Wave function effects and the elastic magnetic form factor of ^{17}O

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We determine the parameters of an algebraic radial wave function (a solution of the Ginocchio potential) corresponding to the $d_{5/2}$ valence neutron orbital of ^{17}O by making a fit to the high q^2 magnetic electron scattering data. The wave function so determined is similar to other finite well wave functions also fitted to these data. It has, however, an rms radius about 6–10% larger than the corresponding Woods-Saxon wave functions. We suggest from this result that (i) the determination of rms radii of valence nucleon radial wave functions from magnetic electron scattering is more model dependent than previously realized, and (ii) the Okamoto-Nolen-Schiffer anomaly in $A=17$ may be underestimated by previous calculations with finite wells.

I. INTRODUCTION

Electron elastic determines the spatial distribution of the charge and magnetization density of the nucleus in its ground state [1]. Elastic charge scattering arises from coherent contributions to the charge density operator and therefore cannot tell us about the contribution of the individual proton orbits to the total charge density. In contrast, elastic magnetic scattering is capable of providing information on the radial distribution of valence orbits for both protons and neutrons. In particular, a microscopic nuclear theory explanation of the magnetization data [2,3] of ^{17}O begins with the simple shell-model picture that the entire magnetic cross section originates from the $d_{5/2}$ neutron outside a doubly closed ^{16}O core. This simple picture is not adequate to explain the entire range in q of the data and many other effects have been examined [4]. In spite of this plethora of effects, magnetic electron scattering stands out as a relatively clean means of probing fundamental aspects of the strong interaction. It was early realized that at high q the highest-order allowed multipole ($M5$) is dominant and configuration-mixing effects are minor in this region. This means that individual shell-model orbitals can be obtained from the high q data [1,10,13]. The primary purpose of this paper is to determine from the ^{17}O data an algebraic radial wave function for the $d_{5/2}$ neutron orbital of the independent particle shell model. Having obtained this wave function, we then compare with other studies of the magnetization density of ^{17}O to draw the tentative conclusions: (i) the determination of rms radii of valence nucleon radial wave functions from magnetic electron scattering is more model dependent than previously realized and (ii) the Okamoto-Nolen-Schiffer (ONS) anomaly [14] for the case of $A=17$ may have been underestimated by previous calculations. A precise knowledge of the systematics of the ONS anomaly is needed to assess the recent flurry of explanations of the anomaly which appeal to traditional sources of nuclear charge asymmetry (primarily $\rho-\omega$ mixing) [15] or which attribute charge asymmetry to a partial restoration of chiral symmetry in nuclei [16].

To arrive at these results we begin with the finite well, proposed by Ginocchio, which allows analytic bound-state and scattering solutions [17]. The Ginocchio potential is well suited for nuclear physics applications: To an excellent approximation, the parameters can be separately tuned to reproduce specific features of nuclear charge form factors and the Ginocchio potential reproduces the charge form factors of ^{16}O as well as a Woods-Saxon potential. We have reexamined the radial distribution of the $d_{5/2}$ orbital in the extreme single-particle model of ^{17}O assuming the Ginocchio potential describes the nuclear mean field. Because the orbitals are analytic, one can refine the calculation of the ^{17}O form factor to the sophistication desired. Because the orbitals arise from a finite well, one can examine the single-particle picture (and eventually higher-order effects) accurately and transparently.

Of those higher-order effects which are important in the $M5$ multipole, we neglect three-body force and relativistic effects, but do pay careful attention to meson-exchange currents and the nucleon electromagnetic form factor. A consistent treatment of three-body force, relativistic, and exchange current effects has been described [18], but the primary aim of this work is a study of realistic wave function effects so we follow the usual approach in order to compare with other calculations. Furthermore, we leave configuration-mixing effects (which largely effect $M3$) for a future effort [19].

