Pion absorption on ^{3}He: Absorption amplitude in the Faddeev-quasiparticle scheme

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The absorption channel in π-^{3}He collisions is investigated. Amplitudes are derived for the pion-induced breakup of the target into both two clusters and three free nucleons. We employ elementary absorption mechanisms of one- and two-body type, which are presently considered to be important for the description of π-nucleus collisions from the low-energy region up to the region dominated by the Δ resonance. We put the major emphasis on the few-body aspects of the process, such as the three-nucleon dynamics in the final state, and the initial-state breakup of the bound nuclear system in presence of the pion. On the other hand, we neglect pion-nucleus correlations in the initial state, which would entail a complete four-body treatment of the process. For both one- and two-body elementary absorption, the full amplitudes are explicitly represented through quasiparticle (Sturmian) expansions. The entire set of graphs corresponding to the present model is given and discussed.

I. INTRODUCTION

Pionic reactions on nuclei represent a fundamental tool for understanding low- and intermediate-energy nuclear physics, because of the privileged role played by pions inside the nuclear systems. An important aspect of the pionic probe is related to the presence of the absorption channel (i.e., no pions in the final state), and since a large momentum is transferred to the nuclear system in pion absorption, short-range correlations in nuclei are probed.

To get reliable information from pion absorption one has to individuate the various reaction mechanisms underlying the absorption process, and first of all, how many nucleons are directly involved in the process. It is generally understood that the dominant pionic absorption in nuclei involves two nucleons, while the remainder nucleons act as spectators. One-nucleon absorption is strongly suppressed in nuclei and completely forbidden on free nucleons, because of the energy-momentum mismatch.

A particularly promising system to probe with pions is the $A=3$ nucleus (^{3}He,^{3}H). On one hand, this system is sufficiently complicated to undergo absorption on both isoscalar and isovector pairs, or to support absorption mechanisms where all the three nucleons are involved. On the other hand, the same system is simple enough so that the conventional nuclear aspects can be treated exactly by means of Faddeev-type theories. Detailed experimental data are now available through exclusive experiments, where two-nucleon coincidence measurements have been performed for the $(\pi^+,2p)$ and (π^-,np) reactions on helium targets with energies of the incoming pions spanning from the region dominated by the Δ resonance to lower-energy regions, dominated by absorption in s wave [1–8]. From these measurements, the quasifree two-body absorption peaks are clearly discernible on the edge of the Dalitz plot [9] whereas the background which covers the whole phase space can be ascribed to three-nucleon absorption.

Up to now the two-body quasifree absorption has been the subject of most of the theoretical investigations. Particular attention has been devoted to the isospin ratio $R=\sigma[^3\mathrm{He}(\pi^+,pp)]/\sigma[^3\mathrm{He}(\pi^-,np)]$ and to the asymmetry of the differential cross section for π^- absorption in ^{3}He. With reference to the isospin ratio R, absorption via Δ formation only overestimates R because the formation of an intermediate s-wave N-Δ state is inhibited when pions are absorbed on an isovector pair. Therefore, to get a better agreement with experimental results, a careful treatment of other mechanisms must be performed, such as the NN' intermediate propagation [10] and mechanisms occurring via an s-wave $\pi\pi NN$ vertex [11].

It is generally assumed that the asymmetry in the (π^-,np) differential cross section is a consequence of the odd and even partial-wave interference [4,11–15]. In spite of the theoretical efforts, a satisfactory agreement with both the angular distribution and the overall normalization of the integrated cross section has not yet been achieved. This can be ascribed to an inherent ambiguity in the distinction between two- and three-body absorption modes [11,16]. In the case of π^- absorption this ambiguity can be quite important because the ratio of peak to background is considerably smaller than in the π^+ case.

Three-body absorption has been theoretically investigated to a lesser extent with respect to the two-body mechanism. Existing models [17,18] describe three-nucleon absorption through multistep reactions with multiple Δ excitation or rescattering. As a first step towards the identification of such three-body mechanisms, one has to ascertain the relative importance of more conventional reaction modes which lead to the same kinematical signature. These could be, for instance, conventional three-nucleon correlations in the final state, following an elementary absorption process.

In this paper we describe how three-body correlations following a one- or two-body absorption process can be theoretically treated in an exact way. Our starting point consists in the general representation of the absorption amplitude as a matrix element of an absorption operator between initial and final-state scattering wave functions. The initial and final states describe the π-^{3}He system before absorption, and the three outgoing nucleons, respectively. Under a theoretical point of view, an exact description of the pion-nucleon correlations in the initial state would require the unitary treatment of the πNNN-NNN system. The relevant formalism has been provided in Ref. [19], where connected-kernel equations have been derived through the rigorous Grassberger-Sandhas method. The numerical solution of these equations, however, represents a formidable task, even if the nuclear two-body interactions are reduced to a schematic separable form. We neglect in our formalism the pion-nucleon initial-state correlations, and focus our attention on the final-state interactions among the three outgoing nucleons. Since these latter effects can be realistically evaluated through three-body techniques, this seems to us the natural way of proceeding towards a comprehensive treatment of the absorption process, which is in itself very complicated. Obviously, it remains to be ascertained to what extent initial-state interactions (ISI) can be ignored. With the lack of a unitary πNNN calculation, this question is still controversial in the literature. On one hand, several authors assume that ISI are rather weak for $A=3$ nuclei, in view of the small size of the target [1,2,9]. The role of ISI in two-step absorption mechanisms has been carefully investigated for ^{3}He in exclusive experiments [6,8,9]. Experimental data show no clear kinematic signature of an ISI mechanism followed by two-nucleon absorption. On the other hand, for the (inverse) production reaction (p,π) on deuterons, distortion effects have been found to be important in determining the normalization of the cross sections [20].

By resorting to the Faddeev formalism we separate the plane-wave contribution to the absorption process from the contribution in which the nucleons undergo a complete three-body final-state interaction (FSI). This latter contribution is expressed in terms of the Faddeev-Alt-Grassberger-Sandhas (F-AGS) $3\to3$ or $3\to2$ transition operators, where the three-body dynamics is taken into account to all orders. The above transition operators are related, respectively, to processes with three nucleons or with one nucleon and the deuteron in the final state.

The basic ingredient for the explicit representation of the absorption amplitude is given by the Sturmian quasiparticle method [21–26]. By resorting to this method the initial total breakup of the target, in presence of the pion, can be rewritten as a sum over sequential two-body decays into one nucleon plus a correlated pair, which subsequently decays into two free (off-shell) particles. The same method together with the F-AGS equations [27,28] allows one to express the $3\to3$ and $3\to2$ amplitudes in terms of rearrangement-type amplitudes, representing transitions between different quasiparticle configurations and satisfying F-AGS-Lovelace (F-AGS-L) coupled-channel equations.

As for the elementary absorption modes, we shall consider a one-body absorption mechanism through the Galilei-invariant πNN vertex, together with a two-nucleon absorption process through a $\pi N\Delta$ vertex, intermediate ΔN propagation, and the subsequent decay of the isobar via a ΔN interaction. In addition to the above-mentioned mechanisms, occurring predominantly in p-wave, a two-body s-wave absorption through a phenomenological $\pi\pi NN$ Lagrangian of the Koltun-Reitan type is also considered [29]. In the spirit of Refs. [11,30–33], all these mechanisms concur to form the *rescattering model* once they are employed within the plane-wave approximation. However, care must be exercised in comparing the rescattering model of Refs. [11,30–33] with the one discussed here, since, whereas for the s-wave $\pi\pi NN$ contribution and for the p-wave rescattering term with Δ excitation our plane-wave contribution is completely equivalent to that model, for the one-body πNN term our plane-wave amplitude represents what is called the impulse approximation, which is a one-body mechanism. To have in our formalism a two-body absorption mode via the πNN vertex, one has to consider the first and most elementary contribution referring to the FSI. Had we embedded *tout court* the rescattering model in a three-body framework, we would have encountered double counting problems, since the NN interaction, which is responsible for the three-body dynamics in the final state, is also taken into account in the NN rescattering graph. If a more general coupled-channel model were employed, where the three nucleons in the final state are allowed to interact also via ΔN coupling, one would have to modify in the same way also the p-wave ΔN rescattering term. The presence of both one- and two-body elementary absorption naturally fits in the quasiparticle representation, since in the former case the elementary process occurs on one nucleon alone, with the remainder correlated pair acting as a spectator, whereas in the latter case the pion is absorbed on the correlated pair with the third nucleon being the spectator.

