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# Multi-lambda matter in a derivative coupling model

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We have investigated, in a mean-field approximation, the properties of multi-lambda matter within a highly nonlinear $\sigma$-$\omega$ obtained from a derivative coupling between the scalar and fermion fields. The matter composition and saturation properties are calculated. We also discuss the spin-orbit properties of the effective Lagrangian.

## I. INTRODUCTION

In recent years, the question of the existence of strange matter has received increasing interest. Besides exotic baryonic matter carrying an appreciable amount of strange quarks, which is supposed to be more stable than common $d$-$u$ matter at high density [1–3], the possibility of the existence of hypernuclei containing several lambdas [4, 5] has been investigated. Because such systems decay primarily by weak processes or low-rate nuclear collisions, their lifetimes are long enough to ensure their stability against particle emission over times characteristic of strong interaction processes. Consequently, the search for such systems possesses an intrinsic interest, and may have relevant consequences in cosmology.

From a theoretical point of view, the discussions rely essentially on models at our disposal for the many-body problem in the presence of strong interactions. Attempts by Ikeda, Bando, and Motoba [4] were based on various Nijmegen potentials and an extension of the Brueckner theory. The results arising from these different potentials vary so much that they merely indicate the basic interactions to be insufficiently determined.

On the other hand, relativistic mean-field models appear to yield reasonable answers within a restricted parameter space. Calculations along this scheme have been performed by Rufa, Stoecker, Reinhard, Maruhn, and Greiner [5]. They used a nonlinear $\sigma$-$\omega$ model to determine the properties of systems containing several lambdas embedded in a $^{16}O$ core.

A recent work by Zimanyi, and Moszkowski [6] has proposed a new kind of nonlinear model. They replace the usual scalar coupling by a derivative form. When compared to the ordinary $\sigma$-$\omega$ model of Serot and Walecka [7] the derivative coupling succeeds in decreasing the incompressibility of nuclear matter considerably and brings the effective mass to a value of 0.85.

The aim of the present work is to extend the calculation of Ref. [6] to hypernuclear matter and to compare the results with those derived from the linear $\sigma$-$\omega$ model of Walecka. There exists a number of different approaches concerning hypernuclei based on potential or relativistic models. However, they usually consider only the properties of a single hyperon bound to the nucleus. Very few studies have dealt with multi-hyperon systems, hence the present study.

This paper is organized as follows. The model is introduced in Sec. 2. The way the coupling constants are fixed is explained in Sec. 3, which summarizes also the results of numerical calculations. Because the spin-orbit splitting has been used in many works to fix the coupling constants in the lambda-nuclei calculations, in Sec. 4 we discuss its value in the Zimanyi and Moszkowski model. The conclusions are drawn in Sec. 5.

## II. THE MODEL

The model of Zimanyi and Moszkowski (ZM) involves a derivative coupling of the fermion fields to the scalar meson field $\sigma$, in addition to the standard coupling to vector field. According to the ZM work, we consider the following Lagrangian:

$$\mathcal{L}=-\bar{\psi}M\psi+\left[1+\frac{g_\sigma\sigma}{M}\right](\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)$$
$$-\bar{\psi}_\Lambda M_\Lambda\psi_\Lambda+\left[1+\frac{g_{\sigma\Lambda}\sigma}{M_\Lambda}\right](\bar{\psi}_\Lambda i\gamma_\mu\partial^\mu\psi_\Lambda-g_{\omega\Lambda}\bar{\psi}_\Lambda\gamma_\mu\psi_\Lambda\omega^\mu)\ . \tag{1}$$

We have used the notation of Ref. [6] for the $\sigma$-$\omega$ nucleon part, and we have added the lambda contribution in a self-explanatory way. This Lagrangian is Lorentz invariant but is not renormalizable. It contains also a coupling between scalar and vector mesons. Following the prescription of ZM, we rescale the fermion fields

$$\psi \rightarrow \left[1+g_\sigma \frac{\sigma}{M}\right]^{-1/2} \psi, \quad \psi_\Lambda \rightarrow \left[1+g_{\sigma\Lambda} \frac{\sigma}{M_\Lambda}\right]^{-1/2} \psi_\Lambda .$$