We find, in agreement with earlier results [10], that the set of well parameters fitted to charge scattering, which is somehow "averaged" over all the occupied proton orbits of ^{16}O does not agree with the parameters which best fit the magnetic scattering of a single valence neutron. We show that the $d_{5/2}$ wave function of the Ginocchio parametrization of the nuclear mean field compares well with the only self-consistent microscopic calculation available [20]. We extract a $d_{5/2}$ radius with a smaller uncertainty than before because of our use of analytic orbitals and because data at high q has become available since the earlier calculations. The obtained radius is about 10% larger than that of the most recent Woods-Saxon parametrization; a result which has implications for Coulomb-energy differences of mirror nucleus pairs [14] and thus for nuclear charge asymmetry.

II. FORMALISM AND RESULTS

The evaluation of the transverse form factor in terms of single-particle reduced matrix elements of the multipole operators is well known [21,22]. Convenient tables exist which summarize the matrix elements in terms of angular momentum factors times radial integrals of single-particle wave functions [23]. We assume the independent particle shell model (i.e., no configuration mixing) and use closed-form single-particle wave functions derived from Ginocchio's finite well. This well and its orbitals are a little complicated so we refer to the original article [24] for a description of the general potential. We employ a limiting case which is finite at the origin and the effective-mass parameter is set to zero so that the mass does not depend on the radial coordinate (i.e., the well is local in each partial wave). The Ginocchio potential is similar in shape to a Woods-Saxon potential but it is angular momentum dependent. Spin-orbit splitting is incorporated by having the strength parameters depend on the spin alignment relative to the orbital angular momentum. The potential is characterized by the parameters s, λ, and ν_{lj}. The scale parameter s, in units of inverse femtometers, defines a dimensionless radial coordinate and effectively determines the depth of the potential at the origin for some fixed value of l and j. The potential is actually a function of a dimensionless coordinate y which varies from 0 to 1 as the radial coordinate varies from zero to infinity. The shape of the potential is determined by the parameter λ which appears in the implicit definition of y. For given values of s and λ, the spin-orbit parameters ν_{lj} are then fixed by the single-particle binding energies in the nucleus. These analytic bound-state wave functions, unlike the orbitals of the harmonic-oscillator potential, thus have the correct behavior at large r. They also differ at large r from the wave functions of the microscopic theory of Ref. [20] which had Gaussian tails for $r \geq 10$ fm.

Because the potential is given as a function of y, it is convenient to use $y(r)$ as the independent variable of integration in the radial integrals. The transformation to $z=\ln[2/(1-y)]$ as the independent variable is helpful numerically [25] because of the peaking of the integrands near $y=1$. There are no problems with granularity in the integral, as could happen with the usual finite well solutions when the form factor is evaluated at high momentum transfer q. For example, Woods-Saxon wave functions exist only on a finite mesh from numerical integration of Schrödinger's equation or as a set of expansion coefficients in a suitable basis. The Ginocchio wave function is readily available for every value of y, allowing adaptive techniques to evaluate the required integrals for any desired q. We used an algorithm based upon composite trapezoid sum with cautious Romberg extrapolation [26].

For orientation we first fitted the charge form factor of ^{16}O with the values $1/s=7.75$ fm and $\lambda=2.75$ and the ν_{lj}'s appropriate to the three proton single-particle energies of ^{16}O. There is a rather definite physical significance of each parameter for the charge distribution, as noted in Ref. [25]. An adjustment in the depth of the $p_{1/2}$ potential at the origin (obtained by varying s) changes primarily the location of the first diffraction minimum and hence the size of the calculated nucleus. An adjustment of the shape parameter λ changes primarily the height of the second diffraction maximum. The fit by eye for $q \leq 3$ fm^{-1} appears equal in quality to earlier fits with Woods-Saxon potentials [27]. The Woods-Saxon potential has four parameters: an overall strength V_0, a well radius r_0, a diffuseness parameter a, and the spin-orbit strength $V_{\text{s.o.}}$. Electric charge scattering is sensitive to only the two parameters a and the ratio V_0/r_0^3 and is virtually independent of $V_{\text{s.o.}}$ [28]. Other experimental probes of the nuclear wave function are then needed to learn about the remaining parameters of the Woods-Saxon potential.