The distinctive feature of the present analysis is that the few-body aspects of the problems are treated correctly, while they are either ignored or treated in a very schematic way in the existing literature, where the analysis of the elementary absorption mechanism is emphasized. From the general F-AGS expression for the final-state transition amplitude it is possible to extract the various reaction mechanisms and to classify them according to the number of participating nucleons. For instance, all the connected terms coming from the F-AGS equations represent mechanisms in which the pion's energy and momentum are shared among all three nucleons. Indeed, as a general result of this F-AGS quasiparticle approach, it is possible to associate a well-defined graphical representation to all the mechanisms consistent with three-body dynamics and the assumed elementary absorption model.

In Sec. II we present the general F-AGS formalism for the absorption process. The quasiparticle approach is developed in Sec. III, where the relevant reaction mechanisms are also identified. In Sec. IV we discuss the physical aspects of the results and we outline possible approximations for practical purposes.

II. GENERAL FORMALISM FOR THE ABSORPTION PROCESS

A. The transition operators

The transition amplitude for meson absorption is generally expressible as a matrix element of an absorption operator $\mathcal{A}$ [34,35]. In the case of pion absorption on ^{3}He (or ^{3}H), the transition amplitudes for the complete disintegration of the target into three nucleons or for the dissociation into a deuteron plus a nucleon, $A_{3N}^{\rm tot}$ and $A_{dN}^{\rm tot}$, respectively, are

$$A_{3N}^{\rm tot}=\langle \psi_{\mathbf{pq}}^{(-)}|\mathcal{A}|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ , \tag{2.1}$$

$$A_{dN}^{\rm tot}=\langle \psi_{d\mathbf{q}}^{(-)}|\mathcal{A}|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ . \tag{2.2}$$

Here $\mathbf{p}$ and $\mathbf{q}$ are the Jacobi momenta of the pair and the spectator, respectively (these are determined once the role of spectator has been chosen for one of the three nucleons), and $\mathbf{P}_\pi^{(0)}$ represents the pion momentum with respect to the target c.m. We have assumed that the initial four-body state can be expressed approximately as a diadic combination of a pion plane wave and the ^{3}He (^{3}H) ground state, so that meson-nucleus correlations in the initial state are ignored (note, however, that nucleon-nucleon correlations in the initial state are fully taken into account through the exact bound-state wave function). As already noticed, the consistent treatment of ISI would entail the solution of two four-body problem coupled to all orders to the absorption channel.

The final-state interactions are exactly taken into account through the scattering states $\langle \psi_{\mathbf{pq}}^{(-)}|$ and $\langle \psi_{d\mathbf{q}}^{(-)}|$, which asymptotically describe three free nucleons, and a deuteron and a nucleon, respectively, with ingoing boundary conditions. These states contain the full three-body dynamics in the final state.

To write the absorption amplitude (2.1) as a plane-wave matrix element of a suitable transition operator we first express the scattering state $\langle \psi_{\mathbf{pq}}^{(-)}|$ in terms of the corresponding asymptotic state $\langle \mathbf{p},\mathbf{q}|$:

$$\langle \psi_{\mathbf{pq}}^{(-)}|=\lim_{\epsilon\to 0^+} i\epsilon\langle \mathbf{p},\mathbf{q}|G(E+i\epsilon) \ . \tag{2.3}$$

Here G is the total resolvent operator for the three-nucleon system evaluated at the total energy E. One can now rewrite G by introducing the AGS transition operator $U_{\beta\alpha}$, namely [27,28],

$$G=G_\beta\delta_{\beta\alpha}+G_\beta U_{\beta\alpha}G_\alpha \ , \tag{2.4}$$

with $\alpha,\beta=0,1,2,3$. In Eq. (2.4) G_β represents the resolvent operator referring to channel β; in particular, for $\beta=0$, G_β is the free three-body Green's function. Using the above equation with $\alpha=\beta=0$ in (2.3), one gets

$$\langle \psi_{\mathbf{pq}}^{(-)}|=\langle \mathbf{p},\mathbf{q}|+\langle \mathbf{p},\mathbf{q}|U_{00}G_0(E) \ , \tag{2.5}$$

once the identity $\lim_{\epsilon\to 0^+} i\epsilon\langle \mathbf{p},\mathbf{q}|G_0(E+i\epsilon)=\langle \mathbf{p},\mathbf{q}|$ has been employed.

Similarly, to transform the amplitude (2.2) we rewrite $\langle \psi_{d\mathbf{q}}^{(-)}|$ in the form

$$\langle \psi_{d\mathbf{q}}^{(-)}|=\lim_{\epsilon\to 0^+}\langle \mathbf{q}|\langle \psi_d|G(E+i\epsilon) \ , \tag{2.6}$$

where $|\psi_d\rangle$ and $|\mathbf{q}\rangle$ represent the deuteron bound state and the spectator plane wave, respectively.

We use in the above equation the relation (2.4) with $\alpha=0$ and $\beta=\beta_d\neq 0$, where β_d refers to the final two-cluster channel. Then, by means of the identity $\lim_{\epsilon\to 0^+} i\epsilon\langle \psi_d|\langle \mathbf{q}|G_{\beta_d}=\langle \psi_d|\langle \mathbf{q}|$, one gets

$$\langle \psi_{d\mathbf{q}}^{(-)}|=\langle \psi_d|\langle \mathbf{q}|U_{\beta_d 0}G_0 \ . \tag{2.7}$$

In order to separate in the above equation the plane-wave term from the final-state correlations, as in Eq. (2.5), we isolate the disconnected part in the $3\to 2$ transition operator $U_{\beta_d 0}$, namely,

$$U_{\beta_d 0}=G_0^{-1}+\tilde{U}_{\beta_d 0} \ . \tag{2.8}$$

We shall see in Sec. III A that the Faddeev theory provides an explicit representation of $\tilde{U}_{\beta_d 0}$. Once inserted into (2.7), Eq. (2.8) yields

$$\langle \psi_{d\mathbf{q}}^{(-)}|=\langle \psi_d|\langle \mathbf{q}|+\langle \psi_d|\langle \mathbf{q}|\tilde{U}_{\beta_d 0}G_0 \ . \tag{2.9}$$

In view of further developments, we transform also the initial-state part of the absorption amplitudes by introducing the homogeneous integral equation for the three-nucleon bound state in the total four-body space:

$$|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle=G_0^4(E)V|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ , \tag{2.10}$$

where G_0^4 is the free four-body propagator and V represents the total interaction among the three nucleons. In the usual three-body notation, one has $V=V_1+V_2+V_3$, V_1 being the pair interaction between particles 2 and 3. As we shall see in Sec. III A, this transformation allows one to exhibit the vertex function describing the virtual breakup of the target nucleus.

Inserting the representations (2.5) and (2.9) for the final scattering states into Eqs. (2.1) and (2.2), we get

$$A_{3N}^{\rm tot}=A_{3N}^{\rm PW}+A_{3N}^{\rm FSI} \ , \tag{2.11}$$

$$A_{dN}^{\rm tot}=A_{dN}^{\rm PW}+A_{dN}^{\rm FSI} \ , \tag{2.12}$$

where

$$A_{3N}^{\rm PW}=\langle \mathbf{p},\mathbf{q}|\mathcal{A}|\psi_{\rm BS}\rangle\mathbf{P}_\pi^{(0)}\rangle \ , \tag{2.13}$$

$$A_{dN}^{\rm PW}=\langle \psi_q|\langle \mathbf{q}|\mathcal{A}|\psi_{\rm BS}>|\mathbf{P}_\pi^{(0)}\rangle \ , \tag{2.14}$$

and

$$A_{3N}^{FSI}=\langle \mathbf{p},\mathbf{q}|U_{00}G_0\mathcal{A}G_0^4V|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ , \tag{2.15}$$

$$A_{dN}^{\rm FSI}=\langle \psi_d|\langle \mathbf{q}|\tilde{U}_{\beta_d 0}G_0\mathcal{A}G_0^4V|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ . \tag{2.16}$$

In writing down the last pair of equations use has been made of the identity (2.10). We have thus separated the absorption amplitude into a plane-wave part (without initial and final-state interactions) and a correlated part which properly takes into account the three-body dynamics in the final state.

B. Graphical representation of the absorption mechanism

We focus our attention on the transition amplitudes with final-state correlations [Eqs. (2.15) and (2.16)]. Their physical meaning is quite transparent, since after insertion of complete sets of plane-wave states, the term $\langle \mathbf{p},\mathbf{q}|V|\psi_{BS}\rangle$ describes the virtual decay of the target nucleus into three free particles in presence of the incoming pion. Therefore, the pair correlations between the three nucleons in the initial four-body state are fully taken into account through this form factor. Once the pion has been absorbed, the final-state interactions are described by the operators U_{00} and $\tilde{U}_{\beta_d 0}$ for the $3\rightarrow3$ and $3\rightarrow2$ transitions, respectively. The two propagators on the right and the left of the absorption operator $\mathcal{A}$ describe the intermediate free propagation of the four and three particles, respectively.