Inserting this transformation into (1), the Lagrangian becomes:

$$\mathcal{L}=\bar{\psi}(i\gamma_\mu \partial^\mu - Mm^* - g_\omega \gamma_\mu \omega^\mu)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \tfrac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma) + \bar{\psi}_\Lambda (i\gamma_\mu \partial^\mu - M_\Lambda m_\Lambda^* - g_{\omega\Lambda}\gamma_\mu \omega^\mu)\psi_\Lambda , \tag{2}$$

where we have introduced the fermion effective masses

$$m^* = \left[1+g_\sigma \frac{\sigma}{M}\right]^{-1}, \quad m_\Lambda^* = \left[1+g_{\sigma\Lambda}\frac{\sigma}{M_\Lambda}\right]^{-1} . \tag{3}$$

Thus

$$m_\Lambda^* = \left[1+\frac{g_{\sigma\Lambda}M}{g_\sigma M_\Lambda}\left[\frac{1}{m^*}-1\right]\right]^{-1} . \tag{4}$$

It is worth noting that nucleons and lambdas are interacting through the same meson fields, which are also responsible of the nucleon-nucleon and $\Lambda$-$\Lambda$ interactions. As a consequence, $m_\Lambda^*$ is not independent of $m^*$ [cf. Eq. (4)], since both effective masses have the same meson-field origin. The same occurs in the case of the Walecka model if its extension to hypernuclear systems is carried out as in Ref. [5], i.e., by introducing the standard lambda-scalar and -vector field couplings.

From Eq. (2), one can see that the couplings between the scalar meson and fermions are finally written as

$$\mathcal{L}_{\sigma\text{-}N} = m^* g_\sigma \sigma \bar{\psi}\psi , \qquad \mathcal{L}_{\sigma\text{-}\Lambda} = m_\Lambda^* g_{\sigma\Lambda}\sigma \bar{\psi}_\Lambda \psi_\Lambda . \tag{5}$$

As a consequence, the couplings between the fermions and the scalar meson involve higher powers of sigma, as can be seen from the Taylor expansion of $m^*$ and $m_\Lambda^*$.

Within the mean-field theory (MFT), it is straightforward to obtain the field equations of motion and energy per unit volume corresponding to a uniform system. In terms of the nucleon and lambda densities $\rho_N$ and $\rho_\Lambda$, this energy is given by

$$\epsilon(\rho_N,\rho_\Lambda) = \frac{4}{(2\pi)^3}\int_0^{k_{F_N}} d^3k[\mathbf{k}^2+(Mm^*)^2]^{1/2} + \frac{2}{(2\pi)^3}\int_0^{k_{F_\Lambda}} d^3k[\mathbf{k}^2+(M_\Lambda m_\Lambda^*)^2]^{1/2} + \frac{1}{2}\left[\frac{g_\omega}{m_\omega}\right]^2\left[\rho_N+\frac{g_{\omega\Lambda}}{g_\omega}\rho_\Lambda\right]^2 + \frac{1}{2}\left[\frac{m_\sigma}{g_\sigma}\right]^2 M^2\left[\frac{1}{m^*}-1\right]^2 , \tag{6}$$

where $m^*$ is determined from the consistency condition

$$m^* = 1 - \frac{4}{(2\pi)^3}\left[\frac{g_\sigma}{m_\sigma}\right]^2 m^{*4}\int_0^{k_{F_N}}\frac{d^3k}{[\mathbf{k}^2+(Mm^*)^2]^{1/2}} - \frac{2}{(2\pi)^3}\left[\frac{M_\Lambda}{M}\right]\left[\frac{g_\sigma}{m_\sigma}\right]^2 \frac{g_{\sigma\Lambda}}{g_\sigma} m_\Lambda^{*3} m^* \times \int_0^{k_{F_\Lambda}} \frac{d^3k}{[\mathbf{k}^2+(M_\Lambda m_\Lambda^*)^2]^{1/2}} . \tag{7}$$

