PHYSICAL REVIEW C VOLUME 44, NUMBER 5 NOVEMBER 1991

Proton 4p-4h intruder excitations in heavy even-even nuclei

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(Received 25 July 1991)

Proton 4p-4h intruder excitations can be formed in even-even nuclei with a large neutron excess. The excitation energy and the systematics of such 4p-4h intruder excitations is discussed for the even-even Sn and Pb nuclei.

A quite unexpected consequence of the strongly attractive proton-neutron interaction is the observation of very-low-lying $J^\pi = 0^+$ excitations near single-closed-shell nuclei. Normally, closed shells form an inert core which stabilizes the motion of the remaining valence nucleons. If, now, single-closed-shell nuclei are studied (Sn nuclei, Pb nuclei, etc.) in which the number of valence neutrons is maximal, the closed shell becomes unstable against 2p-2h excitations across the closed shell [1–3]. Even though the unperturbed energy for such excitations is very high and of the order of 7–10 MeV in medium-heavy and heavy nuclei, the proton-neutron interaction modifies the nucleon motion in an important way so as to give rise to very-low-lying $J^\pi = 0^+$ excitations. A most dramatic illustration of these "intruder" excitations was shown to exist in the Pb region by the LISOL group in Leuven [3,4]. Such excitations correspond also to states with a large quadrupole-deformed shape, which quite often can give rise to shape coexistence.

The detailed mechanism for describing these intruder excitations has been discussed in Refs. [5] and [6]. The basic effect, though, is illustrated in Fig. 1 in a schematic way. Starting from an unperturbed energy $2(\varepsilon_{j_p} - \varepsilon_{j_h})$ needed to form the 2p-2h excitation, which is taken constant over a given mass region at a single closed shell, various energy corrections have to be considered. The first correction ΔE_{pair} takes into account the extra pairing-correlation energy among the 0^+ coupled particle and hole pair. This energy gain is also taken as constant over the given mass region. There are some typical shell-model effects, modifying the single-particle energies with changing nucleon number, i.e., the relative self-energy corrections or monopole energy shift ΔE_M with

$$\Delta E_M = 2 \sum_{j_\nu} (2j_\nu + 1) v_{j\nu}^2 [(\bar{E}(j_\pi j_\nu) - \bar{E}(j'_\pi j_\nu)] \ . \tag{1}$$

This describes the shift in proton single-particle energy for the 2p-2h excitation energy due to the filling of neutron orbitals [with occupation probability $v_{j_\nu}^2$ and average proton-neutron interaction matrix elements $\bar{E}(j_\pi j_\nu)$]. This energy correction does not change the 2p-2h 0^+ excitation energy in a major way and has therefore not been indicated in Fig. 1. The attractive proton-neutron force, though, gives the dominant energy correction. Because of polarization effects of the proton-neutron force, changing 0^+ coupled pairs into 2^+ coupled pairs, both the ground-state and intruder pair distributions become modified as follows:

$$|\widetilde{0_\pi^+ \otimes 0_\nu^+}\rangle \Longrightarrow |0_\pi^+ \otimes 0_\nu^+\rangle + \alpha |2_\pi^+ \otimes 2_\nu^+\rangle + \cdots \ . \tag{2}$$

Using now a residual quadrupole-quadrupole proton-neutron interaction and 0^+ ground-state and intruder wave functions that are approximated by SU(3) wave functions, one derives the quadrupole binding-energy gain ΔE_Q as [5]

$$\Delta E_Q \cong 2\kappa \Delta N_\pi N_\nu \ . \tag{3}$$

It is now in particular the latter term, though the $\Delta N_\pi N_\nu$ dependence ($\Delta N_\pi = 2$ for a proton 2p-2h 0^+ excitation and N_ν the number of valence neutron pairs), which is causing the intruder 0^+ state to occur at such low energies as is observed in the various single-closed-shell regions, e.g., the $Z = 50$ (Sn) [2], $Z = 82$ (Pb) [3,4], $N = 20$ nuclei [7].