The ^{16}O Ginocchio well parameters determined by the charge distribution, however, yield a very bad representation of the ^{17}O magnetic form factor. The $M5$ contribution at large $q>2.5$ fm^{-1} falls lower than the data by an order of magnitude. In this respect, the Ginocchio potential is no different than the harmonic-oscillator potential [5] nor a Woods-Saxon potential [10].

To determine the parameters of the Ginocchio potential which best fit the ^{17}O data above 2.0 fm^{-1}, we assumed no quenching of the $M5$ multipole (consistent with the quenching factor 1.03 ± 0.11 found by Hicks [10] and 0.92 ± 0.09 obtained in another early analysis [29]). The best fit to the high q data is shown in Fig. 1 as the dash-dotted line. It was obtained with the parameters $\lambda=2.975$ and $1/s=7.35$ fm (corresponding to a well depth at the origin of 50 MeV, or about 10 MeV greater than the ^{16}O well). The $d_{5/2}$ wave function of this potential appears as the dash-dotted line of Fig. 2(a); it has an rms radius of 3.68 ± 0.04 fm. The errors associated with the rms radius correspond to an increase in the χ^2 per degree of freedom (equals 1.17 in our displayed best fit) from χ^2 to χ^2+1 as the input parameters are varied. This is the point-neutron rms radius of the $d_{5/2}$ wave function as corrections for the finite size of the neutron, center-of-mass corrections, and meson-exchange currents have already been included in the fit. The meson-exchange current contributions [30] and center-of-mass corrections [31] were calculated for a harmonic-oscillator (HO) wave function ($b=1.76$ fm). The quality of the description of the data by the HO wave function with the same three corrections included can be inferred from the short dashed line of Fig. 1.

The point neutron rms radius of the $d_{5/2}$ orbital extracted from the data was not affected greatly by small changes in the input. For example, omitting the three highest data points gave an rms radius of 3.66±0.04 fm with a lower $\chi^2=0.45$ corresponding to well parameters $\lambda=3.025$ and $1/s=7.425$ fm. A center-of-mass correction with a different oscillator parameter ($b=1.72$) gave an rms of 3.70±0.04 fm. It was difficult to fit the data when meson-exchange currents were not included in the fit. The best representation was with well parameters $\lambda=2.975$ and $1/s=7.05$ fm corresponding to an rms of 3.59±0.04 fm. This was a poor fit with $\chi^2=3.4$; the theoretical curve was below the points $q=2$–2.7 fm^{-1}.

The finite size of the neutron's magnetization was parametrized by the latest information available to us. That is, we combined the Sach's form factors G_E obtained from a recent determination of the deuteron structure function [32] and G_M from inelastic electron scattering at higher momentum transfers [33] to form the $F_2(q^2)$ needed in the calculation. This seemingly *ad hoc* parametrization does not differ much, in the momentum range $q=2$–3 fm^{-1}, from the Karlsruhe parametrization [34] which was the last modern dispersion relation fit to use all available data subjected to analyticity and unitary constraints. The fitted $d_{5/2}$ wave function is not sensitive to a choice between these two neutron form factors, but an often used dipole ($\Lambda=855$ MeV) forces a new fit at $\lambda=2.85$ and $1/s=7.15$ fm or rms radius of 3.73±0.04 fm. This is because the dipole is about 5% higher than the more accurate parametrizations at $q\approx 3$ fm^{-1}.

To maintain a good fit to the data, a small change in the shape parameter λ must be accompanied by a compensating change in $1/s$. To a good approximation, the region of parameter space which fits the data takes the shape of long ellipses with major axis along the line $(\lambda+0.7)=(1/s)(0.5\ \mathrm{fm}^{-1})$. This relationship is reminiscent of the "V_0/r_0^3" ambiguity of the Woods-Saxon potential for scattering from the charge density, and has the same explanation [28], which we adapt here. That is, magnetic electron scattering from the highest multipole of ^{17}O depends upon the amount of intrinsic magnetization, and hence on the wave function, at a given radius. Thus, for example, if we deepen the well (by decreasing $1/s$) we must also extend it (by decreasing λ) in order to keep the spatial distribution of the wave function approximately the same. However, in the case of magnetic electron scattering, this argument can be sharpened by the observation that the high q fit is most sensitive to the radial position of the peak of the wave function [10,13,22]. This is easily seen from the relation between the contribu-