Introducing the baryon density $\rho_B=\rho_N+\rho_\Lambda$ and the ratio $Y_\Lambda=\rho_\Lambda/\rho_B$, we define the energy per baryon in the following way:

$$\frac{E}{B} = \frac{\epsilon(\rho_B,Y_\Lambda)}{\rho_B} - M(1-Y_\Lambda) - M_\Lambda Y_\Lambda . \tag{8}$$

## III. RESULTS

The energy density given by Eq. (6) is completely specified once the four constants $(g_\sigma/m_\sigma)^2$, $(g_\omega/m_\omega)^2$, $g_{\sigma\Lambda}/g_\sigma$, and $g_{\omega\Lambda}/g_\omega$ are fixed. The first two are taken from Ref. [6]:

$$g_\sigma^2(M/m_\sigma)^2=169.2, \quad g_\omega^2(M/m_\omega)^2=59.10 .$$

For ordinary nuclear matter ($Y_\Lambda=0$) at saturation, these constants yield a saturation density $\rho_0=0.16$ fm$^{-3}$, $E/B=-16.0$ MeV, $m^*=0.855$ and an incompressibility $K=225$ MeV. For the original $\sigma$-$\omega$ model that we are using, for the sake of comparison, the corresponding values are

$$g_\sigma^2(M/m_\sigma)^2=267.1, \quad g_\omega^2(M/m_\omega)^2=195.9 ,$$

$$\rho_0=0.193 \text{ fm}^{-3}, \quad E/B=-15.75 \text{ MeV} ,$$

$m^*=0.547$, and $K=560$ MeV.

In order to fix the two other parameters of the model, we have used only one physical constraint, namely, the energy $E_\Lambda$ of one single lambda in symmetric nuclear matter at saturation. According to the analysis of Bouyssy [8] and of Hausmann and Weise [9], we have taken the value $E_\Lambda=-28$ MeV. Consequently, we are left with one single free parameter.

As can be seen in Fig. 1, the set of $g_{\omega\Lambda}/g_\omega$ and $g_{\sigma\Lambda}/g_\sigma$ values leading to the chosen value of $E_\Lambda$ are lying on straight lines, both in the ZM and Walecka models. In other words, given a value of $g_{\omega\Lambda}/g_\omega$ the corre-

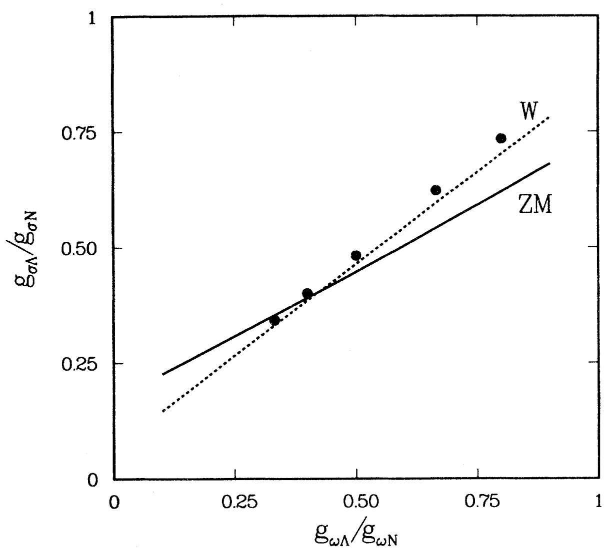


FIG. 1. Relationship between the two meson-lambda coupling constants found in the present work for the Zimanyi and Moszkowski (solid line) and the Walecka model (dashed line), respectively. The dots correspond to a similar quantity found by Mares and Zofka [10].

sponding value of $g_{\sigma\Lambda}/g_\sigma$ ensuring $E_\Lambda=-28$ MeV is given by a linear relationship.