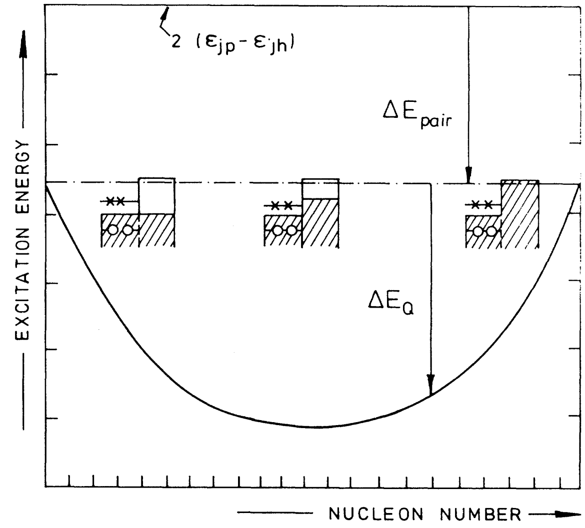

FIG. 1. Schematic illustration of the various binding-energy terms contributing to the low-lying $J^\pi = 0^+$ 2p-2h intruder excitations. The unperturbed energy $2(\varepsilon_{j_p} - \varepsilon_{j_h})$, pairing energy gain ΔE_{pair}, and proton-neutron interaction energy $\Delta E_Q = 2\kappa \Delta N_\pi N_\nu$ are given as a function of the number of valence nucleons.

An interesting question naturally centers around the possible occurrence and excitation energy of the next class of proton intruder configurations, i.e., the 4p-4h excitations. Analyzing the basic contributions to the energy of such 4p-4h proton intruder excitations, one obtains the expression

$$E_{\text{intr}}(4\text{p-}4\text{h}) \cong 4(\varepsilon_{j_{\text{p}}} - \varepsilon_{j_{\text{h}}}) + \Delta E_M(4\text{p-}4\text{h}) - \Delta E_{\text{pair}} + \Delta E_Q(4\text{p-}4\text{h}) \ . \quad (4)$$

Comparing with the evaluation of the proton 2p-2h 0^+ intruder excitations, one observes that both the unperturbed energy, monopole energy shift ΔE_M, and proton-neutron interaction energy shift ΔE_Q scale with the number of particles and holes, i.e.,

$$\begin{aligned} &\Delta E_M(4\text{p-}4\text{h}) = 2\Delta E_M(2\text{p-}2\text{h}) \ , \\ &\Delta E_Q(4\text{p-}4\text{h})(\Delta N_\pi = 4) = 2\Delta E_Q(2\text{p-}2\text{h})(\Delta N_\pi = 2) \ . \end{aligned} \quad (5)$$

Only the pairing-energy correction ΔE_{pair} for $n/2$ 0^+ coupled pairs is reduced over the energy gain $G\Omega$ for a single pair times the number of pairs according to the expression [8]

$$\Delta E_{\text{pair}} = -\frac{n}{2} G\Omega \left[1 - \frac{n}{2}\frac{1}{\Omega} + \frac{1}{\Omega} \right] , \quad (6)$$

with Ω the shell degeneracy and $n/2$ the number of pairs. Combing the above results, one gets as a quite general result that

$$E_{n\text{p-}n\text{h}}(0^+) \gtrsim \frac{n}{2} E_{2\text{p-}2\text{h}}(0^+) \ , \quad (7)$$

where the approximate equal sign is best fulfilled for large Ω and relatively small n.

We have applied the above method to obtain the energy behavior of the lowest 4p-4h 0^+ intruder excitations. We have used the unperturbed energy, monopole correction and quadrupole correction energy values as discussed in Refs. [5] and [6]. Only the pairing correction needs some more attention. The pairing energy for four identical particles $-2G\Omega(1-1/\Omega)$ needs to be evaluated for particles and holes separately. By fitting the two-particle pairing energy $-G\Omega$ to experimental pairing energies, obtained from proton and two-proton separation energies and presented in Figs. 2(a) and 2(b) of Ref. [6] for the Pb and Sn regions, respectively, one can deduce a pairing strength G (Pb) and G (Sn), respectively. For degeneracy we choose the following: (i) In the Sn region, particles (p) moving in the nearby $1g_{7/2}$ and $2d_{5/2}$ proton orbitals, giving rise to $\Omega_{\text{p}}=7$ and holes (h) moving in the nearby $1g_{9/2}$ and $2p_{1/2}$ proton-hole orbitals, resulting in a value of $\Omega_{\text{h}}=6$. This results in the values G_{p} (Sn) $\cong G_{\text{h}}(\text{Sn}) = -0.3$ MeV. (ii) In the Pb region, particles (p)