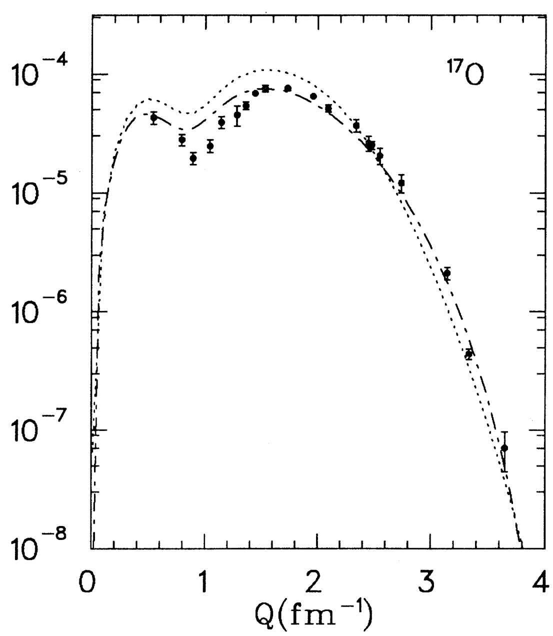

FIG. 1. Magnetic form factor squared of ^{17}O versus momentum transfer. The short dashed curve gives the harmonic-oscillator ($b=1.76$ fm) results while the dot-dashed curve represents the Ginocchio potential result discussed in this paper. Corrections for neutron finite size, center of mass motion, and meson-exchange currents are included in both curves. The data are taken from Ref. [2,3].

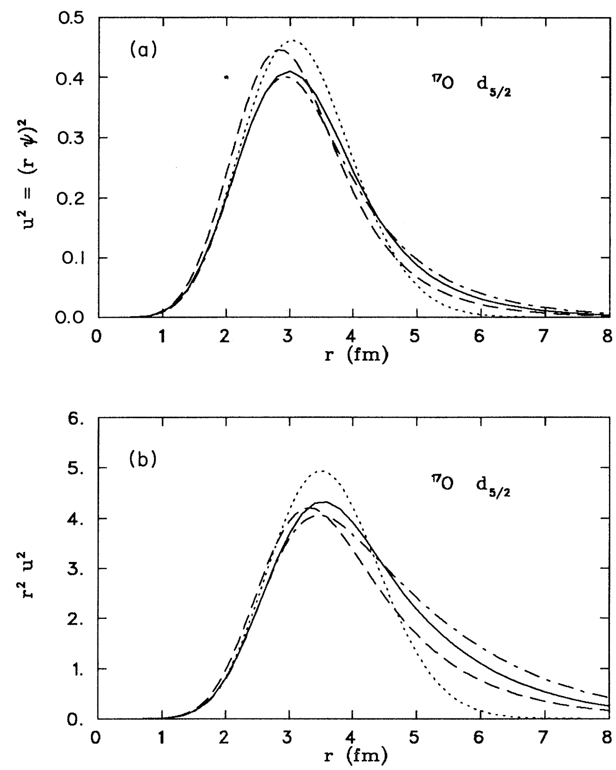

FIG. 2. Wave functions (squared) for $d_{5/2}$ neutron valence orbitals of ^{17}O. The curves represent the harmonic oscillator (short dashes), Woods-Saxon (long dashes) with parameters from Ref. [35,3], a Brueckner calculation (solid line), and Ginocchio (dots-dashes) wave functions. The harmonic-oscillator and Ginocchio wave function were used to obtain the magnetic form factor results presented in Fig. 1. Integrands for the point neutron rms radius calculation. Curves are as in short dashes-harmonic oscillator (rms radius=3.29 fm), long dashes-Woods-Saxon (rms radius=3.37 fm) [3], solid-Brueckner calculation (rms radius≈3.5 fm), dots-dashes-Ginocchio wave function (rms radius=3.68 fm).

tion to the highest multipole and the radial wave function $u=r\psi$:

$$F_{M5} \propto q \int_0^\infty u^2(r) j_4(qr) dr \ . \tag{1}$$

At high q, the smaller the r at which the peak of u^2 sits, the more support for F_{M5}. The single-particle density ($\propto u^2$) at very large radius does not significantly affect F_{M5}, but this density is heavily weighted at large r in the rms integral. Thus it is possible for $d_{5/2}$ wave functions of two different nuclear mean fields to both fit the data, but have slightly different rms radii.