As pointed out by Rufa *et al.* [5], the spectroscopic data on lambda nuclei are not able to determine the coupling constants univocally. A similar conclusion can be drawn from the work of Mares and Zofka [10] in their lambda-nucleus calculations using a $\sigma$-$\omega$ model. In this latter case, the authors have used several pairs of coupling constants, which yield equivalent spectroscopic results. These pairs are shown as dots on Fig. 1; they also lie on a straight line. It is interesting to note that these values are close to the ones obtained in our standard $\sigma$-$\omega$ calculations.

On the basis of the quark model, one would have $g_{\omega\Lambda}/g_\omega=g_{\sigma\Lambda}/g_\sigma$ [11]. This ratio has been fixed to $\frac{1}{3}$ by Boguta and Bohrmann [11], on the grounds of lambda single-particle energies in lambda nuclei. More recent fits decrease it up to 0.21 [5]. We shall display results of the ZM model for two sets of parameters:

$$g_{\omega\Lambda}/g_\omega=1/3,\quad g_{\sigma\Lambda}/g_\sigma=0.353$$

and

$$g_{\omega\Lambda}/g_\omega=2/3,\quad g_{\sigma\Lambda}/g_\sigma=0.542\ .$$

For the sake of comparison, calculations in the Walecka model have been made with

$$g_{\omega\Lambda}/g_\omega=g_{\sigma\Lambda}/g_\sigma=1/3\ .$$

Before discussing the results, we would like to draw attention on the following point. The total energy of multi-lambda hypernuclear matter would be given by $\epsilon(\rho_N,\rho_\Lambda)/\rho_B$, Eq. (6), whereas we have chosen to subtract the fermion masses and to study the binding energy per particle, according to Eq. (8). This is legitimate when looking, for a fixed amount of lambdas $Y_\Lambda$, at the properties of the system as a function of the baryon density. Similarly, the mass energy does not contribute to the determination of the lambda-drip line, i.e., to the limit of stability against lambda emission.

However, the fermion masses are relevant when discussing the stability against $Y_\Lambda$, i.e., for a given $\rho_B$ which is the ratio $Y_\Lambda$ that yields the absolute minimum of $\epsilon(\rho_N,\rho_\Lambda)/\rho_B$. As in the case of ordinary nuclei, the maximum stability does not correspond to the maximum of binding energy, but is modified by the mass differences of the constituents. In this respect, the results displayed below have to be interpreted with caveats. Actually, when taking into account the lambda-nucleon mass difference, we find $Y_\Lambda=0$ to be the most stable situation up to 2–3 times the saturation density of ordinary nuclear matter. In other words, no hypernuclear matter is found absolutely stable up to densities where the present models become questionable.

The results of the present calculations are summarized in Figs. 2 and 3. In Fig. 2, the lower part displays for each value of $\rho_B$ the minimum of $E/B$ [Eq. (8)] for each value of $\rho_B$, whereas the upper part gives the corresponding lambda ratio $Y_\Lambda$. The results obtained with the ZM model indicate that up to normal density, the properties of the nucleon-lambda mixture are not very dependent on the coupling constants once the energy of a single lambda has been fixed. The Walecka model yields qualitatively similar results for the bulk properties.

In view of the caveats developed above, the interpretation of Fig. 2 may be somewhat ambiguous. At best, it shows the optimum amount of binding energy and lambdas in a metastable situation. It could be relevant, however, for the discussion of high-energy lambda-production experiments well above threshold, a situation in which one assumes the system to quickly evolve towards a maximal binding energy state.