The absorption mechanism is completely specified once a particular model for the $\mathcal{A}$ operator is finally assumed. As is well known, two-nucleon absorption plays a major role in pion absorption in nuclei, with respect to the one-body mechanism (the so-called impulse approximation). A simple model for the $\mathcal{A}$ operator embodying the main dynamical features of $2N$ absorption is given by the rescattering model. We briefly recall its basic mechanisms, referring the reader to the relevant literature for details [11,30–33,36]. Since in the π-N dynamics a dominant role is played by the Δ excitation in p wave, a first contribution to be considered is given by the Δ-rescattering graph, along with the corresponding crossed graph. The former is shown in Fig. 1(a). In addition to Δ rescattering, other two-body absorption contributions to the cross section have been considered. These include a phenomenological s-wave rescattering term which can be constructed starting from the Koltun-Reitan $\pi\pi NN$ vertex [see Fig. 1(b)]. Moreover, the direct and crossed p-wave rescattering graphs with intermediate nucleon propagation should also be included; however, attention has to be paid here to possible double counting problems, since the static nucleon-nucleon interaction, which is responsible for the direct and crossed rescattering mechanisms, is already considered to all orders in the final-state interaction U_{00} or $\tilde{U}_{\beta_d 0}$ for the direct term, or in the nuclear form factor of the target in the crossed case. Therefore, for p-wave rescattering with intermediate nucleon propagation, the $\mathcal{A}$ operator must contain the impulse contribution only [Fig. 1(c)], being the rescattering already included through initial or final-state correlations between the nucleons.

In light of the above considerations, the contribution to the full absorption process with FSI included [see Eq. (2.15)] can be associated to the graphs shown in Fig. 2. In Fig. 2(a) the p-wave Δ-rescattering contribution is represented, to which the corresponding crossed term has to be added; the s-wave $\pi\pi NN$ vertex contribution as well as the p-wave nucleon rescattering contribution are exhibited in Figs. 2(b) and 2(c), respectively. We do not represent in a new figure the diagrams with two fragments, namely, a deuteron plus a free nucleon, in the final asymptotic state. These can be easily obtained by substituting $\tilde{U}_{\beta_d 0}$ for U_{00} in the diagrams of Fig. 2.

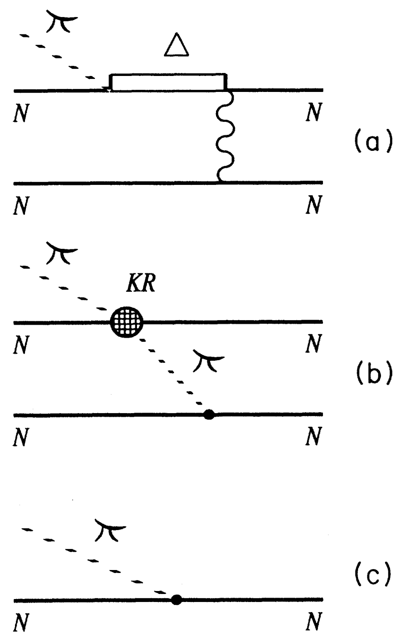

FIG. 1. Elementary absorption mechanisms. (a) The rescattering graph involving an intermediate isobar propagation, and (b) the rescattering graph generated from the $\pi\pi NN$ vertex of Koltun-Reitan. (c) The graph represents the one-body absorption via the πNN vertex.

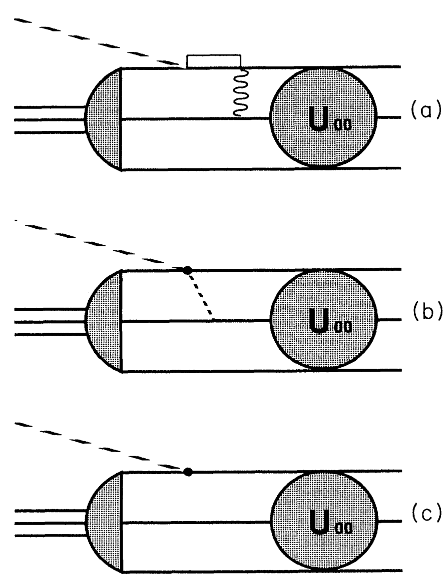

FIG. 2. Complete absorption processes with three free nucleons in the final state. Full lines are nucleons and the dashed line is the pion. The vertex functions of the virtual target breakup are depicted as semicircles, and the $3\rightarrow3$ F-AGS amplitudes are represented as full circles.

III. THE ABSORPTION PROCESS IN THE FADDEEV-AGS QUASIPARTICLE SCHEME

A. The quasiparticle representation

The evaluation of the amplitudes (2.15) and (2.16) is a very difficult task since, as can be seen from the corresponding graphs in Fig. 2, it entails integration over momenta associated with up to three different loops. Moreover, the Faddeev amplitudes for the $3\to 3$ and $3\to 2$ final-state scattering depend upon one of the integration momenta, and have to be evaluated therefore half-off the energy shell.

In this section we shall use the F-AGS theory to express U_{00} and $\tilde{U}_{\beta_d 0}$ in terms of the rearrangement transition operator $U_{\beta\alpha}$ with $\alpha,\beta\neq 0$. Furthermore, we shall introduce the quasiparticle representation for the two-body T matrices T_α; this leads to a new explicit representation for the absorption amplitudes which allows one to simplify the integration over the intermediate momenta on the basis of reasonable kinematical assumptions. At the same time the quasiparticle approach exhibits the correlated nature of the nuclear pairs which are responsible for the dominant absorption process.

In order to apply the quasiparticle scheme, it is convenient to rewrite the vertex function for the virtual target breakup. To this end, we use the obvious identity $V=\sum_{\gamma=1}^{3}V_\gamma$, and the homogeneous equations

$$|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle=G_\gamma^4(E)V^\gamma|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \tag{3.1}$$

for the three-nucleon bound state in the full four-body space (E being the total four-body energy). Here we have introduced the interactions external to the partition γ, namely, $V^\gamma=V_\alpha+V_\beta$. By resorting to (3.1) and to the well-known identity $V_\gamma G_\gamma=T_\gamma G_0$, one gets

$$V|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle=\sum_{\gamma=1}^{3}T_\gamma(E)G_0^4(E)V^\gamma|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ . \tag{3.2}$$

The aim of this transformation is to represent the virtual breakup of the target into three free nucleons as a sum over three decays into a two-nucleon pair and a spectator, plus the subsequent breakup of the pair through the two-body T matrix T_γ.

To treat the final-state correlations we start from the AGS relations among transition operators:

$$U_{\beta\alpha}=\bar{\delta}_{\beta\alpha}G_0^{-1}+\sum_{\gamma=1}^{3}\bar{\delta}_{\beta\gamma}T_\gamma G_0 U_{\gamma\alpha} \ , \tag{3.3}$$

$$U_{\beta\alpha}=\bar{\delta}_{\beta\alpha}G_0^{-1}+\sum_{\gamma=1}^{3}U_{\beta\gamma}G_0 T_\gamma\bar{\delta}_{\gamma\alpha} \ , \tag{3.4}$$

which hold for $\alpha,\beta=0,1,2,3$. Here $\bar{\delta}_{\beta\alpha}=1-\delta_{\beta\alpha}$. As is well known, the $3\to 3$ and $3\to 2$ transition operators can be expressed as a linear combination of the rearrangement transition operators $U_{\gamma\delta}(\gamma,\delta=1,2,3)$ by resorting to Eqs. (3.3) and (3.4). One has

$$U_{00}=\sum_{\gamma}T_\gamma+\sum_{\gamma,\delta=1}^{3}T_\gamma G_0 U_{\gamma\delta}G_0 T_\delta \tag{3.5}$$

and

$$U_{\gamma_d 0}=G_0^{-1}+\sum_{\delta=1}^{3}U_{\gamma_d\delta}G_0 T_\delta \ . \tag{3.6}$$

Insertion of Eqs. (3.5) and (3.2) into (2.15) allows one to write the absorption amplitude with three free nucleons in the final state in the form

$$A_{3N}^{\rm FSI}=\sum_{\alpha,\beta=1}^{3}\langle\mathbf{p},\mathbf{q}|T_\alpha G_0\mathcal{A}G_0^4T_\beta G_0^4V^\beta|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle+\sum_{\alpha,\beta,\gamma=1}^{3}\langle\mathbf{p},\mathbf{q}|T_\alpha G_0U_{\alpha\beta}G_0T_\beta G_0\mathcal{A}G_0^4T_\gamma G_0^4V^\gamma|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \ , \tag{3.7}$$

where the former term on the rhs represents the disconnected contribution associated with the $3\to 3$ amplitude, while the latter one takes into account the complete three-body dynamics in the final state.