We illustrate the wave function part u^2 of the integrand of Eq. (1) in Fig. 2(a). The short dashed (dot-dashed) lines represent the harmonic-oscillator (Ginocchio) orbitals which produce the magnetic form factors of Fig. 1. For comparison, we display as long dashes a Woods-Saxon wave function which was phenomenologically successful in a fit to the ^{16}O(d,p) ^{17}O experiment at three energies [35]. The parameters of this Woods-Saxon potential (first line in Table 2 of Ref. [35]) give a good accounting [3] of the ^{17}O magnetic scattering form factor if meson-exchange currents are neglected [36]. The solid line is the result of a systematic approach (in the context of Brueckner theory) to the calculation of self-consistency effects in core plus valence nucleon systems [20]. This microscopic $d_{5/2}$ wave function is underbound by approximately 1.5 MeV compared to experiment ($E_{5/2}=-4.15$ MeV). Therefore, for $r \leq 10$ fm, the long-range tail of the "Brueckner" wave function falls off slightly less rapidly than those of the Woods-Saxon or Ginocchio parametrizations which do incorporate $E_{5/2}$. After $r \approx 10$ fm the microscopic wave function decays as a Gaussian because of the large but still modest harmonic-oscillator basis space used to expand the wave function. (The harmonic-oscillator orbital has, of course, an incorrect Gaussian tail due to the extreme truncation of a harmonic-oscillator basis space.) Except for this small difference in the binding energy, the two finite well orbitals of Fig. 2(a) are in quite good agreement with the best microscopic calculation available [20].

It is, however, not the tail but the position of the peak of $u^2(r)$ which determines the high q behavior of the magnetic form factor. We note that the Woods-Saxon orbital peaks at a smaller value of r than the other three orbitals shown in Fig. 2(a). This follows from Eq. (1): the orbital must peak at smaller r to fit the data because the Fourier-Bessel transform does not include the additive (positive) contribution from meson-exchange currents which were included in the HO and Ginocchio fits. If we had refit the Woods-Saxon orbital in the same way as the latter two orbitals, it would have peaked at nearly the same place. On the other hand, if one weights $u^2(r)$ by r^2, one has the rms integrand plotted in Fig. 2(b). A comparison of Figs. 2(a) and 2(b) shows clearly the major difference between a Woods-Saxon and a Ginocchio $d_{5/2}$ orbital to be, not in the location of the peak, but on the large r side of the peak. It is not surprising that the rms radius of a Ginocchio orbital fit to the data is larger than that of a Woods-Saxon fit. Indeed this is the case: our best fit Ginocchio orbital has a point rms radius of 3.68 ± 0.04 fm which is about 6–10 % larger than comparable Woods-Saxon radii. Those point nucleon radii in the literature which (we believe) also include the three (finite nucleon size, center of mass motion, and meson-exchange current) corrections in the fit to the data are 3.49 ± 0.09 fm [10], 3.35 ± 0.03 fm [29], and 3.46 fm [3]. (We have scaled the values 3.36 fm of [3] and 3.37 fm obtained by us [36] to roughly account for meson-exchange-current corrections.) Alternatively, one could take the point of view that meson-exchange-current corrections are nearly canceled by core polarization effects [37] so one should leave them out for a better estimate. This procedure, advocated in Ref. [3], yields the smaller rms radii 3.39 ± 0.09 fm [10], 3.36 fm [3], and 3.59 ± 0.04 of the present investigation. Again the best-fit Ginocchio orbital has a point rms radius which is about 7% larger than comparable Woods-Saxon radii.