We remark that beyond a critical value of $\rho_B$, the minimum of the binding energy is obtained with a nonzero amount of lambdas. This is astonishing at first glance, in view of the fact that the lambdas endure a weaker interaction than the nucleons. Actually, this is reflecting the absence of the Pauli blocking for the lambda particles, which leads to a higher energy per particle than for the nucleons even if the coupling constants are smaller.

At low density, the ZM and Walecka models yield the same approximate expansion to Eq. (8):

$$\begin{aligned}\frac{E}{B}=&\frac{3\hbar^2}{10M}\left[\frac{6\pi^2}{4}\right]^{2/3}\rho_B^{2/3}(1-Y_\Lambda)^{5/3}\\&+\frac{3\hbar^2}{10M_\Lambda}\left[\frac{6\pi^2}{2}\right]^{2/3}\rho_B^{2/3}Y_\Lambda^{5/3}\\&+\frac{1}{2}\left[\frac{g_\omega}{m_\omega}\right]^2\left[1-Y+\frac{g_{\omega\Lambda}}{g_\omega}Y_\Lambda\right]\rho_B\\&+\frac{1}{2}\left[\frac{g_\sigma}{m_\sigma}\right]^2\left[1-Y_\Lambda+\frac{g_{\sigma\Lambda}}{g_\sigma}Y_\Lambda\right]\rho_B\ .\end{aligned}\tag{9}$$

At very low density, the kinetic-energy terms dominate and the minimum of $E/B$ corresponds to finite values of $Y_\Lambda$, as can be checked from the above expression. This explains the occurrence of a small lambda contribution in the low-density part of the results displayed in Fig. 2. However, we do not consider these kinds of models to be reliable in the very-low-density region.

In Fig. 3 we have plotted for different $Y_\Lambda$ values the energy per baryon versus the baryon density for the ZM model with $g_{\omega\Lambda}/g_\omega = 1/3$. We note that as $Y_\Lambda$ increases, the saturation density increases from 0.16 $\text{fm}^{-3}$ for normal nuclear matter to a maximum value $\sim 0.26$, which is reached for $Y_\Lambda \simeq 0.5$. From this value on, the saturation point moves downward in density, disappearing at $Y_\Lambda \lesssim 0.75$.

We note from Fig. 3 that $E/B$ at fixed values of $Y_\Lambda$ has a negative minimum at least up to $Y_\Lambda = 0.6$, a feature which is also valid for the other cases studied in this work. In an oversimplified picture it means that systems containing up to 60% of lambdas will still be stable against particle emission. This result can be compared to the semiempirical bounds for the ratio of neutron number to nucleon numbers [12] observed in very neutron-rich light nuclei:

$$2/3 \le N/A \le 3/4 \ .$$

Our results are qualitatively in agreement with those of Bando, derived from Brueckner calculations using the $D$ version of the Nijmegen potential. However, in our case the gain in binding energy from $Y_\Lambda = 0.0$ to $Y_\Lambda = 0.2$ is much less than the 4 MeV obtained by Ikeda, Bando, and Motoba [4].