Similarly, use of Eqs. (3.6), (3.2), and (2.8) leads to the expression

$$A_{dN}^{\rm FSI}=\sum_{\alpha,\beta=1}^{3}\langle\psi_d|\langle\mathbf{q}|U_{\beta_d\alpha}G_0T_\alpha G_0\mathcal{A}G_0^4T_\beta G_0^4V^\beta|\psi_{\rm BS}\rangle|\mathbf{P}_\pi^{(0)}\rangle \tag{3.8}$$

for the FSI absorption amplitude (2.16) for the two-cluster breakup. The amplitudes (3.7) and (3.8) are finally amenable to be represented by means of two-nucleon quasiparticles. We observe that the two-nucleon quasiparticles have to be introduced in a three-body context ($3N$) on the left-hand side of the operator $\mathcal{A}$, and in a four-body context ($3N+\pi$) on the right-hand side. In order to introduce the quasiparticle representation, we need to specify the kinematical aspects for the process. Following the standard three-body notation we call $\mathbf{p}_\gamma$ the relative momentum of the pair (α,β), and $\mathbf{q}_\gamma$ the relative momentum of the spectator γ with respect to the c.m. of the pair (α,β). By $\mathbf{P}_\pi$ we denote the relative momentum of the pion with respect to the three-nucleon center of mass. Before absorption, the energy of the system in the total c.m. frame can be written in the form

$$E=\frac{\mathbf{p}_\gamma^2}{2\mu_\gamma}+\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}+\frac{\mathbf{P}_\pi^2}{2\mu_\pi}+m_\pi c^2 \ , \quad \gamma=1,2,3 \ . \tag{3.9}$$

After absorption, the energy of the system in the total c.m. frame is

$$E=\frac{\mathbf{p}_\gamma'^2}{2\mu_\gamma}+\frac{\mathbf{q}_\gamma'^2}{2\nu_\gamma}\ ,\quad \gamma=1,2,3\ . \tag{3.10}$$

In Eqs. (3.9) and (3.10), μ_γ, ν_γ, and μ_π are the well-known Jacobi reduced masses of the pair, of the spectator nucleon, and of the pion, respectively, while m_π represents the pion rest mass.

The approach here proposed is suited to describe the full final-state correlations, in the low-energy domain, where a nonrelativistic description of the three nucleons is quite reasonable. On the contrary, much more delicate is the pion, which one can treat as a nonrelativistic particle only for very low energy. Minimal relativistic-kinematic corrections can be introduced along the lines developed for πd scattering by Thomas [37], namely, the expression $\sqrt{\mathbf{P}_\pi^2c^2+m_\pi^2c^4}-m_\pi c^2$ is used for the pion kinetic energy; in so doing, the expression (3.9) for the energy of the four-body system is replaced by

$$E=\frac{\mathbf{p}_\gamma^2}{2\mu_\gamma}+\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}+\frac{\mathbf{P}_\pi^2}{6m_N}+\sqrt{\mathbf{P}_\pi^2c^2+m_\pi^2c^4}\ , \tag{3.11}$$

m_N being the nucleon rest mass. Moreover, in the usual expressions for the reduced masses, the pion rest mass is replaced by the pion relativistic energy. For the sake of simplicity, we do not consider relativistic effects in this paper, although minimal relativistic corrections can be accommodated without major modifications of the formalism. More demanding relativistic prescriptions, such as the use of relativistic Jacobi momenta [38,39] and of covariant dynamical equations [40], are beyond the scope of the present approach.

The homogeneous Sturmian problem defining the two-nucleon quasiparticles can be written in the three-body space as follows:

$$G_0(E)V_\gamma\left|\phi_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right]\right\rangle|\mathbf{q}_\gamma\rangle =\eta_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right]\left|\phi_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right]\right\rangle|\mathbf{q}_\gamma\rangle\ . \tag{3.12}$$

Here we have exhibited the dependence of the Sturmian eigenvalues $\eta_{\gamma i}$ and eigenvectors $|\phi_{\gamma i}\rangle$ upon the energy $E-\mathbf{q}_\gamma^2/2\nu_\gamma$ of the two-body subsystem (α,β). On the other hand, in the four-body space the Sturmian problem becomes

$$G_0^4(E)V_\gamma\left|\phi_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right]\right\rangle|\mathbf{q}_\gamma\rangle|\mathbf{P}_\pi\rangle =\eta_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right]\left|\phi_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right]\right\rangle|\mathbf{q}_\gamma\rangle|\mathbf{P}_\pi\rangle\ , \tag{3.13}$$

where, again we have exhibited the dependence upon the energy of the two-nucleon subsystem.

By means of well-known properties of these Sturmian functions [21–26], the two-body T matrices T_γ acting in three- and four-body space are found to be

$$T_\gamma(E)=\sum_{i=1}^{\infty}\int d\mathbf{q}_\gamma\left|\Gamma_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right],\mathbf{q}_\gamma\right\rangle\tau_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right]\left\langle\mathbf{q}_\gamma,\Gamma_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}\right]\right| \tag{3.14}$$

and

$$T_\gamma(E)=\sum_{i=1}^{\infty}\int d\mathbf{q}_\gamma d\mathbf{P}_\pi\left|\Gamma_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right],\mathbf{q}_\gamma,\mathbf{P}_\pi\right\rangle \times\tau_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right]\left\langle\mathbf{P}_\pi,\mathbf{q}_\gamma,\Gamma_{\gamma i}\left[E-\frac{\mathbf{q}_\gamma^2}{2\nu_\gamma}-\frac{\mathbf{P}_\pi^2}{2\mu_\pi}-m_\pi c^2\right]\right|\ , \tag{3.15}$$

respectively. For convenience, the three- and four-body states

$$|\Gamma_{\gamma i}(X),\mathbf{q}_\gamma\rangle\equiv V_\gamma|\phi_{\gamma i}(X),\mathbf{q}_\gamma\rangle\equiv V_\gamma|\phi_{\gamma i}(X)\rangle|\mathbf{q}_\gamma\rangle \tag{3.16}$$

and

$$|\Gamma_{\gamma i}(X),\mathbf{q}_\gamma,\mathbf{P}_\pi\rangle\equiv V_\gamma|\phi_{\gamma i}(X),\mathbf{q}_\gamma,\mathbf{P}_\pi\rangle\equiv V_\gamma|\phi_{\gamma i}(X)\rangle|\mathbf{q}_\gamma\rangle|\mathbf{P}_\pi\rangle \tag{3.17}$$

have been defined, and the quantity

$$\tau_{\gamma i}(X)=\frac{1}{\eta_{\gamma i}(X)[1-\eta_{\gamma i}(X)]} \tag{3.18}$$

has also been introduced. Here, the Sturmian orthonormality with the normalization

$$\langle \phi_{\gamma i}(X)|V_\gamma|\phi_{\gamma j}(X)\rangle = \eta_{\gamma i}(X)\delta_{ij} \tag{3.19}$$

has been employed. Note that, when using Sturmians, no complex conjugation is implied in passing from ket to bra vectors.