The results to this stage raise the issue of whether the Ginocchio wave function is physically acceptable since it has a somewhat larger radial extension than those of the traditional Woods-Saxon well. The full test of the usefulness of Ginocchio's potential in nuclear physics must come from a variety of experimental probes, such as those of Ref. [29, 35]. At the moment we have evidence of its successful description of scattering from the charge densities of ^{12}C [25], ^{16}O (this paper and [38]), and ^{208}Pb [17], and the magnetization density of ^{17}O (this paper). In addition, the $B(E2)$ electromagnetic transition rates of ^{16}O are quite well reproduced by a large shell-model calculation with the Ginocchio potential wave functions [38].

We close by briefly commenting on the comparison of binding-energy differences of mirror nuclei (such as ^{17}O-^{17}F) with calculated electromagnetic effects [14]. The binding-energy difference should mainly be determined by the direct Coulomb interaction of the odd nucleon with the core. Many other correction terms modify this direct Coulomb interaction contribution only slightly. The total calculated electromagnetic energy differences in light mirror pair ($A \leq 41$) are about 5–6 % smaller than experimental measured values [14,39]. This "Okamoto-Nolen-Schiffer anomaly" has often been characterized by the rms radius of the odd nucleon–the larger the rms radius, the smaller the direct Coulomb interaction. Experience gained from shell-model calculations with Woods-Saxon potentials indicate a nearly linear dependence between the value of the valence-orbit rms radius and the Coulomb energy difference. Increasing the rms radius of the odd nucleon by 2% results in a 1% decrease of the theoretical direct Coulomb energy difference [40]. A similar trend (but with different numbers) has been obtained with single-particle wave functions deduced from Hartree-Fock calculations [41,42]. If (i) this linear dependence upon the rms radius also holds for the Ginocchio orbitals and if (ii) they provide a correct description of ^{17}O, then the Okamoto-Nolen-Schiffer anomaly can be estimated for the $A=17$ pair by interpolation of the results of Table 4 of Ref. [40]. The anomaly then increases from 310 keV of Table 6 (quoted, for example, in the third of Refs. [16]) to 420 keV or about 35% over Woods-Saxon estimates for the $A=17$ pair. (If one does not correct the data for meson-exchange-current contributions, the finite size rms radius of the Ginocchio orbital is 3.68 fm, and one estimates a discrepancy of 360 keV or 15% more than the Woods-Saxon estimate.) More work is needed to substantiate assumptions (i) and (ii).

In conclusion, we have successfully described the ^{17}O magnetic form factor at high q by orbitals of a finite well which are available in closed form. The $d_{5/2}$ Ginocchio wave function has a point rms radius of 3.68 ± 0.04 fm. This radius is about 6–10 % higher than that of Woods-Saxon orbitals which describe the same data. Yet, both finite well orbitals bracket the one obtained from a microscopic Brueckner calculation of ^{17}O. If one assumes that the Ginocchio potential provides a good description of the mean field of the nucleus, two conclusions can be drawn from this result: (i) the determination of rms radii of valence nucleon radial wave functions from magnetic electron scattering is more model dependent than previously realized, and (ii) the Nolen-Schiffer anomaly may be underestimated by current calculations. A need for further experience with this interesting new potential is suggested by our results.

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$$\Phi_{nl}(r)=N(n,l)\frac{\{1+\lambda^2(1-a)-[1-\lambda^2(1-a)]x\}^{1/2}}{\{2[1+\lambda^2-(1-\lambda^2)x]\}^{1/4}}\times\left[\frac{1+x}{2}\right]^{\beta_{nl}/2}\left[\frac{1-x}{2}\right]^{(2\alpha_l+1)/4}P_n^{(\alpha_l,\beta_{nl})}(x)\;.$$

We set $a=0$ and $\alpha_l=l+1/2$ as explained in the text.
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tential of [35] via an expansion in a harmonic oscillator basis according to, for example, the formalism presented in S. A. Coon and O. Portilho, *Lecture Notes in Physics,* Vol. 273 (Springer-Verlag, Berlin, 1986), p. 219. We thus verified that the long dashes of Fig. 2(a) do indeed adequately describe the high q data of Fig. 1 in agreement with [3]. Our slightly different treatment of the center-of-mass correction and neutron finite size have a negligible effect at this qualitative level of comparison.

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