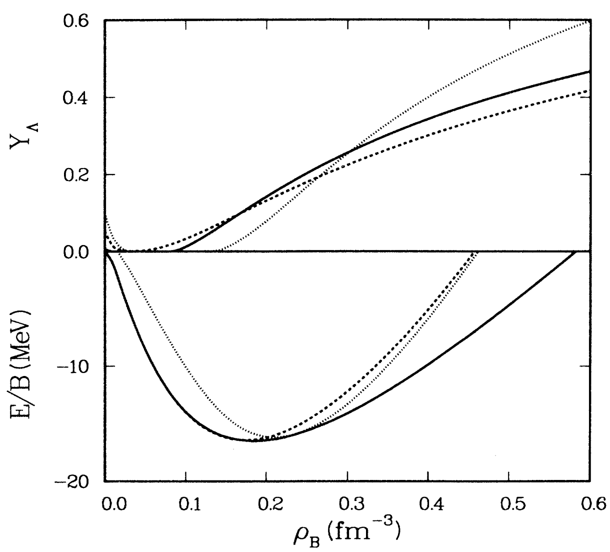


FIG. 2. Minimum of the energy per baryon $E/B$ [Eq. (8)] and corresponding fraction of lambdas in the medium $Y_\Lambda$ plotted against the baryon density. The solid line is the result of the ZM model for $g_{\omega\Lambda}/g_\omega = 1/3$. The dashed line is from the same model with $g_{\omega\Lambda}/g_\omega = 2/3$. The dotted line is obtained with the Walecka model with $g_{\omega\Lambda}/g_\omega = 1/3$.

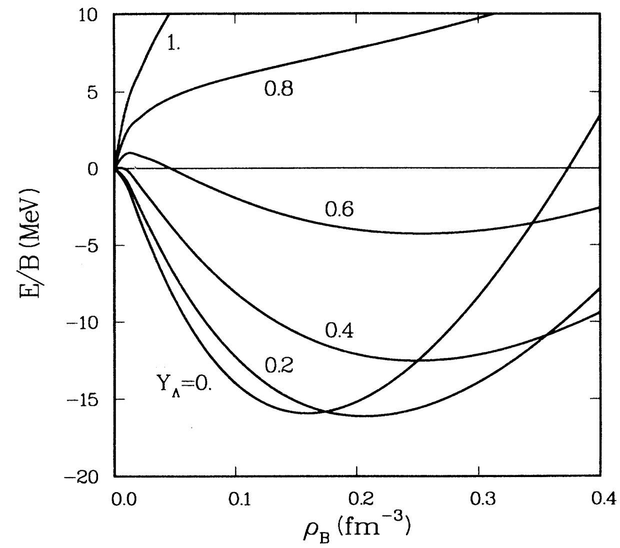


FIG. 3. Energy per baryon $E/B$ [Eq. (8)] versus baryon density for fixed values of $Y_\Lambda$. The results correspond to the ZM model with $g_{\omega\Lambda}/g_\omega = 1/3$.

## IV. REMARKS ON THE NUCLEON AND LAMBDA SPIN-ORBIT SPLITTINGS

As stated in the preceding section, most authors have fixed $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ (or their ratio to meson-nucleon coupling constants) by considering spectroscopic data of lambda hypernuclei. In this respect, the spin-orbit splitting is of key importance. For this reason we have investigated the value of this splitting in the ZM model.

For that purpose, it is sufficient to carry out the calculations in an approximate way. We first examine the case of ordinary nuclei. We start from the Dirac equation

$$(\boldsymbol{\alpha}\cdot\mathbf{p} + \gamma_0 M m^* + g_\omega \omega_0)\psi = E\psi \ , \tag{10}$$

which can be written

$$(\boldsymbol{\alpha}\cdot\mathbf{p} + \gamma_0 M + \gamma_0 U_s + U_\omega)\psi = E\psi \ . \tag{11}$$

The scalar potential $U_s$ is related to the scalar field sigma by

$$U_s = -m^* g_\sigma \sigma = -\left[1 + g_\sigma \frac{\sigma}{M}\right]^{-1} g_\sigma \sigma \ ,$$

where

$$(\nabla^2 - m_\sigma^2)\sigma(r) = -g_\sigma m^{*2}\rho_s \ . \tag{12}$$

To approximately solve these equations, use is made of the local-density approximation for the product $m^{*2}\rho_s$.