Finally, by substitution of Eqs. (3.14) and (3.15) into (3.7) and (3.8), it is possible to exhibit in these amplitudes the full dependence upon the intermediate momenta. For instance, for the disconnected part of the amplitude (3.7) we obtain

$$A_{3N}^{\mathrm{FSI,dis}} = \sum_{\alpha,\beta=1}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{q}'_\alpha d\mathbf{q}''_\beta d\mathbf{P}_\pi \langle \mathbf{p},\mathbf{q}|V_\alpha|\phi_{\alpha i}(\xi'_\alpha),\mathbf{q}'_\alpha\rangle \tau_{\alpha i}(\xi'_\alpha)$$
$$\times \eta_{\alpha i}(\xi'_\alpha)\langle \mathbf{q}'_\alpha,\phi_{\alpha i}(\xi'_\alpha)|\mathcal{A}|\phi_{\beta j}(\xi''_\beta),\mathbf{q}''_\beta,\mathbf{P}_\pi\rangle \eta^2_{\beta j}(\xi''_\beta)\tau_{\beta j}(\xi''_\beta)\langle \mathbf{P}_\pi,\mathbf{q}''_\beta,\phi_{\beta j}(\xi''_\beta)|V^\beta|\psi_{\mathrm{BS}},\mathbf{P}_\pi^{(0)}\rangle , \tag{3.20}$$

where ξ'_α and ξ''_β are $E-\mathbf{q}'^2_\alpha/2\nu_\alpha$ and $E-\mathbf{q}''^2_\beta/2\nu_\beta-\mathbf{P}^2_\pi/2\mu_\pi-m_\pi c^2$, respectively. This amplitude, after a few straightforward manipulations, can be written as follows:

$$A_{3N}^{\mathrm{FSI,dis}} = \sum_{\alpha,\beta=1}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{p}'_\alpha d\mathbf{p}''_\beta d\mathbf{q}''_\beta \frac{\langle \mathbf{p}_\alpha|V_\alpha|\phi_{\alpha i}(\xi'_\alpha)\rangle\langle\phi_{\alpha i}(\xi'_\alpha)|\mathbf{p}'_\alpha\rangle}{1-\eta_{\alpha i}(\xi'_\alpha)}$$
$$\times \langle \mathbf{p}'_\alpha,\mathbf{q}_\alpha|\mathcal{A}|\mathbf{p}''_\beta,\mathbf{q}''_\beta,\mathbf{P}_\pi^{(0)}\rangle\langle \mathbf{p}''_\beta|\phi_{\beta j}(\xi''_\beta)\rangle \frac{\eta_{\beta j}(\xi''_\beta)}{1-\eta_{\beta j}(\xi''_\beta)}\langle \mathbf{q}''_\beta,\phi_{\beta j}(\xi''_\beta)|V^\beta|\psi_{\mathrm{BS}}\rangle , \tag{3.21}$$

with $\mathbf{q}'_\alpha=\mathbf{q}_\alpha$ in ξ'_α. The corresponding expression for the connected part of this amplitude is somewhat more complicated, but can be derived in the same way.

In Eq. (3.21), the treatment of the initial target breakup deserves particular attention. On the right of the absorption operator one recognizes the decay of the target into three particles, as expressed by Eq. (3.2). Indeed, projecting Eq. (3.2) onto plane-wave states and employing the quasiparticle expansion, one has

$$\langle \mathbf{p}'',\mathbf{q}''|V|\psi_{\mathrm{BS}}\rangle = \sum_{\beta=1}^{3}\sum_{j=1}^{\infty}\langle \mathbf{p}''_\beta|\phi_{\beta j}\rangle \frac{\eta_{\beta j}}{1-\eta_{\beta j}}\langle \mathbf{q}''_\beta,\phi_{\beta j}|V^\beta|\psi_{\mathrm{BS}}\rangle . \tag{3.2'}$$

On the left of Eq. (3.2′), one has the standard vertex function for the three-body decay of the bound state [41]. On the right, the vertex function for (2)+1 target decay is exhibited, as defined in the four-body literature [41,42]. With respect to the standard definition, the two-body bound state is here replaced by the Sturmian state $\phi_{\beta j}$, describing a correlated pair which propagates through $\eta_{\beta j}/(1-\eta_{\beta j})$, and decays via the two-body vertex function $\langle \mathbf{p}''_\beta|\phi_{\beta j}\rangle$.

For the sake of simplicity, we do not consider here the partial-wave analysis of the amplitudes; this can be accomplished by taking into account the actual structure of the basic absorption models [36], and by resorting to standard few-body techniques for the three-nucleon bound-state and scattering problems [43].

The final form for the amplitudes here discussed can be obtained once the assumed absorption model specifies the kinematical dependence for the operator $\mathcal{A}$. Under this point of view, we shall now distinguish between two-nucleon absorption, as described by the mechanisms shown in Figs. 1(a) and 1(b), and the one-body absorption due to the πNN vertex [Fig. 1(c)].

B. The two-nucleon rescattering contribution

If one introduces the rescattering model outlined in Sec. II, the relevant momenta are $\mathbf{K}_\pi$, $\mathbf{p}''_\beta$, and $\mathbf{p}'_\beta$, where $\mathbf{K}_\pi$ is the Jacobi momentum of the pion with respect to the absorbing pair β, and $\mathbf{p}''_\beta$, $\mathbf{p}'_\beta$ are the relative momenta of the pair β before and after the absorption process, respectively. Hence, the two-nucleon absorption mechanism implied by this model allows one to factorize a delta function $\delta(\mathbf{q}'_\beta-\mathbf{Q}''_\beta)$ from the matrix elements of $\mathcal{A}$, where $\mathbf{q}'_\beta$ is the relative momentum of the spectator nucleon in the final state, while $\mathbf{Q}''_\beta$ is defined as the Jacobi momentum of the spectator nucleon with respect to the πNN c.m. One can write therefore

$$\langle \mathbf{p}'_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha),\mathbf{q}'_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha)|\mathcal{A}|\mathbf{p}''_\beta,\mathbf{K}_\pi(\mathbf{q}''_\beta,\mathbf{P}_\pi^{(0)}),\mathbf{Q}''_\beta(\mathbf{q}''_\beta,\mathbf{P}_\pi^{(0)})\rangle$$
$$=\delta(\mathbf{q}'_\beta-\mathbf{Q}''_\beta)\mathcal{A}(\mathbf{p}'_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha);\mathbf{p}''_\beta,\mathbf{K}_\pi(\mathbf{q}''_\beta,\mathbf{P}_\pi^{(0)})) . \tag{3.22}$$

To evaluate the amplitude (3.21) it is now convenient to distinguish between the terms with $\alpha\neq\beta$ and the contributions with $\alpha=\beta$. In the former case we integrate directly with respect to $\mathbf{p}'_\alpha$ in virture of (3.22), which implies that $\mathbf{p}'_\alpha$ has to be evaluated at a value $\mathbf{p}^*_\alpha$ given by

$$\mathbf{p}'_\alpha(\mathbf{q}''_\beta,\mathbf{q}_\alpha,\mathbf{P}_\pi^{(0)})=\mathbf{q}''_\beta+\tfrac{1}{2}\mathbf{q}_\alpha-\tfrac{1}{3}\mathbf{P}_\pi^{(0)}\equiv\mathbf{p}^*_\alpha . \tag{3.23}$$

Let's call $E_{3N}^{\mathrm{FSI,dis}}$ the contribution to (3.21) with $\alpha\neq\beta$; one then gets

$$E_{3N}^{\mathrm{FSI,dis}}=\sum_{\alpha,\beta=1}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{p}_\beta'' d\mathbf{q}_\beta'' \frac{\langle \mathbf{p}_\alpha | V_\alpha | \phi_{\alpha i}(\xi_\alpha')\rangle\langle \phi_{\alpha i}(\xi_\alpha')|\mathbf{p}_\alpha^*\rangle}{1-\eta_{\alpha i}(\xi_\alpha')}$$
$$\times\mathcal{A}(\mathbf{p}_\beta^*;\mathbf{p}_\beta'',\mathbf{K}_\pi)\langle \mathbf{p}_\beta''|\phi_{\beta j}(\xi_\beta'')\rangle\frac{\eta_{\beta j}(\xi_\beta'')}{1-\eta_{\beta j}(\xi_\beta'')}\langle \mathbf{q}_\beta'',\phi_{\beta j}(\xi_\beta'')|V^\beta|\psi_{\mathrm{BS}}\rangle \ . \quad (3.24)$$

Here, the energies of the two-body subsystems in three- and four-body spaces have been denoted as

$$\xi_\alpha'(q_\alpha)=E-\frac{3\mathbf{q}_\alpha^2}{4m_N} \ , \quad (3.25a)$$

$$\xi_\beta''(q_\beta'',P_\pi^{(0)})=E-\frac{3\mathbf{q}_\beta''^2}{4m_N}-\frac{\mathbf{P}_\pi^{(0)2}}{2\mu_\pi}-m_\pi c^2 \ , \quad (3.25b)$$

respectively. The momenta $\mathbf{K}_\pi$ and $\mathbf{p}_\beta^*$ are related to the external and loop momenta as follows:

$$\mathbf{K}_\pi(\mathbf{q}_\beta'',\mathbf{P}_\pi^{(0)})=\frac{r}{r+2}\mathbf{q}_\beta''+\frac{6+2r}{6+3r}\mathbf{P}_\pi^{(0)} \ , \quad (3.26)$$

$$\mathbf{p}_\beta'(\mathbf{q}_\beta'',\mathbf{q}_\alpha,\mathbf{P}_\pi^{(0)})=-\tfrac{1}{2}\mathbf{q}_\beta''-\mathbf{q}_\alpha+\tfrac{1}{6}\mathbf{P}_\pi^{(0)}\equiv\mathbf{p}_\beta^* \ , \quad (3.27)$$

where $r=0.149$ is the pion to nucleon mass ratio.

