Low energy behavior of ^{11}Li dissociation cross section

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The Coulomb dissociation cross section for ^{11}Li on ^{197}Au is calculated for different models of the distribution of dipole response strength in ^{11}Li. All the available models fail in accounting for the low energy behavior of the cross section.

The dipole strength distribution of the ^{11}Li nucleus has been the subject of intense scrutiny in recent years [1–4]. A calculation of the distribution is needed in order to reproduce theoretically the experimental data on Coulomb dissociation of ^{11}Li incident on a large-Z target, e.g., ^{197}Au. Although only a few experimental data [5,6] exist up to now, an enormous number of theoretical studies of the subject have been reported recently. This fact shows clearly that this problem is not as definitely well settled as many authors claim it is. Different models have been shown to reproduce, either partially or "successfully," the Coulomb dissociation cross section of ^{11}Li incident on heavy targets at 800 MeV/nucleon [5]. Despite that, it is important to study the sensitivity of the cross section at low energies to the model used for the ^{11}Li dipole response. This can be done by comparing the predictions of the different models to the experimental data at 30 MeV/nucleon [6]. It is worth mentioning that a previous work by Sustich [7] has assessed the same problem. He compared the prediction of the single particle [1], correlated state [2], and cluster model [8]. In this Brief Report we include an analysis of the recently developed hybrid-RPA-cluster model [3] and the quasiparticle random-phase-approximation (RPA) method of Lenske and Wambach [9].

The Coulomb interaction between colliding nuclei contains Fourier components which are dominated by low energy components [10], i.e., virtual photons of low energies. This fact is more accentuated the lower the colliding energy is. Therefore, the Coulomb dissociation of ^{11}Li in low energy collisions probes more efficiently the dipole response of this nucleus at lower excitation energies.

Coulomb excitation cross section within the dipole approximation is given by

$$\sigma_c = \frac{15\pi^3}{9}\alpha \int dE\, n_{E1}(E) \left[\frac{dB(E1)}{dE} \Big/ e^2 \right] , \tag{1}$$

where E is the excitation energy, α is the fine structure constant, and $dB(E1)$ is the dipole response function of the nucleus. Expressions for $n_{E1}(E)$ appropriate for high energy collisions can be found in textbooks [11]. At lower energies, Winther and Alder [10] have shown that such expression may be used in a good approximation, with a rescaling of the impact parameter of the form

$$b \rightarrow b + \frac{\pi}{2}\,\frac{Z_1 Z_2 e^2}{m_0 v^2 \gamma} \tag{2}$$

is done. In fact, as shown by Aleixo and Bertulani [12], such an approximation yields only a 10–20 % discrepancy with an exact numerical calculation. It was also shown that the approximation is worse if one goes to lower excitation energies. Since the important part of the dipole response function is located at very low energies for unstable nuclei such as ^{11}Li, it is therefore appropriate to discuss this question more closely.

In this Brief Report we are more interested in giving a more exact description of the Coulomb excitation of unstable nuclei, which is important the lower the binding energy of the excited nucleus is. We do not want to promote one or the other nuclear model which enters Eq. (1) through the dipole response $dB(E1)/dE$. Indeed, we find it much more exciting when the discrepancies between the models show up in certain situations so that a chance is given to a deeper physical understanding of the phenomena under scrutiny.

In Ref. [13] it was shown that an analytical expression for $n_{E1}(E)$, which is valid for all bombarding energies, is given (we observe that the original formula for the dipole case appearing in Ref. [12] has a misprinted sign in one of its terms)

$$n_{E1}(E) = \frac{2}{\pi} Z_1^2 \alpha e^{-\pi\eta} \left[\frac{c}{v}\right]^2 \left[-\xi K_{i\eta} K'_{i\eta} - \frac{1}{2}\left[\frac{c}{v}\right]^{-2} \xi^2 \left\{ K_{i\eta+1} K_{i\eta-1} - K_{i\eta}^2 - \frac{i}{\varepsilon_0}\left[K_{i\eta}\left[\frac{\partial K'_\mu}{\partial\mu}\right]_{i\eta} - K_{i\eta}\left[\frac{\partial K_\mu}{\partial \mu}\right]_{\mu=i\eta}\right]\right\}\right] , \tag{3a}$$

where Z_1 and v are the target charge and projectile (^{11}Li) velocity, respectively. α is the fine structure constant, and ε_0 is the eccentricity factor of the lowest allowed Coulomb trajectory, that is,

$$\varepsilon_0 = \begin{cases} 1 \quad \text{for} \quad 2a > R \ , \\ \dfrac{R}{a} - 1 \quad \text{for} \quad 2a < R \ , \end{cases} \tag{3b}$$

where $R = R_T + R_p$ is the sum of the target and projectile matter radius. The quantities η and ξ are defined by

$$\eta = \frac{\omega a}{\gamma v} \quad \text{and} \quad \xi = \varepsilon_0 \eta \ , \tag{3c}$$

where ω is the excitation frequency, $a = Z_1 Z_2 e^2 / 2E_{\text{c.m.}}$ is half the distance of closest approach for a head-on collision, and $\gamma = (1 - v^2/c^2)^{-1/2}$.