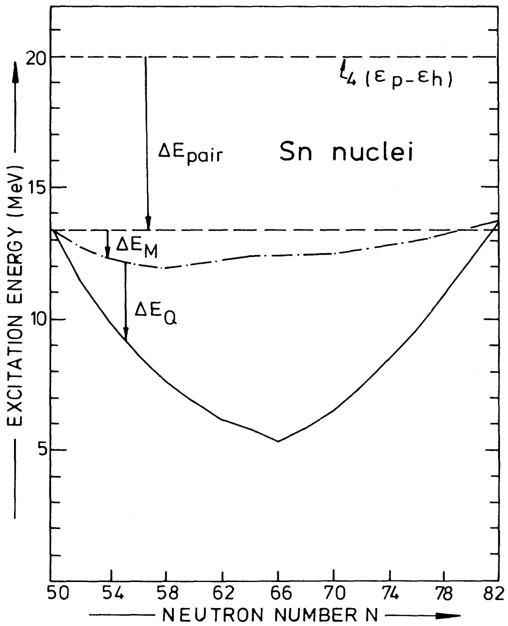

FIG. 2. Variation of the lowest intruder 4p-4h 0^+ excitation across the $Z=50$ closed shell as a function of neutron number ($50 \le N \le 82$). Besides the unperturbed 4p-4h energy, the pairing-energy correction ΔE_{pair} [Eq. (6)], monopole correction ΔE_M, and quadrupole-energy correction ΔE_Q [see Eqs. (5)] are shown. The final curve (solid line) then gives the energy variation for the lowest-lying 4p-4h 0^+ intruder excitation.

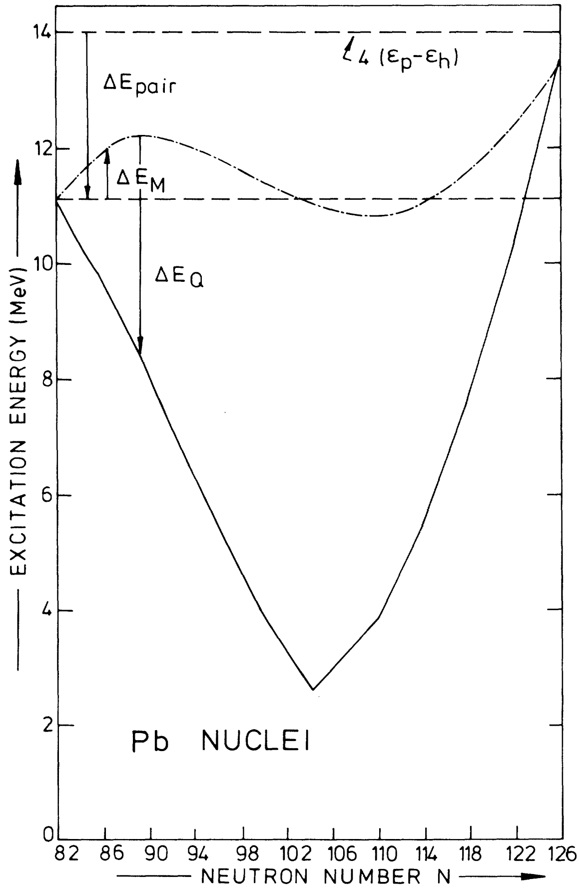

FIG. 3. See caption to Fig. 2, but now for the Pb region (with $82 \le N \le 126$).

moving in the proton $1h_{9/2}$ orbital, giving rise to $\Omega_p = 5$ and holes (h) moving in the close-lying $2d_{3/2}$ and $3s_{1/2}$ orbitals with $\Omega_h = 3$ as a result. Here we obtain the estimates G_p (Pb) $= -0.2$ MeV, G_h (Pb) $= -0.33$ MeV.