In the case $\alpha=\beta$ one exploits Eq. (3.22) to integrate with respect to $\mathbf{q}_\beta''$ to get

$$\mathbf{q}_\beta''(\mathbf{q}_\beta,\mathbf{P}_\pi^{(0)})=\mathbf{q}_\beta+\tfrac{1}{3}\mathbf{P}_\pi^{(0)}\equiv\mathbf{q}_\beta^* \ . \quad (3.28)$$

One thus obtains the following expression for the direct ($\alpha=\beta$) contribution $D_{3N}^{\mathrm{FSI,dis}}$ to the disconnected part of the absorption transition amplitude A_{3N}^{FSI}:

$$D_{3N}^{\mathrm{FSI,dis}}=\sum_{\beta=1}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{p}_\beta' d\mathbf{p}_\beta'' \frac{\langle \mathbf{p}_\beta | V_\beta | \phi_{\beta i}(\xi_\beta')\rangle\langle \phi_{\beta i}(\xi_\beta')|\mathbf{p}_\beta'\rangle}{1-\eta_{\beta i}(\xi_\beta')}$$
$$\times\mathcal{A}(\mathbf{p}_\beta',\mathbf{p}_\beta'',\mathbf{K}_\pi)\langle \mathbf{p}_\beta''|\phi_{\beta j}(\xi_\beta'')\rangle\frac{\eta_{\beta j}(\xi_\beta'')}{1-\eta_{\beta j}(\xi_\beta'')}\langle \mathbf{q}_\beta^*,\phi_{\beta j}(\xi_\beta'')|V^\beta|\psi_{\mathrm{BS}}\rangle \ . \quad (3.29)$$

Here the two-body energy in the four-body space ξ_β'' is given again by (3.25b), where $\mathbf{q}_\beta''$ is now replaced by $\mathbf{q}_\beta^*$, and by means of Eqs. (3.26) and (3.28) $\mathbf{K}_\pi$ can be expressed exclusively in terms of the external momenta

$$\mathbf{K}_\pi(\mathbf{q}_\beta,\mathbf{P}_\pi^{(0)})=\frac{r}{2+r}\mathbf{q}_\beta+\mathbf{P}_\pi^{(0)} \ . \quad (3.30)$$

It is interesting at this point to provide a graphical representation of the direct and exchange amplitudes, $D_{3N}^{\mathrm{FSI,dis}}$ and $E_{3N}^{\mathrm{FSI,dis}}$, respectively; in so doing one can more easily identify the reaction mechanism underlying each contribution. The direct contribution $D_{3N}^{\mathrm{FSI,dis}}$, given by Eq. (3.29), corresponds to the virtual decay of the target nucleus into a nucleon and a correlated pair, which successively absorbs the incoming pion and undergoes a FSI, the third nucleon always acting as a spectator. In the exchange contribution (3.24), on the contrary, one of the nucleons of the pair which absorbs the pion exchanges energy and momentum with the third nucleon through a two-body FSI. In Fig. 3(a) we exhibit the diagram corresponding to the direct contribution with p-wave absorption and Δ excitation. In Fig. 3(b) we give the p-wave absorption graph with Δ excitation, corresponding to the exchange term $E_{3N}^{\mathrm{FSI,dis}}$. This graph represents the simplest three-body mechanism, where the incident-pion energy and momentum are shared among all three nucleons. On the contrary, the D term has to be

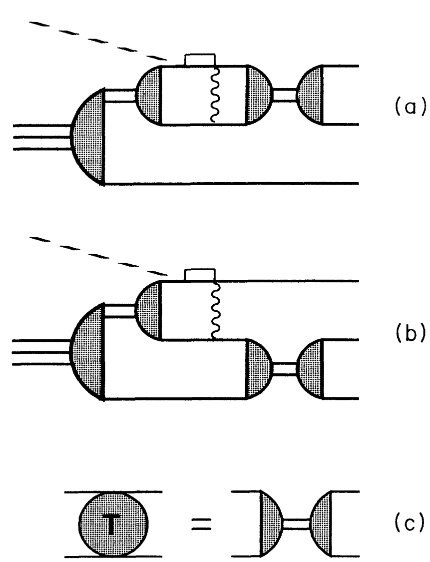

FIG. 3. Graphs (a) and (b) are, respectively, the amplitudes (3.29) and (3.24) in the case of Δ rescattering. The initial-target breakup and the two-body FSI are represented in the quasiparticle scheme. The quasiparticle representation of the two-body T matrix is given in Fig. 3(c).

associated with a genuine two-body absorption mechanism, where the third nucleon has a momentum distribution according to an unperturbed nucleon in ^{3}He (quasifree absorption). The graphs associated with the $\pi\pi NN$ s-wave vertex have the same structure and are not shown for shortness. For the same reason we do not represent the relevant crossed diagrams. Of course, for each contribution mentioned above, there is a corresponding term associated with the absorption through the other nucleon in the active pair.

In Fig. 3(c) we have graphically exhibited the Sturmian representation of the two-body T matrix T_β, with the corresponding intermediate propagation of the quasiparticles [see Eqs. (3.14) and (3.15)].

The amplitudes for the connected contribution to the absorption process can be explicitly given following a similar procedure. The new ingredient is now represented by the F-AGS-L amplitudes $X_{\beta i,\alpha j} = \langle \phi_{\beta i}, \mathbf{q}_\beta | U_{\beta\alpha} | \phi_{\alpha j}, \mathbf{q}'_\alpha \rangle$, which appear once the Faddeev equations (3.3) and (3.4) are folded between quasiparticle states. Insertion of the Sturmian representations (3.14) and (3.15) for the two-body T matrices into the connected part of the absorption amplitude (3.7), with the condition

$$\mathbf{p}'_\beta(\mathbf{q}''_\gamma, \mathbf{q}'_\beta, \mathbf{P}_\pi^{(0)}) = \mathbf{q}''_\gamma + \tfrac{1}{2}\mathbf{q}'_\beta - \tfrac{1}{3}\mathbf{P}_\pi^{(0)} \equiv \mathbf{p}^*_\beta \ , \qquad (3.31)$$

implied by the two-body absorption model (3.22), leads to the expression

$$E_{3N}^{\mathrm{FSI,conn}} = \sum_{\substack{\alpha,\beta,\gamma=1 \\ \beta\neq\gamma}}^{3} \sum_{i,j,k=1}^{\infty} \int d\mathbf{q}'_\beta d\mathbf{q}''_\gamma d\mathbf{p}''_\gamma \langle \mathbf{p}_\alpha | V_\alpha | \phi_{\alpha i}(\xi'_\alpha) \rangle \frac{1}{1-\eta_{\alpha i}(\xi'_\alpha)} X_{\alpha i,\beta j}(\mathbf{q}_\alpha, \mathbf{q}'_\beta) \frac{\eta_{\beta j}(\xi'_\beta)}{1-\eta_{\beta j}(\xi'_\beta)} \langle \phi_{\beta j}(\xi'_\beta) | \mathbf{p}^*_\beta \rangle$$
$$\times \mathcal{A}(\mathbf{p}^*_\gamma, \mathbf{p}''_\gamma, \mathbf{K}_\pi) \langle \mathbf{p}''_\gamma | \phi_{\gamma k}(\xi''_\gamma) \rangle \frac{\eta_{\gamma k}(\xi''_\gamma)}{1-\eta_{\gamma k}(\xi''_\gamma)} \langle \mathbf{q}''_\gamma, \phi_{\gamma k}(\xi''_\gamma) | V^\gamma | \psi_{\mathrm{BS}} \rangle \qquad (3.32a)$$

for the connected exchange ($\gamma\neq\beta$) contribution. Here, the momentum $\mathbf{p}^*_\gamma$ depends upon the three momenta $\mathbf{q}''_\gamma$, $\mathbf{q}'_\beta$, and $\mathbf{P}_\pi^{(0)}$ as given by Eq. (3.27). Similarly, the relative momentum $\mathbf{K}_\pi$ of the pion with respect to the c.m. of the pair γ is given in terms of $\mathbf{q}''_\gamma$ and $\mathbf{P}_\pi^{(0)}$ by Eq. (3.26). The two-body energies $\xi'_\alpha(q_\alpha), \xi'_\beta(q'_\beta)$, and $\xi''_\gamma(q''_\gamma, P_\pi^{(0)})$ are given through Eqs. (3.25).