The function $K_{i\eta}$ is the modified Bessel function with imaginary order. $K'_{i\eta}$ means the derivative of $K_{i\eta}$ with respect to the argument.

Before presenting our calculation of σ_c based on Eq. (1) for the different models of $dB(E1)/dE$, we first discuss the behavior of the function $K_{i\eta}$ given by the integral [14]

$$K_{i\eta}(\xi) = \int_0^\infty e^{-\xi \cosh x} \cos \eta x \, dx \ . \tag{4}$$

These functions are not tabulated and have to be obtained by means of the numerical evaluation of the integral at the right-hand side of (4). The functions $K_{i\eta+1}$ and $K_{i\eta-1}$ are not needed since

$$K_{i\eta+1}(\xi) K_{i\eta-1}(\xi) = \frac{\eta^2}{\xi^2} K_{i\eta}^2(\xi) + K_{i\eta}'^2 \ . \tag{5}$$

In Fig. 1 we show the functions $K_i(\xi)$ and $K_{5i}(\xi)$ vs ξ. It is easy to understand the oscillatory behavior of $K_{i\eta}(\xi)$ vs ξ for small values of ξ by using the stationary phase method. By writing $\cos \eta x = \frac{1}{2}(e^{i\eta x} + e^{-i\eta x})$ and since the integral of Eq. (4) is even in x, one may take only the $e^{i\eta x}$ branch of the cosine and extend the lower limit of integration to $-\infty$. Changing x to $x + i\pi$ and using the stationary phase method, we find

$$\begin{aligned} K_{i\eta}(\xi) &\cong \frac{\pi e^{-\pi\eta/2}}{2} \left[\frac{4Y}{\xi^2 - \eta^2} \right]^{1/4} \text{Ai}(Y) \ , \\ Y &= - \left[\eta \cosh^{-1} \frac{\eta}{\xi} - (\eta^2 - \xi^2)^{1/2} \right]^{2/3} \left[\frac{3}{2} \right]^{2/3} , \quad \eta > \xi \ , \\ Y &= \left[\xi \left[1 - \frac{\eta^2}{\xi^2} \right]^{1/2} - \eta \sin^{-1} \left[1 - \frac{\eta^2}{\xi^2} \right]^{1/2} \right]^{2/3} \\ &\quad \times \left[\frac{3}{2} \right]^{2/3} , \quad \eta < \xi \ . \end{aligned} \tag{6}$$

In Eq. (6), Ai(Y) is the Airy function. This function oscillates for negative values of its argument ($\xi < \eta$) and dies out as $e^{-\xi}$ for large positive values of Y, just as Fig. 1(b) shows. Further, the local period of the oscillations goes as $\Delta\xi \simeq 2\pi\xi/\eta$. Thus, even for small values of η, the function $K_{i\eta}(\xi)$ oscillates at very small values of ξ. In Fig. 1(a) these oscillations are not shown.

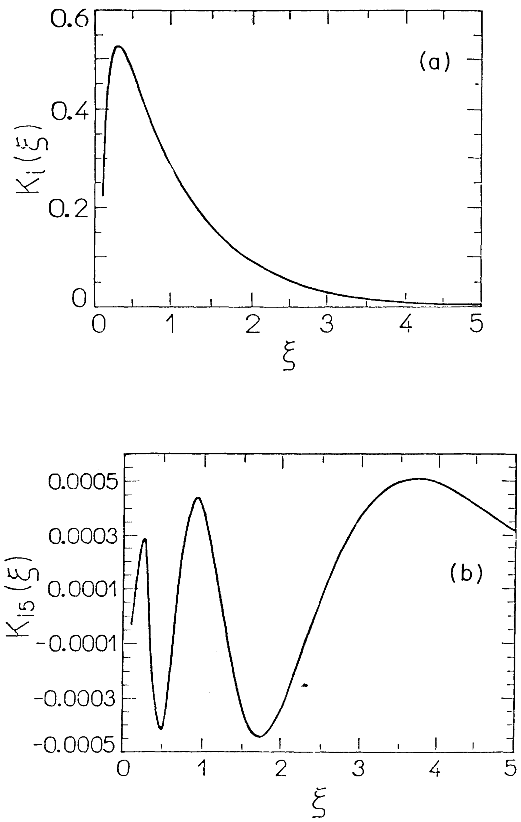

FIG. 1. Function $K_{i\eta}(\xi)$ vs ξ. (a) $\eta=1$ and (b) $\eta=5$.