Combining these pairing corrections, one finally obtain the values

$$\Delta E_{\text{pair}}(\text{4p-4h})_{\text{Sn}} = 6.6 \text{ MeV} \quad (<8 \text{ MeV}) ,$$

$$\Delta E_{\text{pair}}(\text{4p-4h})_{\text{Pb}} = 2.9 \text{ MeV} \quad (<4 \text{ MeV}) ,$$

where the values between brackets indicate $2\Delta E_{\text{pair}}(\text{2p-2h})$ for the Sn and Pb regions, respectively.

The energy systematics for the lowest proton 4p-4h 0^+ excitations is illustrated on Figs. 2 and 3. For the Sn region, those types of configurations come as low as $E_x \approx 5$ MeV at midshell. In the Pb region, however, near neutron number $N = 104$, excitation energies near to $E_x \approx 2.5$ MeV result, excitation energies that should allow for possible observation and population of such 4p-4h intruder excitations in future experiments.

In bringing the new class of proton intruder 4p-4h 0^+ excitations in perspective, relative to other particle-hole excitations across the major closed-shell configuration, we compare in Fig. 4, for the cases of ^{116}Sn and ^{192}Pb, the relative position of the lowest observed 2p-2h 0^+ intruder excitations, 1p-1h excitations, and lowest-lying 4p-4h intruder configurations [9]. The experimental study of 1p-1h excitations in ^{116}Sn was done using single-nucleon transfer reactions starting from the odd-mass Sb nuclei (pick up) or In nuclei (stripping), respectively, with a large density of 1p-1h states starting near $E_x \geq 4$ MeV. The expected position of the proton 4p-4h 0^+ intruder excitations is slightly higher still. In ^{192}Pb we expect, however, the 4p-4h 0^+ intruder excitations to occur relatively low in excitation energy ($E_x \approx 2.5$ MeV).

We finally point out that, in order to populate selectively such 4p-4h states, four-proton transfer reactions would be needed with possible candidates like (^{40}Ca,^{36}S), (^{54}Fe,^{50}Ti), and (^{52}Cr,^{48}Ca) reactions starting from the most neutron-rich Pd nuclei (in studying even-even Sn nuclei) and the most neutron-deficient Pt nuclei (in studying even-even Pb nuclei).

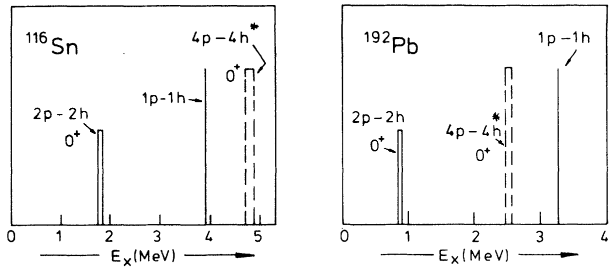

FIG. 4. Calculated energy for the lowest 4p-4h 0^+ proton intruder configurations in ^{116}Sn and ^{192}Pb. The observed 2p-2h 0^+ proton intruder energy [2,3] and a typical value for the lowest-lying 1p-1h excitations across the $Z = 50$ and 82 shells, respectively, are also given for comparison.

In conclusion, proton 4p-4h 0^+ intruder excitations can be constructed in medium-heavy and heavy even-even nuclei. The energy systematics have been studied for the even-even Sn and Pb nuclei with a prediction of $E_{\text{intr}}(\text{4p-4h}) \approx 2.5$ MeV near $N = 104$ in the Pb nuclei. Here the 4p-4h intruder excitations even come lower in energy compared with the 1p-1h excitations across the $Z = 82$ closed shell. Possible experiments for populating these 4p-4h 0^+ intruder excitations are suggested.

The authors are grateful to J. L. Wood and R. F. Casten for early discussions on the possibilities of describing proton 4p-4h excitations. The authors also thank the NFWO and IIKW for financial support as well as support from NATO through research Grant No. NATO RG-86/0452.

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