As for the direct contribution ($\gamma=\beta$) to the connected amplitude one gets in a similar way

$$D_{3N}^{\mathrm{FSI,conn}} = \sum_{\alpha,\gamma=1}^{3} \sum_{i,j,k=1}^{\infty} \int d\mathbf{p}'_\gamma d\mathbf{q}'_\gamma d\mathbf{p}''_\gamma \langle \mathbf{p}_\alpha | V_\alpha | \phi_{\alpha i}(\xi'_\alpha) \rangle \frac{1}{1-\eta_{\alpha i}(\xi'_\alpha)} X_{\alpha i,\gamma j}(\mathbf{q}_\alpha, \mathbf{q}'_\gamma) \frac{\eta_{\gamma j}(\xi'_\gamma)}{1-\eta_{\gamma j}(\xi'_\gamma)} \langle \phi_{\gamma j}(\xi'_\gamma) | \mathbf{p}'_\gamma \rangle$$
$$\times \mathcal{A}(\mathbf{p}'_\gamma, \mathbf{p}''_\gamma, \mathbf{K}_\pi) \langle \mathbf{p}''_\gamma | \phi_{\gamma k}(\xi''_\gamma) \rangle \frac{\eta_{\gamma k}(\xi''_\gamma)}{1-\eta_{\gamma k}(\xi''_\gamma)} \langle \mathbf{q}^*_\gamma, \phi_{\gamma k}(\xi''_\gamma) | V^\gamma | \psi_{\mathrm{BS}} \rangle \ . \qquad (3.32b)$$

Here, the momenta $\mathbf{q}^*_\gamma$ and $\mathbf{K}_\pi$ are expressed in terms of $\mathbf{q}'_\gamma, \mathbf{P}_\pi^{(0)}$ through equations of the form (3.28) and (3.30), respectively. The graphical representation for the two contributions (3.32a) and (3.32b) is given in Fig. 4.

As is well known, the dynamical equations for the F-AGS-L amplitudes $X_{\beta i,\alpha j}$ can be derived by folding Eq. (3.3) for the rearrangement transition operators $U_{\beta\alpha}$ between quasiparticle states, and introducing the representation (3.14) for the two-body T matrices. In a shorthand notation one has

$$X_{\beta i,\alpha j} = Z_{\beta i,\alpha j} + \sum_{\gamma=1}^{3} \sum_{k=1}^{\infty} Z_{\beta i,\gamma k} \frac{\eta_{\gamma k}}{1-\eta_{\gamma k}} X_{\gamma k,\alpha j} \ , \qquad (3.33)$$

where the driving term is defined according to

$$Z_{\beta i,\alpha j} = \eta_{\beta i}^{-1} \langle \phi_{\beta i} | V_\beta G_0 V_\alpha | \phi_{\alpha j} \rangle \eta_{\alpha j}^{-1} \bar{\delta}_{\beta\alpha} \ . \qquad (3.34)$$

In practical calculations, the number of coupled equations to be solved can be reduced once nucleon identity is taken into account.

C. The one-body πNN contribution

As already mentioned in Sec. II, for the p-wave absorption with intermediate nucleon propagation the rescatter-

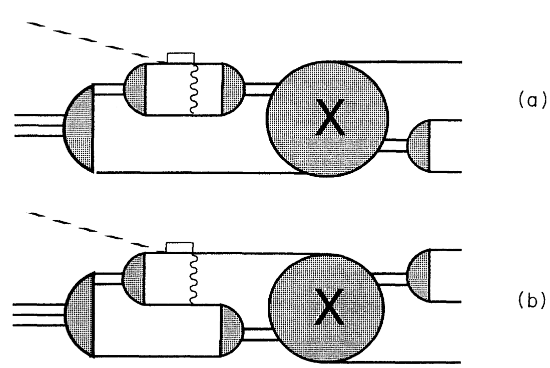

FIG. 4. (a) Direct and (b) exchange amplitudes with three-body FSI (quasiparticle representation). The full circles indicate the rearrangement-type Lovelace amplitudes.

ing graphs are taken into account, to all orders, by the exact treatment of initial- and final-state correlations. Consequently, in this case, the absorption operator has a genuine one-body nature, and will depend upon the relative momentum between the incoming pion and the absorbing nucleon. Therefore, differently from the previous section, the quasiparticle representation has to be applied to the spectator pair, which does not participate to the absorption process.

It is here convenient to describe the four-body kinematics before absorption through the Jacobi momenta $\mathbf{p}_\beta$, $\mathbf{Q}_{\beta\pi}$, and $\mathbf{p}_{\beta\pi}$, where $\mathbf{p}_\beta$ and $\mathbf{p}_{\beta\pi}$ are the relative momenta for the pairs $\alpha\gamma$ and $\beta\pi$, and $\mathbf{Q}_{\beta\pi}$ is the momentum between the centers of mass of the two pairs.

The kinematical dependence of the operator $\mathcal{A}$ in Eq. (3.21) will be now

$$\langle \mathbf{p}'_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha),\mathbf{q}'_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha)|\mathcal{A}|\mathbf{p}''_\beta,\mathbf{Q}_{\beta\pi}(\mathbf{q}''_\beta,\mathbf{P}^{(0)}_\pi),\mathbf{p}_{\beta\pi}(\mathbf{q}''_\beta,\mathbf{P}^{(0)}_\pi)\rangle=\delta(\mathbf{p}'_\beta-\mathbf{p}''_\beta)\delta(\mathbf{q}'_\beta-\mathbf{Q}_{\beta\pi})\mathcal{A}(\mathbf{p}_{\beta\pi}) , \tag{3.35}$$

where $\mathbf{Q}_{\beta\pi}$ and $\mathbf{p}_{\beta\pi}$ are expressed in terms of the momenta $\mathbf{q}''_\beta$ and $\mathbf{P}^{(0)}_\pi$, by

$$\mathbf{Q}_{\beta\pi}=\mathbf{q}''_\beta+\tfrac{2}{3}\mathbf{P}^{(0)}_\pi , \tag{3.36a}$$

$$\mathbf{p}_{\beta\pi}=\frac{r}{1+r}\mathbf{q}''_\beta-\frac{3+r}{3(1+r)}\mathbf{P}^{(0)}_\pi . \tag{3.36b}$$

Inserting (3.35) into the terms with $\alpha\neq\beta$ of (3.21) and integrating over $\mathbf{q}''_\beta$ and $\mathbf{p}''_\beta$ we have

$$E^{\mathrm{FSI,dis}}_{3N}=\sum_{\substack{\alpha,\beta=1\\ \alpha\neq\beta}}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{p}'_\alpha\frac{\langle\mathbf{p}_\alpha|V_\alpha|\phi_{\alpha i}(\xi'_\alpha)\rangle\langle\phi_{\alpha i}(\xi'_\alpha)|\mathbf{p}'_\alpha\rangle}{1-\eta_{\alpha i}(\xi'_\alpha)}\mathcal{A}(\mathbf{p}_{\beta\pi})\langle\mathbf{p}^*_\beta|\phi_{\beta j}(\xi''_\beta)\rangle\frac{\eta_{\beta j}(\xi''_\beta)}{1-\eta_{\beta j}(\xi''_\beta)}\langle\mathbf{q}^*_\beta,\phi_{\beta j}(\xi''_\beta)|V^\beta|\psi_{\mathrm{BS}}\rangle \tag{3.37}$$

for the exchange disconnected transition amplitude.

Here, the two momenta $\mathbf{p}^*_\beta$ and $\mathbf{q}^*_\beta$ are related to the external and loop momenta through

$$\mathbf{p}''_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha)=-\tfrac{1}{2}\mathbf{p}'_\alpha-\tfrac{3}{4}\mathbf{q}_\alpha\equiv\mathbf{p}^*_\beta , \tag{3.38a}$$

$$\mathbf{q}''_\beta(\mathbf{p}'_\alpha,\mathbf{q}_\alpha,\mathbf{P}^{(0)}_\pi)=\mathbf{p}'_\alpha-\tfrac{1}{2}\mathbf{q}_\alpha-\tfrac{2}{3}\mathbf{P}^{(0)}_\pi\equiv\mathbf{q}^*_\beta , \tag{3.38b}$$

whereas $\mathbf{p}_{\beta\pi}$ is obtained once the expression (3.38b) for $\mathbf{q}''_\beta$ is inserted into Eq. (3.36b). As usual, ξ''_β and ξ'_α represent the energies of the two-body subsystems before and after absorption.