We further verified that the representation (6) is also approximately valid for $K_0(\xi)$. Finally, we remark that, for our purpose here, the argument of the modified Bessel function is related to its order through $\xi = \varepsilon_0 \eta$ and since $\varepsilon_0 \geq 1$, ξ is equal or larger that η, and thus the low-ξ oscillations are not relevant.

Since we want to give a description of the Coulomb excitation process which will be useful for the analysis of

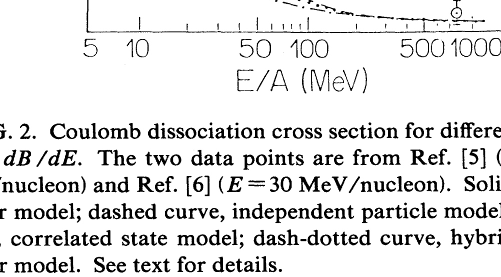

FIG. 2. Coulomb dissociation cross section for different models of dB/dE. The two data points are from Ref. [5] ($E=790$ MeV/nucleon) and Ref. [6] ($E=30$ MeV/nucleon). Solid curve, cluster model; dashed curve, independent particle model; dotted curve, correlated state model; dash-dotted curve, hybrid-RPA-cluster model. See text for details.

Coulomb dissociation of unstable nuclei in general, we observe that for collisions of tens of MeV per nucleon and above, $R \gg a$, that is, $\xi = \varepsilon_0 \eta \gg \eta$. In this case one may use the approximation

$$K_{i\eta}(\xi) \cong K_0(\xi) - \eta^2[K_1(\xi) - K_0(\xi)] , \tag{7}$$

which simplifies Eq. (3a) enormously since the $K_0(\xi)$ and $K_1(\xi)$ functions are much easier to handle than the $K_{i\eta}(\xi)$ functions.

Further simplifications can be done by noting that

$$\xi = \varepsilon_0 \eta \cong \frac{\hbar\omega R}{\gamma \hbar v} = \frac{E_x R}{\gamma \hbar v} \ll 1 ,$$

for excitation energies $E_x \ll \gamma \hbar v / R$, which are the important energies involved in the excitation process of very unstable nuclei. Using the approximation $K_1 \simeq 1/x$ and $K_0 \simeq \ln(\delta / x)$, where $\delta = 1.123 \ldots$, one gets

$$n_{E1}(E) \cong \frac{2}{\pi} Z_1^2 \alpha e^{-\pi\eta} \left[\frac{c}{v}\right]^2 \left\{ \left[1 - 2\eta^2 + \frac{\eta^2}{\xi}\right] \ln\left[\frac{\delta}{\xi}\right] - \xi\eta^2 \left[\frac{1}{\xi^2} + \ln^2\left[\frac{\delta}{\xi}\right]\right] \right.$$
$$\left. - \frac{1}{2}\left[\frac{c}{v}\right]^{-2} \xi^2 \left[\frac{1}{\xi^2} - \ln^2\left[\frac{\delta}{\xi}\right] + \eta^2\left[\frac{1}{\xi^2}\ln^2\left[\frac{\delta}{\xi}\right] + 2\ln^2\left[\frac{\delta}{\xi}\right] + 2\frac{\xi - 1}{\xi}\right]\right]\right\} .$$

We turn now to the results obtained for σ_c [Eq. (1)] using for the dipole strength distribution $dB(E1)/dE$ different models discussed recently in the literature. In Fig. 2 we show a comparison among the cross sections obtained with the modified independent particle model [1], the correlated state model [2], the hybrid-RPA-cluster model [3], and the cluster model [8]. Our result shows that none of the models account for the low energy data point ($E_{\text{lab}} = 30$ Mev/nucleon). Actually, the data point at $E_{\text{lab}} = 30$ MeV/nucleon was extracted by Sustich [7] from the experiment [6], after a separation of the calculated nuclear part contribution to the total cross section. Whereas the cluster model overestimates the cross section, the other models fall short in value. The recent calculation of Lenske and Wambach [9], using the quasiparticle RPA method, also falls short in value (the cross section for this case is not shown in Fig. 2 as it almost coincides with the independent particle result).

In conclusion, we have calculated this Brief Report the Coulomb dissociation cross section for ^{11}Li on ^{197}Au using different models for the dipole strength distribution of ^{11}Li. All available models fail in accounting for the low energy cross section. Further theoretical studies and experiments are clearly needed to settle the matter. In particular, we have discussed in detail the necessary corrections in the theory of Coulomb excitation in order to calculate the low and high energy cross sections for the Coulomb dissociation of unstable exotic nuclei. These corrections are of increasing relevance with the decrease of the binding energy of the nuclei.

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