The vector potential $U_\omega \equiv g_\omega \omega_0$ is obtained from

$$(\nabla^2 - m_\omega^2)\omega_0(r) = -g_\omega \rho_B(r) \ . \tag{13}$$

For a rough estimate, it is sufficient to take for $\rho(r)$ a two-parameter Fermi function, normalized to the nucleon number $N$:

$$\rho_B(r)=\rho_0\left[1+\exp\left[\frac{r-R}{a}\right]\right]^{-1}, \quad \int \rho_B(r)\,d^3r=N . \tag{14}$$

The two parameters are fixed at standard values:

$$R=1.1N^{1/3}\ \text{fm}, \quad a=0.5\ \text{fm} .$$

Reducing the Dirac equation to a Schrödinger-like equation by means of the usual Foldy-Wouthuysen transformation, we can identify a spin-orbit term, which, to order $1/M^2$, reads [13, 14]

$$W_{\text{s.o.}}=\alpha(r)\mathbf{L}\cdot\mathbf{S} , \tag{15}$$

where

$$\alpha(r)=\frac{\hbar^2}{2M^2}\frac{1}{r}\frac{d}{dr}(U_\omega-U_s) . \tag{16}$$

The calculations have been performed for $^{40}$Ca, in order to compare directly with the Walecka model calculations of Ref. [7]. In the case of the standard $\sigma$-$\omega$ model, our approximate calculations agree within 20% with the Thomas-Fermi estimate of Ref. [7], the discrepancy being somewhat larger in the center of the nucleus due to the high saturation density of the Walecka model.

The results obtained with the ZM model are displayed in Fig. 4. Although they are qualitatively similar, the scalar field is reduced by a factor of 2.5, and the vector field by 3.5. Consequently, the spin orbit is also reduced, the reduction factor being of the order of 3. Thus the ZM model predicts a rather small spin-orbit splitting, which forbids its use for describing spectroscopic data in its present form.

On the other hand, recent works have stressed the dependence of spin-orbit splitting on quark degrees of freedom and medium effects [15, 16]. In particular, if one agrees with Brown [16] that the effective mass of the scalar and vector mesons scales as the nucleon effective mass, there is no need for large scalar and vector fields, and the situation may change. Such possibilities, together with more conventional ones based on the role of $\pi$ and $\rho$ meson at the Hartree-Fock approximation [17], are presently being investigated.

For the case of single lambda hypernuclei, we can follow a similar approach [18–20]. The lambda is submitted to the $\sigma$ and $\omega$ fields generated by the nucleons, which are proportional to the coupling constants $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$, respectively. Consequently, the potentials experienced by the lambda $U_{s\Lambda}$ and $U_{\omega\Lambda}$ in the corresponding Dirac equation are proportional to $g_{\sigma\Lambda}g_\sigma$ and $g_{\omega\Lambda}g_\omega$. We can write the following relationships:

$$U_{s\Lambda}=g_{\sigma\Lambda}/g_\sigma U_s, \quad U_{\omega\Lambda}=g_{\omega\Lambda}/g_\omega U_\omega .$$

If, according to Brockmann and Weise [19], we take $\frac{1}{3}$ for the ratio of the coupling constants, we get $W_{\text{s.o.}\,\Lambda}=\frac{1}{3}(M/M_\Lambda)^2 W_{\text{s.o.}}$; that is $W_{\text{s.o.}\,\Lambda}=0.25W_{\text{s.o.}}$. We want to stress that the tensor coupling introduced by Jennings [15] in the $\sigma$-$\omega$ model to explain the small value of the spin-orbit splitting in lambda nuclei could be applied in a similar way to the ZM model. We conclude that the ZM model is not in contradiction with the experimental spectroscopic data of lambda hypernuclei, which constitutes the important point for the present study.

