected to be good and this may explain partly the poor agreement of the calculated results for potential *C* with data. We note that (Fig. 1) our semiclassical results for case A agree qualitatively with the calculated results of Halbert *et al.* [2]. The quantitative disagreement may be caused by the approximate nature of the semiclassical method followed as well as by the slightly different Coulomb fields considered. We also note from Fig. 2 that the potential *B* appears to give better agreement than potential *A* at least for higher energies. The number of peaks is less here and the overall trends of results for case B follow those of the experimental data. The complexity of the inelastic processes involved can be seen as none of the parametrizations fit data over a wide range of energies. The method of calculation, however, leaves scope for refinement. The imaginary part of the potential has not been dealt with properly. Note that an evaluation of the first-order correction term in $\hbar^2$ becomes difficult if one chooses complex trajectories. In a semiclassical treatment, each radial equation has to be treated separately for its special features, and it may not be worthwhile to go for a semiclassical calculation in all cases. The perturbative semiclassical method, nevertheless, can play a useful role within its range of applicability.

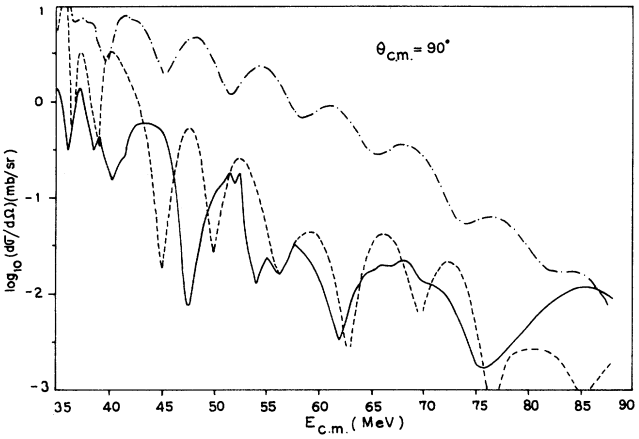

FIG. 2. $O^{16}+O^{16}$ elastic-scattering excitation functions. The solid line shows the experimental results. The semiclassical results are shown by dashed curve (potential *B*) and dash-dotted curve (potential *C*).

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[1] J. V. Maher, M. W. Sachs, R. H. Siemssen, A. Weidinger, and D. A. Bromley, Phys. Rev. **188**, 1665 (1969).
[2] M. L. Halbert, C. B. Fulmer, S. Raman, M. J. Saltmarsh, A. H. Snell, and P. H. Stelson, Phys. Lett. **51B**, 341 (1974).
[3] A. Gobbi, R. Wieland, L. Chua, D. Shapira, and D. A. Bromley, Phys. Rev. C **7**, 30 (1973).
[4] R. A. Chatwin, J. S. Eck, D. Robson, and A. Richter, Phys. Rev. C **1**, 795 (1970).
[5] D. Pocanic, K. Van Bibber, J. S. Dunham, W. A. Seale, F. Sperisen, and S. S. Hanna, Phys. Rev. C **30**, 1520 (1984).
[6] G. Pantis, K. Ioannidis, and P. Poirier, Phys Rev. C **32**, 657 (1985).
[7] S. Mukherjee, D. C. Agrawal, C. Maheswari, and P. C. Sood, in *Proceedings of the International Conference on Nuclear Interactions, Canberra, Australia,* 1978, *Lecture Notes in Physics,* Vol. 92, edited by B. A. Robson (Springer, New York, 1978).
[8] S. C. Miller, Jr. and R. H. Good, Jr., Phys. Rev. **91**, 174 (1953).
[9] M. Rosen and D. R. Yennie, J. Math. **5**, 1505 (1964).
[10] M. V. Berry and K. E. Mount, Rep. Prog. Phys. **35**, 315 (1972).
[11] P. Lu and E. M. Measure, Phys. Rev. D **5**, 2514 (1972).
[12] S. S. Wald and P. Lu, Phys. Rev. D **9**, 2254 (1974).
[13] S. Mukherjee and S. S. Chandel, J. Phys. A **11**, 1257 (1978).
[14] S. S. Chandel and S. Mukherjee, J. Phys. A **12**, 329 (1979).
[15] C. K. Chan, P. Subeka, and P. Lu, Phys. Rev. C **24**, 2035 (1981).
[16] P. Subeka, C. K. Chan, Z. C. Kang, and P. Lu, Phys. Rev. C **29**, 1088 (1984).
[17] R. K. Samanta and S. Mukherjee, Phys. Rev. D **26**, 2916 (1982).
[18] R. K. Samanta, Ph.D. thesis, North Bengal University, 1986.
[19] J. Knoll and R. Schaeffer, Ann. Phys. **97**, 307 (1976).
[20] R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956).
[21] T. Ericson, Adv. Phys. **9**, 425 (1960).