Similarly, for the direct contributions ($\alpha=\beta$), integration over $\mathbf{p}''_\beta$ and $\mathbf{q}''_\beta$ leads to

$$D^{\mathrm{FSI,dis}}_{3N}=\sum_{\beta=1}^{3}\sum_{i,j=1}^{\infty}\int d\mathbf{p}'_\beta\frac{\langle\mathbf{p}_\beta|V_\beta|\phi_{\beta i}(\xi'_\beta)\rangle\langle\phi_{\beta i}(\xi'_\beta)|\mathbf{p}'_\beta\rangle}{1-\eta_{\beta i}(\xi'_\beta)}\mathcal{A}(\mathbf{p}_{\beta\pi})\langle\mathbf{p}'_\beta|\phi_{\beta j}(\xi''_\beta)\rangle\frac{\eta_{\beta j}(\xi''_\beta)}{1-\eta_{\beta j}(\xi''_\beta)}\langle\mathbf{q}^*_\beta,\phi_{\beta j}(\xi''_\beta)|V^\beta|\psi_{\mathrm{BS}}\rangle , \tag{3.39}$$

with

$$\mathbf{q}''_\beta(\mathbf{q}_\beta,\mathbf{P}^{(0)}_\pi)=\mathbf{q}_\beta-\tfrac{2}{3}\mathbf{P}^{(0)}_\pi\equiv\mathbf{q}^*_\beta , \tag{3.40a}$$

$$\mathbf{p}_{\beta\pi}=\frac{r}{1+r}\mathbf{q}_\beta-\mathbf{P}^{(0)}_\pi . \tag{3.40b}$$

The exchange and direct disconnected amplitudes for the one-body absorption mechanism are graphically represented in Figs. 5(a) and 5(b).

In addition to the disconnected contributions discussed above, to have a complete picture of the one-body absorption process, the connected contributions too have to be considered. This can be achieved much in the same way as done in the previous section, provided that the kinematics introduced here is employed. For brevity, we do not give the explicit results, and we limit ourselves to observe that one more loop momentum and the half-off-shell F-AGS-L amplitudes $X_{\beta j,\alpha i}$, given by Eq. (3.33), would now appear in the relevant absorption amplitudes.

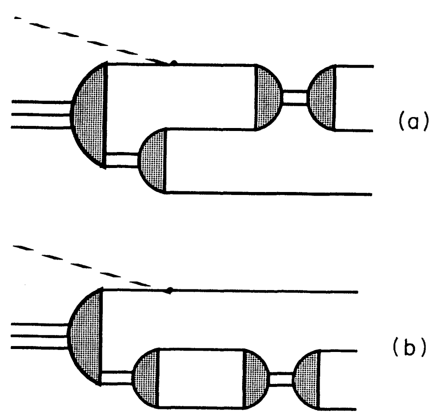

FIG. 5. Quasiparticle representation of the (a) exchange and (b) direct disconnected amplitudes in the case of one-body elementary absorption.

IV. DISCUSSION

The evaluation of the connected amplitudes (3.32) is quite a difficult task, since it requires the integration over three loop momenta, one of which occurs in the F-AGS-L amplitudes $X_{\beta j,\alpha i}$. These amplitudes, therefore, must be determined half-off the energy shell over a whole range of values for the spectator momentum. It can be expected that significant information on the FSI can already be obtained with suitable kinematical assumptions which lead to much simpler calculations. As a first reasonable approximation, one may assume that absorption takes place on a correlated pair with zero relative momentum between the constituent nucleons, and the pion's impulse is entirely given to the pair c.m. We shall refer to this basic approximation as the rigid pair mechanism (RPM). As a consequence, two loop integrations are immediately removed from the absorption amplitudes (3.32). In particular, in the direct term (3.32b), only the integration over the spectator momentum $\mathbf{q}'_\gamma$ survives, so that one can graphically interpret this term as a typical triangular-diagram contribution (see Fig. 6). As for the exchange term (3.32a), on the other hand, the condition $\mathbf{p}''_\gamma=\mathbf{p}^*_\gamma=0$ implies a well-defined relation between $\mathbf{q}'_\beta$ and $\mathbf{q}''_\gamma$, which leads again to one integration only over a spectator momentum.

The RPM leads to a complete factorization of the disconnected amplitudes (3.24) and (3.29). The remaining integration in the connected terms, on the other hand, can be removed introducing the further approximation that in the virtual breakup of the target, the three nucleons are produced at rest, so that one has *both* $\mathbf{p}''_\gamma=0$ *and* $\mathbf{q}''_\gamma=0$. Within more phenomenological approaches to pion absorption on ^{3}He, approximations of this kind have been successfully employed in the literature [11].

Up to now we have discussed absorption referring to the two-nucleon-rescattering diagrams (Sec. III B), where the RPM assumption allows one to perform most of the loop integrals involved in the calculation of the amplitudes. For the case of one-body πNN contribution, the nucleon, being treated as elementary, already undergoes a rigid one-body absorption process. In that case, therefore, the assumption that the three nucleons are produced at rest in the virtual breakup of the target is enough to eliminate all the integrations over loop momenta appearing in the connected one-body amplitudes. The same assumption ($\mathbf{q}''_\beta=\mathbf{p}''_\beta=0$), when applied to the disconnected one-body amplitudes (3.37) and (3.39), not only eliminates the integration over the only loop momentum, but also selects the phase space for the three outgoing nucleons; indeed the momentum of the outgoing spectator nucleon $\mathbf{q}_{\rm out}$ has to be related to incoming pion momentum by $\mathbf{q}_{\rm out}=\frac{2}{3}\mathbf{P}^{(0)}_\pi$, for both the direct and the exchange term.

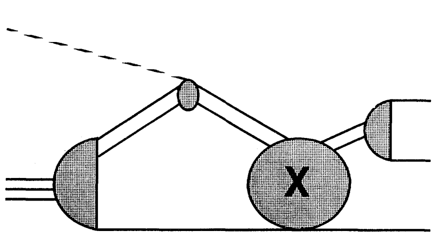

FIG. 6. Triangular diagram occurring when the RPM is assumed. Representation of the direct amplitude with three-body FSI.

It is possible, at this stage, to classify the terms which contribute to the total absorption amplitude according to the number of nucleons sharing energy and momentum oı the incoming pion. The reaction mechanisms where one nucleon only receives the pion momentum are given by the plane-wave πNN contribution [see Eqs. (2.13) and (3.35)] as well as by the direct one-body disconnected term [see Fig. 5(b)]. On the other hand, the two-body plane-wave amplitude [see Eqs. (2.13) and (3.22)], together with the exchange disconnected one-body term [see Fig. 5(a)] and the direct two-body one [Fig. 3(a)], yields contributions in which two nucleons actively participate to absorption. Finally, in the other mechanisms we have discussed all the three nucleons are directly involved in pion absorption. With reference to this point, it is worth noting that the three-body absorption diagrams considered in the present model involve pure nucleonic degrees of freedom. Other three-body absorption mechanisms, such as those proceeding through multiple Δ excitation or rescattering, are presently not contemplated. These contributions, however, can be incorporated in this model through a suitable redefinition of the absorption operator $\mathcal{A}$.

The general features of the present model do not depend in an essential way from the assumption of a Sturmian representation for the two-body amplitudes. Actually, once the two-body T matrices have been systematically exhibited in the absorption amplitudes [see Eqs. (3.7) and (3.8)], any separable representation would lead to three-body equations of the Faddeev-Lovelace type. Under the formal point of view the Weinberg's ideal choice leads to a particularly simple form for the effective propagators $\tau_{\gamma i}$ [see Eq. (3.18)], and stresses the two-body correlations in the absorption process. In particular, it emphasizes the contributions due to two-body quasiparticles corresponding to Sturmian eigenvalues near to 1 (bound states or resonances). At the same time it allows one to deal with realistic nucleon-nucleon interactions—at least if one can afford the required rank—thereby avoiding the further uncertainties introduced by the use of *ad hoc* separable potentials. An efficient numerical algorithm for the momentum-space evaluation of the Weinberg states in correspondence to modern nucleon-nucleon interactions will be given in a forthcoming paper [44].

Finally, the inclusion of pion-nucleon ISI requires a nontrivial generalization of the coupled-channel "πNN-NN" unitary models [45–47] to the four-body case. Such an extension has been formulated within the isobar model in Ref. [19] but its detailed discussion is beyond the scope of the present paper.

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