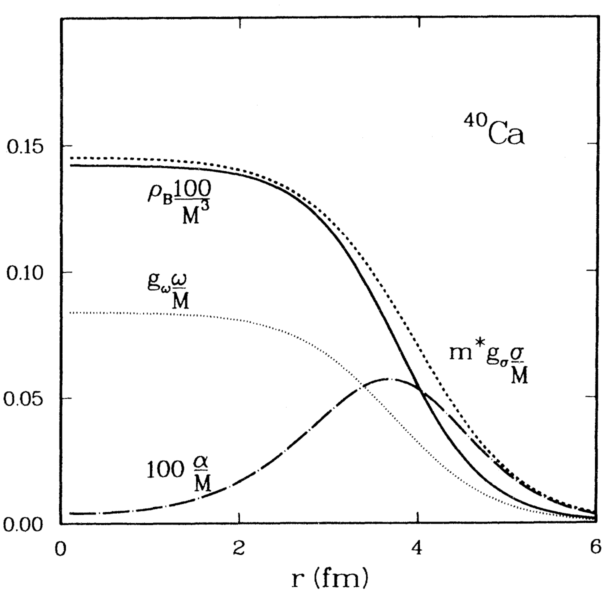


FIG. 4. Approximate baryon density $\rho_B$, scalar field $\sigma$, vector field $\omega$, and spin-orbit potential $\alpha(r)$ calculated for $^{40}$Ca in the ZM model for $g_{\omega\Lambda}/g_\omega=1/3$. The various quantities are scaled to allow for a direct comparison with the calculation of Ref. [7].

## V. CONCLUSIONS

In the present work we have extended the derivative coupling model of Zimanyi and Moszkowski [6] to multi-lambda hypernuclear matter. It results in a highly nonlinear $\sigma$-$\omega$ model, which we have investigated in the mean-field approximation for the homogeneous medium.

The calculations have been carried out for two sets of coupling constants, chosen in such a way to ensure $-28$ MeV for the energy of a single lambda in ordinary symmetric nuclear matter. For the sake of comparison, we have also considered the standard $\sigma$-$\omega$ model of Serot and Walecka [7].

The results suggest that multi-lambda systems are stable against particle emission. In particular, the calculations presented here indicate that above a certain baryonic density (between 0.5 and 0.7 times the saturation density) the inclusion of a few lambdas in the medium lowers the binding energy per baryon. However, as shortly stated, lambda hypernuclear matter is unstable against slow processes at normal nuclear matter density $\rho_0$. It is only at baryon densities 2–3 times $\rho_0$ that the absolute ground state will contain a finite amount of lambdas. We also conclude, in agreement with Rufa *et al.* [5], that a large number of lambdas can be added to the nuclear medium before reaching the particle emission threshold.

Finally, we have examined the question of the spin-orbit splitting, which is especially relevant for finite systems. At the present stage, the ZM model predicts a rather small nucleon spin-orbit splitting. It is worth mentioning, however, that the $\sigma-\omega$ model yields a reasonable splitting at the price of yielding a rather small nucleon effective mass, $m^* \sim 0.60-0.65$, as compared with the empirical value $\sim 0.80-0.85$ [21, 22]. For the scope of the present application, we consider that good values of $m^*$ and of the nuclear incompressibility are far more relevant than an accurate description of the spin-orbit splitting.

Incidentally, the ZM model yields a very small lambda spin-orbit coupling. This, in conjunction with the fact that it yields a reasonable description of finite nuclei properties of global character such as binding energies, charge radii, and density [23], implies that the model may yield single-lambda spectroscopic results in agreement with the experimental findings.

As this work was submitted for publication, we became aware of a paper by Rufa *et al.* [24], on the same subject, which extends their previous work on multi-lambda systems [5]. From their results on $^{208}Pb_{\Lambda}$, where neutrons are successively substituted by lambdas, we can infer qualitative similarities with the present results displayed in Fig. 3. As few lambdas are added (or substituted), the system gains binding energy, and becomes smaller in size (higher saturation density), up to a point where the situation reverses. The changes occur roughly at $Y=0.15$ for $E/B$ and 0.30 for the radius in their case, to be compared to $Y=0.2$ and 0.4, respectively, in the present calculations. More detailed comparisons are postponed to future work devoted to finite systems in the ZM model. We dedicate this work o the late Jan Zofka, friend and teacher.

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